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VIRTUE THEORY OF MATHEMATICAL PRACTICES

Moral improvement through mathematics: Antoine Arnauld and Pierre Nicole's *Nouveaux éléments de géométrie*

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Abstract
This paper examines the ethical and religious dimensions of mathematical practice in the early modern era by offering an interpretation of Antoine Arnauld and Pierre Nicole's *Nouveaux éléments de géométrie* (1667). According to these important figures of seventeenth-century French philosophy and theology, mathematics could achieve extra-mathematical or non-mathematical goals; that is, mathematics could foster practices of moral self-improvement, deepen the mathematician's piety and cultivate epistemic virtues. The *Nouveaux éléments de géométrie*, which I contend offers the most robust account of the virtues cultivated by mathematics in the period, was envisaged by its authors to cultivate moral, Christian and epistemic virtues that could serve in the fulfilment of moral and Christian obligations. In this paper, I set out the goals of mathematical inquiry for the Port-Royalists and describe the specific virtues they believed a revised edition of the *Elements* of Euclid could foster. I show that Arnauld and Nicole believed that an acquaintance with mathematics could render a student of Euclid more just, truth-loving, attentive and humble, and better able to discern truth from falsity.

Keywords Early modern mathematics · Early modern moral philosophy · *Elements* · Euclid · Arnauld · Moral virtue · Epistemic virtue

1 Introduction

There was little consensus in the seventeenth century about the aim and use of mathematics. Pierre de Fermat, for instance, was interested solely in the solution of mathematical problems and focussed his ambitions for his intellectual work on the discovery of mathematical truths. Radically unlike him, Benedict de Spinoza made virtually no contributions to pure mathematics. Instead, Spinoza drew on the tools

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of mathematics, namely the geometrical method of Euclid, in the presentation of his masterpiece of metaphysics, moral philosophy and moral psychology, the *Ethics*. Just as there was significant diversity in the aims and uses of mathematics, there was no consensus about whether these mathematical developments of the seventeenth century were to be lauded or deplored. Little agreement existed about what counted as an achievement in the mathematical arts which in the early modern era included geometry, arithmetic, astronomy, mechanics, and optics. Hobbes, for instance, was antagonistic towards the algebraic symbolism of innovators such as Viète and Descartes. Though the developments they inaugurated were "necessary scaffolds of demonstration", he writes, "they ought no more to appear in public, than the most deformed necessary business which you do in your chambers" (1845, p. 248).

Not only was there no widespread agreement about the intellectual value of these mathematical achievements, there was also considerable uncertainty about their moral value. This latter worry culminated in arguments aimed at showing how mathematics could lead an individual to abandon their moral responsibilities and meaningful engagement with the world. For Saint-Evremond, whose remarks first appeared in 1666, "[mathematics] pulls you away from actions and pleasures and occupies you entirely" (1705, p. 138).[1] Mathematical achievement, in his view, came at too high a cost. Others defended mathematics in terms drawn from Plato, arguing that mathematics could, in fact, draw one closer to an understanding of the world and aid in the execution of one's moral and spiritual responsibilities.[2] From the Pythagoreans to Origen, Boethius to Gerbert, Regiomontanus to Ramus, mathematics was described in terms of its ennobling effects. Centuries after Plato, Boethius writes that man is scarcely able to attain philosophical wisdom without a study of the quadrivial arts (Boethius 1983, p. 71). In his *De Institutione Arithmetica*, Boethius prescribes the reading of mathematics in Pythagorean terms, writing that God considered arithmetic the first discipline and exemplary of his own thought, and so created nature in accordance with this reason (1983, p. 74.) But evident in the comments of Boethius' contemporaries is the fact that these prescriptions about the propaedeutic value of mathematics were not heard. As Ullman put it, "Cassiodorus is disconsolate about the low status of the Quadrivium: a course in arithmetic is announced and the lecture halls are empty. Or a course in geometry, which deals to such an extent with important matters pertaining to heaven, is followed only by specialists" (1964, p. 266). Before the Renaissance and early modern era, "the quadrivium was largely ignored" (Wagner 1983, p. 22). The decline of mathematical culture after the fall of Alexandria and the rediscovery of classical works in the Renaissance bookend a period of scant mathematical innovation and learning of mathematics for propaedeutic ends in Europe. Only the rediscovery of mathematical works lost to European culture and the revival of Pythagorean and Platonic philosophy in the Renaissance would inaugurate a process of philological, philosophical and mathematical restoration of ancient works that culminated in their

1 For a discussion, see Jones (2006, p. 1).

2 For representative passages of this view, see Plato (2006, p. 527b).

broader study and brought to the fore arguments about their ennobling effects in the Renaissance and early modern era.[3]

But while many philosophers and mathematicians of the era endorsed the general view that mathematics had moral and cognitive value for its students, few works of mathematics were expressly authored to nurture these virtues.[4] For instance, Descartes writes of the extra-mathematical value of geometry in the *Regulae*, particularly its capacity to teach clear and indubitable cognition and to help students and mathematicians discern truth from falsity (CSM I 1985, pp. 10–11; p. 59).[5] Yet this propaedeutic attitude did not inform his ambitions in the *Géométrie* which was incompatible with the methodological and propaedeutic claims outlined in both the *Regulae* and later in the *Discours*.[6] For Arnauld and Nicole, however, not only were moral, spiritual and intellectual virtues, including those mentioned by Descartes in the *Regulae,* worthy consequences of mathematical learning, they motivated the composition of their mathematical treatise. That is, providing a context for the development of virtues cultivated by mathematics largely exhausted Arnauld and Nicole's aims in writing the *Nouveaux éléments de géométrie.*

Written by Antoine Arnauld in the mid-1650s and prefaced by Pierre Nicole, the *Nouveaux éléments de géométrie* (2009) is a textbook for the study of Euclid's *Elements*. Created at the instigation of Pascal, Arnauld penned the *Nouveaux éléments* to remedy the methodological defects that he believed inhered in contemporary editions of the *Elements*, especially the edition written by Pascal. According to Arnauld, most editions of Euclid's *Elements*, including the volume produced by Pascal, accustomed the mind to disorder and confusion. In rewriting the *Elements* of Euclid, Arnauld

3 As Angela Axworthy argues in her discussion of the propaedeutic value of Renaissance mathematics, "thanks to the rediscovery of the works of Plato and the commentary of Proclus on the first book of Euclid's *Elements*, the reassertion of the propaedeutic status of mathematics went hand in hand with the *restauratio mathematicarum*, which was undertaken especially in Italy in the mid-sixteenth century" (2009, p. 33).

4 In *The Good Life in the Scientific Revolution: Descartes, Pascal, Leibniz, and the Cultivation of Virtue*, Jones argued that the idea that mathematics has propaedeutic value is found in the works of early modern mathematical thinkers such as Descartes, Pascal and Leibniz. Briefly, Jones' view is that Descartes, Pascal, and Leibniz's "efforts to improve techniques for living and thinking well helped shape the kind of mathematics and experimentation they did, what they thought made good mathematics and method, and what they saw as the lessons illustrated by their innovations" (2006, p. 2). In addition to this being a contentious interpretation of Descartes, Pascal and Leibniz, a claim I defend in another paper, I argue in this paper that the view ascribed to these figures is more accurately attributed to Arnauld and Nicole. This issue, as it emerges in Descartes' philosophy, has more recently been treated by Nelson (2019).

5 I use the following standard abbreviations to refer to works:

Nouveaux éléments: *Nouveaux éléments de géométrie* [Edited by D. Descotes]
Logique: *Logique, ou l'art de penser* [Edited by J. V. Buroker]
OA: *Oeuvres de Messire Antoine Arnauld*. 1–43 vols
CSM: *The philosophical writings of Descartes*.

