



The paradox paradox

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Abstract

In this paper we argue that our conception of and intuitions about paradoxes are themselves paradoxical. Specifically, we argue that our commitment to the existence and nature of paradoxes is inconsistent with a norm of rationality—which is a paradox.

Keywords Credence · Paradox · Rationality

Consider the following three theses.

The Ontological Thesis: There are paradoxes.

The Conceptual Thesis: A paradox is a set¹ of inconsistent propositions such that it is rationally permissible to find the conjunction of any proper subset of them plausible even while being aware that the set as a whole is inconsistent.

The Normative Thesis: If A entails B, then it is rationally impermissible for an agent aware of the entailment to find A more plausible than B.

These three theses jointly constitute what we call *the paradox paradox*: the conjunction of any two of them is plausible, but the Conceptual and Normative Theses

¹Strictly speaking, the set of propositions comprising the paradox has to be *countable*, and so we should restrict our analysis to paradoxes of this kind. This is because normal probability functions are only countably additive. They can't handle sums of more than countably many propositions. For ease of exposition, we have avoided making this restriction explicit throughout. Most paradoxes discussed in the literature, including the Paradox Paradox, are paradoxes of this kind.

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together entail that there are no paradoxes, a conclusion that directly contradicts the Ontological Thesis. That's paradoxical. Our aim in this paper is to defend this claim.

1 The ontological thesis

Not only do paradoxes exist, there is an abundance of them. Our favourites include Zeno's paradoxes of motion, the Liar paradox, Russell's paradox, Newcomb's paradox, the St. Petersburg paradox, the Two-Envelope paradox, the paradox of fiction, the paradox of analysis, the surprise examination paradox, and the paradox of the question. But there are many more—you might even have your own favourites. Those interested in a catalogue of the various paradoxes would be well served by consulting such aptly titled volumes as *Paradoxes* by R.M. Sainsbury (1995), *Paradoxes from A to Z* by Michael Clark (2015), *Paradoxes: Their Roots, Range and Resolution* by Nicholas Rescher (2001). All of these monographs are excellent recent discussions that give a more complete list of the different paradoxes than we can give here. But, it is worth noting, even these catalogues are incomplete. There are just too many paradoxes.²

2 The conceptual thesis

Among the many ways of characterizing what a paradox is, we agree with Rescher in so far as he suggests that paradoxes are, *inter alia*, sets of inconsistent propositions (where an inconsistent set is one that either includes an explicit contradiction, or entails one). Although this is a relatively uncontroversial claim, some philosophers who work on paradoxes don't explicitly analyse them in terms of an inconsistency. But their alternatives are often compatible with the Rescherian conception. For example, R.M. Sainsbury defines a paradox as 'an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises' (Sainsbury, 1995, p. 1). But notice that if Sainsbury is right, there is apparently a set of inconsistent propositions comprising the premises and the negation of the conclusion. Likewise, van Heijenoort (1967, p. 45) defines a paradox as two 'contradictory propositions to which we are led by apparently sound arguments.' We simply note that the union of van Heijenoort's contradictory propositions, to which the apparently sound arguments lead us, is where we can find the inconsistency.

As these alternative analyses suggest, though, there is more to a paradox than an apparent inconsistency. The set of propositions {Jacinda Ardern is a politician; it's not the case that Jacinda Ardern is a politician} is inconsistent, but there is nothing paradoxical about it. What, then, is the missing element? Rescher suggests that the propositions that comprise the set must also be 'individually plausible.' Sainsbury

² Presumably there are some paradoxes that have not been discovered yet. There are also paradoxes discovered some time ago, but (for whatever reason) do not receive extended discussion in these monographs. And there are certainly paradoxes that were discovered only after the publication of these books—for example, the Paradox Paradox!

suggests that the propositions must be ‘acceptable’ (or ‘apparently so’).³ Likewise, Thomas Bolander’s (2017) SEP entry suggests that a paradox must be comprised of ‘apparently true assumptions.’ We take these suggestions to be roughly equivalent; the inconsistent propositions comprising the paradox must also be plausible or (apparently) acceptable, or apparently true.⁴

While we agree that something like this plausibility constraint is a necessary condition for a set of inconsistent propositions to count as paradoxical, the suggestion needs amending if we are to properly distinguish paradoxes from mere inconsistencies, for the account is vulnerable to two kinds of counter-example.

