



The evidence approach to paraconsistency versus the paraconsistent approach to evidence

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Abstract

In this paper, we analyze the *epistemic approach to paraconsistency*. This approach is advanced as an alternative to dialetheism on what concerns interpreting paraconsistency and contradictions; instead of having to accept that there are *true contradictions* (as dialetheists argue), it is suggested that we may understand such situations as involving only *conflicting evidence*, which restricts contradictions to a notion of evidence weaker than truth. In this paper, we first distinguish two conflicting programs entangled in the proposal: (1) *interpreting* paraconsistency in general through the notion of evidence, and (2) modeling reasoning with evidence by *using* paraconsistent logic. The first part of the program, we argue, does not succeed, on the grounds that it does not lead to a uniform proposal to the understanding of paraconsistency, and fails to engage with dialetheism in a legitimate dispute about interpretation of paraconsistency. Also, when seen through the lights of the second kind of approach, a ‘logic as modeling’ approach, weaknesses of dealing with evidence through paraconsistency come to light, basically because evidence does not seem to suggest the need of a paraconsistent treatment. As a result, one can neither approach paraconsistency in general through evidence, nor approach evidence with the use of paraconsistent logics.

Keywords Paraconsistent logics · Contradictions · Evidence · Dialetheism · Logic as modeling

1 Introduction

Dialetheism is the thesis that some contradictions, understood as propositions of the form A and $\neg A$, may both be true *sometimes* (see Priest 2006 for an articulated defense;

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Priest et al. 2018 for general exposition). Understanding the falsity of proposition A as the truth of $\neg A$ (the negation of A), that leads us to the thesis that some propositions are both true and false at the same time (i.e., they bear *truth-value gluts*). Now, supposing that some proposition B is also just false, one has a direct case against the so-called *rule of explosion*, that is, from A and $\neg A$ one is not allowed to validly infer every B. Under these conditions, not every proposition follows from a contradiction. This restriction on explosion characterizes *paraconsistent logics* (see da Costa et al. (2007), Barrio and da Re (2018), Barrio et al. (2018) advance further discussion). Dialetheism, then, is closely related to paraconsistency.

Philosophically, dialetheism must be justified by its own arguments, and that is an issue we shall not examine here. The fact is that being a dialetheist, one has good reasons to adopt a paraconsistent logic: it pops out as a result of the intuitive semantics that a dialetheist advances, and those facts are then codified in a system of logic, of course (again, see Priest 2006a). Recently, however, Carnielli and Rodrigues (2015, 2019a, b), and Carnielli et al. (2018), have advanced a related question: can one be a paraconsistent logician and *not* be a dialetheist? Is it possible to explain in an intuitively satisfactory way the failure of the rule of explosion without embracing a version of dialetheism? How to make sense, in a non-dialetheist context, of the idea that we may ‘have a contradiction’, but that not every proposition follows from it?

It is to this question that Carnielli and Rodrigues (2015, 2019a, b) and Carnielli et al. (2018) have turned themselves. Their proposal, in a nutshell, is to shift from the ‘ontological’¹ dialetheistic approach, to a more epistemic-oriented view of paraconsistency. They defend the claim that one can ‘interpret’ paraconsistent contradictions and paraconsistent logics as dealing not with *truth*, but rather with an intuitive notion weaker than truth, a notion of *evidence*. To have evidence for the truth of a proposition is not the same as to *have conclusive evidence* for the truth of such a proposition. In this case, one may have non-conclusive evidence for A, non-conclusive evidence for $\neg A$, and still have evidence for the falsity of B, for some B. As a result, evidence is not transmitted in this kind of inference, and one is justified in avoiding the corresponding version of the explosion rule, without having to accept true contradictions (although having to accept contradictory or conflicting evidence in its place).

The so-called *epistemic approach* to paraconsistency is explicitly offered as an alternative to the dialetheist picture. In fact, Carnielli and Rodrigues (2019a, p. 3790) list three opposing views on paraconsistency:

- (1) dialetheism, which accepts that there are true contradictions, and implies that contradictions are ‘ontological’, or relative to the world (see our previous footnote);
- (2) a pragmatic approach, which claims that commitment to paraconsistency may be carried on without having to deal with the nature of contradictions (i.e., the view proposing that it is a fact that contradictions appear in some contexts, so we have to apply a paraconsistent consequence relation to deal with these contexts);

¹ The claim that dialetheism involves ontological contradictions is made by the mentioned authors. We shall not discuss in this paper whether that attribution is appropriate or not. See Priest (2006a, b) and Priest et al. (2018).

- (3) an epistemic approach, according to which *no contradiction is ontological*; contradictions are artifacts of language and thought, resulting from cognitive problems such as defective empirical or mathematical theories, or malfunctioning of measuring apparatuses, among others.

Now, although one could very well discuss whether dialetheism is the best approach to the nature of truth, in this paper we shall argue that *evidence* is not completely satisfactory when used as a substitute for truth to understand contradictions in paraconsistent logics. In fact, we shall argue that there is something deeper going on here: the very idea that contradictions should be *philosophically interpreted* sets the problem in the wrong perspective to begin with. Contradictions have an intuitive meaning that logical systems attempt to capture, and it is not as if they needed to be interpreted with a further layer of philosophical content over each system of logic. In this sense, the idea that contradictions may receive three rival possible interpretations is ill-grounded. So, after putting the problem in what seems to us to be the correct perspective (a ‘logic as modeling approach’), we shall be in a better position to understand the weaknesses of the epistemic approach. To put the results we shall achieve in a nutshell, we identify two major difficulties: i) the idea that a logic needs an interpretation in terms of evidence or dialetheism fails to generate a legitimate debate about the understanding of paraconsistency, and it is confused with the idea of application of logic, and ii) once properly understood as a case of application of logic, it becomes clear that evidence (as understood by Carnielli and Rodrigues) does not require a paraconsistent treatment. These two tasks are related in our approach, and part of our effort consists in showing that they are entangled in Carnielli and Rodrigues’ papers, and also, to disentangle them accordingly.

This paper is structured as follows. In Sect. 2, we briefly explain the key ideas behind the epistemic approach, as advanced by Carnielli and Rodrigues (2015, 2019a, b) and Carnielli et al. (2018). In Sect. 3, we advance what is thought to be a better perspective from which to understand what has been advanced by Carnielli and Rodrigues. Basically, our plan is to argue that there is a confusion between interpreting paraconsistency in terms of evidence, and studying what may be called ‘a logic for reasoning with evidence’ in terms of paraconsistency (i.e. applying a paraconsistent system in order to deal with reasoning about evidence). We argue that the project may be seen as an attempt to model reasoning with evidence in a paraconsistent system, setting the approach inside a ‘logic as model’ perspective. Section 4 explores the ‘logic of evidence’ proposal from this ‘logic as modeling’ perspective, and locates some of the weaknesses of the proposal, which are made salient once seen through this specific perspective. We conclude in Sect. 5 by summing up and recalling the arguments presented.

2 The epistemic approach

As we have already mentioned, the epistemic approach is advanced as an alternative to dialetheism for a possible understanding of paraconsistency and contradictions, given that Carnielli and Rodrigues (2015, 2019a, b) and Carnielli et al. (2018) believe

dialetheism to have ontological features, and, as a consequence, to go too far (we come back to this issue soon). Recall that in order for us to have a paraconsistent logic, we need to invalidate the rule of explosion, and doing that, in technical terms, seems rather simple; it just requires that we provide a model M for both A and $\neg A$, that is not also a model of some B . From a mathematical point of view, this is not a problem: choose a semantics in which A and $\neg A$ may be attributed designated values, and where B is not designated, for some B . The problem, however, consists in providing for an account of the “intuitive meaning” of the designated values for the set of propositions $\{A, \neg A\}$ (or $\{A \& \neg A\}$, given that most paraconsistent systems are adjunctive), without having this value meaning “true”. Carnielli and Rodrigues (2019a, p. 3791) put it in terms of a question:

Q: what property are we going to ascribe to a pair of accepted contradictory propositions such that it would be possible for a proposition to enjoy it without being true?

