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The logic of coherence

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Abstract
To remedy the lack of precision attached to the concept of coherence, a plethora of probabilistic measures has been developed. To broaden the perspective, we do not focus on the differences between these quantitative but the differences between qualitative approaches to coherence by comparing three probabilistic definitions for the relation denoted by 'coheres with'. To reveal the different logics underlying these relations, we introduce a considerable number of formal properties and examine whether the given coherence relations possess them. Among these properties are not only the classics reflexivity, symmetry and transitivity, but also a variety of features that concern propositions containing negation, conjunction and disjunction, as well as features that concern inconsistency, entailment and equivalence.

Keywords Probability · Formal epistemology · Coherence

1 Introduction

Coherence is usually understood as a property of sets of propositions hanging together or dovetailing with each other. Apart from this minimal condition, however, the concept of coherence is notorious for its elusiveness. To gain precision, there has been developed a vast number of probabilistic measures in the last 20 years. Among them are measures quantifying coherence in terms of deviation from probabilistic independence (cf. Schupbach 2011; Shogenji 1999), relative set-theoretic overlap (cf. Glass 2002; Meijs 2006; Olsson 2002) or degree of mutual confirmation (cf. Douven and Meijs 2007; Fitelson 2003, 2004; Roche 2013). What is more, within each of these families there are, often a good many, measures that are not even ordinally equivalent.

In this paper, we will not dwell on the differences between these *quantitative* accounts but broaden the view by studying the differences between three *qualitative*,

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yet probabilistic approaches to coherence. More exactly, we offer three probabilistic definitions for the relation denoted by ‘coheres with’, which is assumed to hold between two propositions, and study the different logics underlying these relations.

The structure of the paper is as follows. Section 2 presents some formal preliminaries and our three relations of coherence: *incremental* coherence, *absolute* coherence and *strong* coherence. In Sect. 3, various formal properties are introduced in order to examine whether the given coherence relations possess them. Among these properties are the standard properties of binary relations, viz. reflexivity, symmetry and transitivity. Since we define coherence for a *propositional* language, we also include a variety of properties that concern the coherence of propositions containing Boolean connectives. For example, we go into the question whether two conjunctions $x \wedge z$ and $y \wedge z$ cohere if the conjuncts they differ in, i.e. x and y, cohere. Our study is confined to negation, conjunction and disjunction. Conditionals are implicitly treated insofar as $x \rightarrow y$ is logically equivalent to $\bar{x} \vee y$ and such propositions can be substituted salva validitate in probabilistic formulas. Moreover, we address some properties to which attention was already directed within the debate on probabilistic measures of coherence, namely, properties concerning inconsistency, entailment and equivalence. One of the questions in this context is whether x coheres with z if x coheres with a proposition y that entails z. Finally, Sect. 4 offers a brief summary and outlook.

Our survey takes 28 properties into account. Some readers may find some of the results unexciting because they are in line with their expectations. However, aside from the fact that there will hopefully remain enough unexpected results for everyone, we can at least take up the cause of providing mathematical proofs for all results, whether unexpected or not.

2 Coherence as a relation

Let L be a finite propositional language, i.e. a set of formulae closed under some functionally complete selection of classical connectives. Let $P: L \rightarrow [0, 1]$ be a probability function over L, i.e. a non-negative, real-valued function such that $P(x)=1$ if $x \in L$ is a tautology, and $P(x \vee y)=P(x)+P(y)$ if $x, y \in L$ are inconsistent. By L_c we denote the set of all *contingent* propositions in L, viz. propositions that are neither logically true nor logically false. Furthermore, let $\mathbf{P}$ be the set of all *regular* probability functions over L, i.e. probability functions such that $P(x)=1$ iff $x \in L$ is a tautology, and $P(x)=0$ iff $x \in L$ is a contradiction. Then coherence can be construed as a relation on contingent propositions $\sim \subseteq L_c \times L_c$ such that $x \sim y$ holds if x and y are coherent, and $x \nsim y$ otherwise.[1]

[1] Such a relation must not be confused with the binary coherence relation that is in the focus of Bovens and Hartmann’s (2003, chs. 1.4 and 2.2) account. Bovens and Hartmann are interested in the relation ‘is no less coherent than’, while we are interested in ‘coheres with’. Even if the latter relation’s scope is extended to *sets* of propositions, these relations remain independent of each other. For a set that is no less coherent than another set need not cohere with it, and a set that coheres with another set need not be no less coherent than it.

When explicating coherence in probabilistic terms, the majority of approaches appeals to C.I. Lewis' (1946, p. 338) characterisation in terms of mutual *confirmation*: "A set of statements […] will be said to be congruent if and only if they are so related that the antecedent probability of any one of them will be increased if the remainder of the set can be assumed as given premises." The notion of confirmation applied here is the one that is most prominent in Bayesian confirmation theory, namely, confirmation as probability-*raising* (cf. Earman 1992; Fitelson 1999; Howson and Urbach 2006). In this *incremental* sense, evidence y confirms a hypothesis x just in case y increases the firmness of x, which in probabilistic terms means that $P(x|y) > P(x)$. Since $P(x|y) > P(x)$ is equivalent to $P(x|y) > P(x|\bar{y})$, the corresponding relation of incremental coherence can also be defined as follows:

(IC) $x \sim_i y$ iff $P(x|y) > P(x|\bar{y})$ and $P(y|x) > P(y|\bar{x})$.

Note that it is possible to dispense with one of the conditions because $P(x|y) > P(x|\bar{y})$ is equivalent to $P(y|x) > P(y|\bar{x})$. Since it is not possible to simplify the next definition in this way, however, we adhere to our 'over-explicit' wording in order to perspicuously stress the differences.

It often goes unnoticed that 'confirmation' is an ambiguous term even when understood in a probabilistic manner. There is an increase in probability if y raises x's probability merely to, say, .01, so that x is still highly improbable. Carnap (1962, xvf.) has thus distinguished confirmation as increase in firmness from confirmation as firmness. The latter is sometimes called 'absolute confirmation', and it holds if the posterior probability $P(x|y)$ exceeds a threshold t beyond which x receives sufficient firmness. In order to let a hypothesis be firm just in case its negation is not firm, the threshold t should be identified with .5.

These reflections on confirmation have a bearing on coherence because they show that coherence as mutual confirmation can be spelled out in at least two ways. Apart from mutual *incremental* confirmation, which was defined in (IC), there is also mutual *absolute* confirmation. It obtains among two propositions x and y if both $P(x|y)$ and $P(y|x)$ exceed .5. Since $P(x|y) > .5$ is equivalent to $P(x|y) > P(\bar{x}|y)$, coherence in this absolute sense can also be introduced as follows:

(AC) $x \sim_a y$ iff $P(x|y) > P(\bar{x}|y)$ and $P(y|x) > P(\bar{y}|x)$.

