Disjunction and distality: the hard problem for purely probabilistic causal theories of mental content

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Abstract

The disjunction problem and the distality problem each presents a challenge that any theory of mental content must address. Here we consider their bearing on purely probabilistic causal (ppc) theories. In addition to considering these problems separately, we consider a third challenge—that a theory must solve both. We call this "the hard problem." We consider 8 basic ppc theories along with 240 hybrids of them, and show that some can handle the disjunction problem and some can handle the distality problem, but none can handle the hard problem. This is our main result. We then discuss three possible responses to that result, and argue that though the first two fail, the third has some promise.

1 Introduction

Causal theories of mental content come in many varieties, but they are all based on the same motivating idea—that the content of a given mental representation type is determined by what causes tokens of that type.[1] If, say, the content of perceptual belief type b is the proposition that *that's a dog*, then, the story goes, this is because tokens of b are caused by the presence of dogs. This is just part of the picture, however, since tokens of b are also caused by foxes-at-a-distance, by retinal states of various sorts,

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[1] The extant literature on causal theories of mental content is huge. See Adams and Aizawa (2017, sec. 4.4) for a helpful overview.

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and by lots of other things. The general challenge is to say why b's content is one proposition rather than another, and to spell out an answer in the language of causality, perhaps employing ideas from logic and probability theory along the way.[2]

This challenge has many facets. Suppose, for simplicity, that tokens of b occur only when they are caused by dogs or by foxes-at-a-distance. If a given causal theory T implies that b's content is the proposition that *that's a dog-or-fox-at-a-distance* and isn't the proposition that *that's a dog* or the proposition that *that's a fox-at-a-distance*, then T entails that beliefs of this type never misrepresent—they are never false. It may be acceptable that a theory should occasionally judge that some belief types have contents that are never false. However, any theory that goes farther, and judges that misrepresentation is impossible, has gone too far. Misrepresentation is ubiquitous and theories of content, whether they are causal or not, must explain why beliefs have contents that are sometimes false. This is the point of the infamous "disjunction problem."[3]

The disjunction problem has a cousin, the so-called "distality problem."[4] Suppose now that tokens of b are caused by dogs, by retinal states of type s, and by nothing else, where dogs cause retinal states of type s, and the latter, in turn, cause tokens of b. If b's content is the proposition that *that's a dog*, and a given causal theory T says that b's content is the proposition that *a token of s is occurring* (which concerns the more proximate cause), and isn't the proposition that *that's a dog* (which concerns the more distal cause), then T has made a mistake. Here again, there's nothing wrong with a theory that entails that some beliefs are about retinal states, but a theory that says beliefs are never about events in the external world has gone too far.

The disjunction problem and the distality problem both concern ways in which a theory of content can go wrong. To cleanly separate these two problems, it helps to think of the disjunction problem as a synchronic problem and the distality problem as diachronic (Stampe 1977, 44); see Fig. 1. The arrows from C_d to X, from X to C_p, and from C_p to B are causal; they indicate that C_d (the distal cause in the chain) causes X, X causes C_p (the proximate cause in the chain), and C_p causes B, where B is the proposition that this or that organism has a token of b at a given time.[5] The arrow from X to $X \vee Y$, in contrast, is logical; it indicates that X logically entails $X \vee Y$. The issue raised by the disjunction problem concerns whether b's content is X and isn't the

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[2] Here and throughout our focus in on causal theories of *the contents of perceptual belief types*. How exactly such theories should be incorporated into *fully general* theories of the contents of belief types is a difficult but separate issue (as is the issue of how they should be incorporated into theories of the contents of mental state types other than beliefs). See Adams and Aizawa (2017, sec. 4.4), Buras (2009), and Gerken (2014) for relevant discussion.

[3] The disjunction problem is widely associated with Fodor. This makes some sense, since Fodor makes heavy use of it in his 1984 paper "Semantics, Wisconsin style" and in many subsequent works (e.g., Fodor 1987, Ch. 4; 1990a, Ch. 3; b). But there are works prior to Fodor (1984) in which the essence of the disjunction problem is discussed (though not under that name). See, e.g., Stampe (1977, 44) and Dretske (1983, p. 89). For further discussion and references, see Adams and Aizawa (2017).

[4] The distality problem goes back (at least) to Armstrong (1968, Ch. 11) and Dretske (1981, Ch. 6). See Adams and Aizawa (2017) for further discussion and references.