The answer needs to indicate some property weaker than truth, so that we do not have to accept dialetheism as a default option. The suggested answer comes by noticing that *acceptance of a contradiction does not mean acceptance of the truth of the contradiction*. There may be good reasons for accepting both members of a pair of contradictory propositions, although it needs not imply that both are true. We need also not even *believe* that both are true. There may be positive evidence for the truth of a proposition, even if the proposition is false, and we may wish to reason with that proposition. *Evidence*, then, understood as ‘reasons for believing/accepting a proposition’ plays the role of the weaker property sought by Carnielli and Rodrigues, with these reasons being *not conclusive*. They complement the explanation of evidence with a follow up in more recent papers: evidence is information *plus* an epistemic ingredient, a justification for the proposition. Information, to be sure, is merely the objective content of a sentence, without requiring truth, belief, or justification. Furthermore, non-conclusive evidence is characterized more precisely as “*bare-boned information plus a justification that might be wrong*” (Rodrigues et al. 2020, p. 12, italics in the original; see also Carnielli and Rodrigues 2019b, pp. 11–12). Also, as put by Carnielli and Rodrigues:

The acceptance of A and $\neg A$ can be understood as some kind of ‘conflicting information’ about A , in the sense of having *non-conclusive reasons* for accepting the truth as well as the falsity of A . This kind of ‘conflicting information’ we call here *conflicting evidence*. (Carnielli and Rodrigues 2019a, p. 3792)

Under the typical definition that $\neg A$ represents the falsity of A , we may have, intuitively, then, the following semantic understanding:

‘evidence that A is true’ means ‘reasons for believing/accepting A ’
 ‘evidence that A is false’ means ‘reasons for believing/accepting $\neg A$ ’

Applying the terminology proposed, notice that conflicting evidence occurs when *there is evidence for the truth of both A and $\neg A$* , and both reasons or evidences are *non-conclusive* (the fact that evidence is to be understood as *evidence for the truth* of a proposition will be important later, in Sect. 4). Some examples may help us getting the idea clearer. Suppose a person visits two distinct doctors, each of which provides

for a distinct diagnosis about a very dangerous disease. Say, doctor D_1 claims that the patient is infected with the disease, while doctor D_2 claims that the patient is not infected. The verdicts of the doctors count as conflicting evidence for the patient, so that there is evidence for the truth of each member of a pair of contradictory propositions. Something similar is said to happen on a trial, where distinct witnesses may provide for conflicting claims as to the whereabouts of a person accused of a crime (for further examples and an analysis, see Arenhart 2015).

Under these circumstances, it may seem pretty natural to think of conflicting evidence as a source of a kind of contradiction. Also, as already mentioned, it seems (at least *prima facie*) to be the case that we could have evidence that some proposition is false. It results that we have a case against the law of explosion, and all of that without having to embrace dialetheism; that is, the semantic understanding of contradictory propositions is advanced without having to appeal to alethic concepts. As Carnielli and Rodrigues put it, the source of the contradiction in this case is the same as the source of the conflicting information, e.g., malfunctioning in theories, problems with witnesses, confusion, and other epistemic shortcomings, but never the world itself:

A third position in paraconsistency, antagonistic to dialetheism, claims that *no* contradiction is ontological but, rather, *all* contradictions that occur in scientific theories, belief systems, a number of situations in informal reasoning, and even in semantic and set theoretical paradoxes—that are, strictly speaking, results about languages with certain characteristics—have epistemic character in the sense that they are related to thought and language. This is the position endorsed by us. (Carnielli and Rodrigues 2019a, p. 3790)

This, then, in a nutshell, characterizes the epistemic approach to contradictions. The plan, if it is to be an alternative to dialetheism, and an interpretation of contradictions, is that it should provide for a general account of the nature of contradictions. Once one is faced with a contradiction, the advice is not to think of it in terms of dialetheias, but rather in terms of conflicting evidence, not reaching reality (see also the overview in Rodrigues et al. 2020, Sect. 2.2.1).

Notice that the claims that dialetheism is dispensable, at first, could be compatible with a kind of pluralism concerning approaches to paraconsistency. One could adopt the epistemic approach in some cases, and dialetheism in others, depending on the circumstances. This is not the position advanced by Carnielli and Rodrigues, though, and this should be clear by now (see also Barrio and da Re 2018 for this reading of the proposal by Carnielli and Rodrigues). They clearly see their approach as dispensing dialetheism, to say the least (as the previous quote makes clear). On the way to ground their position, we find the claim that dialetheism is inadequate, because it advances the wrong account of paraconsistency. In a nutshell, their claim is that “dialetheism does not provide a sustained account of paraconsistency” (Carnielli and Rodrigues 2019b, p. 11), mostly due to its commitment to understanding validity as truth-preservation, which requires true contradictions about reality (precisely in accounting for the invalidity of the rule of explosion). Now, Carnielli and Rodrigues (2019b, p. 14) claim that there is absolutely no evidence that contradictions exist in reality, so, dialetheism has nothing in its favor, to say the least. It results then that “a paraconsistent logic *has to* abandon logical consequence as truth preservation” (Carnielli and Rodrigues 2019b,

p. 11) i.e., paraconsistent logics have to embrace validity as dealing with preservation of evidence. In this sense, then, there is a clear claim to the effect that dialetheism fails to account for paraconsistency, while the epistemic approach succeeds. This is clearly a matter of a dispute between what Carnielli and Rodrigues take to be two excluding proposals, and only one of them is appropriate, according to them.

3 Disentangling two proposals

Now, the idea that paraconsistent logics must be *interpreted*, and that dialetheism and the epistemic approach are rival views on that issue, seems plausible at first, but it does not completely resist a closer examination, as we shall argue in this section. The claim by Carnielli and Rodrigues, as we have seen, is that there are at least two competing approaches to interpret paraconsistency (dialetheistic and epistemic), and one must choose between them (the epistemic one, of course). We shall argue that the proposal, however, despite its *prima facie* clear message, is ambiguous between two distinct kinds of claims: one claim according to which evidence is used to read, interpret, or confer meaning, to paraconsistency (this is what we call *approaching paraconsistency with evidence*), and another, distinct one, according to which paraconsistency is a formal tool that may be applied for formalizing reasoning with evidence (this is what we call *approaching evidence with a paraconsistent system*). Both claims are entangled in the proposal by Carnielli and Rodrigues, but we are better advised to distinguish them, given that they do pose distinct requirements on what is expected from the epistemic approach. The first claim, that evidence is used to understand or interpret paraconsistency, fails to account for paraconsistency, and is not in complete touch with what Carnielli and Rodrigues claim on other parts of their work; in particular, it fails to really motivate a debate between evidence and dialetheism. The second idea, on the other hand, allows that we properly access the merits of the epistemic proposal in its relation with contradictions, and we shall check whether the reasoning with evidences should be accounted for paraconsistently. The point is that Carnielli and Rodrigues (2015, 2019a, b), as well as Carnielli et al. (2018), alternate between these two views when it comes to explain what an evidence approach to contradictions should mean. We separate these issues and approach them one at each time. In this section we begin with the claim that evidence may be used to interpret or confer meaning to paraconsistency.