Here it is not possible to dispense with one of the conditions because $P(x|y)$ may be greater than .5 while $P(y|x)$ is smaller.

The incremental coherence relation $\sim_i$ corresponds directly to the measures obtainable from Douven and Meijs' (2007) recipe by utilising a measure of incremental confirmation, as well as to the measures that quantify coherence in terms of deviation from probabilistic independence. More exactly, $x \sim_i y$ holds if and only if the set $\{x, y\}$ is assessed coherent by any of these measures because these measures assign coherence just in case $P(x|y)$ exceeds $P(\bar{x}|y)$ and thus $P(y|x)$ exceeds $P(\bar{y}|x)$. But what about the measure advocated by Roche (2013) and the overlap measures proposed by Glass (2002), Meijs (2006) and Olsson (2002)?

The overlap measures agree on taking the coherence of two propositions x and y to be $P(x \wedge y)/P(x \vee y)$. These measures are not provided with a neutral point where incoherence turns into coherence. Since their values range from 0 to 1, however, one may give .5 a trial. Then it would not be possible that x and y are coherent on the overlap measures whereas they are incoherent in the sense of (AC). For if $P(x \wedge y)/P(x \vee y) > .5$, then $2\ P(x \wedge y) > P(x \vee y)$, and therefore $2\ P(x \wedge y) > P(x)$ and $2\ P(x \wedge y) > P(y)$. But this is neither compatible with $P(x|y) = P(x \wedge y)/P(y) < .5$, nor is it compatible with $P(y|x) = P(x \wedge y)/\mathrm{P}(x) < .5$, because the former implies $2\ P(x \wedge y) < P(y)$ and the latter $2\ P(x \wedge y) < P(x)$. Conversely, there are propositions that are coherent on (AC) while the overlap measures offer a value smaller than .5. The conditions of (AC) could thus be deemed necessary but not sufficient for coherence in the overlap sense.

However, there is also an argument for a threshold of 1/3. The overlap for two propositions can be rewritten as $(P(x|y)^{-1} + P(y|x)^{-1} - 1)^{-1}$ (cf. Glass 2002). Hence, if both $P(x|y)$ and $P(y|x)$ are greater than .5, then the overlap measures take on a value greater than 1/3. Given this threshold, quite the reverse is true. Then the conditions of (AC) are sufficient but not necessary for coherence in the overlap sense.

Things are clearer when we turn to Roche's measure because it is supplemented with a natural threshold. While Roche applies Douven and Meijs' recipe for construing measures of mutual confirmation, he does not insert a measure of *incremental* confirmation but the posterior $P(x|y)$ as a measure of how much y confirms x in the *absolute* sense. One may thus think that the verdicts of Roche's measure coincide with the relation of absolute coherence defined by (AC). And indeed, if x and y are coherent according to (AC), i.e. $P(x|y)$ and $P(y|x)$ are both greater than .5, then Roche's measure registers coherence because the neutral point .5 is exceeded on average. Conversely, however, x and y can be coherent on Roche's measure but incoherent in the absolute sense because the averaging integrated into this measure allows for compensation. If $P(x|y)$ is much greater than .5 while $P(y|x)$ is only a bit below .5, then, although the conditions of (AC) are not satisfied, the neutral point of Roche's measure is exceeded.

Even though (AC)'s conditions are thus stronger than the ones of Roche's measure, we adhere to our definition of absolute coherence in order to obtain a one-to-one counterpart to the definition of incremental coherence (IC). Remember that (IC) specifies coherence in the sense of *mutual* incremental confirmation, i.e. incremental confirmation of *both* x by y and y by x. To parallel this approach, (AC) provides the conditions for *mutual* absolute confirmation, entailing *both* that y has to absolutely confirm x and that x has to absolutely confirm y.

To round off the presentation of qualitative approaches to coherence, note that it is possible to introduce a relation combining the previous proposals. A third notion of confirmation merges firmness and increase in firmness by stating that evidence y confirms hypothesis x just in case x is not only firm in the light of y but is also firmer than it was before. In probabilistic terms, this means that $P(x|y)$ is both greater than $P(\bar{x}|y)$ and $P(x|\bar{y})$.[2] The resulting *strong* coherence relation $\sim_s$ obtains just in case both $\sim_a$ and $\sim_i$ obtain:

[2] Cf. Achinstein's (2001, chs. 3f.) discussion of these three approaches to confirmation.

(SC) $x \sim_s y$ iff $x \sim_a y$ and $x \sim_i y$.

If $x \sim_s y$, then most probabilistic measures of coherence agree that the set $\{x, y\}$ is coherent. Again, the overlap measures constitute an exception if tentatively supplemented with a neutral point of .5. Consider a random cast of a dodecagonal dice. Let x be the proposition that a 3, 4, 5, 6 or 7 was thrown and y the proposition that it was a 5, 6, 7, 8 or 9. Then $P(x|y) = P(y|x) = 3/5$, in order that $x \sim_a y$. Furthermore, since $P(x) = P(y) = 5/12$, we also get $x \sim_i y$ and hence $x \sim_s y$. But $P(x \wedge y)/P(x \vee y) = 3/7$ and thus smaller than the threshold .5. However, if we deploy the threshold 1/3, the overlap measures agree with assigning coherence if (SC) is satisfied. For this threshold is exceeded if the propositions are coherent in the sense of (AC), i.e. $P(x|y)$ and $P(y|x)$ are greater than .5.

3 The logic of coherence

3.1 Basic properties

We start our investigation into the logic of coherence by examining some of the most prominent properties of relations, to wit, *reflexivity*, *symmetry* and *transitivity*. The upshot is that, although the coherence relations previously defined are reflexive and symmetric, they are not transitive. This, we will argue, is as it should be. Here are the formal characterisations of the first two properties:

(REF) For any $x \in L_c$ and any $P \in \mathbf{P}$: $x \sim x$.
(SYM) For any $x, y \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$, then $y \sim x$.

The coherence relations defined by (IC), (AC) and (SC) are reflexive and symmetric. (All proofs are given in the "Appendix".) Whereas the latter result seems uncontroversial, the former might be considered problematic. For it is usually assumed that coherence is a property of *non-singleton* sets of propositions, in order that 'self-coherence' is regarded as a degenerate case at best (cf. Akiba 2000; Fitelson 2003). However, there are situations in which the coherence of mental or linguistic occurrences with identical contents enters the limelight. Just consider two witnesses having the same belief and thus issuing the same statement, or one person exactly repeating something she asserted before. It is quite natural to describe the scenario involving the two witnesses by the qualitative claim that what they testify to coheres for the very reason that it is identical. It is even natural to proceed with the quantitative claim that, due to this identity, the testimonies are maximally coherent. And the same is true, mutatis mutandis, for other scenarios with identical contents.