First, it seems to us that it is too easy to find counter-examples to the plausibility constraint as stated above. The discovery of someone dim enough to find both of the above propositions about Ardern plausible is not enough to make the set paradoxical. Exterminating those who find plausible each of the propositions within an inconsistent set is not a way of solving a paradox. One friendly amendment is the following: a set of inconsistent propositions is a paradox just in case *it is rationally permissible* to find the propositions that compose it individually plausible.

This amended proposal, though, does not avoid trouble altogether. It—along with the original proposal—is vulnerable to a second kind of counter-example. Consider the following thought experiment. A friend has three qualitatively identical empty cups in front of her. She turns them all over and places a ball under one. She then shuffles the cups around so you lose track of which one the ball is under. Now consider the following four propositions.

- (1) The ball is under one of the three cups,
- (2) The ball is not under the first cup,
- (3) The ball is not under the second cup,
- (4) The ball is not under the third cup.

It should be clear that the set is inconsistent. Moreover, unless you—a rational agent—have some further evidence available to you, you will presumably be confident of the first proposition, and give credence 0.67 or so to each of the remaining propositions. And so, it seems, the propositions that comprise this set are individually plausible. Nonetheless, our intuitions tell us that most people would not be tempted to call this set of propositions a paradox. For, *in most cases*, the plausibility of a conjunction of propositions will be less than the plausibility of the conjuncts. And so the discovery in this case that the conjunction of propositions (1) through (4) is less plausible than each of the individual conjuncts shouldn’t be at all paradoxical—it’s just what we should expect. Moreover, the discovery that the conjunction of (1) through (4) is certainly false is at worst interesting and informative—but it hardly counts as paradoxical.

On the original proposal, though, the fact that propositions (1) through (4) are individually plausible but jointly inconsistent conceptually guarantees that the set

³ More specifically, Sainsbury says the premises of the argument that partially constitute the paradox must be ‘apparently acceptable’ and presumably the negation of the conclusion is apparently acceptable as well, given that the actual conclusion is ‘apparently unacceptable’.

⁴ But see Rescher 2001, footnote 8 page 6. There Rescher explains why these suggestions are not strictly equivalent. However, these particular differences in characterisation make no substantive difference to the argument presented here (but cf. Section 5).

comprised of these propositions is a paradox. Furthermore, the suggested amendment proposed above will not help us avoid trouble. For not only do we *in fact* find propositions (1) through (4) individually plausible, we are also *rationally permitted* to do so. Indeed, if we have no further evidence available to us, it may even be rationally required of us that we do!

How then are we to adequately explain the difference between paradoxes and other inconsistencies? The conceptual thesis articulated in the introduction is one very natural way to do just that. We claim that *a set of inconsistent propositions is a paradox if and only if it is rationally permissible to find the conjunction of any proper subset of them plausible even while being aware that the set as a whole is inconsistent*. Call this the **strengthened analysis of a paradox**. The inconsistent set of propositions about Ardern doesn't fit this strengthened analysis because it is not rationally permissible to find the second proposition—that Ardern is not a politician—plausible. And the inconsistent set of propositions about the ball also fails to fit this strengthened analysis. For the conjunction of, say, propositions (2) and (3)—that the ball is not under the first cup and is not under the second cup—is not plausible.⁵

3 The normative thesis

The Normative Thesis claims that if A entails B, then it is rationally impermissible for an agent aware of the entailment to find A more plausible than B. We take this constraint on rational belief to be nearly self-evident, but for those who don't, it will be instructive to contrast this thesis with a couple of closely related, but implausible principles.