The idea of philosophically interpreting a system of logic is not easily defined, and an exact definition of what interpreting a logic means cannot be found in Carnielli and Rodrigues (2015, 2019a, b) and in Carnielli et al. (2018). What we find, however, are explicit claims that paraconsistent logics must be interpreted in terms of evidence. Besides the mentioned sources of the epistemic approach, we also find it explicitly in Rodrigues et al. (2020) p. 11 the claim that evidence is “well-suited to a non-dialetheist reading of paraconsistency” (we take it that “reading” and “interpreting” do the same work here). Along with Carnielli and Rodrigues, Barrio and da Re (2018) also advance the claim that systems of logic must be interpreted in a philosophical way, although they also do not explicitly define what that would mean. A good indication, however, appears in Barrio and da Re (2018) p. 159, where we find a distinction between pure

logic, applied logic, and *philosophical interpretations of logic*. Pure logic concerns the systems of logic understood as purely mathematical objects of study. From a pure point of view, there seems to be very little to discuss on the legitimacy or correction of a system of logic: all are on equal footing, more or less as non-Euclidean geometry is a legitimate geometry, just as Euclidean geometry. Applied logic selects a context of application, such as the study of electrical circuits, computer programming, inference in natural languages, among others, so that a system of logic may be used to study the context selected. According to Barrio and da Re, even if we select a single field of application, such as reasoning in natural language, a system of logic may have distinct interpretations:

It is clear that one can use philosophical interpretations to try to obtain additional understanding of certain pure logical theories (from a proof-theoretical or model-theoretical point of view). It is also true that at least in some cases pure logics are developed to find an answer to a philosophical interpretation. For example, pure modal logics have been motivated by our interest in explaining what is metaphysically necessary or possible. Nevertheless, we would like to argue that there are no intrinsic connections between pure logics and one standard philosophical interpretation, even when one is considering these systems applied to human reasoning. (Barrio and da Re 2018, p. 159)

For instance (the examples are provided by Barrio and da Re), one may use Kleene's weak three-valued logic, and interpret the third truth-value as 'meaningless' (Bochvar) or as 'off-topic' (Beall). For another example, one may use Priest's LP to study inferences, and follow Priest's understanding of the third truth-value as a glut (true and false), or, instead, follow Beall and Ripley (2004), who provide an interpretation for it where the third truth-value is a gap (i.e., neither true nor false, provided that the language is extended with a truth predicate; we shall not be concerned here with the details). In the end, then, the authors reject the view "that there is a single canonical philosophical interpretation for some pure paraconsistent logic even when one is interested in the standard application of pure logics: the reasoning in natural languages." (Barrio and da Re 2018, p. 160).

It seems that it is in the context of this tripartite distinction that Carnielli and Rodrigues advance their epistemic interpretation of paraconsistency. As a result of such a multiplicity of interpretations, one may either follow a *conciliatory or pluralistic approach*, as suggested by Barrio and da Re (2018) themselves, or a *monistic approach to interpretations*, and then face a dispute to determine which single interpretation available, if any, is the correct one. Carnielli and Rodrigues, as we have already indicated, seem to suggest that only one of them is legitimate, by advancing their belief that *all contradictions are epistemic* (Carnielli and Rodrigues 2019a, p. 3790), and that there is no evidence for the possibility of existence of ontological contradictions, the ones required for dialetheism (2019b, p. 14). This leads one to a dispute on whether contradictions, tout court, are epistemic, resulting from conflicting evidence, or else may acquire the status of being sometimes true. That is, one has to choose between an epistemic approach on the one hand, or a more ontological approach on the other.

The major problem with this idea of interpreting paraconsistent systems, and in generating an opposition between epistemic and dialetheist approaches, concerns the

scope of the claim involving the need of interpretation. It seems to suggest that each and every paraconsistent system must be understood in one or the other interpretation. Carnielli and Rodrigues suggest this in their characterization of dialetheism: “[d]ialetheism is a *view on paraconsistency* according to which some contradictions are true” (2019b, p. 2, our emphasis). Then, dialetheism is one of the options for understanding paraconsistency. It is not the only one, of course, when one considers the claims that there are *three* interpretations of paraconsistency, as we mentioned in the introduction (it appears in Carnielli and Rodrigues 2019a, p. 3790). The idea that one of the approaches must be chosen for understanding of paraconsistency in general is also evident in (2019b, p. 1), when it is claimed that the conceptual significance of paraconsistent logics has not been discussed yet, in the sense that it is not clear what one accepts when one considers that some contradictions are acceptable without entailing triviality. So, we clearly find in the works of Carnielli and Rodrigues a claim to the effect that *paraconsistent logics* must be understood according to one of the approaches in the game. They suggest that the epistemic approach is the more appropriate one.

But, besides the lack of clarity on the idea of philosophical interpretation of a system of logic, the idea of a single appropriate conceptual framework (in the lack of a better word) according to which the paraconsistent systems should be interpreted is in conflict with the best practices of most paraconsistent logicians. First of all, the idea that pure systems of paraconsistent logic are awaiting to receive one of the interpretations (epistemic or dialetheic) fails to do justice to most systems of paraconsistent logic, given that they may not even be compatible with such readings. For an example, consider da Costa’s hierarchy of paraconsistent calculi C_n (for details, see da Costa et al. 2007). These calculi invalidate the law of non-contradiction (LNC) $\neg(A \ \& \ \neg A)$, being inappropriate for Priest’s dialetheistic interpretation, for instance (curiously enough, it should be remembered that under Priest’s dialetheistic interpretations, this law is valid; see Priest (2006a) chap. 4, for instance). Furthermore, these calculi validate the law of excluded middle (LEM) $(A \vee \neg A)$. This makes them inappropriate for the kind of evidence interpretation advanced by Carnielli and Rodrigues, which allows that one may have propositions with no evidence for their truth and also no evidence for their falsity, resulting in an alleged failure of LEM (see Carnielli et al. 2018, p. 55; Carnielli and Rodrigues 2019a, p. 3794). Given that none of the two approaches for interpreting paraconsistency seems to apply across the board, supposing that we must use one of them to understand contradictions, how to account for the contradictions in these calculi? Should another interpretation be given? Or perhaps these calculi are, for this very reason, wrong, according to both perspectives? Even a conciliatory approach to interpretations would have problems here, given that none of the options on offer seem to be appropriate for the calculi at hand. Seeing paraconsistent logics as dealing either with evidence or else with true contradictions would result in that most paraconsistent systems simply do not fit such interpretations, due to their failure to be interpreted in one or the other of the proposed ways. Many systems of the LFI (Logics of Formal Inconsistency) family seem to suffer the same fate (see Carnielli and Coniglio 2016). Then, it seems, even supposing that we could make a clear sense of interpreting paraconsistent logics in terms of one of these approaches, *these are not interpretations of paraconsistent systems of logic in general*, but rather, approaches closely tied to very

specific systems of paraconsistent logic (and, as we shall soon argue, they are to be viewed as target concepts that a paraconsistent system may attempt to model).

So, the problem of providing interpretations for paraconsistent systems, as advanced by Carnielli and Rodrigues, has just a too broad scope. It certainly cannot be the case that we have two options on the table and they do provide for *the* reading or understanding of paraconsistent systems. It is in this sense that the claim that paraconsistent logics must be interpreted is quite problematic. Perhaps this is not the major moving force behind Carnielli and Rodrigues' work, but it is a major claim found there, as we have seen, being the one that is used to oppose the epistemic approach to dialetheism.

