Interpreting philosophical interpretations of paraconsistency

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Abstract

In this paper, we critically discuss the idea of a 'philosophical interpretation' of para-consistent logics. We do so by considering the *epistemic approach to paraconsistency*, by Carnielli and Rodrigues (2019a), according to which paraconsistent logics *should* be interpreted exclusively in terms of non-conclusive evidence, and also, by considering counter-arguments by Barrio (2018) and Barrio and Da Re (2018), according to whom paraconsistent logics are not specially tied to any specific interpretation. We begin by presenting the positions involved, and by arguing that the debate may be profitably understood in terms of the distinction between pure and applied logics. We argue that *dialetheism is not an interpretation of paraconsistent logics*, but rather a view on truth and negation which requires *use* of paraconsistent logics. Something similar may be said about the epistemic approach itself. The result is that there is nothing distinctive to be called *a philosophical interpretation* of paraconsistent logics, but rather what we have are distinct *applications* of paraconsistent logics. Arguments by Barrio (2018) and Barrio and Da Re (2018) may be then re-framed more effectively.

1 Introduction

In a series of papers, Carnielli and Rodrigues (2015, 2019a, 2019b, 2020; see also Carnielli et al., 2018; Rodrigues et al., 2020; Rodrigues & Carnielli, 2022) have advanced the claim that paraconsistent logics require *an interpretation* in epistemic terms. Although the meaning of 'interpretation' is not explicitly defined in those

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papers,[1] the reason for such a claim is simple to motivate, at first sight. Recall that in paraconsistent logics the *rule of explosion* does not hold: from formulas of the form A and $\neg A$ (where '$\neg$' is a paraconsistent negation), it is not the case that any formula B whatever follows (see da Costa et al. 2007). Now, from a technical point of view, for such a failure to occur, it is required that one produces a model M (or, more generally, a 'circumstance' or a 'case') in which both A and $\neg A$ hold, and in which B does not hold. The problem that demands an interpretation, according to Carnielli and Rodrigues (see in particular Carnielli & Rodrigues, 2015, pp. 59–60), may now be stated as: how to make sense of the claim that A and $\neg A$ both *hold*, for some sentences A? This is the "central question for the philosophy of paraconsistency: how to explain, in a plausible and intuitive manner, the nature of contradictions tolerated by paraconsistent logics?" (Carnielli & Rodrigues, 2020, p. 576). Does the holding of A and $\neg A$ commit one to saying that *some contradictions are true*? If that is the case, then, any paraconsistent logician would also be a dialetheist, which, by definition, is someone holding that some sentences of the form A and $\neg A$ are true (such sentences are the *dialetheias*; for the basics of dialetheism, see Priest et al. 2018).

To avoid such a commitment, Carnielli and Rodrigues (C&R from now on, when their proposal is referred to in general) hold that paraconsistent logics can and should be interpreted in *epistemic* terms, rather than in *alethic* terms. In a nutshell, the idea is that *acceptance of a contradiction does not mean acceptance of its truth* (Carnielli et al., 2018, p. 54). There may be good reasons for accepting both propositions A and $\neg A$, although this needs not imply that both are actually true. We need not even *believe* that both are true. Rather, we may take this circumstance to indicate that we accept both propositions in the specific sense that we have *evidence* for them, where evidence is understood as providing 'reasons for believing/accepting a proposition', with these reasons being, perhaps in most cases, *not conclusive*. There may be evidence for a proposition even if the proposition is false, and we may wish to reason with that proposition; this leads us to cases where there may be evidence for both A and $\neg A$. C&R go on to offer two paraconsistent and paracomplete systems, *BLE* and *LET$_J$* whose aim is to codify the inferences that are legitimate when it is such notion of non-conclusive evidence that is preserved from premises to conclusion, instead of truth (see in particular Carnielli & Rodrigues, 2019a).

Now, Barrio and Da Re (2018) and Barrio (2018) have understood the epistemic approach to paraconsistency, as formulated by C&R, as involving also an *exclusivity claim*: paraconsistent logics should be interpreted *only* in terms of evidence, with no place for dialetheism. Such a claim for the existence of a *canonical interpretation* of paraconsistent logics, Barrio and Da Re (2018) and Barrio (2018) argue, is ill-grounded: a paraconsistent system of logic may be compatible with a variety of mutually incompatible interpretations (the epistemic one included, as well as dialetheistic ones), and none of them is to be thought of as 'the' canonical interpretation. The

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[1] Also, it is clear that 'interpretation', in this context, is not what is typically called an interpretation in logical textbooks, that is, a formal semantics developed inside some set theory employed in the metalanguage (see, in particular, the discussion in Carnielli and Rodrigues 2019b). In the present paper, 'interpretation' always refers to the idea of informal interpretation as advanced by Carnielli and Rodrigues; whenever we have to refer to the usual notion of interpretation as provided in logic textbooks, we use the word 'formal semantics'.

formalism of paraconsistent systems simply cannot fix any interpretation as the single intended interpretation; worst yet, there are no purely logical grounds to hold that one interpretation excludes the others as the only correct one. Priest (2019a) has also advanced similar claims, and argued that the existence of epistemic interpretations does not represent any problem for dialetheism.

Seen from that point of view, it may be thought that the issue now concerns a dispute between some kind of pluralism and some kind of monism concerning interpretations of paraconsistent logics, with C&R defending one single epistemic interpretation, Barrio and Da Re defending a plurality of interpretations as legitimate. In this paper, however, we do not enter *that* debate. Rather, we argue that the very idea that a logic must be philosophically interpreted, if 'interpretation' is understood in the sense of C&R (and of Barrio (2018) and Barrio and Da Re (2018)), is an unnecessary addition to the already well-known distinction between pure and applied logic. Furthermore, once one keep restricted to the distinction between pure and applied logic, some of the confusions that appear in the debate may be properly addressed by disentangling two distinct senses of *interpretations* used in the discussion: i) 'interpretation' in the sense of providing a specific formal semantics, and ii) 'interpretation' in the sense of providing an application for the logic. We shall argue that, seen through these lights, the claims by Barrio (2018) and Barrio and Da Re (2018) may be properly qualified, and these qualifications give a more powerful meaning to their objections to C&R.

This paper is structured as follows. In Sect. 2, we present the main details of the epistemic interpretation, and recall some of the main objections by Barrio (2018) and Barrio and Da Re (2018). In Sect. 3, we recall the well-known distinction between pure and applied logics, and spell in details how it can account for the positions in the discussion without the need of an additional 'philosophical interpretation'. In Sect. 4, we advance the claim that under such a perspective, there are no reasons to believe that paraconsistent systems of logic could have anything like a 'canonical interpretation', in any of the possible senses of 'interpretation'. We will see that the arguments on the table, then, for both parties of the debate, lead us to well-known themes, and hinge either on a fact of life (a system of logic may have distinct applications) or else on a fact already much discussed in classical logic, which is the difficulty in fixing an intended formal semantics by means of the formalism. A skeptical argument against the possibility of fixing a semantic arises also for paraconsistent logics. We finish with a recollection of the arguments in Sect. 5.