[5] Here, for ease of presentation, we're being a bit sloppy. C_d, X, C_p, and B are propositions, not events, but causation is a relationship among events, not propositions. We trust that readers will not be thrown into confusion by this.
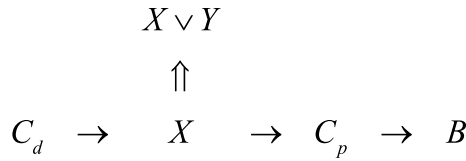
$$X \vee Y$$

$$\Uparrow$$

$$C_d \quad \rightarrow \quad X \quad \rightarrow \quad C_p \quad \rightarrow \quad B$$

Fig. 1 The synchronic disjunction problem and the diachronic distality problem

"simultaneous" proposition $X \vee Y$. The issue in the distality problem concerns whether b's content is X and isn't the "earlier" proposition C_d or the "later" proposition C_p.[6]

Our interest here is in purely probabilistic causal (ppc) theories. Suppose that tokens of b are caused by X_1, by X_2, …, and by X_n. Let T be a ppc theory. If T entails that b's content is X_1 and isn't X_2, …, or X_n, then this is solely because of B's probabilistic (and causal) profile with respect to X_1, X_2, …, and X_n. Perhaps, for example, it's because B *indicates* that X_1 is true in that $\Pr(X_1 \mid B) = 1$, but does not do the same for any of X_2, …, or X_n. More specifically, a ppc theory T takes as "input" a set (perhaps infinite) of candidate propositions for b's content (where, since a ppc theory is a *causal* theory, these propositions are restricted to propositions about things that can cause tokens of b), a probability distribution defined over B and those propositions, and nothing else, and then "outputs" a verdict on b's content—for instance, the verdict that b's content is X and isn't any other proposition.[7]

Some causal theories, in contrast, are probabilistic but only "partially" so. They take as input not just a set of candidate propositions for b's content and a probability distribution defined over B and those propositions, but also something else.[8]

We will focus on ppc theories, but this isn't because we're convinced that an adequate theory of content should be purely probabilistic. We harbor no such conviction. Our motivation, rather, is that the prospects of ppc theories in the context of the disjunction problem and the distality problem have been underexplored in the literature, and that this gap is unfortunate since a more thorough examination might be significant.[9] If it turns out that some ppc theories are able to cope with the disjunction problem and the distality problem, and if ppc theories are nonetheless problematic as

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[6] We aren't assuming that causes *always* precede their effects; it suffices that this is *often* the case with causal chains leading to belief states.

[7] We put "input" and "output" in scare quotes because we aren't saying that theories of content are decision procedures in the sense of providing an algorithm.

[8] An anonymous reviewer asks whether partially probabilistic causal theories are logically stronger than their corresponding purely probabilistic causal theories. Take some ppc theory T and some partially probabilistic variant of it T*, and suppose that each theory has the form "For any b and X, b's content is X if and only if …" where the right-hand side of T* is logically stronger than the right-hand side of T. It doesn't follow that T* is logically stronger than T. In fact, it might well be that T and T* are mutually exclusive. Compare: the right-hand side of "S is a bachelor if and only if S is an unmarried adult male" is logically stronger than the right-hand side of "S is a bachelor if and only if S is an adult male", but the first biconditional is inconsistent with and thus isn't logically stronger than the second.

[9] We aren't claiming here that there has been absolutely no discussion of ppc theories in the context of the disjunction problem and the distality problem. Artiga and Sebastián (forthcoming) examine the theories of Eliasmith (2005), Rupert (1999), and Usher (2001) in that context. We will discuss these theories in what follows. However, the present point is that the class of ppc theories in logical space goes well beyond these examples. This should be abundantly clear by the end of Sect. 3.

a whole, then this isn't because of the disjunction and distality problems. If, instead, it turns out that no ppc theory can handle both problems, then this provides additional motivation for partially probabilistic theories and also for non-probabilistic theories (i.e., theories that don't take probability distributions as relevant inputs). Furthermore, it might be that a more thorough examination of ppc theories will suggest novel theories in the partially probabilistic camp, and perhaps some such theories will compare favorably with extant partially probabilistic theories.