Perhaps the case of dialetheism could enlighten further our claim that the idea of *interpretation* is not completely appropriate in this situation, and that what is really relevant is *application* and *modeling* (which would shift the focus from approaching paraconsistency with evidence to approaching evidence with paraconsistency, as we shall argue). Let us confine ourselves to Priest's (2006a, b) version of dialetheism, which is perhaps the better known version in the literature. According to the idea that one must interpret paraconsistency (following Carnielli and Rodrigues' claim), dialetheism is a way to understand or attribute meaning to contradictions. Given a paraconsistent logic with its contradictions expressed syntactically, dialetheism should be an option for the interpretation of such contradictions. However, that is clearly *not* how dialetheism is typically advanced: it is not an added content on the top of any paraconsistent logic. Rather, when advancing dialetheism, Priest proposes a paraconsistent logic as a tool for modeling inferences in natural language in their totality, and, given that he considers that people must reason with propositions that are true and false without triviality, a specific paraconsistent logic is recommended, he believes. That is, true contradictions are there to begin with, and modeling them in a system of logic is set as a goal for Priest. He approaches true contradictions with a specific paraconsistent logic, given that he believes that other systems (such as classical logic and intuitionist logic, and even da Costa's C_n paraconsistent hierarchy) are not up to the task. What Priest *does not do* is to interpret paraconsistent logics with true contradictions. In other words, the order of priorities is different, according to Priest: we do not adopt paraconsistent logics, and read them, as it were, as if they were speaking about true contradictions; no, one first has a dialetheist view, and then investigates which logic is more appropriate for it:

If dialetheism is correct, then —unless one is a trivialist— one must subscribe to some form of paraconsistent logic. The orthodox logic of our time is not a paraconsistent logic, and so dialetheism requires us to revise our beliefs about which logic is correct. (Priest 2006a, p. 155)

This idea that paraconsistency is not approached by the use of a background conceptual apparatus of epistemic contradictions, or of dialetheism, but rather that one first fixes the kind of contradictions one is dealing with, and *then* decides which paraconsistent logic is more appropriate, suggests that the claim by Carnielli and Rodrigues about interpreting paraconsistency is not completely correct. Perhaps the most appropriate approach consists in *applying* paraconsistent logics to deal with specific situations that we believe to involve contradictions (and Routley 1980 presents a nice case where this kind of methodology is applied on a step by step basis).

In this shift from an ‘interpretational picture’ to an ‘application picture’, and still considering Priest’s dialetheism to get the issue clearer, one may more clearly see that we have to deal with *two* separate problems: the problem of whether there are dialetheias (true contradictions) to be modeled on the one hand, and the problem of whether the system advanced by Priest deals appropriately with his stated goal (modeling deductive inferences in natural language when true contradictions are involved), on the other. The *first problem* is dealt with by Priest through the use of his well-known arguments concerning the semantic paradoxes, for instance (see again Priest (2006a, b) for further arguments, and Priest et al. (2018) for a battery of further arguments). The *second problem* is more delicate, and requires a discussion of the adequacy of LP (Priest’s *Logic of Paradox*) to natural language deductive inferences in general. Basically, one has to judge whether the inferences allowed by LP are good enough to capture deductive reasoning, and whether the rules not allowed by LP are in fact not valid. The controversy, of course, turns around the dispute on the most appropriate system of logic for natural language reasoning, but we don’t have to enter in such disputes here (see also Priest 2006a, chap. 4, for arguments favoring the adequacy of a system basically along the lines of LP, and leading away from C_n kind of systems). What is relevant for us is that, despite what Carnielli and Rodrigues claim, *dialetheism is not a thesis on the interpretation of contradictions*; rather, it is a thesis about the existence of some true contradictions and about the nature of truth itself. Paraconsistency appears as a result of such a thesis: the fact that some contradictions will be present requires that a paraconsistent logic *be applied* to describe the valid inferences we are allowed to do under such circumstances.

Now, that does not mean that dialetheism is true, or that paraconsistent logics can only be applied to deal with dialetheism. Rather, it indicates that the idea of interpreting paraconsistency in terms of evidence, as a rival to dialetheism, misconceives dialetheism. That is, the epistemic approach fails to engage in a dispute with dialetheism, given that dialetheism is not an approach to interpreting paraconsistency and contradictions in general (as the definition of dialetheism advanced by Carnielli and Rodrigues above seems to indicate). Furthermore, the epistemic approach, formulated in terms of an interpretation of paraconsistency, misconceives the role evidence is playing even in Carnielli and Rodrigues’ own account. This becomes clearer when we examine what Carnielli and Rodrigues (2015, 2019a, b) and Carnielli et al. (2018) have done in terms of applied logic and logic as modeling. In this new setting, the evidence approach should not be understood as an abstract thesis about the nature of contradictions in general, or as a general interpretation of contradictions in paraconsistent systems, as suggested by Carnielli and Rodrigues (2015, 2019a, 2019b) themselves. Rather, we should see them as providing for the goal of capturing the idea that one can reason with evidence, and that this kind of reasoning requires a paraconsistent treatment.

Carnielli et al. (2018) indeed come very close to stating literally that what they are doing is to offer a model for reasoning when what is at the center of the stage is transmission of evidence: “we want to express how people actually, and naturally, draw inferences, when the criterion is preservation of evidences” (Carnielli et al. 2018, p. 55). Carnielli and Rodrigues (2019a, p. 3790) are also clear that a system of logic where it is evidence that is preserved, and not truth, is still lacking, and they set their

aim as *presenting one such system*; the result is BLE, the basic logic of evidence, and LET_J, a logic of evidence and truth:

It is likely that no paraconsistent logician would be opposed to epistemic contradictions. However, to the best of our knowledge, a paraconsistent formal system suited to an intuitive reading according to which only epistemic contradictions are allowed (and true contradictions are ‘prohibited’) is still lacking. Our aim here is to present a system of this kind. In order to work out such an account of paraconsistency we have to explain what it means to say that a pair of propositions A and $\neg A$ simultaneously ‘hold’, or ‘may be accepted’, without being true. (Carnielli and Rodrigues 2019a, p. 3790)

That is, here, as they explain, *their aim is to present specific systems to capture reasoning with evidence* (approaching evidence with paraconsistency); this task is different than the task of providing interpretations of paraconsistent systems in general, in terms of evidence (approaching paraconsistency with evidence). This shift in focus is also clear in a more recent paper, where they claim that

The inference rules of BLE were obtained by asking whether an inference rule preserves evidence. Since evidence can be incomplete (no evidence at all) and contradictory (conflicting evidence), explosion and excluded middle do not hold. (Rodrigues et al. 2020, p. 3)

So, let us emphasize it: as goals of the epistemic approach, these—concerning modeling reasoning with evidence—are very far from the previous one, advancing a general approach to interpreting paraconsistency and contradictions! Rather, they indicate that a specific system (or systems) will be devised, with specific and restricted target phenomena to be modeled (just as in the case of dialetheism, as we presented before, although one could rightly claim that dialetheists have a more ambitious aim). These facts point to the following alternative understanding of the ‘epistemic approach’ project: there is a target phenomena that one wants to capture, viz. reasoning in circumstances where evidence is preserved; once that is done, a system should be developed to codify features of such kind of reasoning. From this perspective, instead of attempting to interpret paraconsistent contradictions *tout court*, what is being done is to offer a model (a system of logic) which will have to present features that capture the workings of the notion of evidence in natural reasoning. Carnielli et al. (2018) (just as Carnielli and Rodrigues 2015, 2019a, b) offer a new paraconsistent system, BLE, the basic logic of evidence, which is alleged to formalize the features of reasoning with evidence (given that BLE is equivalent to Nelson’s N4, we shall not present it here). That is, from this latter perspective, the authors are really attempting to provide a *paraconsistent model of evidence*, and not an *evidence interpretation of paraconsistency*. So, instead of understanding paraconsistency from the point of view of evidence, they are approaching evidence with the use of a paraconsistent system, which allegedly captures important features of the reasoning with evidence.

It results from the previous discussions that the epistemic approach, as currently presented, is an entanglement of two distinct goals, interpreting paraconsistency and contradictions in general on the one hand, and modeling a particular kind of reasoning, viz. reasoning with evidence, on the other, by using a particular paraconsistent

system. As we have argued, the goal of interpreting paraconsistency from an epistemic perspective is clearly not successful. It fails to do justice to paraconsistency as part of pure logic, and also fails to make a case that there should be a dispute between dialetheistic and epistemic approaches to contradictions, given that dialetheism itself is not a thesis on the interpretation of contradictions and paraconsistency (whatever ‘interpreting paraconsistency and contradictions’ could mean more specifically). In this sense, the epistemic approach is not really an alternative to dialetheism, and fails in this front. We suggest, then, that we restrict ourselves now to examine the epistemic approach exclusively from the model-building approach, as an attempt to model reasoning with evidence. From this perspective, the idea is that Carnielli and Rodrigues (2015, 2019a, b) and Carnielli et al. (2018) have provided for an applied logic, and their success may be judged on how well they fare in achieving the goals proposed for the formalization. In this perspective, systems may be ill-suited or well-suited for their goals. The goals themselves, of course, may vary, and be as general as ‘capturing deductive inferences in natural language’, or more restrict, as ‘capturing inferences when the notion of evidence is at stake’. Before doing that, a few short remarks are in order.