2 Philosophical interpretations: one or many?

The idea of interpreting paraconsistent logics in epistemic terms to avoid dialetheism goes back, at least, to Rahman and Carnielli (2000, p. 202), where it is claimed that there are two possible *interpretations* for paraconsistency:

> The one, which we call the *compelling interpretation*, based on a naive correspondence theory, stresses that paraconsistent theories are ontologically committed to inconsistent objects. The other, which we call the *permissive interpretation* does not assume this ontological commitment of paraconsistent theories. In the

permissive interpretation, (for example) lack of information prevents us from rejecting prima facie either *A* or ¬*A*. (Rahman & Carnielli, 2000, p. 202)

The message is clear: the contradictions required to invalidate explosion may have two readings, one ontologically loaded (which is seen as bad), in terms of *true contradictions*, and another, less philosophically demanding interpretation, involving lack of information (which is good).[2] This is the basic background to the claim by Carnielli and Rodrigues in their many papers, where they contrast their own epistemic interpretation with dialetheism, proposing that an epistemic approach is to be adopted, while the dialetheist approach is to be rejected.

In Carnielli and Rodrigues (2019a, p. 3790), the same issue is presented, but now including a middle ground option (which is not really discussed). According to them, on what concerns paraconsistency, one may adopt one of the following positions: 1) be a *dialetheist*, holding that there are true contradictions, implying that contradictions are 'ontological',[3] 2) adopt a *pragmatic approach*, claiming that commitment to paraconsistency may be carried on without having to worry with the nature of contradictions, or else, 3) one may adopt a third position (the *epistemic approach*), described as follows:

> A third position in paraconsistency, antagonistic to dialetheism, claims that *no* contradiction is ontological but, rather, *all* contradictions that occur in scientific theories, belief systems, a number of situations in informal reasoning, and even in semantic and set theoretical paradoxes — that are, strictly speaking, results about languages with certain characteristics — have epistemic character in the sense that they are related to thought and language. This is the position endorsed by us. (Carnielli & Rodrigues, 2019a, p. 3790)

This means that one must completely reject dialetheism when it comes to paraconsistency (as they put it in the quoted passage, "*no* contradiction is ontological"); all that is needed is a kind of epistemic understanding of paraconsistency. Contradictions, whenever they appear, always arise as a result of problems in our theories or our representational apparatuses of the world, they concern merely the evidence available for the propositions involved, and never reach reality:

> The acceptance of *A* and ¬*A* can be understood as some kind of 'conflicting information' about *A*, in the sense of having *non-conclusive reasons* for accepting the truth as well as the falsity of *A*. This kind of 'conflicting information' we call here *conflicting evidence*. (Carnielli & Rodrigues, 2019a, p. 3792)

Then, assuming that ¬*A* represents the falsity of *A*, we have that 'evidence that *A* is true' means 'reasons for believing/accepting *A*', while 'evidence that *A* is false' means

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[2] Curiously, Rahman and Carnielli seem to suggest that *lack of information* requires a *paraconsistent* treatment. Latter on, in Carnielli and Rodrigues (see Carnielli and Rodrigues 2015), the distinction between 'lack of evidence' (requiring paracompleteness) and 'conflicting evidence' (requiring paraconsistency) is made clearer and properly distinguished. See also Arenhart and Krause (2019) for further discussion of the limits of applying paraconsistency to discuss *lack of information*.

[3] The claim that true contradictions are 'ontological', and that the truth theory adopted by dialetheists is a correspondence theory, is common in Carnielli and Rodrigues' papers. This is certainly more than most dialetheists would claim. See in particular Priest (2019a) for a reply on these issues.

'reasons for believing/accepting ¬A'. The very idea of evidence for *A* as 'reasons to believe in *A*' is further explained in the following terms:

> (i) these reasons may be non-conclusive or wrong, (ii) evidence for *A* does not imply the truth of *A*, (iii) there may be simultaneous conflicting evidence for a pair of propositions *A* and ¬A, (iv) evidence for *A* is objective in the sense that it is independent from the belief of an agent in *A*, and for this reason, a logic of evidence is not a logic of propositional attitudes. (Rodrigues & Carnielli, 2022, p. 316)

Two formal systems, *BLE* and *LET$_J$* are advanced as formal tools to logically address *how people actually reason about evidence*, that is, actual reasoning when preservation of evidence is concerned, and not preservation of truth (Carnielli et al., 2018, p. 54). For evidence, both systems allow for four scenarios, when it comes to a proposition *A*:

a) There is evidence for the truth of *A*,
b) There is evidence for the falsity of *A* (*i.e.*, for the truth of ¬A)*,*
c) There is no evidence for the truth of *A*,
d) There is no evidence for the falsity of *A* (*i.e.*, for the truth of ¬A).

These cases account for situations where one may have evidence for the truth of *A* and for its falsity (i.e., for the truth of ¬A), resulting in a paraconsistent logic, and also account for situations where one may have evidence neither for *A*, nor for ¬A, resulting in a paracomplete logic. Formally, *BLE* is both paraconsistent and paracomplete. It is logically equivalent to Nelson's *N4*, although Carnielli and Rodrigues present it by a natural deduction system with distinct motivations. *LET$_J$* is a LFI, a system that extends *BLE* by the addition of a well-behavior operator allowing for the recapture of classical logic. In particular, the well-behavior operator allows C&R to define what it means for a proposition to have conclusive evidence, which then is treated as truth or falsity *simpliciter*. These well-behaved propositions obey classical logic, and *LET$_J$* is a logic of both evidence and truth (for details on the formal systems, see Carnielli & Rodrigues, 2019a, Carnielli et al., 2018).

A further issue remains: according to C&R, in order to be of any interest, one has to advance a system with an interpretation and "show that such an account is committed to real situations of reasoning" (Carnielli & Rodrigues, 2015, p. 72). By advancing the claim that contradictions may be understood as representing cases of conflicting evidence, Carnielli and Rodrigues argue that they "have thus given an intuitive interpretation of the paraconsistent negation that justifies the invalidity of explosion" (2015, p. 74). Furthermore, this would be evidence that dialetheism is not required, which is illustrated by the quote of Carnielli and Rodrigues (2019a p. 3790) presented above, and many others, including the claim that "[s]ince we think that dialetheism does not provide a sustained account of paraconsistency, a paraconsistent logic *has to* abandon logical consequence as truth preservation" (Carnielli & Rodrigues, 2020, p. 579). That is, dialetheism is wrong, because it interprets paraconsistency in terms of preservation of an inconsistent notion of truth, not of evidence. This is claimed, but not really argued for (more on this soon).