CSMK: *The philosophical writings of Descartes: The correspondence.*

6 According to correspondence with Vatier, Descartes was not convinced that the ideas on method articulated in the *Discours* were made use of in the *Géométrie.* He writes, "I couldn't show the usage of the method in the three treatises which I gave because [the method] requires an order for investigating things that is very different from that which I thought necessary to use to explain them" (CSMK, 1991, p. 85). I develop this line of inquiry in full elsewhere. See also Gaukroger (1995) and Garber (2002, chapter 2) for a complimentary assessment.

had hoped to ground demonstrations on clear and certain principles and present the *Elements* in a true and natural order so as to accustom the mind to the clarity and certainty required of a demonstrative science, specifically, and our everyday affairs more generally. Arnauld intended the work for students of elementary mathematics, mathematicians, readers of their *Logique ou l'art de penser* (for which the *Nouveaux éléments* was to serve as a companion since it exemplified the central precepts of the *Logique*), and members of the learned public for whom the cultivation of virtue was an aim. From its inception to the final edition produced by Arnauld (1683), methodological rather than mathematical considerations guided the writing of the textbook. According to plans for a new elements of geometry set out in an early manuscript of the *Logique,* the task was to present geometry according to its true and natural order, and avoid indirect methods of demonstration (n.d., p. 198). The first published edition of the *Nouveaux éléments* (1667), which is vastly expanded from their original plans, reflects Arnauld and Nicole's desire to ground the *Elements* on the principles of a demonstrative science. Like the first edition, the second edition published in 1683 underscores Arnauld's commitment to improving the method of the *Elements.* In a prefatory *avertissement*, Arnauld writes that becoming accustomed to the true and natural method has propaedeutic value (1683, n.p.). The mathematical improvements of the second edition, he continues, were pursued for the benefits they wrought in propaedeutic terms. From Arnauld and Nicole's remarks in the *Logique* and *Nouveaux éléments* it is clear that purely mathematical considerations were secondary to Arnauld's reasons for producing the geometry textbook. In fact, the *Nouveaux éléments* cautions the reader against pursuing mathematical achievement too eagerly. Much like Saint-Evremond, Nicole writes in his prefatory essay that mathematics can cause a person to withdraw from worldly concerns.

As I will show in this paper, Nicole could be reconciled to the value of mathematics if its practice was put to extra-mathematical use. In this paper, I argue that the *Nouveaux éléments* contained moral injunctions and recommendations that could help the student of the *Elements* to learn about virtue and piety. In the first section, I examine Arnauld and Nicole's remarks about the futility of mathematical study and the disutility of mathematical achievement. According to Arnauld and Nicole, mathematics cannot make one happy and constitutes a trivial misuse of one's time. But if mathematics was so bad, why did Arnauld write the *Nouveaux éléments*? In section two, I offer an account of flourishing that can be developed on Arnauld and Nicole's behalf and connect it to their recommendation to study the *Nouveaux éléments*. Mathematics, I show, is valuable to the extent that its practice can contribute to an individual's moral and spiritual improvement. This is true even for the epistemic virtues described by Arnauld and Nicole since the *Nouveaux éléments* teaches the cultivation of epistemic virtues for moral and religious ends. I precisify this account in section three wherein I describe the moral and Christian virtues Arnauld and Nicole argued could be cultivated by the study of mathematics and justified an individual's acquaintance with the *Elements*. In their view, mathematics could help dispose the mind to love the truth and teach a student of the *Elements* to recognise truths by cultivating reason. This was a valuable lesson, I argue, since the goal of all inquiry for the Port-Royalists was truth and not knowledge. Second, I show that mathematics could teach its reader to be attentive and offered lessons in humility. In the course of so doing, I show how

Arnauld and Nicole intended for the *Nouveaux éléments* to reveal the limits of natural knowledge and the demands of faith. Lastly, I explore how these recommendations about the moral and religious value of mathematics are evidence of an underlying account of education and moral psychology.

It was not uncommon for works of mathematics to pursue extra-mathematical goals in the early modern era. As Axworthy (2009), Goulding (2010), Jones (2006) and Rose (1975) have shown, these extra-mathematical goals sought by mathematicians could be disciplinary, historical, philological, moral, philosophical, methodological, pedagogical, or theological. This paper develops an account of the moral and spiritual goals of early modern mathematicians and philosophers by examining the virtues cultivated by mathematics as they are presented in the *Nouveaux éléments*. Few mathematical works of the seventeenth century offer such detailed reflections on the connection between mathematical practice and virtue, and in particular the moral and Christian virtues Arnauld and Nicole thought could be cultivated by mathematics. Scholarship on early modern mathematics and accounts of virtue in the early modern era, I therefore argue, benefits from examining how works of mathematics were understood to foster a life of virtue and participated in the achievement of such a life. The *Nouveaux éléments*, I argue, is central to understanding this propaedeutic process since Arnauld and Nicole's outlook on mathematics more generally, and their pedagogically significant edition of the *Elements*, in particular, presented the cultivation of virtue as a central and important goal of mathematics.

2 The disutility of mathematical achievement

In the treatise for which the Jansenist community at Port-Royal is best known—the *Pensées*—Pascal offered reflections on topics such as human nature, virtue, grace, salvation, the problem of human knowledge, and the unhappy state of man without God. These concerns are one, for Pascal, since "[m]an's true nature, his true good, true virtue, and true religion, are things that cannot be known separately" (2004, S12/L393, p. 3). Pascal opens his discussion of these issues in the *Pensées* by meditating upon possible sources of human happiness: "[t]he Stoics say, 'Withdraw into yourself, where you will find peace.' And this is not true. Others say 'Go out, seek happiness in diversion.' And this is not true. Illnesses arise" (2004, S26/L407, p. 5). In his introduction to the *Pensées*, Popkin argues that Pascal aims to show that common, ordinary, worldly proposals for happiness produce only the opposite outcome because the goals individuals typically seek tend to be worthless (1989, p. 11). For Pascal, the only path to happiness lay outside any secular or terrestrial ambition. Reflecting on mathematics, in particular, Pascal wrote to Fermat that though it was a stimulating intellectual exercise, mathematics "is at the same time so useless" (1922, p. 300).

This was just as true for Arnauld and Nicole, according to their *Nouveaux éléments*. Nicole begins his prefatory remarks to the *Nouveaux éléments* with arguments about the disutility of mathematics pursued for purely mathematical ends. The knowledge mathematics gives rise to, that is the "lines, angles, and proportions" that occupy geometers, cannot bring about happiness. As Nicole puts it "these sterile speculations contribute nothing to making us happy" (*Nouveaux éléments*, p. 95). The arts of mathematics, Nicole writes, "do not relieve our miseries, they do not cure our pains; they cannot give us any real and solid contentment"[7] (*Nouveaux éléments*, p. 95). In his view, the knowledge taught by mathematics cannot make one happy or alleviate one's burdens. For this reason, he states that geometry is not a worthwhile pursuit. In this way, Nicole's views are utterly unlike those of Descartes who was, for the Port-Royalists, an important influence in epistemological matters. For Descartes, "[t]rue philosophy […] teaches that even amidst the saddest disasters and most bitter pains we can always be content, provided that we know how to use our reason" (CSMK 1991, p. 272).