At the utmost, one might argue that, while this holds for contingent propositions, it does not hold for contradictions. The latter lack 'self-coherence' because of the inner tension they are exposed to. But first, our considerations have been restricted to contingent propositions. And secondly, we are concerned here with a relation of agreement between whole propositions. Although contradictions consist of *parts*

Table 1 Basic properties

	(REF)	(SYM)	(TRA)
$\sim_i$	+	+	−
$\sim_a$	+	+	−
$\sim_s$	+	+	−

that do not fit together, this is far from implying that these *wholes* do not fit together. One may even maintain that they match in an important respect because they share the feature of self-contradiction. Hence, we consider it appropriate to impose symmetry on coherence.[3]

The third basic property is transitivity:

(TRA) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: If $x \sim y$ and $y \sim z$, then $x \sim z$.

At first sight, it is tempting to require that a coherence relation is transitive. The proposition that someone's pet, Tweety, is a bird coheres with the proposition that it has wings; the latter proposition coheres with the proposition that Tweety can fly; and so do the propositions that Tweety is a bird and can fly. However, a slight variation shows that transitivity does not hold in general. Although the proposition that Tweety is a penguin coheres with the proposition that it is a bird, which in turn coheres with the proposition that it can fly, the propositions 'Tweety is a penguin' and 'Tweety can fly' are highly incoherent (cf. Bovens and Hartmann 2003, ch. 2). Hence, it is an advantage that the coherence relations $\sim_a, \sim_i$ and $\sim_s$ violate transitivity.[4]

Up to now, the logic underlying these relations is the same because they coincide in being symmetric and reflexive but not transitive. However, differences will emerge in the next sections when, among other things, the interplay between coherence and Boolean connectives is examined. Table 1 is a summary of the relations' performances with respect to the basic conditions considered thus far.

3.2 Coherence, negation and inconsistency

In this section, we dwell on some constraints involving negation and inconsistency. From a quantitative perspective, it is usually assumed that a proposition x and its negation $\bar{x}$ constitute a case of maximal incoherence (cf. Fitelson 2003; Roche 2013; Schippers and Siebel 2015). If we dispense with quantitative

[3] In the last resort, it is possible to add the constraint that coherence relations obtain between x and y only if $x \neq y$. This would merely entail that reflexivity is no longer among the possible properties of these relations. Note also that, to model cases of identical contents in a set-theoretical manner, one may draw on multisets because they can contain the same proposition more than one time.

[4] As shown by Schippers (2014), there are nonetheless *screening-off* conditions that guarantee transitivity for each of the above relations.

verdicts, there remains the qualitative requirement that such a pair of contradictory propositions is always assessed *in*coherent:

(INC) For any $x \in L_c$ and any $P \in \mathbf{P}$: $x \nsim \bar{x}$

This principle can be extended to *all* pairs involving inconsistent propositions. For any pair of propositions, let $x \perp y$ denote that x and y are inconsistent. Then we may add the following strengthened condition for coherence relations:

(INC′) For any $x, y \in L_c$ such that $x \perp y$ and any $P \in \mathbf{P}$: $x \nsim y$.

Both conditions are easily seen to be satisfied by all qualitative coherence relations defined above. In this regard, these relations are again on a par.

A different picture emerges with respect to the following negation symmetry condition:

(NSC) For any $x, y \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$, then $\bar{x} \sim \bar{y}$.

According to (NSC), the coherence of a pair of propositions spreads to their negations: if x and y are coherent, then so are $\bar{x}$ and $\bar{y}$. While this condition is satisfied by $\sim_i$, it is violated by $\sim_a$ and hence also by $\sim_s$. The former is a straightforward consequence of the fact that probabilistic relevance is inherited to negations. If x is probabilistically relevant to y (and vice versa), then so is $\bar{x}$ to $\bar{y}$ (and vice versa). In contrast, this does not hold for absolute confirmation. Even if $P(x|y) > P(\bar{x}|y)$ and $P(y|x) > P(\bar{y}|x)$, the values of $P(\bar{x}|\bar{y})$ and $P(\bar{y}|\bar{x})$ are in no way determined. They may exceed $P(x|\bar{y})$ and $P(y|\bar{x})$, respectively, or fall short of them.

Here is an example illustrating that the absolute coherence relation violates (NSC). Consider a standard card deck. Let x be the proposition that the drawn card is either a number or a court card, and let y be the proposition that the card is either a number card or an ace. Then both $P(x|y) > P(\bar{x}|y)$ and $P(y|x) > P(\bar{y}|x)$, and hence $x \sim_a y$. On the other hand, $\bar{x}$ is the proposition that the drawn card is an ace while $\bar{y}$ is the proposition that it is a court card. Since these negations are inconsistent, they are to be regarded as incoherent. Note also that this example does not prove *incremental* coherence to violate (NSC) because x and y are *not* coherent in the incremental sense. The reason is that these propositions are *subcontrary*, i.e. cannot be false together. Since the negation of one of these propositions therefore implies the other proposition, they are negatively relevant to each other, with the result that $x \nsim_i y$ (cf. Siebel 2004). Thus, the example cannot be used to cast doubt on $\sim_i$ but is to be considered an example revealing the different logics underlying the relations $\sim_i$ and $\sim_a$.

No such disagreement occurs with respect to the following consistency condition involving the pairs (x, y) and $(x, \bar{y})$. According to this condition, a proposition does not cohere both with another proposition and its negation:

(CON) For any $x, y \in L_c$ and any $P \in$ P: if $x \sim y$, then $x \nsim \bar{y}$.

Table 2 Negation and inconsistency properties

	(INC)	(INC′)	(NSC)	(CON)	(CON′)
$\sim_i$	+	+	+	+	−
$\sim_a$	+	+	−	+	+
$\sim_s$	+	+	−	+	+

While this constraint is met by all three coherence relations, disagreement arises when we, as we have already done in the case of (INC), extend it to *all* cases of inconsistency. (CON) states that no proposition x coheres with another proposition y and its negation $\bar{y}$. But straightforward negation is only one type of inconsistency. We might thus assume that no proposition x coheres both with y and z if the latter are inconsistent, regardless of whether the inconsistency arises from straightforward negation or something else:

(CON′) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$ and $y \perp z$, then $x \nsim z$.

While both $\sim_a$ and $\sim_s$ satisfy (CON′), it is violated by $\sim_i$. This should come as no surprise given that (CON′) closely resembles a condition that keeps apart the underlying confirmation relations. According to this condition, no proposition x can both confirm a proposition y and another proposition z that is inconsistent with y. This condition is satisfied by absolute confirmation while being violated by incremental confirmation (cf. Crupi and Tentori 2015). A summary of the results for the constraints involving negation is given in Table 2.