The problem is that, in the absence of convincing arguments for the selection of a single interpretation as the correct one (or more: in the absence of reasons for us to adopt the epistemic approach as the sole correct interpretation for paraconsistency), what C&R have delivered is, perhaps, one further possible interpretation of paraconsistency, one that is on offer just as dialetheism is. It is still an open issue whether one will find arguments for the choice of just one of them, or whether both are perfectly legitimate (or whether both fail). It is in this sense, it seems to us that Barrio and Da Re (2018) advance their claim that distinct systems may be advanced for distinct purposes, and that they may be legitimately used to enlighten distinct situations. In particular, given the distinct contexts in which human beings reason, and that logic is concerned with the study of inferences, distinct kinds of interpretations are allowed in distinct cases. They claim that.

> It is clear that one can use philosophical interpretations to try to obtain additional understanding of certain pure logical theories (from a proof-theoretical or model-theoretical point of view). It is also true that at least in some cases pure logics are developed to find an answer to a philosophical interpretation. For example, pure modal logics have been motivated by our interest in explaining what is metaphysically necessary or possible. Nevertheless, we would like to argue that there are no intrinsic connections between pure logics and one standard philosophical interpretation, even when one is considering these systems applied to human reasoning. (Barrio & Da Re, 2018, p. 159)

Basically, it is a fact of life that distinct interpretations are employed for distinct systems, and also, that the same system may have distinct incompatible interpretations, with no logical feature or restriction pointing to a canonical interpretation. In the end, then, the authors reject the view "that there is a single canonical philosophical interpretation for some pure paraconsistent logic even when one is interested in the standard application of pure logics: the reasoning in natural languages." (Barrio & Da Re, 2018, p. 160). That is, even if one is concerned with logics applied to the study of inferences in natural language, there is no preferred interpretation that could be singled out as the only correct one; many of them are available for distinct situations. The logical structure of a paraconsistent system is not determinate enough to restrict the scope of available interpretations to only one. Barrio is emphatic on this point:

> No semantic structure or inference rule system necessarily leads to a particular interpretation. There are usually multiple interpretations for each logical system. There is not a single privileged interpretation. (Barrio, 2018, p. 101)

In this sense, to claim that there is a single interpretation allowed for a system of paraconsistent logic is basically to go against the facts. And the facts underlying the above claim are telling the following story (see Barrio, 2018, p. 102): although *BLE* may be interpreted in terms of non-conclusive evidence, as C&R are proposing, it may also have a perfectly coherent dialetheistic interpretation. It is simple to see that. Recall that *BLE* is formally equivalent to Nelson's *N4*, whose semantics is typically presented in terms of valuations attributing one of *four truth-values* to propositions, along with a set of possible worlds. In these logics, according to *this* formal semantics, for any proposition *A*, under any valuation in a given world *w*, we may attribute one of

the following four values: just true, just false, both true and false (glut), or neither true nor false (gap; see Priest, 2008, chap. 9 for the details). So, in formal terms, nothing suggests that one interpretation is to be preferred over the other, they are all available and plausible.

This is not a phenomenon restricted to *BLE* (*i.e.*, *N4*), of course. Barrio and Da Re (2018, p. 162) point to the fact that even *LP*, Priest's *Logic of Paradox* (see Priest, 2008, chap. 7)*,* although it may be interpreted most of the times as dealing with truth-value gluts (sentences that are at the same time true and false), may also have alternative, non-dialetheistic, interpretations, in terms of gaps (as suggested by Beall & Ripley, 2004). Barrio (2018, p. 98) emphasizes the case of this system: "adopting *LP-valuations does not mean accepting dialetheia*". So, even *LP* is not committed exclusively with a dialetheistic interpretation. That does not mean, however, that dialetheism should be abandoned, but only that there is no single interpretation for a given system that is selected by a formal system. If dialetheism is going to fail, that will happen for different reasons, not for the mere existence of rival interpretations for paraconsistent systems.

Priest (2019a, p. 588) also makes similar points concerning *LP*:

> Carnielli and Rodrigues [...] note that the use of a paraconsistent logic (such as *N*4) does not commit one to dialetheism. Indeed not. This is something that I have pointed out many times. Nor does using *LP* imply a commitment to dialetheism either. One may hold that the actual world is consistent, and that the interpretations in which contradictions hold represent impossible situations. Indeed, there are interpretations of *LP* which dispense with the middle 'contradictory' value altogether. The interpretation of paraconsistent logics where the semantic values are given informational interpretations ('told true' and 'told false') are also well known. Having said that, the mere fact that there are interpretations of a paraconsistent logic, such as theirs, which does not endorse dialetheism is not an argument against it. There may be interpretations of it which do; or, alternatively, if dialetheism is correct, this may simply be the wrong logical system. C&R do not engage with the arguments for dialetheism at all.

Notice that Priest lists three *non-dialetheistic* interpretations for *LP:*

(a) in some interpretations*,* one may dispense the additional truth value (for an example, consider the semantics of bivaluations, as those presented for *BLE* by C&R, which are also available for *LP*; they are even available for *FDE*, see Rodrigues et al., 2020), which would presumably deliver a non-dialetheistic interpretation of *LP* (more on this soon), or else

(b) one may interpret the third truth-value as not involving true contradictions, but impossible situations. Finally, one may also

(c) take the system as dealing with information that is fed to a machine, so that for any proposition *A,* the machine is told that *A* is true, or it is told that *A* is false, or else it is told that *A* is true and also told that it is false (most likely by different sources).

The dispute, then, as framed by the passages just quoted, seems to indicate that the problem concerns whether we could live with these distinct interpretations, or

whether we should argue for one of them as the correct or true one. Notice that we have been following the debate, without specifying *what is meant by 'interpretation'* in the intended sense employed here. The attentive reader may have already noticed that two senses of 'interpretation' are called forth in the quotes and examples presented: 'interpretation' is sometimes understood as a formal semantics (so that a system may have distinct formal semantics, bivalent or non-bivalent, bivalued or many-valued, and so on), and 'interpretation' is at other times used as an informal meaning attributed to the semantic components of a formal semantics (so that the formal semantics for a system may be understood as carrying distinct informal meanings, *i.e.*, representing distinct notions, such as truth-value gluts in one reading, or contradictory *information* in another, for an example). Our claim, in the following section, is that when one distinguishes such meanings, they seem to indicate that a pluralistic reading of interpretation is to be preferred, and the lessons to be learned go even deeper than suggested by Barrio (2018) and Barrio and Da Re (2018). Let us check.