First, the Normativity Thesis is not the principle that if A entails B, it is rationally impermissible for an agent to find A more plausible than B. For there are many cases in which rational agents do not see that A entails B, and in such circumstances they may reasonably find A more plausible than B. But when agents *are* aware of the entailment, the canons of rationality require that they do not find A more plausible than B. The Normativity Thesis captures this idea.

Second, the Normative Thesis is not the principle that when A entails B, it is impossible to find A more plausible than B, even when the agent is aware of the entailment from A to B. Indeed, there are good reasons to think that it is in fact possible for human subjects to find themselves in this exact situation. Tversky and Kahneman (1982), for example, outline the results of a number of experiments that suggest that most university students fail to conform to the normativity thesis (see, for example, their discussion of “the Conjunction Fallacy”). But when they do so, to that extent at least, they are being irrational.

For those who need an argument, though, here is one, drawing out an absurd consequence of the contrary view. In order for the Normativity Thesis to be false, there must be a cases where A entails B *and* it is rationally permissible for an agent *x* who

⁵ So, the strengthened analysis does not count as paradoxical sets of inconsistent propositions we ordinarily would not count as paradoxes. To that extent, it is an improvement on other analyses. However, it might be objected that the strengthened analysis is *too* strong in that it rules out sets of propositions that are traditionally categorised as paradoxes. See Sect. 5 for an extended consideration of this concern.

is aware of the entailment to find A more plausible than B. In order for x to find A more plausible than B, though, it must be subjectively possible for x that A and not-B. That is, it must be subjectively possible for x that A does not entail B. But given that x is, *ex hypothesi*, aware that A does entail B, this seems to us to be irrational in the extreme. Those who deny the Normativity Thesis cannot agree.

4 The inconsistency

Consider any arbitrary set of inconsistent propositions S . Next arbitrarily separate any one of its constituent propositions p from the conjunction of the remaining propositions r . Either (a) rational agents can ascertain that S is inconsistent and find r plausible, or (b) they can't. Suppose (a) they can't ascertain that S is inconsistent and find r plausible. In that case, S is not a paradox (from the Conceptual Thesis).

Suppose, instead, that (b) a rational agent can ascertain that S is inconsistent and also find r plausible. In that case, if the agent finds r plausible, then the subjective probability of r for the agent is greater than 0.5. After all, if the subjective probability of r were lower than 0.5, a rational agent would find its negation more plausible. And if S is inconsistent, and if the agent is aware of this, she will see that r entails not- p . And so the subjective probability for the agent of not- p will also be greater than 0.5 (from the Normative Thesis). Therefore, the subjective probability for the agent of p will be less than or equal to 0.5, which means the agent will not find p plausible. But, then, since S includes p , S is not a paradox (from the Conceptual Thesis).

Either way, then, S is not a paradox. And because S is any arbitrary set of inconsistent propositions, there are no paradoxes. But there are paradoxes (from the Ontological Thesis). That's paradoxical!⁶

5 The conceptual thesis revisited

One complaint against our characterization of a paradox might be that it assumes that all paradoxes are structured in the same way—or can be reduced to such a structure. If it can be shown that some paradoxes cannot fit our characterization, this undermines the conceptual thesis.

Doris Olin, for example, (2003, pp. 6–7) suggests that there are two fundamentally different kinds of paradox. Like Sainsbury's analysis, Olin's 'Type I' paradoxes are arguments 'in which there appears to be correct reasoning from true premises to a false conclusion.' Like van Heijenoort's analysis, Olin's 'Type II' paradoxes arise 'when

⁶ This is not the only way we might draw out the paradoxical conclusion that there are no paradoxes. Consider the following principle:

If a conjunction is implausible, it's plausible that there's a false proposition among the conjuncts.

If there is such a thing as a paradox, the conjunction of the propositions that constitute it is inconsistent. An inconsistent set is one whose corresponding conjunction is false, so it must contain at least one false proposition. All this is rationally knowable. So it's plausible that it contains at least one false proposition. So there can be no paradoxes. Thanks to an anonymous reviewer for posing this alternative argument to us for the same conclusion.

there is one argument in which there appears to be correct reasoning leading from true premises to a conclusion *A*, and another argument in which there appears to be correct reasoning leading from true premises to a conclusion *B*, and *A* and *B* appear to be inconsistent.’