3.3 Coherence, conjunction and disjunction

This section focuses on the interplay between coherence and conjunction on the one hand, and coherence and disjunction on the other hand. The first two principles are sometimes called 'weak ∧-composition' and 'weak ∨-composition' (cf. Huber 2007):

(WAC) For any $x, y \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$, then $x \sim x \wedge y$.
(WOC) For any $x, y \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$, then $x \sim x \vee y$.

According to these principles, if a proposition x coheres with another proposition y, then it also coheres with the conjunction and disjunction of x and y. Both constraints are met by all coherence relations at issue. But what about the inverse direction? Do x and y cohere if x coheres with $x \wedge y$ or $x \vee y$, respectively? The first of these principles is sometimes labelled 'weak consequence' (cf. Huber 2007). We thus use the titles 'weak consequence for conjunctions' and 'weak consequence for disjunctions':

(WCC) For any $x, y \in L_c$ and any $P \in \mathbf{P}$: if $x \sim x \wedge y$, then $x \sim y$.
(WCD) For any $x, y \in L_c$ and any $P \in \mathbf{P}$: if $x \sim x \vee y$, then $x \sim y$.

Table 3 Conjunction and disjunction properties (part I)

	(WAC)	(WOC)	(WCC)	(WCD)	(EOC)	(EOD)
$\sim_i$	+	+	−	−	+	+
$\sim_a$	+	+	−	−	−	−
$\sim_s$	+	+	−	−	−	−

Both constraints are violated by all coherence relations, and there is a good reason for this behaviour. After all, x's cohering with $x \wedge y$ or $x \vee y$ may be due to the fact that it coheres only with the x-part of the conjunction or disjunction while the y-part is irrelevant to x.

Next, consider a random pair of propositions (x, y). Regardless of whether x and y themselves fit together, it might be tempting to assume that x coheres with the conjunction $x \wedge y$ and the disjunction $x \vee y$:

(EOC) For any $x, y \in L_c$ such that $x \wedge y \in L_c$ and any $P \in \mathbf{P}$: $x \sim x \wedge y$ or $y \sim x \wedge y$.
(EOD) For any $x, y \in L_c$ such that $x \vee y \in L_c$ and any $P \in \mathbf{P}$: $x \sim x \vee y$ or $y \sim x \vee y$.

The incremental relation of coherence $\sim_i$ conforms to these principles. Since $x \wedge y$ implies x, it increases x's firmness; and since x implies $x \vee y$, it increases the disjunction's firmness. Hence, due to the fact that increase in firmness is symmetric, we obtain coherence in the incremental sense. By contrast, $x \wedge y$ may be highly improbable in the light of x, and x may be highly improbable in the light of $x \vee y$. Therefore, these pairs need not be coherent in the absolute or the strong sense. Table 3 is a summary of the results so far.

There is another 'disjunctive weakening' property that is satisfied by $\sim_a$. Again, we introduce it together with its conjunctive counterpart. Suppose that there is a pair of propositions (x, y) such that x coheres with y. The idea then is that the coherence of these propositions should spread to the pair that can be obtained from (x, y) by adding an arbitrary proposition z as a conjunct or disjunct to each element? Here are the corresponding 'conjunctive strengthening' and 'disjunctive weakening' conditions:

(CSC) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$, then $x \wedge z \sim y \wedge z$.
(DWC) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$, then $x \vee z \sim y \vee z$.

While both constraints are violated by $\sim_i$ and $\sim_s$, the latter is satisfied by $\sim_a$. That is, on the absolute interpretation of coherence, although strengthening coherent propositions by conjunction may destroy their coherence, weakening them by disjunction is a coherence-preserving procedure.

The disjunctive weakening condition can be reframed as follows. We know that all three coherence relations are reflexive so that $z \sim z$ for each $z \in L_c$. Hence, if also $x \sim y$, the question is whether merging these coherent pairs of propositions either by conjunction or disjunction preserves coherence. The answer of the incremental account is twice no while the absolute account at least allows for disjunctive merging. But why is disjunctive merging allowed in such a case? Does this rest on the reflexivity property, in order that merging is coherence-preserving only if the same proposition z is added? Or is it possible to generalise (DWC) to any coherent pair of propositions (z, z'), regardless of whether z' is identical with z or not? For reasons of completeness, we will again introduce the generalised merging properties for conjunctions *and* disjunctions:

(CMC) For any $x, y, z, z' \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$ and $z \sim z'$, then $x \wedge z \sim y \wedge z'$.
(DMC) For any $x, y, z, z' \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$ and $z \sim z'$, then $x \vee z \sim y \vee z'$.

Given the previous considerations, it probably comes as no surprise that $\sim_i$ does not meet these constraints. What is more interesting is that the same holds for $\sim_a$. Although (DMC) is satisfied by $\sim_a$ in the special case $z = z'$, the general condition is violated. But this is quite reasonable. Consider the pairs (x, $\bar{y}$) and (y, $\bar{x}$). Probability distributions making these pairs coherent in the absolute or the incremental sense abound. By (CMC), however, it would be allowed to conclude that the conjunctive proposition $x \wedge y$ coheres with the conjunctive proposition $\bar{x} \wedge \bar{y}$, which is highly counterintuitive. Analogously, (DMC) entails the counterintuitive consequence that the disjunctive propositions $x \vee y$ and $\bar{x} \vee \bar{y}$ are coherent. Hence, the coherence relations' refusal to comply with these principles agrees with intuition.

For a similar pair of principles, consider three propositions x, y and z such that x and y both cohere with z. The question then is whether the conjunction or disjunction of x and y also coheres with z. We call the corresponding constraints the 'left and' and 'left or' condition[5]:

(LAC) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: if $x \sim z$ and $y \sim z$, then $x \wedge y \sim z$.
(LOC) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: if $x \sim z$ and $y \sim z$, then $x \vee y \sim z$.

Analogously, let x cohere both with y and z. Here the question is whether x also coheres with the conjunction or disjunction of y and z. These conditions will be labelled 'right and' and 'right or':

(RAC) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$ and $x \sim z$, then $x \sim y \wedge z$.
(ROC) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$ and $x \sim z$, then $x \sim y \vee z$.

Thanks to the symmetry of the coherence relations, (LAC) and (RAC), as well as (LOC) and (ROC), amount to the same thing. For example, by replacing in (LAC) x by y, y by z and z by x, and by exchanging what is to the left and what is to the right of the relation sign $\sim$, we reach (RAC). We thus confine our argumentation to (LAC) and (LOC).