3 Logic: pure, applied, and philosophical interpretation

Barrio and Da Re (2018, p. 161) state that there is a very common confusion between the distinction 'pure versus applied' logic on the one hand, and a philosophical interpretation on the other. If one keeps the distinctions properly in mind, one will notice, they claim, that dialetheism is one possible interpretation of *LP* as *a pure logic*. As we have seen, according to them, there are other legitimate non-dialetheist interpretations of the same pure logic. C&R argue, on the other hand, that paraconsistent logics should have their *correct* interpretation in terms of non-conclusive evidence. This is where we left the discussion in the previous section. Both parties are thinking of logic as dealing with inference in natural language. While Barrio and Da Re indicate that it is pure logics that have philosophical interpretations, C&R do not introduce the distinction between pure and applied logics explicitly. Our contention in this section is that the idea of 'philosophical interpretation', as an addition to either pure or applied logic, may be dispensed with, in terms of the distinction of 'pure vs applied' logic, *pace* Barrio and Da Re (2018) and Barrio (2018). Let us see.

We start by recalling the distinction between pure and applied logics, which is reasonably well known. Graham Priest (2006, chap.10 and chap.12) introduces the distinction through an analogy between logic and geometry. Just as there are many pure geometries, all of them mathematically legitimate, there are many pure logics, all of them mathematically legitimate. There is no rivalry between logics at a pure level (this is *theoretical pluralism*). However, once an *application* is selected, there may be the question of which logic is the correct one, or the most appropriate, for the application in hand. This is similar to what happens to geometry: once one is attempting to describe physical space, let us say, there is an issue of whether Euclidean geometry is up to the task, or whether a non-Euclidean geometry fares better. When the subject concerns application of logic, Priest famously selects reasoning in natural language as the *canonical application*. As Priest explains it, there are many pure logics,

> [e]ach is a well-defined mathematical structure with a proof theory, model theory, etc. There is no question of rivalry between them at this level. This can occur only when one requires a logic for application to some end. Then the question of which logic is right arises. (Priest, 2006, p. 195)

Given some goal, which is attached to an application, one is able to judge on the adequacy of a system of logic. But where does a *philosophical interpretation,* as a third horn on these distinctions, enter the stage, according to this picture? Presumably, if the suggestion by Barrio and Da Re (2018, p. 161) is to be adopted, one has a *pure* logic, and then adds an additional layer of philosophical interpretation over it. For instance, there is Priest's *LP,* with a system of proof and formal semantics, and one may add dialetheism as a further information over it, by interpreting, perhaps, the third truth value in a three-valued semantics as involving truth-value gluts, besides the usual, classical, truth-values truth and falsity.

The problem with this account is that dialetheism, understood as a philosophical interpretation, is not doing any work here. Seeing this is a matter of putting the issue in the appropriate perspective. To begin with, this is not how dialetheism should be seen, and that is not even how Priest sees it, it seems to us (see also Arenhart, 2021, p. 11548).That is, dialetheism is not a thesis about how paraconsistent logics are to be read or understood, but it "is a quite general metaphysical/semantic view about truth and negation" (Priest, 2019b, p. 48). That is, using dialetheism to interpret *LP* is to see things in the wrong perspective; one confuses the priorities. Paraconsistent logics *are applied* to the study of reasoning in circumstances where dialetheism is true (if it is true at all, of course), and Priest subscribes to a very specific paraconsistent logic because the field of application with which he is concerned, reasoning, is deeply impacted if dialetheism is correct. He does not advance dialetheism as an interpretation for each and any paraconsistent logic (probably, most paraconsistent logics available in the literature are not suitable for dialetheism, as defended by Priest, and he himself is aware of that). Although Priest expresses himself cautiously, he is not suggesting that dialetheism is an interpretation of paraconsistent logic, but rather that some paraconsistent logic will have to be adopted if one is to embrace dialetheism:

> If dialetheism is correct, then —unless one is a trivialist — one must subscribe to some form of paraconsistent logic. The orthodox logic of our time is not a paraconsistent logic, and so dialetheism requires us to revise our beliefs about which logic is correct. (Priest, 2006, p. 155)

So, contrary to what C&R and Barrio (2018) and Barrio and Da Re (2018) claim, dialetheism is not to be understood as an interpretation of paraconsistent logic (again, see Arenhart, 2021, Sect. 3). Rather, *paraconsistent logics are applied to account for reasoning in situations in which dialetheism is the case*, that is, where truth and negation behave as described by Priest, in particular (see Priest, 2006 chap.4 for an illustration; for a book length discussion of how a logic must be developed to describe the most wild situations of reasoning in natural language, including contradictions and failure of excluded middle, see the pioneer discussion in Routley (1980); there is no talk of 'interpretation' there, but rather of modeling diverse inferences that the author

judge to be valid under some assumptions concerning the behavior of truth, falsity, and non-existent objects in general).

But this is not only true of dialetheism. Something similar can be said to hold when we put C&R own efforts to deal with non-conclusive evidence in the appropriate perspective. Despite their best attempts to describe their work as an interpretation of paraconsistency in general (as many quotes already presented show), *what C&R are really doing is to devise two systems of paraconsistent logic (BLE and LET$_J$) when the field of application is reasoning with evidence*. In their own terms, this is very clear:

> It is likely that no paraconsistent logician would be opposed to epistemic contradictions. However, to the best of our knowledge, a paraconsistent formal system suited to an intuitive reading according to which only epistemic contradictions are allowed (and true contradictions are 'prohibited') is still lacking. Our aim here is to present a system of this kind. In order to work out such an account of paraconsistency we have to explain what it means to say that a pair of propositions A and $\neg A$ simultaneously 'hold', or 'may be accepted', without being true. (Carnielli & Rodrigues, 2019a, p. 3790)

What they are claiming is that, so far, there is no system dealing with reasoning when one is concerned with preservation of evidence and conflicting evidence. This is the *target phenomenon* they will attempt to model through the paraconsistent systems they present; from this perspective, this is a clear case of applied logic: C&R are going to create two systems for a specific application (whether the field of reasoning with evidence really requires a paraconsistent logic is something that may be reasonably doubted, but this is not an issue to be discussed here; see lo Guercio and Szmuc (2018) and Arenhart (2021) for pertinent discussion). Furthermore, to see that this is not a case of a general interpretation of paraconsistency, it is enough to check that 'evidence' does not seem an appropriate understanding of the workings of every paraconsistent logic available (given that, as C&R recognize in the above quote, no system available so far is suited for such cases, and also that this reading may hardly be adequate for systems satisfying the law of excluded middle, provided that evidence requires paracompleteness, according to C&R). We can go even further, noticing that C&R do not even attempt to interpret other available systems of paraconsistency in terms of evidence; they rather go on to develop *new* systems. In this sense, what we have here, both in the case of dialetheism, as in the case of the epistemic approach, is application of logic to the study of deductive inference under specific circumstances (as argued in Arenhart, 2021, what is done is a modeling of the notion of evidence with use of paraconsistent logic, not a use of the notion of evidence to illuminate the idea of paraconsistency). These circumstances are clearly disjoint (inconsistent truth and non-conclusive evidence are treated as disjoint fields of application), indicating that the accounts cannot even be rival.