Worse than simply being an inadequate source for the alleviation of our misery, Nicole believed that mathematical pursuits which aimed at discovery were useless and mathematicians were vainglorious.[8] In the very first lines of his Preface to the *Nouveaux éléments*, Nicole writes that he is "rather inclined to diminish the too lofty idea that some people have about geometry because [he is] convinced that it is far more dangerous to overrate these kinds of things than not to estimate them enough" (*Nouveaux éléments*, p. 94).[9] That is, Nicole writes that he is inclined to be critical about the value of mathematics since the opposite inclination, praising it highly, can have dramatic and unwanted consequences. What danger did Nicole identify in praising mathematical achievement or pursuing mathematical knowledge? According to Nicole, mathematics was no lofty pursuit. Mathematicians, he writes, pride themselves on discoveries, they pursue these single-mindedly and work to achieve them as though they were important contributions. Chiding mathematicians who believe they "oblige the world very much if they share [their discoveries]" and "deserve a very considerable rank among scholars and great minds", Nicole writes that it is a mistake to esteem the pursuit of mathematical discoveries. According to Nicole, the pursuit of discoveries cannot elevate the soul and reveals the baseness of the human mind. As he puts it "since [the human mind] is so vain or so empty of the true good, it is capable of being completely occupied by things so vain and so useless" (*Nouveaux éléments*, p. 95). In his view, the preoccupations of contemporary geometers serve as evidence of how base and worthless is the human mind. Nicole viewed the major mathematical innovations of the seventeenth century with disdain. Though he is nowhere so explicit, Nicole would have considered Descartes' program in analytic geometry, for instance, a vainglorious and unworthy pursuit common among mathematicians in the seventeenth century.

Not only could mathematics not be counted among the worthwhile branches of knowledge, Nicole believed mathematics as it was practised by seventeenth-century mathematicians to be morally bankrupt. In the Preface to the *Nouveaux éléments*,

[7] Translation of "C'est une ignorance très blâmable que de ne pas savoir, que toutes ces spéculations stériles ne contribuent rien à nous rendre heureux; qu'elles ne soulagent point nos misères; qu'elles ne guérissent point nos maux; qu'elles ne nous peuvent donner aucun contentement réel et solide."

[8] Compare the two positions Nicole adopts in the Preface about the disutility of mathematical practice: "et je ne sais si l'on ne peut point dire qu'elles sont toutes inutiles en elles-mêmes" (*Nouveaux éléments*, p. 94) and "puisqu'il est si vain et si vide de vrai bien, qu'il est capable de s'occuper tout entier à des choses si vaines et si inutiles" (*Nouveaux éléments*, p. 95).

[9] Translation of "Je serais plutôt porté à diminuer l'idée trop haute que quelques personnes en pourraient avoir, étant très persuadé qu'il est beaucoup plus dangereux d'estimer trop ces sortes de choses, que de ne les pas estimer assez."

Nicole describes the achievements of mathematics as scarcely worth knowing and the interest in learning mathematics or cultivating mathematical knowledge, which involves "having filled one's head of lines, angles, circles, and proportions", as a moral failing (*Nouveaux éléments*, p. 95). This sentiment is echoed in the First Discourse of the *Logique*, which was also penned by Nicole. Here, he writes, "[n]ot only do these [speculative] sciences have nooks and crannies of very little use, but they are completely worthless considered in and for themselves" (Logique, p. 5). By the qualification "in and for themselves" Nicole means that knowledge of mathematics or the achievements that are the outcome of its pursuit are without value. In Nicole's view, not knowing mathematics was no great weakness ("Ce n'est pas un grand mal") whereas believing that mathematics was of considerable importance constituted a great moral failing. In fact, he describes the belief that being a geometer is a worthy pursuit as a "significant evil" while describing the failure to acknowledge the disutility of mathematics as "a very culpable ignorance." Much like his contemporary Pascal, Nicole believed that the abstract sciences are not the proper occupation of man. Putting it in normative terms, man, writes Nicole, "is not made for this" (*Nouveaux éléments*, p. 95). What emerges in the prefatory remarks of the *Nouveaux éléments* is the view that there was little that made practising mathematics for its own end a worthwhile pursuit. Nicole, long thought to be the author of the First and Second Discourses, writes that man's obligation in this life is not to "spend time measuring lines [or] examining the relations between angles […]. The mind is too large, life too short, time too precious to occupy oneself with such trivial objects" (*Logique*, p. 5).

These are surprising arguments to find at the outset of a lengthy textbook on geometry. How can we reconcile such blistering observations by Nicole about the value of the speculative sciences—which numbered geometry among them but also included astronomy and physics—with the fact that Antoine Arnauld devoted significant amounts of time more than once in his life to the composition of his geometry textbook, to which Nicole and Pascal both contributed? Why would Nicole argue that we should ignore the abstract sciences, which for him included mathematics, if at Port-Royal was being composed a work of mathematics roughly contemporaneous with his most scathing critiques in the *Logique*? More pressingly, why would such comments appear in Nicole's Preface to Arnauld's mathematical work?

One, albeit highly unlikely, explanation is that Nicole's prefatory remarks to the *Nouveaux éléments* appeared unbeknownst to Arnauld and earned his disapproval since Nicole's views were so radically unlike his own. This certainly helps us reconcile the fact that Arnauld writes more accommodatingly of the purely intellectual achievements of contemporary mathematicians and natural philosophers.[10] Arnauld's remarks in

[10] As we will see, Nicole's critical remarks are echoed throughout the *Nouveaux éléments*. Yet Arnauld also praises the purely intellectual achievements of contemporary mathematicians and natural philosophers. Arnauld's remarks to Du Bois in the "Réflexions sur l'éloquence des prédicateurs" and his praise of Descartes in the "Examen d'un écrit qui a pour titre: Traité de l'essence du corps, et de l'union de l'âme avec le corps, contre la philosophie de M. Descartes" show not only that he believes that the mathematical and natural sciences are not entirely useless but also that he believes the discoveries of contemporary natural philosophers and mathematicians were impressive. See particularly Antoine Arnauld, the "Examen d'un écrit qui a pour titre: Traité de l'essence du corps, et de l'union de l'âme avec le corps, contre la philosophie de M. Descartes" (*OA* 38 1775–1783, pp. 96–97) and available in partial translation as "Eulogy on Descartes's Philosophy," in Arnauld 1899, pp. 311–314).

the *Examen d'un écrit qui a pour titre: Traité de l'essence du corps, et de l'union de l'âme avec le corps, contre la philosophie de M. Descartes*, for instance, draw on the discoveries of the seventeenth century, which include the mathematical ideas of Descartes, to defend the view that human corruption does not increase over time (1899, pp. 311–314). What makes this proposal so unlikely is that there is no record of such a disagreement in the correspondence related to the *Nouveaux éléments*. Secondly, the idea is incompatible with the fact that the second edition of the *Nouveaux éléments,* published in 1683 and over which Arnauld had significant control, contains the very same remarks about the disutility of mathematical achievement.

With that, the old question remains—why do Nicole's critical remarks preface the *Nouveaux éléments*—and a new interpretative challenge emerges: how are we to understand Arnauld's more accommodating views about mathematics in light of Nicole's censorious remarks in the Preface? The most likely explanation is this: while Arnauld could praise the discoveries of Descartes, Galileo, Huygens and Cassini, and recognise them as worthwhile (a fact about which he may have disagreed with Nicole), this praise does not preclude his endorsement of the view that not everyone ought to spend the greater part of their time immersed in the study of the natural and mathematical sciences. Since many of the critiques articulated by Nicole find both theoretical and practical expression in the parts of the *Nouveaux éléments* penned by Arnauld, it is more likely that their substantive disagreement is about the value of mathematical achievements, and not about the more general aims of mathematical study. What were these more general aims of mathematical study that Arnauld and Nicole agreed should guide the greater part of one's practice? The answer is not far to seek. As Nicole writes in his prefatory remarks to the *Nouveaux éléments*, the practice of mathematics ought to be subordinated to the non-mathematical ends to which it could be directed. That is, mathematics should serve in the moral, spiritual and intellectual improvement of the individual.