As it turns out, all four conditions are violated by all three coherence relations. As to (LAC), this is quite reasonable. Let x be the proposition that Ann has a season

5 The former constraint is sometimes labelled "cautious monotonicity' (cf. Huber 2007).

Table 4 Conjunction and disjunction properties (part II)

	(CSC)	(DWC)	(CMC)	(DMC)	(LAC)	(LOC)	(RAC)	(ROC)
$\sim_i$	−	−	−	−	−	−	−	−
$\sim_a$	−	+	−	−	−	−	−	−
$\sim_s$	−	−	−	−	−	−	−	−

ticket for Liverpool FC, y the proposition that she has a season ticket for Manchester United, and z the proposition that she is an ultra who supports either Liverpool FC or Manchester United. Taken by themselves, x and y cohere with z. But the traditional hostility between fans of Liverpool FC and fans of Manchester United ensures that a person with season tickets for both teams will hardly be an ultra who supports one of these teams. Hence, the conjunction $x \wedge y$ does not cohere with z.

The case is less clear for (LOC) because it is hard to come up with such a concrete example. However, assume that both pairs (x, z) and (y, z) are only slightly coherent, i.e. close to being neither coherent nor incoherent. Then it could be the case that, by conjoining x and y by disjunction, this slight coherence turns into slight incoherence. Hence, the only conjunction and disjunction conditions on which $\sim_a$ and $\sim_i$ disagree are (EOC), (EOD) and (DWC). The first two are satisfied by $\sim_i$ and violated by $\sim_a$, while for the latter the reverse is the case. A summary of the further results within this section is given in Table 4.

3.4 Coherence, entailment and equivalence

In the last section, we focus on conditions involving logical entailment and equivalence. Let $\vdash$ stand for entailment and $\dashv\vdash$ for equivalence. It is a straightforward consequence of the laws governing probability that the probabilistic coherence relations defined by (IC), (AC) and (SC) are not affected when propositions are replaced by equivalent propositions. That is, all relations satisfy the following conditions on 'left logical equivalence' and 'right logical equivalence':

(LLE) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$ and $y \dashv\vdash z$, then $x \sim z$.
(RLE) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$ and $x \dashv\vdash z$, then $z \sim y$.

Similarly, equivalence itself is a case of coherence. One may quibble over the claim that tautologies or contradictions are always coherent because they are equivalent. But if contingent propositions are equivalent, there is no reason to deny them coherence.[6] The corresponding constraint, which is met by all three coherence relations, reads as follows:

[6] It has also been argued that logically equivalent propositions are *maximally* coherent (cf. Fitelson 2003; Siebel and Wolff 2008; contrast Olsson and Schubert 2007). Since we are interested in coherence only from a *qualitative* perspective, we will not dwell on this and related issues.

Table 5 Entailment and equivalence properties

	(LLE)	(RLE)	(CEQ)	(CEN)	(LMO)	(RMO)
$\sim_i$	+	+	+	+	−	−
$\sim_a$	+	+	+	−	−	−
$\sim_s$	+	+	+	−	−	−

(CEQ) For any $x, y \in L_c$ and any $P \in \mathbf{P}$: if $x \dashv\vdash y$, then $x \sim y$.

But what happens when we replace the equivalence condition in the previous constraints by mere entailment? Starting with (CEQ), is it also true that unilateral entailment implies coherence, in order that the following condition is satisfied by our relations?

(CEN) For any $x, y \in L_c$ and any $P \in \mathbf{P}$: if $x \vdash y$, then $x \sim y$.

It is a well-known fact that entailment among contingent propositions implies probabilistic relevance; and since probabilistic relevance is symmetric, entailment implies incremental coherence. That is, $\sim_i$ satisfies (CEN). But this is *not* true for the absolute coherence relation $\sim_a$. Even if x entails y, and therefore $P(y|x)$ exceeds $P(\bar{y}|x)$, the latter need not be the case for $P(x|y)$ and $P(\bar{x}|y)$. In other words, x's entailing y is compatible with a low conditional probability of x given y; consequently, there need not be absolute coherence (and also no strong coherence).

Finally, let us consider counterparts to the first two equivalence principles. They are sometimes called 'left monotonicity' and 'right monotonicity':

(LMO) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$ and $y \vdash z$, then $x \sim z$.
(RMO) For any $x, y, z \in L_c$ and any $P \in \mathbf{P}$: if $x \sim y$ and $z \vdash x$, then $z \sim y$.

None of these constraints is met by any of our coherence relations. There is ample reason for why they must not meet (RMO). Let z be identical with a conjunction $x \wedge z'$, in order that z entails x. If x coheres with y, (RMO) rules that z, i.e. $x \wedge z'$, also coheres with y. But even if the conjunct x coheres with y, the conjunct z' could be a proposition that does not cohere with y. And if the incoherence of z' and y is much stronger than the coherence of x and y, the overall result might be that $x \wedge z'$ does not cohere with y. An analogous argument undermines (LMO). Let x, y and z be propositions such that x coheres with y and $z = y \vee z'$. Then, although z is entailed by y, it need not cohere with x. For z' could be a proposition that is highly incoherent with x, so that the coherence of x and y is surpassed by the incoherence of x and z'. Table 5 gives a summary for all properties considered within this section.

4 Conclusion

Three probabilistic coherence relations were introduced in order to examine their logical properties. These properties included reflexivity, symmetry and transitivity, features concerning negation, conjunction and disjunction, as well as features involving inconsistency, entailment and equivalence. The upshot of the discussion is twofold. On the one hand, the given characterisations of coherence widely comply with intuitive judgements. On the other hand, the different logical structures underlying these relations were revealed. More exactly, it was shown that there are a number of conditions whose satisfaction depends on the chosen coherence relation.

There are different routes to continue our study. One may integrate further properties to foster insights into the logics of the given coherence relations. Or one may examine alternative coherence relations. Three such possibilities come readily to mind. First of all, remember that the definition of absolute coherence (AC) offers stronger conditions than Roche's measure. The former requires that *both* $P(x|y)$ and $P(y|x)$ exceed .5, whereas the latter merely calls for an *average* that exceeds .5. One may thus weaken (AC) by making use of Roche's condition: $x \sim_R y$ iff $(P(x|y)+P(y|x))/2>.5$. Secondly, there is a variant of strong coherence demanding not only that both $\sim_i$ and $\sim_a$ hold but more specifically that $P(x|y)>.5>P(x)$ and $P(y|x)>.5>P(y)$. This variant rests on a fourth conception of confirmation. Remember that evidence y confirms hypothesis x in the absolute sense if x is firm in the light of y, i.e. $P(x|y)>.5$. According to the fourth conception, x should not only *be* firm in the light of y but rather be *made* firm by y, which means that it has not been firm before. This is captured by ruling that y raises the probability of x from a value smaller than .5 to a value greater than .5. Thirdly, a way of developing further coherence relations is to vary the threshold t beyond which a hypothesis is regarded as sufficiently firm. That is, instead of requiring that the posterior probabilities exceed .5, one could choose another threshold $t \in [0, 1)$ so that the pair of propositions (x, y) is coherent iff both $P(x|y)$ and $P(y|x)$ exceed t.