So, if those remarks are correct, the idea of applied logic is doing all the work in these cases, and there is no room for a further kind of interpretative notion which is distinctively philosophical. With this distinction between pure and applied logic in hand, and with the elusive idea of 'interpretation' out of the way, we can make sense of the cases of distinct interpretations of logic as discussed in the previous section by

Barrio (2018), Barrio and Da Re (2018), and Priest (2019a), as suggested in the end of the previous section. Let us check.

We begin with the cases of distinct interpretations for *BLE*. As we have seen, C&R are really applying this system to model inferences dealing with evidence. They present the system through natural deduction rules, along with a bivaluation semantics (see Carnielli & Rodrigues, 2019a). There is an issue, raised by C&R themselves, of whether the bivaluation semantics which they present is appropriate for the task of codifying the idea of 'evidence-preservation' (see the discussion in Carnielli & Rodrigues, 2019b), but we shall not discuss that here. What is relevant for us is that we can make clear sense of their project in terms of applying paraconsistent logic to a specific area of reasoning, which is reasoning with evidence. No need for a further idea of philosophical interpretation. On the other hand, in order to provide for evidence that there are distinct interpretations for this system, Barrio (2018, p. 102) advanced the claim that a dialetheistic interpretation is available for *BLE*. This consisted in indicating the four-valued formal semantics that is typically presented for *N4*, with possible worlds. Now, this four-valued formal semantics is typically seen as involving dialetheias and gaps (along with truth and falsity *simpliciter*). Here, what we have, is the same formal system with two distinct formal semantics (one with bivaluations, and another, with four-values and possible worlds). But notice that the four-valued semantics by itself does not determine any specific reading of the members of the set of truth-values, so that so far as no intended application is given, the proper understanding of the semantic values is left open. Certainly, one could be interested, as Barrio suggested, in taking those four values as involving the truth-values true, false, gluts, and gaps, so that one could use this formal semantics to account for a kind of dialetheism with gaps (that is, to deal with reasoning in cases where there are gluts and gaps). However, that is not the only application field for this logic with this particular semantics. One could be interested, alternatively, in studying the kind of inferences that are available for a computer that receives information from distinct sources. The same formal semantics could be employed, this time to model the classification of information by the computer (the 'told true', 'told false' semantics, originally suggested by Belnap for *FDE*, now with the addition of modalities).

So, the case of many interpretations of *BLE* is accounted for by the distinction between applied and pure logics as follows: *BLE* may be applied to evidence, having perhaps a proof-theoretic semantics (as suggested by Carnielli & Rodrigues, 2019b), or, it may be applied to deal with dialetheism and modalities, by adopting a four-valued semantics with modalities; additionally, this semantic could help us applying the system to deal with information. 'Distinct interpretations' here mean that the same system may have distinct formal semantics, which are applied to the study of distinct fields of application (and notice that a single system, with the same semantics, could be applied to distinct fields or contexts).

The discussion of the examples for *LP* follows a similar route, but here there are two distinct cases. Let us restrict ourselves to Priest's, 2019a comments quoted above, on the plurality of interpretations of *LP*. First of all, we have the claim that the same system may have distinct formal semantics. This is encapsulated in the fact that although the formal semantics for *LP* is typically provided through valuation functions on three truth values, we also have semantics for *LP* in which the third truth-value is dispensed

with (a bivaluation semantics could be provided, or a relational semantics preserving bivalence). Without any field of application specified, it is not clear that they do indeed represent any kind of approach to the nature of the truth-values involved in each case (this is typically assumed, given that systems of logic are most of the times presented with the canonical application in mind, and with logical consequence framed in terms of truth preservation). Of course, the typical understanding of the three-valued presentation of *LP* (Priest, 2008, chap.7) concerns application to deductive inference when dialetheism holds: the third truth-value is understood as *representing dialetheias*. However, even with this specific formal semantics, the same system can be applied to distinct contexts, such as information with which a computer is fed (as suggested by Priest), where the computer is told that some propositions are true, that some propositions are false, and is also told that some propositions are true and false. In this latter sense, 'distinct interpretations' means that a system, with the same formal semantics, is applied to distinct fields of application (information to a computer, reasoning in the presence of dialetheias). Notice also that it is the field of application that indicates what the truth-values of a formal semantics are going to represent (*e.g.* legitimate truth-values, information available to a machine, evidence…). So, in this case, 'distinct interpretations' was ambiguously oscillating between i) having the same system with distinct formal semantics and ii) having the same system, with the same formal semantics, but applied to distinct contexts. This indicates that 'philosophical interpretation', as an extra layer, is not really needed, and that the distinction between pure and applied logic is enough.

Given these discussions, one may distinguish at least the following possibilities, which were conflated in the term 'interpretation', in the previous discussions by C&R and Barrio and Da Re:

1. one may have the same formal system, with a selected formal semantics used to deal with distinct applications. This is the case, for instance, of the three-valued presentation of *LP* as applied to deal with dialetheism on the one hand, and to information states on the other (or even to the gap approach to truth, as developed by Beall & Ripley, 2004). Something similar could be done for the typical four-valued semantics for *FDE*, dealing on the one hand with reasoning in the presence of *gluts and gaps* (one application), or as dealing with the sates of information of a computer (another application of the same formal semantics). For a more familiar case, one may consider classical logic, with its usual two-valued semantics, applied either to the study of inferences in natural language, or else to the study of the behavior of electric circuits.

2. one may also have the same formal system, but with distinct formal semantics. When one is concerned with applied logic, there are two sub-cases:

   2.1) two distinct semantics for the same logic may be applied for distinct fields of applications. This is illustrated by *N4* as applied to the study of evidence, by C&R (either with a bivalent semantics, or else with a proof-theoretic semantics), or else with a four-valued semantics applied to the study of reasoning with constructible negation (Nelson's original aim), or perhaps, with contradictory and *gappy* information fed to a computer (see Song, Omori, and Tojo (2021), and

Omori and Arenhart (2022) for further examples of alternative semantics for some well-known non-classical systems and discussion of their applications).