The remainder of this paper is divided into five sections. In Sect. 2, we clarify how we mean for the disjunction and distality problems to be understood. We also introduce a third problem, which we call "the hard problem." In Sect. 3, we describe four types of ppc theory, and present two theories of each type. We call the eight theories in question "T1," "T2," and so on. Some of these have been discussed in the extant literature, but others are new. We also note several—240, to be exact—hybrids of two or more of T1–T8. In Sect. 4, we describe which of our candidate theories can handle the disjunction problem, which can handle the distality problem, and which can handle the hard problem. It turns out that though some can handle the first, and some can handle the second, none can handle the third. This is our main result. In Sect. 5, we consider three potential responses to that result, and argue that the first two fail, but the third has some promise. In Sect. 6, we offer some concluding comments.

2 The disjunction problem, the distality problem, and the hard problem

2.1 The disjunction problem

Our target theories of content are *causal*, from which it follows that some candidate propositions for b's content are ruled out from the start. For example, B is ruled out as b's content, on the grounds that b can't be caused by B's being true, and propositions about the future are ruled out as well, and for the same reason.

Consistent with this, consider the following:

(DISJ1)   b's content is X, $X \lor Y$, or Y.
(DISJ2) $1 > \Pr(X \lor Y) > \Pr(X \& Y) = 0$.
(DISJ3) $\Pr(B \& X) > 0$ and $\Pr(B \& Y) > 0$.

We will say that a given ppc theory T can handle the disjunction problem if and only if there is a belief state b, there are propositions X, $X \lor Y$, and Y, and there is a probability distribution such that (i) (DISJ2) and (DISJ3) hold and (ii) given the assumption that (DISJ1) holds, T outputs the result that b's content is X and isn't $X \lor Y$ or Y.[10] We propose that an adequate ppc theory of content should be able to handle the

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[10] Note that the right-hand side of this biconditional leaves it open that there is a belief state b, there are propositions X, $X \lor Y$, and Y, and there is a probability distribution such that (i) (DISJ2) and (DISJ3) hold and (ii) given the assumption that (DISJ1) holds, *it's not the case that* T outputs the result that b's content is X and isn't $X \lor Y$ or Y.

disjunction problem thus understood. A ppc theory that passes this test thus makes room for misrepresentation.

The probabilities deployed in (DISJ2) and (DISJ3), and in what follows, should not be understood as *credences*. That would put the cart before the horse, since we want to consider theories that characterize propositional contents in terms of probabilities; this means that the probabilities themselves should not involve degrees of belief. A broadly "objective" interpretation of probability is needed, but we won't assume any particular objective interpretation here.[11],[12]

Our way of understanding the disjunction problem differs from Fodor's (1987) in that his, but not ours, requires that each of *X* and *Y* be causally sufficient but not necessary for b.[13] We prefer ours because it doesn't involve that requirement and thus is more general, at least in that respect. We leave it open, however, that an adequate theory of content should *also* be able to handle Fodor's version of the disjunction problem. The important point here is just that an adequate theory of content should be able to handle *ours*.[14]

Assumptions (DISJ2) and (DISJ3) are pretty modest; for example, they do not entail that *B* is probabilistically dependent on *X* or on *Y*. The two assumptions therefore fail to reflect a feature of our simple example. We said that tokens of b are caused by dogs and by foxes-at-a-distance, and it is natural to assume that these causes raise the probability of b's occurring. This modesty is all for the good, however, since we are especially interested in finding theories of content that *fail* to solve the disjunction problem. If there is no probability distribution that satisfies our austere requirements, there is no probability distribution that satisfies a logically stronger set of requirements. We grant, though, that exploring different formulations of the disjunction problem is a good project for the future, and we take one step in that direction in Sect. 5.3. These points about the disjunction problem also apply to our formulation of the distality problem, to which we now turn.

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[11] There are numerous interpretations of probability; see Hájek (2012) for helpful discussion.

[12] An anonymous reviewer objects that ppc theories don't apply to cases where we lack sample frequencies, and that this is true in many cases of perceptual beliefs. The issue, though, isn't whether *we have* the requisite sample data. The issue is whether the requisite probabilities *exist*. If they do, then even if we don't have evidence about their values, ppc theories have application.