It is evident that these three ways of developing further coherence relations can be interblended, thereby giving rise to numerous alternatives. We leave the investigation of these and other extensions of the present study to future research.

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Appendix

(REF) $P(x|x)=1$, and $P(x|\bar{x})=P(\bar{x}|x)=0$. Hence, the definientia of (IC), (AC) and therefore also (SC) are satisfied.

(SYM) (AC), (IC) and thus also (SC) require bidirectional confirmation in random order.

(TRA) This is a straightforward consequence of the fact that the underlying accounts of confirmation are not transitive (cf. Douven 2011; Roche and Shogenji 2014; Shogenji 2003).

(INC) $P(x|x)=P(\bar{x}|\bar{x})=1$, and $P(x|\bar{x})=P(\bar{x}|x)=0$. Hence, since x and $\bar{x}$ are contingent and the probability function P is regular, $P(x|\bar{x})<P(x)<P(x|x)$ and $P(\bar{x}|x)<P(\bar{x})<P(\bar{x}|\bar{x})$. The definientia of (IC) and (AC), and thus also (SC), are therefore violated.

(INC′) If $x\perp y$, then $P(x|y)=P(y|x)=0$. Hence, $P(x|y)<P(x)$ and $P(x|y)<P(x|\bar{y})$, as well as $P(y|x)<P(y)$ and $P(y|x)<P(y|\bar{x})$. That is, the definientia of (IC) and (AC), and thus also (SC), are again violated.

(NSC) First, if $x \sim_i y$, then $P(x|y)>P(x)$, which is for all contingent propositions equivalent to $P(\bar{y}|\bar{x})>P(\bar{y})$. Analogously, $P(y|x)>P(y)$ is equivalent to $P(\bar{x}|\bar{y})>P(\bar{x})$. Thus, if $x \sim_i y$, then also $\bar{x} \sim_i \bar{y}$, in order that $\sim_i$ satisfies (NSC). Secondly, probability distribution P_1 in Table 6 shows that neither $\sim_s$ nor $\sim_a$ satisfies (NSC). According to this distribution, $P_1(x|y)\approx.731>.637\approx P_1(x)>.5$, and $P_1(y|x)\approx.812>.707\approx P_1(y)>.5$. Hence, $x \sim_s y$ and therefore $x \sim_a y$. However, $P_1(\bar{y}|\bar{x})\approx.477<.5$ and hence neither $\bar{x} \sim_a \bar{y}$ nor $\bar{x} \sim_s \bar{y}$.

(CON) If $x \sim_s y$, then $P(y|x)>P(y)$ and $P(y|x)>P(\bar{y}|x)$. Hence, $P(\bar{y}|x)<P(\bar{y})$ and $P(\bar{y}|x)<P(y|\bar{x})\}$, and therefore $x \nsim_i \bar{y}$ and $x \nsim_a \bar{y}$ and thus also $x \nsim_s \bar{y}$.

(CON′) Consider probability distribution P_6 in Table 7. Here $P_6(x|y)=1>.5=P_6(x)$ and $P_6(y|x)=.5>.25=P_6(y)$; hence, $x \sim_i y$. Furthermore, $P_6(z|x)=.5>.25=P_6(z)$ and $P_6(x|z)=1>.5=P_6(x)$, so that also $x \sim_i z$. Nonetheless, $y \perp z$ because $P_6(y\wedge z)=0$, with the result that $\sim_i$ violates (CON′). On the other hand, if $x \sim_a y$, then $P(y|x)>.5$ and hence

Table 6 Probability distributions for pairs of propositions

x	y	P_1	P_2	P_3	P_4	P_5
1	1	15/29	3/16	5/34	0	1/10
1	0	3/25	45/256	13/44	1/4	0
0	1	11/58	11/32	13/44	1/4	8/10
0	0	251/1450	75/256	49/187	2/4	1/10

Table 7 Probability distributions for triples of propositions, where $a_i=1-Pi(x\vee y\vee z)$

x	y	z	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}
1	1	1	0	1/691	7/53	0	26/105	4/17	61/213	0
1	1	0	1/4	5/36	29/101	0	10/43	5/67	0	85/169
1	0	1	1/4	1/199	0	1/16	3/40	1/32	0	36/145
1	0	0	0	1/56	0	7/16	0	1/24	3/16	0
0	1	1	0	25/56	9/34	7/16	4/43	1/34	5/69	0
0	1	0	0	14/53	0	1/16	1/70	1/26	0	0
0	0	1	0	6/49	15/67	0	5/37	18/89	44/976	0
0	0	0	2/4	a_7	a_8	0	a_{10}	a_{11}	a_{12}	a_{13}

$P(\bar{y}|x) \leq .5$. Hence, since $P(\bar{y}|x) = P(\bar{y} \wedge z|x) + P(\bar{y} \wedge \bar{z}|x)$, $P(\bar{y} \wedge z|x) \leq .5$. Finally, since $y \perp z$, z implies $\bar{y}$, in order that $P(\bar{y} \wedge z|x) = P(z|x) \leq .5$. That is, $x \nsim_a z$ and $x \nsim_s z$, entailing that both relations satisfy (CON′).

(WAC) As to incremental coherence, since P is regular and x is contingent, $P(x|x \wedge y) = 1 > P(x)$. Due to the symmetry of incremental confirmation, it also holds that $P(x \wedge y|x) > P(x \wedge y)$; hence, $x \sim_i x \wedge y$. As to absolute coherence, since P is regular and x is contingent, $P(x|x \wedge y) = 1 > P(\bar{x}|x \wedge y)$. Moreover, $P(x \wedge y|x) = P(y|x)$ and $P(\overline{x \wedge y}|x) = P(\bar{x} \vee \bar{y}|x) = P(\bar{y}|x)$. Since $x \sim_a y$, we also get $P(y|x) > P(\bar{y}|x)$, so that $P(x \wedge y|x) = P(y|x) > P(\bar{y}|x) = P(\overline{x \wedge y}|x)$. Hence, $x \sim_a x \wedge y$. Therefore, all three accounts satisfy (WAC).

(WOC) As to incremental coherence, since P is regular and x is contingent, $P(x \vee y|x) = 1 > P(x)$. Due to the symmetry of incremental confirmation, we thus get $x \sim_i x \vee y$. As to absolute coherence, $P(x \vee y|x) = 1 > 0 = P(\overline{x \vee y}|x)$. Furthermore, since $x \sim_a y$, $P(x|y) > P(\bar{x}|y)$, and therefore $P(x \wedge y) > P(\bar{x} \wedge y)$. Hence, $P(x) = P(x \wedge y) + P(x \wedge \bar{y}) > P(\bar{x} \wedge y) + P(x \wedge \bar{y}) > P(\bar{x} \wedge y)$. Consequently, $P(x|x \vee y) = P(x)/P(x \vee y) > P(\bar{x} \wedge y)/P(x \vee y) = P(\bar{x}|x \vee y)$. This means that $x \sim_a x \vee y$ and $x \sim_s x \vee y$, so that all three accounts satisfy (WOC).