2.2) two distinct formal semantics for the same system, with the same field of application. A simple case concerns the use of the relational two-valued semantics for *LP*, in opposition to valuational three-valued semantics for the same system. One could also consider bivaluation semantics to FDE, in opposition to four-valued semantics for the same system (see also Rodrigues et al. 2020).

3. the same system, but with distinct formal semantics, understood as pure system (so, this is 2, without the application part). Example: *LP* studied with bivaluations, or with bivalence and relational semantics, or through a three-valued semantics. Intuitionistic logic, studied with Kripke semantics, or by the use of bivaluations. Modal systems, studied with Kripke semantics, or else with topological or algebraic semantics.

4. distinct systems (each with a formal semantics), applied to the same field of application. Here, one could consider discussions on the most appropriate modal system for some kind of modality (epistemic and deontic logic provide interesting cases), or even the use of some paraconsistent logics, in dispute with paracomplete logics, to deal with semantic paradoxes.

Of course, other possibilities still may be available. Our goal was simply to distinguish between the distinct kinds of situations that were smuggled in the term 'philosophical interpretation' when it comes to the cases of paraconsistency discussed. The consequences of this disambiguation, we shall see in the next section, indicate some important lessons for the kind of discussion that is being carried on by those concerned with interpretations of paraconsistency.

Before we do that, however, let us briefly consider one further remark by C&R concerning interpretations. Doing so will reveal again that the 'pure vs applied' distinction is already good enough. In an attempt to defend that one should explicitly recognize that 'philosophical interpretations' are useful additions to the pure vs applied logic divide, they claim:

As we understand it, a philosophical interpretation of a logic is the intended meaning attributed to its expressions motivated by, or connected to, philosophical concepts. In order to be an applied logic, a formal system has to have intended meanings attributed to its expressions. These intended meanings, at first sight, may come without a philosophical interpretation, but it is reasonable to suppose that any applied logic is amenable to a conceptual — and so philosophical — discussion, given that in formalizing a domain a logic says things about that domain. Depending on how we understand the concept of an applied logic, it may be that a formal system has a philosophical interpretation but no application, the latter understood in a strict sense. But in a broad sense, a philosophical discussion about some fragment of natural language formalized by a logic can be regarded an application of a logic. In any case, although there is no clear dividing line between applications and philosophical interpretations, we think that having these two concepts at hand, as B&D [Barrio and Da Re] claim, provides a better understanding of formal systems. (Rodrigues & Carnielli, 2022, p. 326)

What is being suggested, it seems to us, is that a philosophical interpretation is just a formal semantics along with a specific application requiring that the concepts present in the formal semantics should have some philosophical appeal. That, on its turn, means that the system is *applied* to a topic that is open for philosophical discussion, such as truth or evidence. For an instance, consider the bivalent semantics for classical propositional logic, applied to the study of validity in natural language. What C&R suggest is that a philosophical interpretation is the understanding that when such a system is applied to something, the two values in the formal semantics represent classical truth and falsity. But then it seems that what is 'philosophical' about it is merely that truth and falsity are topics of philosophical consideration. The situation is different when the same formal semantics is applied to electrical circuits. The same formal semantics is connected with situations involving the passing of electric current through switches, and that is not philosophical (or at least it seems not to be). If that is all that is meant by 'philosophical interpretation', this distinction is basically accounted by our topic 1 in the previous list: distinct applications of the same formal semantics is all that is needed. The fact that truth and falsity are more philosophically interesting than electrical circuits is no justification for the introduction of a separated category for this kind of application; otherwise, electrical engineers would have to create a category of 'electric engineering interpretations', for those applications that are interesting for electrical engineers and to distinguish them from philosophical interpretations. But that is attributing too much emphasis on the fact that distinct people are interested in distinct applications. In this sense, then, it is difficult to see how these considerations can ground the claim that philosophical interpretations are a useful addition to the already available distinction between pure and applied systems.

4 Semantic underdetermination

We have seen in the previous section that what was taken to be a 'philosophical interpretation' may take different meanings on the discussion of interpretations of paraconsistency between the friends of the epistemic approach and its critics. Also, we have suggested that the idea of an interpretation over and above the distinction between pure and applied logic is not required to make sense of the epistemic approach to paraconsistency, and neither to make sense of the counter-examples provided by Barrio (2018), Barrio and Da Re (2018) and Priest (2019a); rather, the distinction helps us disentangle some distinct meanings of 'interpretation' conflated in the debate. In this section, we will use the results of the previous section to re-frame Barrio's and Barrio and Da Re's arguments in a distinct fashion, so that the suggestion that systems of paraconsistent logics may have distinct interpretations may be understood under the appropriate lights.

Let us start by recalling the claim by Barrio (2018, p. 101), who remarked that *no constraints on a formal system* are available to determine a single interpretation as privileged. Barrio and Da Re (2018, p. 159) hit at the same important point when discussing the possibility of many interpretations for the same system of logic:

"the main question is whether there is the possibility of imposing enough formal constraints on a pure logic in such a way that when we apply it to the inferences we make in language, a single philosophical interpretation is determined".

Of course, they are going to argue that there are no such constraints. The lesson to be learned, we suggest, is contained in these remarks, but they have to be adapted to the two basic senses of 'interpretation' that were uncovered by our previous discussion.

Let us start with one of the results of the previous section, which concerned the claim that, when speaking about distinct interpretations, one is really speaking about distinct applications of the same logic. Clearly, this is not controversial, but we claim, this consideration is enough to bring difficulties to the project of C&R. Understood as an application of paraconsistent logic to the systematization of valid inferences as preservation of evidence, one can clearly see that there is no reason to think that paraconsistent logic can only have *this* application. Not even *BLE*, as Barrio was quick to remind us, has a single application, given that it may be employed to deal with dialetheias of a kind too. Even if one is concerned only with logic applied to deductive reasoning in natural language, when one further restricts the field of study to specific aspects of such reasoning, such as reasoning with non-conclusive evidence, reasoning with information fed to a computer, reasoning by constructive means, and so on, then one can still have distinct applications of the same logic (of course, that does not mean that the same formal semantics will be *used* in distinct applications). So, it is hard to see how C&R could motivate a claim of uniqueness of interpretation if 'interpretation' is understood as a specific field of application.