The first important remark is that, contrarily to what Carnielli and Rodrigues suggest, evidence modeling and dialetheism are not at odds, they are not adversary views on paraconsistency. Rather, they concern *separate issues*. Dialetheism is a thesis about the behavior of the concept of truth in natural languages, claiming that it does overlap with falsity, requiring, for a proper account, that we accept true contradictions, truth-value gluts, and all the like. It says nothing about evidence directly. Priest (2006a, b), for one, provides the formal tools to model these interrelated concepts in a specific system of logic, and the success of the approach may be judged independently of whether contradictions of some kind are required to deal with *evidence* (a notion weaker than truth, recall). The epistemic approach, on the other hand, although closely related to the behavior of truth (as we shall argue in the next section), concerns the behavior of *evidence*, and says nothing about truth, except that it behaves classically. Carnielli and Rodrigues (2019a, p. 3790) state that there are no true contradictions, and advance their approach as an alternative to dialetheism. However, they do not argue for *those* claims concerning truth (that would require a refutation of dialetheism, or something to the same effect). ‘Truth’ enters the approach by Carnielli and Rodrigues (2019a) in a second system, LET_J , the logic of evidence and truth, which is an extension of BLE (their basic system), accounting for a separation between propositions that are true or false (which receive a classical treatment), and propositions for which only inconclusive evidence is available (which receive a paraconsistent treatment). Now, the treatment of truth in LET_J is purely classical, but no specific reason is offered for *that*, except for the claim that contradictions must be restricted to the level of evidence, and never touch the truth, or the claim in (2019b, p. 14) that it is very unlikely that true contradictions exist. Notice that the modeling of truth and the modeling of evidence are then seen as two separated tasks. A dialetheist could also wish to model evidence, and, along with her inconsistent theory of truth, have a system to deal specifically with evidence, if she thought that this should be done. In other words: a dialetheist, given that dialetheism is a theory about truth, could completely agree with Carnielli and Rodrigues that *evidence* requires paraconsistency, but still disagree that truth is

classical. In this sense, there would be a dispute on the notion of truth, and the epistemic approach, by dealing only with evidence, does not touch on the problem of truth directly.

Our second remark is that this shift in focus, from interpretations to application, makes for philosophical interpretations unnecessary. The tripartite distinction offered by Barrio and da Re (2018), and indirectly by Carnielli and Rodrigues (2015, 2019a, b) may be reduced, with benefits to the philosophical study of logical systems, to just two (i.e. pure versus applied logic). The addition of philosophical interpretations leads us to the wrong kind of dispute. All we need is a more refined notion of application of logic. Instead of having paraconsistent logic applied to reasoning in natural languages, with additional interpretations to account for the nature of contradictions, what we suggest is that we have pure logic applied to model distinct aspects of reasoning in natural language. The cases of interpretation of Kleene's weak-matrices as 'non-sense' or as 'off-topic' may be seen as applications of the same system of logic to distinct areas of inferences: in one case, when reasoning with a specific kind of non-sense, and in the other, when off- topic sentences are allowed. This opens the perspective for judging the appropriateness of the modeling, instead of looking for the correct philosophical interpretation of a logic (or, even, having to foment a kind of pluralism regarding philosophical interpretations). Something similar, as we have already commented, concerns dialetheism and the epistemic approach to paraconsistency. Both use paraconsistent logics to model inferences under specific circumstances.

The third remark is a further point in favor of confining ourselves to application of a system, instead of in the idea of philosophical interpretation. It concerns the logical equivalence between BLE, the system advanced to deal with evidence, and N4, Nelson's logic of constructive negation. Clearly, Nelson's motivations and Carnielli and Rodrigues' motivations are distinct, they seem to have distinct understandings of negation, although their investigations end up leading to the same system. One could then think that Nelson's approach to negation provides for still a further rival understanding of it, and that there is a fourth approach to interpret contradictions (along with the three already listed by Carnielli and Rodrigues, mentioned in the introduction). Now, it would be without purpose to keep multiplying interpretations and, consequently, rival accounts of negation. As we are suggesting, it is better to think of it as a case where we have the same formal system being applied to model distinct aspects of natural language reasoning. This should come as no surprise, given that the same mathematical formalism may have a variety of distinct applications.

To resume what we achieved in this section: we propose a shift of perspective in this particular discussion. Instead of thinking in the approach by Carnielli and Rodrigues as an interpretation of paraconsistent logics in general in terms of evidence, we propose to evaluate that what they do, from now on, as a paraconsistent modeling of evidence (recall, they see themselves as doing both things, somehow entangling the tasks). This will bring better prospects to judge the proposal. Further, the distinction between application of logic and philosophical interpretation of a system may then be more clearly identified. When people ask for a philosophical interpretation, they are really asking for an application of the system in cases that may be philosophically interesting (instead of viewing the system as a pure system, or as having purely technical applications in dealing with electrical circuits, let us say). So, we now shift

from an evidence-based account of paraconsistency to a paraconsistent approach to evidence.

4 A paraconsistent approach to evidence

In the previous section, we have disentangled two separate goals of the epistemic approach, as it was originally formulated by Carnielli and Rodrigues (2015, 2019a, b) and Carnielli et al. (2018). We have set aside the part of the project that concerns *interpreting* contradictions and paraconsistency through evidence, given that it is not applicable to every paraconsistent system, and given that it fails to motivate a legitimate debate with dialetheism, as it was intended. Now we shall discuss the second project involved in Carnielli and Rodrigues proposal, the idea that reasoning with evidence is going to be *modeled* by a paraconsistent approach. Once the project is set in these terms, in a ‘logic as modeling’ perspective, we are in a better position to evaluate how far Carnielli and Rodrigues have been successful in dealing with the notion of evidence from a formal point of view. Our claim shall be that this aspect of the project advanced by Carnielli and Rodrigues faces severe difficulties coming from distinct fronts. It is a merit of the change of perspective brought in the previous discussion to allow us to see those difficulties clearer. Let us check.

We have seen that one can indeed find a project of modeling reasoning in terms of evidence in Carnielli and Rodrigues (2015, 2019a, 2019b) and Carnielli et al. (2018). Their plan is to select a notion of evidence in informal discourse and provide a formal system that somehow provides for an attempted formalization of how people reason with it in natural language (as already quoted, see in particular Carnielli et al. 2018, p. 55, where this is explicitly stated). As they claim, advancing a formal system is not enough, one must also “show that such an account is committed to real situations of reasoning” (Carnielli and Rodrigues 2015, p. 72). They do so through the presentation of the system BLE, and, also, of the system LET_J, which, along with the target notion of evidence, deals with a *classical* notion of truth. Later, the systems were extended to account for degrees of evidence in quantitative terms, by the use of a probabilistic semantics (Rodrigues et al. 2020).