3 Geometry and the life of virtue

If mathematics, pursued for its own ends and directed towards achievement and discovery, was not an entirely worthwhile activity, what did the Port-Royalists regard as the fulfilment of an individual's obligations? What, in their view, constituted a praiseworthy action in this life? In sum, what conceptions of a virtuous life informed Arnauld and Nicole's efforts, both critical and positive, in the *Nouveaux éléments*? The answer to these questions is given in the First Discourse of the *Logique* where Nicole writes that individuals are "obligated to be just, fair, and judicious in all their speech, their actions, and the business they conduct. Above all they ought to train and educate themselves for this" (*Logique*, p. 5).[11] The Port-Royalists believed that our basic sociability demands that we behave justly, fairly, and judiciously and that we ought to direct all of our efforts to achieving these moral ends. In Arnauld and Nicole's

[11] Translation of "ils sont obligés d'être justes, équitables, judicieux dans tous leurs discours, dans toutes leurs actions, et dans toutes les affaires qu'ils manient; et c'est à quoi ils doivent particulièrement s'exercer et se former" (Arnauld and Nicole 1683, p. 2). Buroker translates "s'exercer et se former" as "train and educate." In my view, "practice and train" may be a more accurate translation.

view, our actions should aim to cultivate these and other virtues, and avoid vices such as vainglory, laziness, and concupiscence, among others.

While mathematics alone could not train us to avoid these vices or cultivate these virtues, in Arnauld and Nicole's view, mathematics was nevertheless the most efficacious secular practice for disposing the mind to virtue (*Nouveaux éléments*, p. 96). Dedicated readers of Arnauld and Nicole will need no reminding that goodness, and living well more generally, was impossible without God's help (Kilcullen 1988, pp. 8–9).[12] This general concern among the Port-Royalists is expressed in the *Nouveaux éléments*. Here, Nicole puts it thus: "It is true that only grace and exercises of piety […] can truly cure." But, and this is crucial for understanding their efforts in mathematics, Nicole continues, "among the human exercises which can serve to diminish and dispose the mind to receive the Christian truth with less opposition and disgust, there [are] hardly any more appropriate than the study of geometry"[13] (*Nouveaux éléments*, pp. 96–97). While the virtues trained by mathematics could not earn a student of mathematics salvation, no secular practice in his and Arnauld's view could so successfully dispose an individual to praiseworthy action and Christian piety as mathematics. That moral and spiritual advantages could accrue to students of mathematics is clear from Nicole's remark that geometry could be put to use in the training of young people (though he elsewhere includes adults in this propaedeutic process) "not only in the correctness of spirit but furthermore in piety and their moral habits" (*Nouveaux éléments*, p. 96). Cognisant of the real source of goodness, which is God's grace, Nicole writes that it is "[n]evertheless […] impossible to dispense completely with a science that serves as a basis for many arts necessary for human life." The ambition of the *Nouveaux éléments* was, Nicole continues, "to show people what kind [of geometry] they should use, and to render their study as advantageous as possible"[14] (*Nouveaux éléments*, pp. 95–96).

This statement is the clearest and most explicit articulation of Arnauld and Nicole's view that mathematics should be practised because it contributes to an individual's moral and spiritual improvement. Before examining how particular virtues are cultivated by mathematics it is worth addressing Arnauld and Nicole's views about the relationship between what the contemporary virtue epistemologist might now identify as distinctly moral and epistemic virtues. The remarks about the goal of education as justice and judiciousness in speech and action quoted from the *Logique* at the outset of this section makes clear that epistemic virtues are not fundamentally distinct from moral virtues, at least as far as the ultimate end or goal of this process of self-improvement is concerned. Rather than being motivated by epistemic ends like knowledge or understanding, the virtuous agent, in Arnauld and Nicole's view, should

[12] For a more recent discussion of Arnauld's theology which touches on questions of God's will and grace see Nadler (2008).

[13] Translation of "Il est vrai qu'il n'y a que la grâce et les exercices de piété qui puissent la guérir véritablement: mais entre les exercices humains qui peuvent le plus servir à la diminuer, et à disposer meme l'esprit à recevoir les vérités chrétiennes avec moins d'opposition et de dégoût, il semble qu'il n'y en ait guère de plus propre que l'étude de la géométrie."

[14] Translation of "Néanmoins comme il est impossible de se passer absolument d'une science qui sert de fondement à tant d'arts nécessaires à la vie humaine, il peut y avoir quelque utilité à montrer aux hommes de quelle sorte ils en doivent user, et de leur rendre cette étude la plus avantageuse qu'il est possible."

be motivated to cultivate epistemic virtues because our basic sociability demands that we behave morally and that we ought to direct our efforts—moral, spiritual, and intellectual—to the achievement of this outcome.[15]

According to Arnauld and Nicole, our orientation to moral ends demands the most diligent and exacting use of our reason. The Port-Royalists believed that the cultivation of epistemic virtues was demanded by the duties that individuals have to one another and to themselves as moral agents.[16] This is because, in their view, the gravest errors we make relate to the conduct of our lives, and not our pursuit of knowledge in the sciences (*Logique*, p. 203). As Arnauld and Nicole put it:

> The most common use of good sense and the power of the soul that makes us distinguish truth from falsehood is not in the speculative sciences, to which so few persons are obliged to apply themselves; but there are hardly any occasions where it is used more frequently, and where it is more necessary, than in judgments we make about what takes place every day in human affairs. (*Logique*, p. 262)

Given how widely good sense is required, the *Logique* of Port-Royal aims to cultivate this quality, which in the First Discourse they describe as praiseworthy. Geometry participated in this moral process, I argue, since according to their remarks in the First Discourse of the *Logique*, it fortified our powers of reasoning. In Arnauld and Nicole's view, we should not use reason to acquire knowledge of the speculative sciences, but we should use the speculative sciences for the cultivation of our reason. As Nicole puts it "this should move wise persons to engage in speculation only to the extent that it serves this purpose, to make it merely the test and not the main use of their mental powers" (*Logique*, p. 5). Arnauld and Nicole, therefore, understood the intellectual benefit of expanding the mind's capacity to reason in moral and religious terms. In what follows, I examine the role of mathematics in the development of pro-attitudes towards truth and prescriptions about the cultivation of reason, attentiveness and humility in the service of moral and spiritual ends.[17]

[15] This discussion is of interest to contemporary virtue epistemologists, such as Baehr (2011), whose attempt to ground the distinction between moral and intellectual virtues informs the account given here. See Baehr (2011, particularly Appendix).

[16] This is just as true of Malebranche who writes in *The Search After Truth* that as spiritual beings our duties are to use our freedom to avoid vanity and falsity. More than this, we are obliged to cultivate our understanding, urge it towards new knowledge, and knowledge of truths based on meditations on worthy subjects. According to Malebranche, we have a moral duty to perfect our minds (1997, p. 11). Of course, what distinguishes this general attitude from the account developed by Arnauld and Nicole is the extent to which Malebranche's mathematical work constituted the context for the cultivation of these virtues. Unlike Arnauld and Nicole, Malebranche did not author a work of mathematics to facilitate the cultivation of virtue.

[17] The more general question of whether epistemic virtues are to be understood as reducible to moral virtues, a subset of moral virtues, or distinct from moral virtues is beyond the scope of this paper. I have bracketed this discussion since it requires a lengthier treatment of Port-Royalist virtue epistemology, and because the epistemic virtues to be treated in this paper are among the intellectual virtues that are reducible to moral virtues.

4 Truth, attentiveness and humility

Mathematics, as I argued above, was valuable to the Port-Royalists since it developed a student's epistemic capacities in morally and spiritually significant terms. Arnauld and Nicole's account of truth is evidence of this view since Arnauld and Nicole argued that mathematics could play a role in cultivating knowledge of, and attitudes towards, truth. Geometry, could offer the student of mathematics a practice in recognising truths and disposing the mind to love them. It was on the basis of this claim that Arnauld and Nicole encouraged an acquaintance with their *Nouveaux éléments* (p. 97). Before detailing how precisely pro-attitudes towards truth and the ability to discern truths from falsehoods could be cultivated by mathematics in Arnauld and Nicole's view, it is worth exploring the Port-Royalists' account of the relationship between truth and virtue, and the moral, spiritual and epistemic importance of truth.