(WCC) According to probability distribution P_2 in Table 6, $P_2(x \wedge y|x) \approx .516 > .5 > .188 \approx P_2(x \wedge y)$, and $P_2(x|x \wedge y) = 1 > .5 > P_2(x)$. Hence, $x \sim_i x \wedge y$ and $x \sim_a x \wedge y$, so that also $x \sim_s x \wedge y$. However, $P_2(x|y) \approx .353 < 0.363 \approx P_2(x) < .5$. Therefore, neither $x \sim_i y$ nor $x \sim_a y$, entailing that $x \sim_s y$ does not hold, too.

(WCD) Consider again distribution P_2. $P_2(x|x \vee y) \approx .514 > .5 > .363 \approx P_2(x)$, and $P_2(x \vee y|x) = 1 > .707 \approx P_2(x \vee y) > .5$. Thus, $x \sim_i x \vee y$ and $x \sim_a x \wedge y$, implying that also $x \sim_s x \vee y$ But since $P_2(x|y) < P_2(x) < .5$, x and y are assessed incoherent on all three accounts.

(EOC) Since x is contingent and P is regular, $P(x|x \wedge y) = 1 > P(x)$. Therefore, by the symmetry of incremental confirmation, $P(x \wedge y|x) > P(x \wedge y)$, in order that $\sim_i$ satisfies (EOC). On the other hand, as is shown by distribution P_3 in Table 6, there are situations in which $P_3(x \wedge y|x) = P_3(x \wedge y|y) \approx .332 < P_3(x \wedge y) \approx 0.147 < .5$. Hence, neither $x \sim x \wedge y$ nor $y \sim x \wedge y$ for either $\sim_a$ or $\sim_{s.}$

(EOD) Since $x \vee y$ is contingent and P is regular, $P(x \vee y|x) = 1 > P(x \vee y)$. Hence, by symmetry also $P(x|x \vee y) > P(x)$, entailing that $x \sim_i x \vee y$. But distribution P_4 in Table 6 proves possible that $P(x|x \vee y) = P(y|x \vee y) = .5$, so that neither $x \sim x \vee y$ nor $y \sim x \vee y$ for either $\sim_a$ or $\sim_{s.}$

(CSC) Consider probability distribution P_7 in Table 7. $P_7(x|y) \approx .165 > .163 \approx P_7(x)$, in order that by symmetry $x \sim_i y$. However, $P_7(x \wedge z|y \wedge z) \approx .003 < .006 \approx P_7(x \wedge z)$, implying that $x \wedge z \sim_i y \wedge z$ does not hold. Furthermore, according to distribution P_8 in the same table, $P_8(x|y) \approx .613 > .5 > .419 \approx P_8(x)$ and $P_8(y|x) = 1 > .684 \approx P_8(y) > .5$; therefore $x \sim_s y$. But $P_8(x \wedge z|y \wedge z) \approx .333 < .5$, with the result that both $\sim_a$ and $\sim_s$ violate (CSC).

(DWC) To show that $\sim_a$ satisfies (DWC), assume that $x \vee z \nsim_a y \vee z$ because $P(x \vee z|y \vee z) = P(x \wedge y \wedge z|y \vee z) + P(x \wedge y \wedge \bar{z}|y \vee z) + P(x \wedge \bar{y} \wedge z|y \vee z) + P(\bar{x} \wedge y \wedge z|y \vee z) + P(\bar{x} \wedge \bar{y} \wedge z|y \vee z) \leq .5$. Since $P(x \wedge \bar{y} \wedge \bar{z}|y \vee z) = P(\bar{x} \wedge \bar{y} \wedge \bar{z}|y \vee z) = 0$, this implies that the remaining probability $P(\bar{x} \wedge y \wedge \bar{z}|y \vee z) > .5$. Hence, $P(\bar{x} \wedge y \wedge \bar{z}|y) \geq P(\bar{x} \wedge y \wedge \bar{z}|y \vee z) > .5$. But this entails that $P(x \wedge y \wedge z|y) + P(x \wedge y \wedge \bar{z}|y) = P(x|y) \leq .5$, so that $x \nsim_a y$. Since an analogous argument applies if $x \vee z \nsim_a y \vee z$ because $P(y \vee z|x \vee z) \leq .5$, $\sim_a$ satisfies (DWC). Furthermore, distribution P_7 in Table 7 proves that $\sim_i$ does not meet this constraint. Here, $x \sim_i y$ but $P_7(x \vee z|y \vee z) \approx .730 < .732 \approx P_7(x \vee z)$, so that $x \vee z \nsim_i y \vee z$. One can easily find similar distributions showing that $\sim_s$ also violates (DWC).

(CMC) We have seen that (CSC) is violated by all coherence relations, that is, it is possible that $x \sim y$ but $x \wedge z \nsim y \wedge z$. Since reflexivity implies that $z \sim z$, it is also possible that $x \sim y$ and $z \sim z$ but $x \wedge z \nsim y \wedge z$. Hence, if $z' = z$, then it is possible that $x \sim y$ and $z \sim z'$ while $x \wedge z \sim y \wedge z'$. Hence, (CMC) is also violated by all coherence relations.

(DMC) By analogy with (CMC), the fact that $\sim_s$ and $\sim_i$ violate (DWC) entails that they also violate (DMC). To show that, although $\sim_a$ satisfied (DWC), it does not satisfy (DMC), let y be $\bar{y}$, z be y and z' be $\bar{x}$, and assume that $x \sim_a \bar{y}$ and $y \sim_a \bar{x}$. Probability distribution P_9 in Table 7 provides an example because all relevant conditional probabilities for these pairs equal 1. However, it is clearly not the case that $x \vee y \sim_a \bar{y} \vee \bar{x}$.

(LAC) Probability distribution P_{10} in Table 7 shows that none of the coherence relations satisfies (LAC). $P_{10}(x|z) \approx .586 > .555 \approx P_{10}(x) > .5$, $P_{10}(z|x) \approx .581 > .551 \approx P_{10}(z) > .5$, $P_{10}(y|z) \approx .619 > .587 \approx P_{10}(y) > .5$ and $P_{10}(z|y) \approx .580 > .551 \approx P_{10}(z) > .5$. Hence, $x \sim_s z$ and $y \sim_s z$. However, $P_{10}(x \wedge y|z) \approx .450 < .480 \approx P_{10}(x \wedge y|z) < .5$. Therefore, neither $x \wedge y \sim_i z$ nor $x \wedge y \sim_a z$, and a fortiori not $x \wedge y \sim_s z$.