Now, even though it may seem hard to look for motivations for such a uniqueness claim, and although C&R did not frame their project in terms of applied logics, one still finds such claims in C&R. First of all, it should be clear that speaking of application of paraconsistency to the study of reasoning with evidence makes better sense of their own project, in the terms that they develop it. And although one may speak of a canonical application, one cannot claim that a system selects a single application, or that its formalism imposes any kind of constraint that would prevent non-standard or non-canonical applications. But, even so, and contrarily to that, C&R seem even to suggest that not only paraconsistent logics, but also classical and intuitionistic logics, *do have* single preferred applications. This is evident in the claim by Carnielli and Rodrigues (2019b, p. 174) that some logics would have some intended fixed field of application:

> The work on inferential semantics has been motivated by, and developed for, intuitionistic logic. The framework of intuitionistic logic is mathematics, where a constructive proof of *A* implies the truth of *A*. But some contexts of reasoning are concerned neither with truth (classical logic), nor with constructive proofs (intuitionistic logic). This is the case of the paraconsistent logics *BLE* and *LET $_J$*, designed to express *preservation of evidence*.

That is, intuitionistic logics would have constructive proofs as its field of application, while classical logic is concerned with truth preservation, and paraconsistent logics *BLE* and *LET $_J$* are concerned with preservation of evidence (see also Rodrigues & Carnielli, 2022, p. 328, fnt. 11). Curiously, when addressing Barrio's criticism, C&R

(2022, p. 325) seem to recognize that one cannot attribute a single reading for a system of logic. Soon afterwards, however, they just state the same uniqueness requirement and the same thesis that appears in the previous quote. Considering that a logic has a bivalent formal semantic, and that one of such values, the one designated by '1' is preserved, they require that distinct formal systems attribute specific meaning to what is being preserved:

> And a philosophical interpretation of a logic is a conceptual framework that motivates and justifies the meaning attributed to the semantic value 1. In classical logic such a meaning is the classical notions of truth and falsity and, of course, the semantics is deterministic. In intuitionistic logic it is the notion of constructive proof, and in *BLE* it is non-conclusive evidence. (Rodrigues & Carnielli, 2022, pp. 328–329)

That is, again it is being claimed that classical logic is to be applied to truth preservation, intuitionistic logic to deal with mental constructions, and paraconsistent systems should deal with non-conclusive evidence. These are the meanings of such systems. But that claim certainly goes too far, and Barrio (2018) and Barrio and Da Re (2018) are certainly right in claiming that no formal constraint of a system of logic imposes a single field of application (although they do not put the issue in these words).

Notice that this does not mean that a system of logic may not have a motivation or intended field of application as it is developed. Basically, systems of logic are developed with an application in mind, they have an intended domain of application (they are mostly 'born' as applied logics). In terms of approaching logical theorizing as modeling validity, logical systems are typically developed along with their targets, as applied logics. What happens is that nothing in the systems grants that the intended application is the only application they will have in the long run, after they are seen as pure mathematical formalism. In this sense, most systems of logic are created as applied systems, and, when considered as pure logics, when set apart from their intended application, may be applied to other fields.

But perhaps this is not the main sense in which a logic may have distinct interpretations, as the arguments by Barrio (2018) and Barrio and Da Re (2018) suggest. Recall that in the previous section we have discussed the case in which *the same system may have distinct formal semantics,* and these semantics may, some of them, be appropriate for a dialetheist reading, while others may not be so. This is perhaps the most important blow on the project advanced by C&R. As the previous quote suggests, they seem to advance the claim that only one semantics is available, and that a proof-theoretic semantics for *BLE* and *LET$_J$* somehow fixes the meaning of these logics as dealing with evidence. Certainly, as we discussed in the previous section, this is also not the case. The same system (*N4*, *LP*, intuitionistic logics, classical logic), may have many distinct formal semantics; their rules of inference do not single out any preferred semantics. In this sense, it is impossible to fix the intended semantics by any kind of formal rules. This lesson was already learned in classical logic. Provided that all that one has is the descriptive means of the language, then, many distinct formal semantics are available, and no single feature of a language plus inferential apparatus can help us single out the intended classical bivalent semantics.

To make a long story short, the issue for classical logic may be put as follows (the whole story and arguments are found in Button & Walsh, 2018, in particular in chap.13). When rules of inference are provided, we set a corresponding syntactical consequence relation, and also provide for an accompanying formal semantics, with a corresponding notion of semantic consequence. A completeness theorem grants us that the model theoretic consequence and the syntactic consequence extensionally pin down the same relation of consequence. However, there are well-known arguments, in classical logic, to grant that the inference rules, by themselves, cannot pin down the *usual* semantics usually employed to define semantic consequence (Button & Walsh, 2018, p. 296 for a summary). Permutation arguments illustrate the issue; our truth-value sets could be: {true, false}, {moon, sun}, {0, 1}, or whatever two distinct objects. The argument then can be further extended: it is not only which objects as surrogates for the truth-values available which gets underdetermined by the syntactical formalism, but also how many they are, and their relation to the logical vocabulary. In particular, any Boolean algebra in which designated values form a filter will do as a model for the system. This includes Boolean algebras with wildly varying amounts of truth values, and, worst, it includes Boolean algebras violating the expected behavior of the relation between connectives and quantifiers with the truth values. In particular, under some such Boolean algebras, it may not be the case that negation shifts from designated values to non-designated values and vice-versa (it is enough that the filter of designated values is not a principal ultrafilter; the details, again, may be seen in Button & Walsh, 2018, chap.13). The result is *semantic underdetermination:* there is no way to single out a specific semantics for classical logic only in terms of the syntactical behavior of the logical apparatus.

Not only there is a very general lesson about semantic underdetermination to be learned in classical logic, but also in non-classical logics, as applied to the discussion between C&R and Da Re and Barrio. Before delving into that one, let us briefly remark how the previous argument applies to another attempt by C&R to grant uniqueness of interpretation to classical logic. They suggest in Rodrigues and Carnielli (2022, p. 237) that due to results by Lindenbaum and Tarski, given that every consistent theory whose underlying logic is classical propositional logic has a common underlying algebraic structure, one could see there some kind of unique canonical interpretation for classical logic, against the suggestion by Barrio (see in particular Rodrigues & Carnielli, 2022, p. 238, footnote 11). But the above considerations already show that there is nothing even on the classical side; one can consider very different Boolean algebras as providing the semantics for classical logic, including algebras attributing non-standard meanings to the connectives. That is enough to stop an attempt to grant uniqueness of interpretation for classical logic through a detour by Boolean algebra.