In the *Logique*, Arnauld and Nicole offer a famous account of clear and distinct ideas. Less well known is the propaedeutic goal Arnauld and Nicole sought through their account of clarity and distinctness. According to the two Port-Royalists, the consequences of having false ideas were gravest in the practical realm from which nobody could be spared. For Arnauld and Nicole, having confused and obscure ideas was morally and spiritually significant. This is why their theoretical discussion of them in the *Logique* turns immediately to an account of their practical importance. As Arnauld and Nicole put it,

> no one is exempted from forming judgments about good and evil, since these judgments are necessary for conducting our lives, directing our actions, and making ourselves eternally happy or miserable. Because false ideas about all these things are the source of the bad judgments we make about them, it is infinitely more important to apply ourselves to knowing and correcting these ideas. (*Logique*, p. 54)

This idea is echoed elsewhere in the *Logique*, wherein Arnauld and Nicole write that false judgements so often impel great undertakings and provide the aims and motivations that ground the conduct of our lives, and for this reason, require our attention (*Logique*, p. 56). Our ability to discern truth from falsity is, in Arnauld and Nicole's view, necessary to our moral and spiritual flourishing in this life and the next. Mathematics could participate in this process of moral and spiritual cultivation, according to Arnauld and Nicole. As they write in their *Nouveaux éléments*, "geometry teaches also to recognise truth and to not be deceived by the great number of obscure and uncertain statements which serve as principles for false reasoning with which people's speech is always filled" (*Nouveaux éléments*, p. 97). For Arnauld and Nicole, truth and virtue were intimately connected.

One cannot overstate the connection between truth and flourishing for the community at Port-Royal. Truth had religious and political, moral and spiritual, and epistemic significance, and to understand the role to be played by mathematics, it is first necessary to describe truth in terms of the significance it had for the Port-Royalists. For one, truth was a principle around which the Jansenist community rallied. According to Brian Strayer, Antoine Arnauld thought truth comes from God and that for this reason it must be vigorously defended (2008, p. 2). This was echoed by Angélique Arnauld, sister to Antoine Arnauld and abbess of Port-Royal, who wrote widely and avidly about defending the truths of Saint-Cyran, as well as Isaac Arnauld whose writings encapsulate the themes of fighting for the truth, cultivating virtue and submission to God (pp. 109–110). As Strayer puts it in his history of Jansenism, "the most frequently used term in Jansenist writings is Verité" (p. 16).

God was synonymous with truth for the Port-Royalists. Nowhere is this more clearly stated than in the *Logique* where Arnauld and Nicole write that "God is truth itself" (p. 261). For this reason, knowledge of truths and a love of the truth had moral and religious significance. This dedication to truth is one source of influence on the Port-Royalists' epistemological commitments. Knowledge, as I argued earlier, was not an unalloyed epistemic good in the Port-Royalists' view. What, then, was the goal of inquiry? I propose that, for the Port-Royalists, truth was the fundamental epistemic good to which all inquiry should aim and was to be valued non-instrumentally.[18] Arnauld and Nicole's attitude to truth emerges clearly in the *Logique* wherein they write that "[t]he only remedy for [reasoning poorly] is to take as our goal only the truth, and to examine the arguments so carefully that this commitment itself cannot mislead us" (p. 214).[19] On the interpretation I offer of Arnauld and Nicole's writings, truth is an unalloyed and non-instrumental epistemic good. Mathematics is a valuable practice since it cultivates pro-attitudes towards truth and offers us practice in recognising truths.

With these general commitments to truth in view, how does an acquaintance with mathematics contribute to a process of self-betterment? For one, mathematics could help an individual to develop pro-attitudes towards truth. As Nicole puts it in the *Nouveaux éléments*, geometry is useful to the important task of "accustom[ing] oneself to love the truth, and to taste it, and to feel its beauty" (*Nouveaux éléments*, p. 97). In Susan James' view, loving the truth had a non-cognitive, emotional element. Describing the general attitude to loving truth in the seventeenth century, James writes that the love of truth, "a particular emotional disposition […] guide[s] our thoughts and actions" (1997, p. 160). Arnauld and Nicole's explanation for why it is important to develop pro-attitudes to truth is largely characterological. In their view, a well-cultivated person (*honnête homme*) will defend the truth above all else and "put one's weapons in the service of the truth as soon as one perceives it, and to want to hear it even in the mouth of one's adversary" (*Logique*, p. 246). An absence of this pro-attitude towards truth, however, is characterised as an epistemic vice since failing to love truth adequately guarantees the inability to judge what is true or false. As Arnauld and Nicole put it in the preliminary discourse, "[t]he little love people have for the truth causes them not to take the trouble most of the time to distinguish what is true from what is false" (*Logique*, pp. 6–7). This discussion has Augustinian resonances since, according to James, "failing to feel the appropriate loves and hatreds" is an emotional failure with consequences for our will and the understanding (1997, p. 225). Like the Augustinian account described by James, Arnauld and Nicole's recommendation for developing

18 The terms of reference for this discussion are Pritchard, "Truth as the Fundamental Epistemic Good" (2014, pp. 112–115).

19 Elsewhere, Arnauld and Nicole write that "[w]e should also govern ourselves in such a way that we can watch them stray without going astray ourselves, and *without wandering from the goal we ought to set for ourselves, which is to be enlightened by the truth we are investigating*." Emphasis added. *Logique* (p. 211).

pro-attitudes towards truth couples the emotions with epistemic consequences. While the immediate consequences may be epistemological, the moral and spiritual consequences were not far to seek. According to Arnauld and Nicole, God makes use of our love of the truth “to introduce us to the love and practice of these truths which lead to our salvation” and “to render us just and equitable throughout [the conduct of] our life” (*Nouveaux éléments*, p. 97).

In Arnauld and Nicole’s view, geometry could offer practice in recognising truths, including spiritual truths. How geometry achieves this end, they present through complex arguments about the nature of the imagination and the senses. In Book IV of the *Nouveaux éléments*, Arnauld follows an algebraic exposition of the multiplication of two numbers by writing that his treatise on mathematics accustoms the mind to conceive of things in an incorporeal way, without the use of any images. Crucially, he continues, this precept “makes us capable of knowing about God and about our soul” (*Nouveaux éléments*, p. 117). It is the fact that mathematical truths and religious truths are incorporeal that make mathematics, in their view, the most appropriate secular context for the exploration of spiritual matters. Nicole offers a similar rationale in the Preface to the *Nouveaux éléments*, writing that the first four books make use of algebraic notation. Along lines articulated by Arnauld, Nicole writes, “[t]he reason that obliged us to use [the algebraic method] is that treating quantities in general insofar as this word includes all the kinds of quantity, *we could not use figures to help the imagination*” (Emphasis added, *Nouveaux éléments*, p. 103). Mathematics is useful, therefore, in helping students and mathematicians recognise truths without the aid of the imagination.

Mathematics, Arnauld and Nicole believed, could cultivate an appetite for spiritual truths, since spiritual truths, like the truths of mathematics, were utterly unlike knowledge derived from the senses or passions. The object of geometry, writes Nicole, “has absolutely no connection with concupiscence” and contained in it nothing that pleases the senses (*Nouveaux éléments*, p. 97). Moreover, geometry also contains nothing in it that excites unhelpful passions. Unhelpful passions, like the senses and concupiscence, were an undesirable part of everyday life that should be diminished through education. Instead, geometry presents its truths “to the soul nakedly”, unvarnished with those things that distract or entice, and for this reason constituted a useful practice in inspiring a love of truth and disgust of what pleases the senses (*Nouveaux éléments*, p. 97). By diminishing the role of the senses in our cognition, geometry, believed Arnauld and Nicole, could give us a taste of spiritual truths since these were akin to the truths taught by mathematics. This similarity in kind earned mathematics the status of being the right kind of secular practice for the cultivation of virtue.