[13] Here is Fodor's (1987, p. 102, emphasis original) formulation: "… a viable causal theory of content has to acknowledge *two* kinds of cases where there are disjoint causally sufficient conditions for the tokenings of a symbol: the case where the content of the symbol is disjunctive ('A' expresses the property of *being (A v B)*) and the case where the content of the symbol is *not* disjunctive and some of the tokenings are false ('A' expresses the property of *being A*, and B-caused 'A' tokenings misrepresent)".

[14] It should be clear, then, that we are giving a *necessary* condition, *not* a *sufficient* condition, for the adequacy of a theory of content. The same is true with respect to the distality problem and the hard problem as formulated below.

2.2 The distality problem

Let's return to the causal chain depicted in Fig. 1, from C_d to X to C_p to B, and consider:

(DIST1)   b's content is X, C_p, or C_d.
(DIST2) B's probability is increased by each of C_p, X, and C_d, C_p's probability is increased by each of X and C_d, and X's probability is increased by C_d.
(DIST3) C_p screens-off each of X and C_d from B, and X screens off C_d from each of B and C_p.[15]

These last two propositions are based on the assumption that (a) B is caused by each of C_d, X, and C_p, (b) C_p is caused by each of C_d and X, (c) X is caused by C_d, (d) causes (at least typically) increase the probabilities of their effects, and (e) events often screen-off their causes from their effects.[16]

   We will say that a given ppc theory T can handle the distality problem if and only if there is a belief state b, there are propositions X, C_p, and C_d that are causally related in the way just described, and there is a probability distribution such that (i) (DIST2) and (DIST3) hold and (ii) given the assumption that (DIST1) holds, T outputs the result that b's content is X and isn't C_p or C_d.[17] We propose that an adequate ppc theory of content should be able to handle the distality problem thus understood.

   We mean for the distality problem to be understood so that it resembles but is distinct from "the solipsism problem." The latter says that an adequate theory of meaning must allow for the possibility that organisms have beliefs about things outside their own *minds.* The former goes farther and says that an adequate theory of meaning must allow for the possibility that organisms have beliefs about things outside their own bodies; this happens, for example, when you have beliefs about dogs as opposed to your retinal states. Any theory that can handle the distality problem can handle the solipsism problem, but not vice versa.[18]

2.3 The hard problem

We will say that a given ppc theory T can handle the hard problem if and only if T can handle both the disjunction problem and the distality problem. The hard problem is harder to handle than the disjunction problem and the distality problem taken

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[15] Here we have in mind "no-impact" screening-off: For any P, Q, and R, Q screens-off P from R precisely when $\Pr(R|Q \& P) = \Pr(R|Q)$ and $\Pr(R \mid \sim Q \& P) = \Pr(R \mid \sim Q)$. This kind of screening-off is logically stronger than "positive impact screening-off" and "negative impact screening-off" as formulated in Roche and Shogenji (2014).

[16] We aren't assuming that causality is transitive. We're assuming only that *some* effects are caused both by their causes and by the causes of their causes.

[17] The distality problem so understood resembles what Artiga and Sebastián (forthcoming, Sec. 3.2) call the "wrong distality attribution problem," but the two are different.

[18] Here we assume, contrary to Descartes, that minds have spatial locations – they are inside the bodies of minded individuals.

individually. Why we give the hard problem that moniker will be clear by the end of Sect. 4.

3 A gaggle of ppc theories

This section has seven subsections. In the first four, we set out ppc theories T1–T8. In the fifth, we provide a table of those eight theories, and note an important distinction. In the sixth, we relate T1–T8 to various probabilistic theories in the extant literature. In the seventh, we provide two schemas for constructing hybrids of two or more of T1–T8. The result is a total of 240 additional ppc theories.

3.1 Maximum-probability theories

Consider the following:

> T1: For any b and X, b's content is X if and only if $Pr(X|B) = 1$.
> T2: For any b and X, b's content is X if and only if $Pr(B|X) = 1$.

We call these theories "Maximum-Probability Theories," since each says that whether b's content is X is a matter of whether a given probability has the maximum value of unity. The probability at issue in T1 is the probability of X given B. The probability at issue in T2 is the probability of B given X. Since there can be cases where the one probability equals unity but the other one does not, T1 and T2 are logically distinct.