(LOC) An example showing that none of the accounts satisfies (LOC) is given by probability distribution P_{11}. $P_{11}(x|z) \approx .535 > .532 \approx P_{11}(x) > .5$, $P_{11}(z|x) \approx .501 > .5 > .498 \approx P_{11}(z)$, $P_{11}(y|z) \approx .531 > .527 \approx P_{11}(y) > .5$ and $P_{11}(z|y) \approx .502 > .5 > .498 \approx P_{11}(z)$. Accordingly, $x \sim_s z$ and $y \sim_s z$. But $P_{11}(z|x \vee y) \approx .493 < .498 \approx P_{11}(z) < .5$. Hence, it is not the case that $x \vee y \sim z$ for each of the three accounts.

(RAC) Due to the symmetry of the coherence relations, (RAC) is equivalent to (LAC).

(ROC) Due to the symmetry of the coherence relations, (ROC) is equivalent to (LOC).

(LLE) This is a straightforward consequence of the fact that the laws governing probability are not affected when propositions are replaced by equivalent propositions.

(LRE) Ditto.

(CEQ) If x and y are equivalent, then $P(x|y)$ and $P(y|x)$ equal 1 and are thus smaller than .5. Moreover, since x and y are contingent and P is regular, it is also entailed that $P(x|y) > P(\bar{x}|y)$ and $P(y|x) > P(\bar{y}|x)$. Hence, $x \sim y$ for all three relations.

(CEN) If x entails y, then $P(y|x) = 1$. Hence, since y is contingent and P is regular, it is also implied that $P(y|x) > P(y)$. By symmetry we obtain $P(x|y) > P(x)$, with the result that $x \sim_i y$. On the other hand, consider probability distribution P_5 in Table 6. Here x entails y because $P_5(y|x) = 1$ with x and y being contingent. However, $P_5(x|y) = 1/9$, so that neither $x \sim_a y$ nor $x \sim_s y$.

(LMO) According to probability distribution P_{12} from Table 7, $P_{12}(x|y) \approx .798 > .5 > .474 \approx P_{12}(x)$ and $P_{12}(y|x) \approx .604 > .5 > .359 \approx P_{12}(y)$. Therefore, $x \sim y$ for all three coherence relations. Furthermore, $y \vdash z$ because $P_{12}(z|y) = 1$ with z and y being contingent. However, $x \sim z$ holds for no relation because $P_{12}(x|z) \approx .352 < .474 \approx P_{12}(x) < .5$.

(RMO) Consider probability distribution P_{13}. $P_{13}(x|y) = 1 > .751 > .5 \approx P_{13}(x)$ and $P_{13}(y|x) \approx .670 > .503 \approx P_{13}(y) > .5$. Hence, $x \sim y$ on all three accounts. Moreover, $z \vdash x$ because $P_{12}(x|z) = 1$ with x and z being contingent. Nonetheless, we have $P_{13}(y|z) = 0 < .5 < .503 \approx P_{13}(y)$, implying that $y \sim z$ obtains for none of the accounts.

References

Achinstein, P. (2001). *The book of evidence*. New York: Oxford University Press.

Akiba, K. (2000). Shogenji's probabilistic measure of coherence is incoherent. *Analysis, 60,* 356–359.

Bovens, L., & Hartmann, S. (2003). *Bayesian epistemology*. Oxford: Clarendon Press.

Carnap, R. (1962). *Logical foundations of probability* (2nd ed.). Chicago: University of Chicago Press.

Crupi, V., & Tentori, K. (2015). Confirmation theory. In A. Hájek & C. Hitchcock (Eds.), *Oxford handbook of philosophy and probability*. Oxford: Oxford University Press.

Douven, I. (2011). Further results on the intransitivity of evidential support. *Review of Symbolic Logic, 4,* 487–497.

Douven, I., & Meijs, W. (2007). Measuring coherence. *Synthese, 156,* 405–425.

Earman, J. (1992). *Bayes or bust? A critical examination of Bayesian confirmation theory*. Cambridge, MA: MIT Press.

Fitelson, B. (1999). *Studies in Bayesian confirmation theory*. Ph.D. thesis, University of Wisconsin-Madison.

Fitelson, B. (2003). A probabilistic theory of coherence. *Analysis, 63,* 194–199.

Fitelson, B. (2004). Two technical corrections to my coherence measure. http://www.fitelson.org/coherence2.pdf. Accessed 29 Jan 2020.

Glass, D. H. (2002). Coherence, explanation, and Bayesian networks. In M. O'Neill et al. (Eds.), *13th Irish international conference (Proceedings) on artificial intelligence and cognitive science* (pp. 177–182). Berlin and Heidelberg: Springer.

Howson, C., & Urbach, P. (2006). *Scientific reasoning. The Bayesian approach* (3rd ed.). Chicago and La Salle, Illinois: Open Court.

Huber, F. (2007). The logic of theory assessment. *Journal of Philosophical Logic, 36,* 511–538.

Lewis, C. I. (1946). *An analysis of knowledge and valuation*. Chicago: Open Court.

Meijs, W. (2006). Coherence as generalized logical equivalence. *Erkenntnis, 64,* 231–252.

Olsson, E. J. (2002). What is the problem of coherence and truth? *Journal of Philosophy, 94,* 246–272.

Olsson, E. J., & Schubert, S. (2007). Reliability conducive measures of coherence. *Synthese, 157,* 297–308.

Roche, W. (2013). Coherence and probability. A probabilistic account of coherence. In M. Araszkiewicz & J. Savelka (Eds.), *Coherence: Insights from philosophy, jurisprudence and artificial intelligence* (pp. 59–91). Dordrecht: Springer.

Roche, W., & Shogenji, T. (2014). Confirmation, transitivity, and Moore: the screening-off approach. *Philosophical Studies, 168,* 797–817.

Schippers, M. (2014). Structural properties of qualitative and quantitative accounts to coherence. *The Review of Symbolic Logic, 7,* 579–598.

Schippers, M., & Siebel, M. (2015). Inconsistency as a touchstone for coherence measures. *Theoria, 30,* 11–41.

Schupbach, J. N. (2011). New hope for Shogenji's coherence measure. *British Journal for the Philosophy of Science, 62,* 125–142.

Shogenji, T. (1999). Is coherence truth conducive? *Analysis, 59,* 338–345.

Shogenji, T. (2003). A condition for transitivity in probabilistic support. *British Journal for the Philosophy of Science, 54,* 613–616.

Siebel, M. (2004). On Fitelson's measure of coherence. *Analysis, 64,* 189–190.

Siebel, M., & Wolff, W. (2008). Equivalent testimonies as a touchstone of coherence measures. *Synthese, 161,* 167–182.