In addition to teaching an individual to recognise truths, Arnauld and Nicole recognised the power of geometry to teach particular truths of the Christian faith. According to the Port-Royalists, mathematics could be used to teach truths about God and the nature of our soul. According to Nicole, the expansiveness of the mind that was cultivated by geometry was an admirable pedagogical tool since “there is hardly any quality of soul which makes its grandeur more visible and which better ruins the base and crude imaginings of those who would like to make [the soul] material.” If the mind were a physical thing, as materialists claimed, it could not perform, writes Nicole, the many operations of the mind that are demanded in the practice of geometry (*Nouveaux éléments*, p. 100). Nicole was not alone in making something of a leap in arguing from the infinity of possible mental operations to the immateriality of the soul. According to Jones, the Jesuit Ignace-Gaston Pardies made similar claims against libertines and sceptics, arguing that "measuring the infinite requires something beyond the finite, namely, a nonmaterial soul" and "[s]ince human beings can measure the infinite, they must have a nonmaterial soul" (2006, p. 118).[20] Though Arnauld and Nicole were perhaps less sanguine about how well an individual can comprehend infinity, they nevertheless believed that the infinitely many operations demanded of the mind in mathematics showed materialists the inadequacy of their claim that the mind was material. In the course of so doing, geometry taught a number of positive lessons about spiritual truths and the nature of a person's soul.

Like loving truth or cultivating an expansive mind, becoming attentive constituted a moral, Christian and epistemic virtue that could be fortified by mathematical practice. Yet unlike in the preceding discussion where Arnauld and Nicole drew on the similarity between mathematical and religious truths for religious ends, in their account of attention Arnauld and Nicole propose that mathematics fosters a capacity to be attentive that is readily and favourably applied to extra-mathematical ends. In this, the Port-Royalists continued earlier spiritual traditions and presented a view compatible with the emerging secular attitudes to the value of attentiveness. In the reflections of seventeenth-century philosophers following Descartes, attentiveness was an epistemic virtue since, as it is presented in Rule 3 of the *Regulae*, it is the attentive mind that guarantees the testimony of our intuitions (CSM I 1985, pp. 13–14). That is, in Descartes' view, we can be certain of the truth of our intuitions on account of their being the result of having attended carefully to a given matter. A similar account emerges in the *Meditations* wherein attentiveness takes on an epistemic role. Here, and in the Replies, Descartes appears to argue that an idea is immune from doubt if it is the consequence of an attentive mind since, as Mole (2017) puts it, "doubt cannot be maintained by an attentive thinker."[21] Arnauld and Nicole embraced the view that an attentive mind was imperative to reasoning well in both the *Logique* and the *Nouveaux éléments.* As Arnauld and Nicole put it in the *Logique*, in terms largely borrowed from Descartes, "most false judgments [...] are caused only by impetuosity and lack of attention, which make us judge recklessly about things we know only confusedly and obscurely" (*Logique*, p. 6). Geometry could participate in this process of accustoming the mind to form clear and distinct ideas since it both provided a model of clarity and distinctness and "accustoms the mind" to put the rules of clarity and distinctness into practice (*Nouveaux éléments*, p. 98). More than this, Arnauld and Nicole argue in the *Logique* that "attention paid when following these rules" of reasoning employed by geometers "is sufficient to avoid making faulty inferences when we are treating scientific matters" (*Logique*, p. 240). In his *Logica vetus et nova* (1654), Johannes Clauberg also argued that mathematics was a subject whose study gave the mind "the lasting

[20] Mathematics was not the only route by which an individual could counter materialism. For Descartes, language showed us the existence of a soul since only humans can arrange words or other signs in ways that allow them to declare their thoughts to others (CSM I 1985, p. 141).

[21] For a more detailed account, see Brown (2007).

habit of being attentive."[22] In his explanation of why this might be, however, Clauberg differs from the Port-Royalists in arguing that it is by wanting to avoid unnecessarily redoing one's demonstrations that students of mathematics learn the habit of attentiveness. In his view, errors resulting from carelessness require the student to restart a demonstration and "such repetition is so unpleasant that we quickly develop the habit of paying more attention" (Clauberg 2007, p. 68).[23] Though Arnauld and Nicole write that they were unaware of Clauberg's *Logica* until after the publication of the *Logique*, their independent assessment of the value of mathematics is telling (*Logique,* p. 185). What is unique about Arnauld and Nicole, however, is that they transformed these insights about the utility of mathematics into a set of methodological (and, therefore, mathematical) improvements of Euclid's *Elements* to serve extra-mathematical or propaedeutic goals, including the cultivation of attentiveness.

Geometry was valuable, in Arnauld and Nicole's view, since it could also deepen an individual's attentiveness to serve moral and spiritual ends. This is perhaps not surprising since, as David Marno has argued, the transformation of attention into a secular, epistemic virtue took place in the late sixteenth and early seventeenth century. Before this time, attention was regarded as a spiritual ideal essential to religious devotion. Among the Port-Royalists, attention also retained the earlier, ethical and Christian dimension in which it was also a spiritual and devotional tool (2014, pp. 135–136). In the *Nouveaux éléments*, Nicole writes that an attentive mind is crucial to apprehending "important truths for the conduct of life and for salvation" which are difficult to comprehend (*Nouveaux éléments*, p. 99). God, he writes, wanted us to work hard to attain these truths. Yet these religious truths to which Nicole refers will exclude many since the majority of individuals are repelled by hard work due to laziness or are poorly disposed to anything that requires hard work or restraint (*Nouveaux éléments*, pp. 99–100). But in Nicole's view, geometry could offer a useful practice to condition the mind and the soul. Both would be required since inattentiveness was equally a moral, spiritual and epistemic vice. According to Nicole, "the study of geometry is again a remedy for [laziness and inattentiveness]; because by applying the mind to abstract and difficult truths, [geometry] makes it easy for the mind to apprehend all those [truths] which require less effort, as by accustoming the body to carry heavy burdens, we achieve that the body almost does not feel any longer the weight of those burdens which are lighter"[24] (*Nouveaux éléments*, p. 100). Nicole's point can be expressed more felicitously through an example: just as someone hoping to improve their physique will find it easier to do so once they have engaged in some exercise, Nicole believes that by accustoming oneself to being attentive and

[22] Thank you to an anonymous *Synthese* reviewer for drawing my attention to Clauberg's *Logica vetus et nova.*

[23] Translation of "telle répétition est si désagréable que nous prenons bien vite l'habitude d'une plus grande attention." See also Clauberg (1654, p. 12).

[24] Translation of "Or l'étude de la géométrie est encore un remède à ce defaut; car en appliquant l'esprit à des vérités abstraites et difficiles, elle lui rend faciles toutes celles qui demandent moins d'application; comme en accoutumant le corps à porter des fardeaux pesants, on fait qu'il ne sent presque plus le poids de ceux qui sont plus légers."

apprehending difficult truths through geometry, one will be less burdened by tasks that demand attentiveness or the examination of difficult truths.[25]

The last of the virtues cultivated by mathematics to be considered in this paper is humility. In the first chapter of Part IV of the *Logique* wherein Arnauld and Nicole discuss basic questions in epistemology such as what can be known and by what means, the Port-Royalists argue that we ought to develop an understanding of the limits of our cognitive capacities and approach particular kinds of knowledge with humility. In Arnauld and Nicole's view, reason distinguishes between what is known clearly and certainly, what we do not know clearly but could, and lastly, what is impossible to know certainly since we either lack the principles to lead us to this knowledge or because this knowledge is disproportionate to the mind. Countless questions in theology and metaphysics such as the nature of God's power and anything to do with infinity (e.g. whether God could make an infinitely large body or whether one infinity is larger than any other) are of this nature. This is because nothing can be known about these questions and they are too removed from clear and distinct principles ever to be resolved (p. 230). The discussions of infinity that emerge in the context of the *Logique* and *Nouveaux éléments* concern not whether infinity exists, but what its existence reveals about our epistemic grasp and the nature of faith. As we will see, mathematics, and particularly the concept of mathematical infinity, serves both as a source of examples about the kinds of knowledge that may produce feelings of epistemic humility and the context for appreciating the disproportion of our minds. Arnauld and Nicole exploited these lessons in epistemic humility for pious ends in the interpretation of religious truths, and to caution their readers against the application of methods taught by mathematicians to religious and moral matters.