It might seem that T1 and T2 are too demanding in requiring probabilities of 1, and that they should be relaxed so that the probabilities in question need to be high but don't need to be maximally high. However, note that these relaxed versions of T1 and T2 would be open to the worry that there's no non-arbitrary threshold for high probability. Why, for example, set the bar at 0.95 as opposed to 0.949?[19]

Note too that T1 and T2 can be understood so that the probabilities in question are restricted to special circumstances. Dretske (1981), for example, defends a theory in the neighborhood of T1 on which $Pr(X|B)$ is relativized to a certain "training" period. This allows there to be cases *after* the relevant training period where B is true but X is false.

3.2 Increase-in-probability theories

T1 and T2 contrast with the following theories:

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[19] Note too that what we say about T1 and T2 in relation to the hard problem carries over to T1 and T2 when relaxed so that the probabilities in question need to be high but don't need to be maximally high.

T3: For any b and X, b's content is X if and only if $\Pr(X|B) > \Pr(X)$.

T4: For any b and X, b's content is X if and only if $\Pr(B|X) > \Pr(B)$.

These are "Increase-in-Probability Theories." Although neither T3 nor T4 is equivalent with either of T1 and T2, T3 and T4 are equivalent with each other, since increase in probability is symmetric. We therefore count them as a single theory, which we call "T3/T4."

We don't know of any proponents of T3/T4, although Artiga and Sebastian (forthcoming) discuss it. We mention this theory for completeness, but there's a further reason: even if T3/T4 is implausible on its own, maybe that theory can be used to construct a hybrid theory on which b's content is X if and only if the right side of T3/T4 holds and some additional condition does too. We explore this possibility in Sect. 3.7.

T3/T4 resembles T1 and T2: its right-hand side, like T1's right-hand side and T2's right-hand side, is non-contrastive. To see whether proposition X is the content of b, you don't need to consider an alternative proposition Y. T1, T2, and T3/T4 are in that respect unlike the theories to which we now turn.

3.3 Highest-probability theories

Here are two more theories:

T5: For any b and X, b's content is X if and only if $\Pr(X|B) > \Pr(Y|B)$ for any Y distinct from X.[20]

T6: For any b and X, b's content is X if and only if $\Pr(B|X) > \Pr(B|Y)$ for any Y distinct from X.

These are "Highest-Probability Theories." The right-hand side of T5 says that the probability of X given B is greater than the probability of any other proposition given B. The right-hand side of T6 says that the probability of B given X is greater than the probability of B given any other proposition. There are cases where the right-hand side of the one holds but the right-hand side of the other does not, so T5 and T6, unlike T3 and T4, are logically distinct.

3.4 Highest-degree-of-confirmation theories

T5 and T6 are contrastive analogues of the non-contrastive T1 and T2. The following, in turn, are contrastive analogues of T3 and T4:

T7: For any b and X, b's content is X if and only if $\mathrm{DOC}(X, B) > \mathrm{DOC}(Y, B)$ for any Y distinct from X.

T8: For any b and X, b's content is X if and only if $\mathrm{DOC}(B, X) > \mathrm{DOC}(B, Y)$ for any Y distinct from X.

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[20] Here and throughout by "distinct" we mean "logically independent".

These are "Highest-Degree-of-Confirmation Theories," where, for any propositions E and H, DOC(H, E) is the degree to which E confirms H, where confirmation is a matter of increase in probability. The right-hand side of T7 says that the degree to which B confirms X is greater than the degree to which B confirms any other proposition. The right-hand side of T8 says that the degree to which X confirms B is greater than the degree to which any other proposition confirms B.

How is degree of confirmation to be measured? Several *prima facie* plausible answers to this question have been discussed in the literature.[21] One is that the degree to which E confirms H is a matter of the *difference* between H's probability given E (i.e., H's posterior probability relative to E) and H's prior probability:

$$\mathrm{DOC_{DM}}(H, E) = \Pr(H|E) - \Pr(H)$$

This is the "difference measure" of degree of confirmation. Another *prima facie* plausible answer is that the degree to which E confirms H is a matter of the *ratio* of H's probability given E and H's prior probability:

$$\mathrm{DOC_{RM}}(H, E) = \Pr(H|E) / \Pr(H)$$

This is the "ratio measure" of degree of confirmation. $\mathrm{DOC_{DM}}$ and $\mathrm{DOC_{RM}}$ both meet the following minimal adequacy condition on measures of degree of confirmation:

(*) There is a number n such that DOC(H, E) $>$/$=$/$<$ n if and only if $\Pr(H|E) >$/$=$/$< \Pr(H)$.[22]

Here "n" is the neutral point between confirmation and disconfirmation. For $\mathrm{DOC_{DM}}$, the neutral point n is 0; for $\mathrm{DOC_{RM}}$, the neutral point is 1.