The concept of evidence has multiple meanings and uses in natural language. If one is going to model a concept by the use of a formal system, it is useful to start by somehow preparing and elaborating the informal notion to be modeled. This is not a novelty in philosophy of science when discussing the relation of a theory with data, and, we should say, not even in logical practice. Logical textbooks on basic classical logic, for instance, do that when they suggest that students ignore the tense of the verbs when formalizing arguments, and when they recommend that desires, orders, questions, and sentences that are not in declarative form, are not to be taken into account. This puts some order in the field of application (whether this is a good order, or disorder, is an issue we don’t have to worry about now). Something similar happens to the notion of logical consequence in natural language. There is no ready-made informal concept, awaiting to be formalized. Some determinations must be provided first, so that the informal notion acquires more determinate contours ready for formalization. This is illustrated by the following sequence of questions by Peter Smith, when addressing

those who claim that that there could be a well-defined notion of logical consequence in natural language, an informal notion of consequence awaiting to be formally captured:

If you think that there is, start asking yourself questions like this. Is the intuitive notion of consequence constrained by considerations of relevance?—do *ex falso quodlibet* inferences commit a fallacy of relevance? When can you suppress necessarily true premises and still have an inference which is intuitively valid? What about the inference ‘The cup contains some water; so it contains some H₂O molecules’? That necessarily preserves truth (on Kripkean assumptions): but is it valid in the intuitive sense—if not, just why not? (Smith 2011, p. 29)

The suggestion is that intuitive notions may be labored in distinct directions, and that before setting oneself the task of advancing a system to account for an informal notion, one must ‘prepare’ the informal concept by endowing it with some more determinate features, choosing features the formalized version will describe, and those that it will not have to account for. Some decisions must be taken before the formal apparatus sets in. The same holds for those attempting to capture the notion of consequence as ‘preservation of evidence’. How is the intuitive notion of evidence thought to behave in this case?

Carnielli and Rodrigues (2015, 2019a, b) do not offer a complete description of the notion of evidence that they are willing to describe. The general plan is that there is a notion of evidence which may substitute truth when accounting for logical validity: evidence flows from the premises to the conclusion in an ‘evidence-valid’ argument. However, from what we have already explored in Sect. 2, some traits of the concept of evidence involved are salient, and we shall confine our evaluation to them: evidence is understood as information (a proposition) along with some justification (an epistemic ingredient) for the acceptance or belief that a proposition *is true*; evidence typically may not be conclusive (when the justification is not conclusive); one may have non-conclusive evidence for both a proposition and its negation, and when this is the case, one may be willing to reason with both in conjunction; also, for propositions with conclusive evidence, one can assert their truth, and they behave as per classical logic.

Perhaps the first feature that calls the attention of a reader in this list of features of the notion of evidence is the claim that one may be willing to reason with propositions for which we have conflicting evidence. That this feature must be present is confirmed by the case that evidence was introduced with the specific purpose to account for the failure of inferences such as $A \ \& \ \neg A \ \text{entail} \ B$ (the rule of explosion). So, the project initially was targeted as involving cases like this. However, while it may seem natural to reason (perhaps hypothetically) with each of a pair of contradictory propositions separately, to check whether one could find further evidence that could lead one to choose one of them (as in attempts to overrule one of the evidences presented), it is hard to believe, unless one is a dialetheist, that one could take it that there are evidences for the truth of a contradiction (even if the evidence is non-conclusive). Let us spell the details.

The idea that evidence may be contradictory, and that this is not an obstacle for reasoning, so that one may go on and allow for contradictions in our empirical theories or in our arguments in a jury as further premise, seems at odds with the best recommendations of what one should do in the presence of conflicting evidence (represented

in the system by a contradiction). Ever since the ancient skeptics, conflicting evidence (not to say an explicit contradiction) is a sign that something went wrong, and that they should not be embraced together as new information about the world or the subject matter under analysis, but that one should somehow quarantine the information. As already remarked by Io Guercio and Szmuc (2018, p. 162):

Although it is natural to assume that some parts of a total body of evidence might support A while other parts of it might support $\neg A$ (what Carnielli & Rodrigues call conflicting evidence), it is incoherent to claim that the *whole* evidence makes it rational to accept both.

Consider a jury, where conflicting testimonies are presented as evidence as to the whereabouts of a given individual on trial. It would be imprudent for the judge to accept both evidences and both propositions, at the same time. As a practical guide for the kinds of inferences one could make, the idea is really unworkable, and the idea that this is how people reason when dealing with evidence seems to be at odds with the facts. The same could be said for a person receiving conflicting diagnosis from distinct doctors as to whether or not the person has a disease. One could hardly claim that people reason with the premise that they have and do not have the disease. So, although it could well be the case that we do reason with inconsistent information sometimes, it is not at all obvious that we reason with explicit contradictions incorporating conflicting evidence. We seem to take one proposition at each time, at best.

This reasonable description of the behavior of evidence points to the fact that perhaps the notion of *inconclusive evidence* is ill-suited for the purposes of invalidating the rule of explosion. Or, in other terms: evidence, as we currently understand it, does not seem to require a directly paraconsistent approach. One typically does not want to have a pair A & $\neg A$ which means that a conflicting evidence *is accepted*, or that we have evidence for its truth. As Io Guercio and Szmuc (2018, p. 164) also put it, the intuitive notion of evidence is such that, evidence in favor of A is evidence against $\neg A$, and evidence for $\neg A$ is evidence against A . Given evidence k for A , evidence k' for $\neg A$, it may be that $k > k'$ (k surpass k'), and then the total amount of evidence ($k + k'$) supports A , or else it may be the case that $k' > k$, and then the total amount of evidence ($k + k'$) supports $\neg A$, or, finally, it may be that both evidences have equal weight, and then ($k + k'$) simply won't allow us to decide which proposition to give accent to. In this case, we should be led to suspension of judgment on what concerns the truth of A , and not to believe that we have some evidence for A & $\neg A$. Both k and k' neutralize themselves, and one should not accept any member of the pair:

in cases in which there is equally strong conflicting evidence, we claim, one must suspend judgment. What is the epistemic benefit of that? Of course, suspending judgment is not something that will contribute to the goal of accepting what is true. However, by suspending judgment one avoids accepting what is false. In the cases at hand, in which the evidence makes A and $\neg A$ equally likely, following the evidence is not helpful in order to achieve the goal of accepting true propositions, so the best we can do is to avoid accepting false ones. By suspending judgment we do precisely that. (Io Guercio and Szmuc 2018, p. 164)

And they continue:

In contrast, it is not clear what would be the epistemic gain of accepting a contradiction. Accepting a contradiction not only does not contribute to the goal of accepting true things but it conspires against the goal of not accepting false ones, since by accepting a pair of contradictory propositions one is *guaranteed* to accept something false. (Io Guercio and Szmuc 2018, p. 164)

This all points to a simple fact: evidence seems not to require a paraconsistent approach. We simply do not typically reason as if a proposition of the form $A \ \& \ \neg A$ had any evidence for itself, and neither with both of A and $\neg A$ (both cases are equivalent in the system advanced by Carnielli and Rodrigues, so, we don't have to bother with distinguishing the cases). Carnielli and Rodrigues (2015, 2019a, b) and Carnielli et al. (2018) suggest that evidence must be dealt with in paraconsistent terms due to their conflation of two distinct projects, which we have already distinguished in the previous section: to provide an evidence interpretation of paraconsistency, and to provide a paraconsistent treatment of evidence. These projects are in clear conflict, and once one sees that paraconsistency is not required to deal with reasoning with evidence, the very idea of a paraconsistent treatment of evidence loses its momentum. It seems that, in the end, the notion of evidence described intuitively by Carnielli and Rodrigues was tailor made to provide for a failure of explosion, but, as a result, it ended up describing a notion of evidence that is far from the actual accounts of evidence in real life.

Notice also that the treatment of evidence so far is qualitative. In Rodrigues et al. (2020) the approach was complemented to provide also a quantitative account of evidence, with a probabilistic semantics proposed to accommodate the fact that evidence may come in degrees. But this falls prey to just the same kind of reasoning we have presented so far. Given evidence k for a proposition A , and k' for $\neg A$, if $k' > k$, then, we seem required to adopt $\neg A$, and if $k > k'$, then, we are recommended to adopt A . If $k = k'$, we suspend judgment, instead of accepting both A and $\neg A$. That is, a probabilistic semantics helps us not with the fact that evidence does not require paraconsistency.