According to Arnauld and Nicole, the mind in the contemplation of infinity is "lost" and "dazzled" (*Logique*, p. 230). This is because the mind furnishes us with many and contrary thoughts about infinity which we find overwhelming. Arnauld and Nicole intend that two lessons be drawn from the difficulty we experience in understanding infinity, and difficult mathematical concepts more generally. First, we ought to avoid inquiries where it is impossible to succeed. This ensures that we make progress in matters where success is more attainable. Secondly, we ought to recognise that the existence of ideas or objects does not depend on our capacity to understand them. Arnauld and Nicole remark that while infinity and other concepts in mathematics may be incomprehensible, it is nevertheless imperative to believe that they do exist, however difficult it may be to imagine and conceive that this is so (*Logique*, p. 231). In fact, Arnauld and Nicole couch their discussion of ratios in the *Nouveaux éléments* in precisely these terms. In their chapter on proportion, for instance, Arnauld acknowledges that understanding the definitions of the equality of ratios is one of the most challenging tasks in geometry (*Nouveaux éléments*, p. 163). Having then explained that two ratios are proportional on the basis of their parts, Arnauld writes that while this may seem strange "…not only can it be, but that we are assured of it with an absolute certainty" (*Nouveaux éléments*, pp. 163–164). Commensurate with Nicole's basic point that geometry in itself is not particularly useful, Arnauld and Nicole write

[25] The Port-Royalists also recommended non-cognitive routes for the cultivation of moral and epistemic virtues. Working hard through manual labour was itself beneficial, according to the Port-Royalists since they believed slovenliness was a great source of sin. See Barnard (1913, p. 95ff).

that the benefit of knowing these proofs "is not just to acquire this kind of knowledge, which in itself is fairly sterile, but to teach us to recognise the limits of the mind, and to make us admit in spite of ourselves that some things exist even though we cannot understand them" (*Logique*, p. 233). Knowing that the mathematical proofs that challenge us are conclusive yet beyond our grasp teaches epistemic humility. This analysis of the value of mathematics is subtly distinct from the account offered by Pascal. For Pascal, in *De l'esprit géométrique et de l'art de persuader*, while a mathematical demonstration may be comprehensible, the concepts to which it refers are often impossible to comprehend, and for this, faith is required (1989, pp. 180–181).

Arnauld and Nicole believed the lessons in epistemic humility taught by mathematics represented an important analogue to the incomprehensibility of the truths of religion. The authors of the *Logique* write that just as the demonstrations of infinity are inconceivable yet certain, so too are the teachings of the Church which can equally escape comprehension by reason.[26] Just as the mind must assent to the conclusions of geometrical demonstrations of infinity despite their incomprehensibility to the understanding, it is equally "to sin against reason to refuse to believe the marvellous effects of God's omnipotence, which is itself incomprehensible, for the reason that the mind cannot understand them" (*Logique*, p. 233). Arnauld and Nicole believed that the difficult concept of infinity taught a kind of epistemic humility that was applied to the truths of religion. How precisely does the mathematical example teach this? Practising mathematics engages and exhausts the mind and, in the course of so doing thought Arnauld and Nicole, shows clearly the mistake of rejecting precepts simply because they are difficult to understand, including those concerning the church. As Arnauld and Nicole write, "it is good to tire the mind on these subtleties, in order to master its presumption and to take away its audacity ever to oppose our feeble insight to the truths presented by the Church, under the pretext that we cannot understand them." They describe this process as one in which the mind is "abase[d] and humiliate[d]" (*Logique*, p. 233). Doing so is beneficial since it shows the mind its weakness and teaches that we best choose topics proportional to it and amenable to certain demonstration, e.g. natural knowledge. Arnauld and Nicole hewed strongly to the view that not everything was amenable to rational investigation and the truths of religion were certainly among those for which faith, and not reason, was required. Only matters of fact, according to Nadler, and not matters of faith were amendable to ratiocination (1989, p. 26). The separation of these two domains would not only represent a crucial intervention for Arnauld in seventeenth-century philosophy and theology but would

[26] Elsewhere, Arnauld and Nicole put this in slightly different terms. The knowledge that we gain by ourselves depends, they write, on reason. The truths of geometry are ordinarily of this kind since it is we that undergo this process of learning through proofs by demonstration. The truths concerning infinity are unlike this in cases where our reason cannot penetrate these truths. There is another kind of path to knowledge which comes from "the authority of persons worthy of credence who assure us that a certain thing exists, although by ourselves we know nothing about it" which "is called faith or belief." The truths of infinity—like the truths of religion - demand faith since in the case of the person for whom infinity is not amenable to comprehension by reason, authority demands that we believe the proofs to be true. See *Logique*, p. 260ff for Arnauld and Nicole's discussion of those things we know by faith and reason.

also precipitate an important refutation of scepticism about reason in the seventeenth century.[27]

Despite Arnauld and Nicole's arguments for the advantages of an education in mathematics, the Preface to the *Nouveaux éléments* offers a cautionary note about applying some of the precepts taught in mathematics textbooks too liberally. In Nicole's view, it would be a grave mistake to demand of all knowledge that it be as certain and accurate as a geometric demonstration. Geometry teaches that we must only assent to certain demonstrations, namely those that are developed from clear and distinct axioms and are proven conclusively. But it is a mistake, Nicole continues, to demand the kind of certainty, accuracy, and parsimony evident in geometrical demonstrations of moral and religious matters. The reason why these qualities are impossible to demand of non-mathematical matters is that these demonstrations "do not depend on a number of coarse and certain (grossiers et certains) principles like the truths of mathematics" (*Nouveaux éléments*, p. 98). Oftentimes when dealing with moral and human matters, Nicole continues, we are faced with a series of demonstrations which can persuade though they lack the certainty of a single geometrical demonstration. In these contexts, we can be convinced of the probability of a truth when by connecting several individual demonstrations they alight on a similar conclusion when considered together (*Nouveaux éléments*, pp. 98–99). There are different degrees of proof, Nicole argues, and it would be a mistake to apply the standards and forms of proof required of geometry to moral and religious truths, which can only permit of a high probability. In such cases, Arnauld and Nicole write that what is required is considering a matter, paying attention to the circumstances that accompany it and, as Nicole writes in the Preface of the *Nouveaux éléments*, aggregating the results of this thinking (*Logique*, p. 264; *Nouveaux éléments*, p. 99). Adjudicating on who is blameworthy in a fist-fight might be an example of this kind of thinking since doing so would demand examining the motivations of its participants, considering the circumstances and evaluating this evidence in light of other facts. Geometrical thinking might play a role in this context in nurturing a love of truth, by helping us attend to crucial details or by diminishing the import of pernicious passions, but only a limited one given that it would be a mistake to consider the event "nakedly and in itself" (*Logique*, p. 264). In the case of such matters, reasoning geometrically could show that it would be a "folly" to ignore particular truths, even though it would be no aid in proving such truths conclusively (*Nouveaux éléments*, p. 99). The example Nicole gives in the Preface of the *Nouveaux éléments* concerns the likelihood that the sun will not rise tomorrow. Geometrical thinking may not prove conclusively that it will, but it can play some role in showing what great folly it is to believe that it will not.[28] Though the Port-Royalists believed that geometry could be used for moral and spiritual purposes, they nevertheless cautioned

[27] For a more detailed account, see Lennon (1996). A thorough treatment of scepticism in the French context, which includes a discussion of the attitudes of Saint-Cyran, an early Abbot of Port-Royal, is offered by Popkin (1979, particularly chapters 5 and 6). It is worth noting that Saint-Cyran and Arnauld's views differ, though a discussion of their respective attitudes to sceptisim is beyond the scope of the current paper.