$\mathrm{DOC_{DM}}$ and $\mathrm{DOC_{RM}}$ are not equivalent. $\mathrm{DOC_{RM}}$ is symmetric in that $\mathrm{DOC_{RM}}(H, E) = \mathrm{DOC_{RM}}(E, H)$ in all cases. This isn't true of $\mathrm{DOC_{DM}}$, for $\mathrm{DOC_{DM}}(H, E) \neq \mathrm{DOC_{DM}}(E, H)$ in some cases.

We take no stand here on whether one of $\mathrm{DOC_{DM}}$ and $\mathrm{DOC_{RM}}$ is preferable to the other. It turns out, however, that if T7 and T8 are understood in terms of $\mathrm{DOC_{RM}}$, then T6, T7, and T8 are all logically equivalent to each other. We show this in Appendix 1. So, since T6 is already on the table, and since no two of T6, T7, and T8 are logically equivalent when T7 and T8 are understood in terms of $\mathrm{DOC_{DM}}$, we shall understand T7 and T8 in terms of $\mathrm{DOC_{DM}}$.[23] This choice allows an additional ppc theory to be placed on the table.

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[21] See, e.g., Crupi et al. (2007, 2013), Eells and Fitelson (2002), and Roche and Shogenji (2014).

[22] This is a compressed way of saying that there is a degree n such that (i) DOC(H, E) $>$ n if and only if $\Pr(H|E) > \Pr(H)$, (ii) DOC(H, E) $=$ n if and only if $\Pr(H|E) = \Pr(H)$, and (iii) DOC(H, E) $<$ n if and only if $\Pr(H|E) < \Pr(H)$.

[23] In Sect. 5.3, we address variants of T7 and T8 on which neither $\mathrm{DOC_{DM}}$ nor $\mathrm{DOC_{RM}}$ is assumed.

3.5 *B*-to-*X* theories versus *X*-to-*B* theories

T1–T8 are listed in Table 1. Even though we count T3 and T4 as a single theory, we list them separately in the table so as to highlight an important distinction. T1, T3, T5, and T7 are "*B*-to-*X*" theories in that the right-hand side of each involves a conditional probability that "moves" from B as the conditioning proposition to X as the conditioned proposition. T2, T4, T6, and T8, in contrast, are "*X*-to-*B*" theories in that the right-hand side of each involves a conditional probability that "moves" from X as the conditioning proposition to B as the conditioned proposition. T3 and T4 are logically equivalent to each other, but this isn't true in general when it comes to *B*-to-*X* theories and their *X*-to-*B* counterparts.[24]

3.6 Extant probabilistic theories

T1–T8 are all inspired by extant probabilistic theories (whether or not they are pure). First, T1, T2, and T3/T4 are inspired by Dretske's (1981, 1983) theory. This is a theory on which b's content is X only if $Pr(X|B) = 1 > Pr(X)$.[25] T1 and T2 are like Dretske's in requiring a maximal probability of unity, whereas T3/T4 is like Dretske's in requiring a probability increase. Second, T5 and T6 are inspired by Rupert's (1999) theory, on which $Pr(B|X)$ needs to be greater than $Pr(B|Y)$ for any Y distinct from X, but doesn't need to have the maximal value of unity. Rupert restricts his theory to natural kind concepts, and so, strictly speaking, it isn't identical to T6 (which isn't thus restricted). Even so, T6 is obviously similar to Rupert's, and so is T5 in requiring a highest probability as opposed to a maximum probability. Third, T7 and T8 are inspired by Eliasmith's (2005) and Usher's (2001) theories. These are theories on which $DOC_{RM}(B, X)$ needs to be greater than $DOC_{RM}(B, Y)$ for any Y distinct from X, but doesn't need to clear some absolute threshold.[26] They frame their theories in terms of "information" as opposed to "confirmation," yet T8 is nonetheless similar to their theories, and so is T7 in requiring a highest degree of confirmation as opposed to a degree of confirmation greater than some absolute threshold.