Our reply. While we acknowledge that there are important and instructive differences between Olin’s Type I and Type II paradoxes, we have also shown that they have this much in common: they can be reduced to sets of inconsistent propositions, such that it is rationally permissible to find the conjunction of any proper subset of them plausible even while being aware that the set as a whole is inconsistent.

This complaint might be pressed further, however. For example, our broad characterization of a paradox might still be too narrow insofar as it does not include within its scope some puzzles that have traditionally been classified as paradoxes. Consider the Sorites paradox. It consists of a large number of propositions (a collection of one grain of sand is not a heap; a collection of two grains of sand is not a heap; a collection of three grains of sand is not a heap ...). Because the individual plausibility of each constituent proposition does not itself confer plausibility on the conjunction of each proposition in the set, on our view the Sorites cannot count as a genuine paradox. But clearly it is. So we have a counterexample to the conceptual thesis.

Our reply. We are tempted to hold our ground, and insist that this so-called ‘paradox’, is not a genuine paradox after all. There is an alternative response though. Like Olin, we can distinguish between different types of paradoxes. A large number of paradoxes are of the kind identified in the conceptual thesis. It would be just as paradoxical (if slightly less interesting) to restrict our attention to the plethora of (apparent) paradoxes that fit our conception. The overwhelming majority of paradoxes discussed in textbooks are of this kind. But if the normative thesis is correct, this entails that there are no such paradoxes!

One might press us further still by suggesting that what makes the difference between what counts as a paradox and what doesn’t is that paradoxes must contain a suitably large number of plausible conjuncts; otherwise the set of propositions simply does not count as a genuine paradox. This would explain why, for example, the Sorites and the Lottery paradoxes count as paradoxes but the Shell Game does not.

Our reply. If this objection had teeth, it would render the so-called ‘Paradox Paradox’ unparadoxical. This puzzle is composed of just three plausible theses. Unsurprisingly, then, we deny that paradoxes have to be composed of a large number of inconsistent propositions. Many paradoxes require no more than three theses to state—paradoxes of self-reference, or Hemple’s raven paradox are paradigm examples. Moreover, we suspect that even the paradoxes most naturally stated in terms of a large number of propositions—paradoxes like the Sorites and the Lottery Paradox—can be recast (or are relevantly similar puzzles to) puzzles that contain just three theses. The Sorites paradox, for example, can be recast as the following set of independently plausible but inconsistent propositions: (i) one grain of sand is not a heap; (ii) there are heaps of sand; and (iii) adding one grain of sand to any number of grains of sand does not change it from a non-heap to a heap. To claim that this set is not paradoxical in virtue of the number of propositions it contains seems, in our view, too arbitrary to count as a plausible diagnosis of what’s going on.

We conclude that the existence of paradoxes is itself paradoxical.⁷

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Declarations

Conflict of interest Authors declare that they have no conflict of interest.

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References

- Bolander, T. (2017). Self-reference. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy (Fall 2017 Edition)*. <https://plato.stanford.edu/archives/fall2017/entries/self-reference/>.
- Clark, M. (2015). *Paradoxes from A to Z* (3rd ed.). Routledge.
- Olin, D. (2003). *Paradox*. McGill-Queen's University Press.
- Rescher, N. (2001). *Paradoxes: Their roots, range and resolution*. Open Court.
- Sainsbury, R. M. (1995). *Paradoxes* (3rd ed.). Cambridge University Press.
- Tversky, A., & Kahneman, D. (1982). Judgements of and by representativeness. In D. Kahneman, P. Slovic, & A. Tversky (Eds.), *Judgement under uncertainty: Heuristics and biases* (pp. 84–98). Cambridge University Press.
- Van Heijenoort, J. (1967). Logical paradoxes. In P. Edwards (Ed.), *The encyclopedia of philosophy* (pp. 45–51). Macmillan.

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