[28] See *Nouveaux éléments* (p. 99) for this example. What Nicole prescribes in this context is using *reductio* proofs to show how absurd is the alternate case. Though in the *Nouveaux éléments* and *Logique* Arnauld and Nicole argue that direct proofs are to be preferred to indirect (including *reductio*) proofs, they nevertheless endorse indirect proofs in cases where positive proofs are impossible. As I have just shown, Arnauld and

against the too liberal use of its precepts, particularly where this extended to attempting conclusive proofs of moral and, particularly, spiritual matters.

5 Geometry and moral psychology

Not only did the *Nouveaux éléments* describe how an acquaintance with mathematics might cultivate particular moral, religious and epistemic virtues, the geometry of Port-Royal also evidences Arnauld and Nicole's commitments in moral psychology. In their accounts of Nicole's moral psychology, Herdt (2008) and Moriarty (2011, pp. 246–247) have discussed Nicole's ideas on the opacity of the self as it relates to moral action, the practical value of intrinsically vicious motives for praiseworthy action and how vicious intentions can often imitate virtuous ones. To these discussions can be added Nicole's pedagogical commitments which are worth exploring for their ideas about the acquisition of moral knowledge. According to Nicole and the educators at Port-Royal, moral education was most effective when the methods of moral instruction were subtle. While, Nicole writes, the study of morality ought to be "the chiefest and frequentist" study undertaken, it ought to be done in such a way that the student is "not overcharged therewith; nay, that [the prince] even feels it not. Endeavour must be used that he learn all Morality, without knowing almost there is such a science or that there is a design to teach him any such thing" (1678, pp. 16–17). Summarising these ideas, Barnard writes in his book on Port-Royalist pedagogy that moral lessons at Port-Royal were delivered indirectly, eschewing all formality. Not only should children not be burdened with moral lessons, but they should also scarcely be aware that they are being taught them (1913, pp. 83–84). As Nicole puts it:

> We ought not to imagine, that [instruction in what is true] is always done by express reflections, nor that at every turn it makes a stop to instill rules of good and evil, true and false: no, on the contrary it does this almost always in an insensible manner. Tis an ingenious turn it gives to things, which exposes to view those that are great, and deserve to be considered, and hides what ought not to be seen; making vice ridiculous and virtue amenable; and insensibly framing the mind to taste and relish good things, and to have a dislike and aversion from the bad. (Nicole 1678, p. 8)

This moral education, making use of mathematics to serve moral and spiritual ends and to satisfy our moral and Christian obligations in this life, was multi-faceted: it involved facilitating an understanding of what is virtuous and vicious, and inspiring pro-attitudes towards virtue.

Of course, while the desire to offer subtle moral instruction may not have justified rewriting Euclid's *Elements* for a contemporary audience, Arnauld certainly saw the potential to render the text advantageous to children as well as adults. As elsewhere in the oeuvres of Port-Royal, the *Nouveaux éléments* contains proposals for moral

Footnote 28 continued
Nicole believed that matters in morality and religion are of this kind. See *Logique* (p. 255) for Arnauld and Nicole's discussion of demonstrations by impossibility. For Arnauld and Nicole's pragmatic account of how best to direct our reason in matters that concern faith, see the *Logique* (pp. 262–265).

conduct scarcely recognisable as such. For instance, in a chapter which teaches the lessons of algebra, Arnauld offers subtle instruction in the Christian virtue of charity:

> Having encountered some poor and wanting to give them each 5 sols,[29] I found that I had one too few. And so, having given them each 4, I had 6 remaining. How many poor were there and how many sols did I have? (*Nouveaux éléments*, p. 148)

The number of poor is seven and the number of sols available for distribution by the benevolent Arnauld is 34.[30] As further evidence of the commitment to the subtle instruction in charitable giving, Part III of the *Logique*, which deals with the various figures and moods of syllogisms, contains this exemplification of Camestres (a second figure syllogism of the form AEE):

> Every true Christian is charitable.
> No one who is pitiless towards the poor is charitable.
> Therefore no one who is pitiless towards the poor is a true Christian. (*Logique*, p. 151)

The example of charitable giving is among a number of moral precepts explored in the pedagogical works of Port-Royal. Here, as in the *Nouveaux éléments*, Arnauld and Nicole describe the qualities of the virtuous person and show examples of praiseworthy action. These examples, like others contained throughout their significant corpus of pedagogical, theological and philosophical works, are evidence of a commitment to the view that effective moral education must be subtle.

6 Conclusion

Why did Arnauld pen a work of elementary geometry if he and Nicole thought knowledge of mathematics constituted a misuse of precious time? In this paper, I showed that in the *Nouveaux éléments de géométrie*, Arnauld sought to teach students of mathematics and logic what moral and spiritual use could be made of an acquaintance with what they otherwise (and to varying degrees) regarded as a useless science. The aim of the *Nouveaux éléments* was, in Nicole's words, to render the study of geometry as useful and advantageous as possible. In practical terms, the *Nouveaux éléments* would serve as a propaedeutic exercise in the cultivation of moral, spiritual, and epistemic virtues.

A textbook on mathematics is an unlikely site for the cultivation of moral and spiritual precepts, and yet, Arnauld and Nicole include in their *Nouveaux éléments* a number of lessons in moral and Christian virtue. Arnauld and Nicole believed that mathematics could acquaint a student with spiritual precepts necessary for salvation. For one, it cultivated the disposition to love the truth which, they argued, was harnessed by God for the purposes of salvation. Underlying this commitment was the view that

[29] A sol, later sou, was a unit of money.

[30] For a complementary assessment of this example see Descotes' editorial note in the *Nouveaux éléments* (2009, p. 148): "The concern for moral edification is just as present in the *New elements of geometry* as it is in the logic."

truth was an unalloyed good and the ultimate goal of inquiry. The Port-Royalists regularly described the senses as the source of errors in reasoning, a natural condition of postlapsarian man. Mathematics was therefore valuable since it also taught the occlusion of the senses. Lastly, lessons in geometry could also help describe the nature of the soul and mind since the infinity of operations with which the mind was occupied in the study of mathematics could show that the soul was immaterial. In Arnauld and Nicole's view, no secular exercise could prepare the mind for moral and religious lessons better than the study of mathematics.

What is more, I have shown that mathematics cultivated epistemic virtues, which Arnauld and Nicole described in moral and religious terms. For Arnauld and Nicole, accustoming one's mind to the clarity and certainty of a demonstrative science, being reason governed, truth seeking, attentive and humble meant the fulfilment of a moral and Christian duty to be good and just. Arnauld and Nicole believed that we were bound by duty to cultivate epistemic virtues since doing so represented the fulfilment of our moral and Christian obligations. It was in these terms that the renovation of the *Elements* to improve its demonstrative certainty were articulated.

Focussing on Arnauld and Nicole's extra-mathematical ambitions allows us to expand our understanding of what was involved in the study of mathematics in the seventeenth century. Moreover, it shows us that scholarship of early modern moral philosophy and religion would benefit from continued investigation into how the sciences, including mathematics, contributed to the cultivation of virtue. Given their role in arguments about personal betterment, rediscovering these attitudes in mathematical works of the era is a crucial preliminary to deepening our understanding of early modern moral philosophy and religion. This paper has sought this intervention through the *Nouveaux éléments de géométrie*, an understudied but widely known work of seventeenth-century mathematics.

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