The morals of this discussion applies not only to the classical case, but also to the case of the epistemic approach and the criticism that was leveled at it by Barrio (2018) and Barrio and Da Re (2018), and in an important similar way. Recall that C&R expect that a proof-theoretic semantics will deliver the intended meanings of the connectives and somehow fix reasoning with non-conclusive evidence as the single application of *BLE* (see the previous quote by C&R); that would result that the connectives behave as if they were dealing with non-conclusive evidence. What the kind of skeptical argument just presented indicates, and which seems to be precisely the kind of argument

advanced by Barrio (2018) and by Barrio and Da Re (2018), is that systems like *BLE* are just in the same boat as classical logic when it comes to pin down a specific semantics; in particular: the rules of inference employed to characterize *BLE* are not strong enough to sort out precisely a non-dialetheist semantics in terms of non-conclusive evidence. There are, indeed, even semantics for the system which are clearly compatible with dialetheism (something along similar lines is indicated by Priest, when he comments that *LP* does not commit one to a dialetheistic semantics, and indicates that semantics dispensing with the third truth-value are available). The formalism does not single out a semantics as the intended one. In this sense, *BLE* does not deal exclusively with preservation of evidence, whatever that could mean, but may also be seen as dealing with preservation of truth, including dialetheias. Notice that this is not a perfect parallel of the result for classical logic, where the same kind of algebra may be used with distinct truth-values and distinct relations to the logical vocabulary, but it seems to reach the same kind of consequence (viz. semantic underdetermination), which is enough to raise worries for C&R's proposal.

But one could still complain that C&R are ready to avoid this conclusion. They do not completely rely on the syntactical rules of inference by themselves (which would lead to the semantic underdetermination), but rather indicate that a system of rules can only have meaning when an *argumentative context* is selected (this is further evidence that their real intention is to *apply* paraconsistent logic to evidence, instead of use evidence to *interpret* paraconsistent logic, as we have just been claiming, and despite their own description of their project in terms of an epistemic interpretation of paraconsistency; see also Arenhart (2021)). They suggest, then, that we need a context-relative approach to the semantics for their calculus,

> Proof-theoretic considerations, [...] have to be contextualized: the inferences are always relative to some argumentative context that cannot be separated from the rules because the meanings do not, and cannot, depend solely on the rules. A previous condition for the rules and the respective formal system is the kind of argumentative context where such rules are used. Our point is that the property being preserved by propositions in inferences is an essential feature of an argumentative context, if the latter is going to be represented, or formalized, by means of a set of inferences allowed in it. Different properties give rise to different rules, and so to different formal systems. And further, the property being preserved in an argumentative context has a role in establishing the meanings of the linguistic expressions of the respective formal system. (Carnielli & Rodrigues, 2019b, p. 178)

Perhaps this classification could soften the blow of Barrio's and Barrio and Da Re's arguments, which are concerned with the formal constraints on a calculus that could specify the intended semantics for it. But in this case, we are back to the first kind of case we have discussed in this section: we have to select reasoning with evidence as the field of application, and then, it is suggested, the rules dealing with evidence will acquire a specific fixed meaning. However, that is just not the case, if one is suggesting that the connectives of the system will have just that meaning. The system just cannot fix that reasoning with evidence is the only application one may have. So, if one considers that the system has meaning only when a context of argumentation is selected,

and concedes that distinct contexts are available, then, distinct meanings may be conferred to the connectives by the same rules, *depending on the context of application*. But that is not satisfying for C&R's purposes. First of all, this includes allowing that one could still attribute meaning to the connectives where the rules are dealing with dialetheism, if one selects a dialetheistic context (*i.e*., a context involving reasoning with dialetheias). But then, how can the epistemic approach fulfill its promises of being an alternative to dialetheism? Priest, for one, can reasonably claim that he applies the rules of a paraconsistent system to a context involving reasoning with true contradictions. Who can prevent him from doing so? Certainly not non-dialetheists claiming that we may also apply paraconsistent rules to the study of reasoning with evidence. In this sense, if this is going to be a way out of the underdetermination argument, there is no *legitimate rivalry* between the epistemic approach and dialetheism, and Priest was right in claiming that C&R do not address any of the arguments for dialetheism at all, and also do not offer any argument against it too.

But there is also a second difficulty with this suggestion of bringing a context of application together with the system. Notice that talk of a context of reasoning given beforehand already involves an element *extraneous* to the pure formal system (which is confined to the concepts employed to determine syntactical consequence). That requires that one puts, as it were, by hand, the kind of meaning that one is attempting to grant to the logical vocabulary, by selecting a preferred application field. But then, one engaged in a project of arguing that this is the privileged semantics would have to provide motivations and reasons for one to accept that this is indeed the privileged interpretation, and not a mere matter of having specific goals of application, given that other applications (and corresponding meanings) are available. That seems difficult to do, mainly when we have already seen that some people will prefer a dialetheistic semantics, given *their* aims of application. As a result, if one sticks to the formal rules, nothing grants a specific semantics, and neither a specific field or context of application. In particular, it seems, there may be interpretations (in the sense of formal semantics) of paraconsistent logics which are compatible with dialetheism, and others which are not compatible with dialetheism. Logic, by itself, does not decide the issue. This, again, is a block on C&R's arguments to the effect that evidence is the only adequate interpretation of paraconsistency, and somehow, make the dispute between distinct interpretations lose its impetus.

5 Conclusion

We have seen that the idea that paraconsistent logics need a *philosophical interpretation* may be profitably understood only in terms of the distinction between pure and applied logics. This distinction, by itself, sheds some light on what C&R are really doing when they propose that paraconsistent logics deal with evidence, although the result, when seen through these lights, is much less spectacular than what was sought by C&R: they are really applying paraconsistent logics to the study of reasoning with evidence. That makes clear that the epistemic approach is not, as advertised, an *interpretation* of paraconsistency in general, and also, that it is not an alternative to dialetheism (at least not as currently formulated), given that it does not address

dialetheism, and that dialetheism may very well just be another field of application for paraconsistent logics. Of course, we have left open whether applying paraconsistent logics to reasoning with non-conclusive evidence is in fact reasonable (see lo Guercio and Szmuc (2018), Arenhart (2021)).

The same distinction between pure and applied logics also helped us to clarify the arguments advanced by Barrio (2018), Barrio and da Re (2018) and Priest (2019a). Basically, their arguments are intended to show that C&R are wrong in claiming that paraconsistent logics can only be interpreted as dealing with evidence. These arguments are divided into two groups. First, there are arguments where 'interpretation' is meant as 'application', and then the suggestion is that paraconsistent logics may have distinct kinds of applications, not only to reasoning with evidence. Second, there is the claim that paraconsistent logics may have distinct formal semantics, and some of them are compatible with dialetheism, some of them are not. This is a kind of argument for *semantic underdetermination.* There is nothing in the formal systems themselves that establish the kind of semantic that is going to be attributed to a system, so that distinct semantics are available. The conclusion is that the advancement of systems such as *BLE* does not prevent that even *BLE* could be understood through the use of a dialetheistic semantics. In the end, the idea of philosophical interpretation becomes much less useful than it was initially thought, and the very idea of dispute between dialetheistic and epistemic interpretations is ill motivated.

Declarations

Conflict of interest There are no conflicts of interest.

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