A property of the systems BLE and LTE_J , which is related to the previous discussion on having evidence for a contradiction, concerns the rule for the introduction of conjunction. This rule allows us to infer $(A \ \& \ B)$ from premises A and B , separately. In terms of inference, having conflicting evidence, that is, evidence for A and evidence for $\neg A$, we have evidence for $(A \ \& \ \neg A)$, resulting that the system authorizes the kind of behavior of evidence that we have already discussed is against the common use of the concept of evidence. But that rule is even more general than that, given that it allows for arbitrary conjunctions (not only of contradictory propositions). It states that, given evidence k for A , and evidence k' for B , one could have evidence for $(A \ \& \ B)$. Carnielli and Rodrigues (2019a, p. 3795; 2019b, p. 13) and Carnielli et al. (2018, p. 55) motivate the point as follows: one takes the sum of the evidences k and k' together as evidence for $(A \ \& \ B)$. The example they provide (see for instance 2019b, p. 13) is the case of two folders containing evidence, one of them for A , and the other for $\neg A$. We are allowed to bring the folders together and, the resulting folder contains evidence for $(A \ \& \ \neg A)$. However, besides what has already been pointed out, it is not clear that evidences may always be taken together.

If that rule were taken as correct for evidence, then, notice that it would allow for some cases of incompatibility which seem hard to allow in the informal concept of evidence as used in science and in daily life. Consider quantum mechanics. In a double slit experiment, there is evidence that an item is a particle (when only one slit is open), and also, there is evidence that the item is a wave (when both slits are open). So, according to the behavior of evidence as described by BLE, there is evidence that the item is a particle and a wave. But there is no such evidence in quantum mechanics. There is no way of putting the evidences together in this case. And this does not depend on specific weirdness of quantum mechanics. Suppose that someone views a cylinder from a specific position so that it looks like a rectangle, and, from another perspective, like a circle. There is (visual) evidence for ‘object A is a rectangle’ and also for ‘object A is a circle’, but there is no evidence for ‘A is a rectangle and a circle’. It seems hard to make sense of the idea that both visual perspectives (which constitute the evidence in this case) may be lumped together to provide evidence that ‘object A is a rectangle and a circle’. Evidence, it seems, is non-adjunctive in a great many variety of cases of reasoning in real life. A detective investigating the suspect of a crime would not be praised for his work if his conclusions were that there is evidence that the suspect is guilty and not guilty, even if all of the evidence gathered is presented as a single document.

So, there is trouble in making it clear how evidence for a conjunction can be obtained from evidence for the conjuncts. But one could complain that, for instance, observations and experiments, as described in the cases suggested in the previous paragraph, should not be understood as the evidence. Clearly, in some cases one cannot bring observations and experiments together. But, it could be said, the idea gets clearer when we stick to the notion of evidence presented by Carnielli and Rodrigues: evidence is the information, a proposition, added with an epistemic ingredient, a justification (see again Carnielli and Rodrigues 2019b, p. 11, Rodrigues et al. 2020, p. 12). The plan, then, seems to be: given a proposition A, evidence for A is just another proposition, E, conveying the information, along with a justification. So, given proposition A with evidence E, and proposition B, with evidence F, the evidence for A & B would be another proposition, the conjunction E & F (to convey the information), and the conjunction of the epistemic ingredients of E and F. And this brings us back to the same problem we started with, given that it is mysterious how the epistemic ingredients separately are to be brought together. That is, even if we stick to the more specific account of evidence presented by Carnielli and Rodrigues, the difficulties persist.

Now, for another serious difficulty. Granted that evidence is always *evidence for the truth* of a proposition, then the notion of truth will have to be clearly spelled out; evidence, somehow, piggybacks on truth and cannot have a separate treatment. Carnielli and Rodrigues (2015, 2019a, b) and Carnielli et al. (2018) adopt a classical theory of truth, which means that they have classical logic working when one assumes that a proposition is true or false (this is formally codified in the system LET_J). However, that is a big problem for the very idea that one may have evidence for contradictions, because that would mean that one has evidence *for the truth of a contradiction*. Recall that the idea that conflicting evidence occurs when we have evidence for A and evidence for $\neg A$. However, under a classical theory of truth, no evidence in the world could be evidence for (A & $\neg A$), given that, under the usual assumptions of classical

theory of truth, this is never true! That is, from the beginning, one could not expect that some evidence for $(A \ \& \ \neg A)$ should be coming, as a matter of the logic of truth adopted by the authors. What is being confused, we suggest, is that *contradictory or conflicting evidence for the truth of a proposition* is different from *evidence for truth of contradictory propositions*. While one may clearly have conflicting evidence, conflicting evidence can never be evidence for the (classical) truth of conflicting propositions. A switch among both distinct situations is what elicits the going from conflicting evidence to evidence for a contradiction; however, this switching is not allowed by the very theory of truth underlying the approach. Formally, this switch is allowed in BLE and in LET_J by the rule for conjunction introduction, which we have already discussed.

Carnielli and Rodrigues (2019b, p. 13) comment briefly on a related issue, that whenever we have evidence for A and evidence for $\neg A$, we have evidence for $(A \ \& \ \neg A)$ (by conjunction introduction), which shows that something went wrong in the process of gathering evidence, and the agent needs not to belief the contradiction; rather, the agent should revise some of her beliefs. However, this does not address the point we are advancing here. Given a classical theory of truth, and the claim that evidence aims at truth, even if fallibly, we should have, right from the start, that $\neg(A \ \& \ \neg A)$, and so, nothing could count as evidence for $(A \ \& \ \neg A)$, even if we know that evidence in this case is mistaken.

Something similar occurs to the idea that LEM (the law of excluded middle) may fail for some propositions. The intuitive idea is that one may not have evidence neither for the truth, neither for the falsity of a proposition A, and this would result in the failure of $(A \vee \neg A)$. However, again, if evidence is evidence for *the truth* of a proposition, by the very behavior of truth that was chosen, there is logical evidence for $(A \vee \neg A)$. While one may have no evidence for A, and no evidence for $\neg A$, there is simple evidence for $(A \vee \neg A)$: a proof in classical logic, which is accepted by the authors when dealing with truth and falsity. Here, lack of evidence for A and for $\neg A$ is taken as evidence for the failure of $(A \vee \neg A)$; however, the latter never fails to be true (on the classical approach to truth). Consider, for instance, investigations about dinosaur fossils. In most cases, there is no evidence for the sex of a dinosaur just by examining the fossil. However, researchers know that the dinosaur was male or was not male. Lack of evidence does not lead to a failure of excluded middle.

These discussions lead us back to a theme that was touched on above: we have mentioned that one cannot have evidence for the truth of a contradiction, unless one is a dialetheist. Indeed, on what concerns the idea that one may have evidence for the truth of a contradiction, curiously, the idea can be saved, but only if one adopts a dialetheist picture of truth. In fact, according to a dialetheist, some propositions of the form $(A \ \& \ \neg A)$ *may be true*. Whether they are true or not will depend on the evidence available when such cases happen. That means that, at least theoretically, one could have evidence for the truth of $(A \ \& \ \neg A)$ (even if the evidence is non-conclusive and turns out to be overruled later). The point is that only when the possibility of contradictions being true is allowed can one find evidence in favor of their truth! But that goes clearly against the proposal by Carnielli and Rodrigues of avoiding dialetheism with their proposal! Something similar could be said about the failure of

($A \vee \neg A$). Only when the logic of truth allows for the failure of such a proposition can one find evidence for its failure.

What this indicates is that Carnielli and Rodrigues, by choosing classical logic to deal with truth to begin with, have predetermined the space of possibilities for evidence. Given the laws of truth, one may look for evidence for the truth or falsity of contingent propositions, as a matter of material conditions of truth for those propositions. Classical logic, however, requires that contradictions will not have any evidence available for them. A dialetheist's logic with a semantics such as Priest's, involving gluts, will allow for such things (see Priest 2006a, chap. 4). Curiously, then, evidence for contradictory propositions will only be allowed if the truth of such contradictions is an open possibility. And this, Carnielli and Rodrigues do not allow.

For a further difficulty, let us widen the net and try to check what kind of consequence the paraconsistent approach to evidence would have in other areas of philosophy. Typically, if a solution to a philosophical problem ends up interfering with other unrelated problems too, then, this counts as evidence against the proposal. This is what happens to the epistemic approach, we claim. Supposing that one may have conflicting non-conclusive evidence and that such evidence is to be treated in terms of paraconsistency, one must adopt a non-factive view of evidence (this is also briefly discussed by Fitting (2017)). That implies that those sources of conflicting evidence, such as scientific theories, cannot be said to be true. According to Carnielli et al. (2018, p. 54), theirs is a *non-representational* view of scientific theories. Scientific theories may bear contradictions, but these contradictions do not reach reality, because the theories themselves are non-representational. Theories are mere artifacts for the calculations of empirical results and for solving problems (see also Carnielli and Rodrigues 2019b, p. 8). This separation of the theories from reality is what grants that dialetheism won't get in, given that in this context, theories are allowed to contain contradictions.

The reading of contradictions as conflicting evidence fits well with the practices of empirical sciences. There are an extensive literature about contradictions in sciences [...] The notion of contradictions as conflicting evidence is in line with the view that empirical theories are better seen as tools to solve problems, rather than descriptions of the world [...]. Of course, the occurrence of contradictions is a problem for the descriptive view of theories, since the latter requires that such a representation be correct (i.e. true). Once this non-representational view of scientific work is accepted, contradictions in the empirical sciences are better viewed as originated in limitations of our cognitive apparatus, failure of measuring instruments and/or interactions of these instruments with phenomena, stages in the development of scientific theories or even simply mistakes, to be corrected. (Carnielli et al. 2018, p. 54)

(Notice that the quote starts indicating the project of interpreting contradictions as contradictory evidence; this does not interfere in the argument that follows, placed in the context of a logic as model approach). The problem is that this view of evidence as non-factive burdens the philosophy of science with an anti-realist view of science. It simply bars scientific realism from the start. More than that: it burdens common speakers (at least those reasoning with contradictions) with a non-factive semantics.

Given that one of the stated aims of the formal project advanced here is that one is willing to capture how people really reason with the notion of evidence, this restriction limits the scope of application of the system that will be developed to those with anti-realist sympathies. Then, this burdens all those realists dealing with theories affording conflicting evidence with unconscious anti-realism, and burdens common speakers with mistakes on their attempts to speak about the facts when they are involved in contradictions. This puts a solution to a logical problem interfering with problems in philosophy of science and semantics; a burden that does not favor the epistemic approach.

This also trumps the greater project of extending BLE to LET_J and getting truth along with evidence. If truth is a limiting case of evidence, but evidence is not factive, then, we are in trouble in getting to know the truths, after all. Propositions for which we have conclusive evidence are consistent, subjected to classical logic (Carnielli et al. 2018, p. 56). However, that would make classical logic practically inapplicable, given the poor resources we have to determine cases with conclusive evidence. How do we know that any empirical proposition has conclusive evidence? We don't, it seems, given that a bit of skepticism may always infiltrate itself. Or even worse: how do we know that mathematical propositions have conclusive evidence? We cannot have definitive evidence that parts of classical mathematics are consistent (by Gödel's results), so, it seems we are not allowed to assume that they are treated by the classical part of LET_J ! But that would require that we treat those parts of classical mathematics with a paraconsistent logic, which is strange, to say the least. It seems that we take consistency for granted, and isolate possible cases of inconsistency, instead of taking inconsistency for granted, and isolating possible cases of consistency for a special treatment.

The discussion of which logic to use in empirical sciences is made difficult also because there is a kind of conflation of consistency with conclusive evidence in some of the discussions. For instance, Carnielli and Rodrigues (2019b, p. 13) suggest that classical mechanics and relativity theory use classical logic, while the addition of the theory of electromagnetic field with classical mechanics must have a paraconsistent logic, given that it is inconsistent. However, if we consider that classical logic is related with conclusive evidence, and that paraconsistent logic is related with inconclusive conflicting evidence, this attribution of logics is mysterious. Let us consider each case in turn. Perhaps classical mechanics is the only safe case: we may use classical logic here, because we know that it is false (it has been succeeded by better theories). But what of the second case, Newtonian mechanics and electromagnetic field theory, which are thought to be inconsistent? Given the inconsistency, the conflation of inconsistency and inconclusive evidence suggests a paraconsistent treatment. However, the possibility of deriving an inconsistency in the theory is not definitive evidence that it is false? Well, then, according to the standards set, we should use classical logic here! It seems that the very reason advanced for the use of a paraconsistent logic, the derivation of a contradiction, in the epistemic approach, requires that we do not use a paraconsistent logic. To put it again: derivation of a contradiction trumps the use of paraconsistent logic in this case. What of relativity theory? Suppose it is consistent. Should we use classical logic? According to the conflation of consistency and classical logic, yes. But our evidence for relativity theory is not conclusive, given that it is very

likely that it will be replaced by a better theory in the future. Well, then, the logic must be paraconsistent, and not classical. Things get more muddled if we consider the probabilistic semantics advanced in Rodrigues et al. (2020, p. 15). Even if there is a classical attribution of truth values (a limit case of the probabilistic semantics), consistency is not enough to grant conclusive evidence. Then, the logic should be paraconsistent. But this is more trouble for the proposal: modus ponens and deduction theorem do not hold for propositions lacking conclusive evidence (2020, p. 17). But it is not clear how science (relativity theory, at least) can be developed without such resources. Or: it seems clear that even when reasoning with non-conclusive evidence, people do use those rules. The machinery of evidence, as advanced here, then, seems ill-suited to deal with cases of real life.

5 Conclusion

Let us bring together what we have done in this paper. As we have seen, Carnielli and Rodrigues (2015, 2019a, b) and Carnielli et al. (2018) promote their view as an epistemic approach to paraconsistency. Their plan is to avoid dialetheism in understanding the contradictions that must be present in paraconsistent logics in order to invalidate explosion, and substitute the idea of *true contradictions* by an epistemic version, which should be understood in terms of *conflicting evidence*. However, we have argued, this perspective does not make justice to what dialetheists really do when they employ paraconsistent logics, and does not provide for a legitimate understanding of paraconsistent logic as a field of logic. Dialetheists are not offering an interpretation of contradictions, but rather using paraconsistent logics to deal with the very idea of true contradiction they believe to be found in natural languages. Also, approaching paraconsistent logics in terms of evidence would demand that every paraconsistent logic could be understood in terms of evidence (or in terms of dialetheism), and that cannot be done.

The epistemic approach may be better understood, then, *not as an epistemic approach to paraconsistency, but rather as a paraconsistent approach to evidence*. This suits better with the idea that what Carnielli and Rodrigues (2015, 2019a, b) and Carnielli et al. (2018) really do is to present two specific systems of paraconsistent logic aiming to capture the inferences in natural language when we deal with evidence transmission from premises to conclusion. This puts the approach in terms of the ‘logic as modeling’ program, which provides a better background in which to discuss the success or failure of a paraconsistent approach to dialetheism.

We have seen that the notion of evidence does not seem to require and, worst yet, does not seem to even allow, a paraconsistent treatment. The notion of evidence, as traditionally employed, allows that one may have conflicting evidence, but does not seem to elicit that we put such conflicting evidences together to originate evidence for a contradiction. Evidence for each member of a pair of contradictory propositions should not be confused with evidence for a contradictory proposition.

Obviously, the very idea that evidence must be dealt with paraconsistently is a result of the conflation between two projects, which are mixed in the epistemic approach: interpreting paraconsistent negation in terms of evidence, and modeling evidence

in terms of a system of logic. The first project sets the stage in a paraconsistent setting, so that the authors are led directly to an attempt to (now developing the second project), force evidence in a paraconsistent mold. In this second approach, we see that paraconsistent logic is not the appropriate kind of system to deal with evidence, and that evidence does not seem to exhibit contradictory behavior.

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