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[24] Our distinction between *B*-to-*X* theories and *X*-to-*B* theories resembles Field's (1990, p. 106) distinction between "head-world reliability" and "world-head reliability." It also corresponds to Nozick's (1981) distinction between "truth-indicating" and "truth-tracking".

[25] Dretske's theory (1981, 1983) additionally requires, roughly, that X be logically stronger than Y for any Y distinct from X such that $Pr(Y|B) = 1 > Pr(Y)$. See Dretske (1981, Ch. 7) on carrying information in "completely digitalized form" for the official requirement.

[26] Consider the following logarithmic variant of DOC_{RM}:

$$DOC_{LRM}(H, E) = \log[Pr(H|E) / Pr(H)]$$

Strictly speaking, Eliasmith (2005) and Usher (2001) frame their theories so that $DOC_{LRM}(B, X)$ needs to be greater than $DOC_{LRM}(B, Y)$ for any Y distinct from X. Since, however, DOC_{RM} and DOC_{LRM} are ordinally equivalent to each other, it follows that $DOC_{RM}(B, X) > DOC_{RM}(B, Y)$ for any Y distinct from X precisely when $DOC_{LRM}(B, X) > DOC_{LRM}(B, Y)$ for any Y distinct from X.

Table 1 A partial taxonomy of ppc theories

Theory types	Theories	
Maximum-probability theories	T1: For any b and X, b's content is X if and only if $\Pr(X|B) = 1$.	T2: For any b and X, b's content is X if and only if $\Pr(B|X) = 1$.
Increase-in-probability theories	T3: For any b and X, b's content is X if and only if $\Pr(X|B) > \Pr(X)$.	T4: For any b and X, b's content is X if and only if $\Pr(B|X) > \Pr(B)$.
Highest-probability theories	T5: For any b and X, b's content is X if and only if $\Pr(X|B) > \Pr(Y|B)$ for any Y distinct from X.	T6: For any b and X, b's content is X if and only if $\Pr(B|X) > \Pr(B|Y)$ for any Y distinct from X.
Highest-degree-of-confirmation theories	T7: For any b and X, b's content is X if and only if $\mathrm{DOC}(X, B) > \mathrm{DOC}(Y, B)$ for any Y distinct from X.	T8: For any b and X, b's content is X if and only if $\mathrm{DOC}(B, X) > \mathrm{DOC}(B, Y)$ for any Y distinct from X.

3.7 Hybrid theories

There are ppc theories additional to T1–T8. Consider, for example, the following:

> T1&T2: For any b and X, b's content is X if and only if (i) $\Pr(X|B) = 1$ and (ii) $\Pr(B|X) = 1$.
> T1∨T2: For any b and X, b's content is X if and only if (i) $\Pr(X|B) = 1$ or (ii) $\Pr(B|X) = 1$.

Each of these theories is based on T1 and T2. The difference is that the right-hand side of T1&T2 is the *conjunction* of T1's and T2's right-hand sides, whereas the right-hand side of T1∨T2 is their *disjunction*.

This is the tip of the iceberg. T1&T2 is but one of 120 instances of the following conjunctive schema (where each theory in question is one of T1–T8):

> Ti&Tj&…&Tn: For any b and X, b's content is X if and only if Ti's right-hand side holds, Tj's right-hand side holds, …, and Tn's right-hand side holds.

Similarly, T1∨T2 is but one of 120 instances of the following disjunctive schema (where each theory in question is one of T1–T8):

> Ti∨Tj∨…∨Tn: For any b and X, b's content is X if and only if Ti's right-hand side holds, Tj's right-hand side holds, …, or Tn's right-hand side holds.

These schemas yield a total of 240 ppc theories in addition to T1–T8. That is a lot.

We will mention additional ppc theories in Sect. 5, but we now have enough theories to get started. There are seven "basic" (non-hybrid) theories (T1, T2, T3/T4, T5, T6, T7, and T8) and 240 hybrids formed from these basics (T1&T2, T1&T3, etc.).

4 How theories T1–T8 and their hybrids fare

T1–T8 are a mixed bag when it comes to the disjunction problem. T1 and T5 fall prey to the the disjunction problem, whereas the remaining theories—T2, T3/T4, T6, T7, and T8—do not. These results are established in Appendix 2. A mix of good news and bad also arises in connection with the distality problem, though here the pattern is different. T1, T2, T3/T4, T6, T7, and T8 all succumb to the distality problem, whereas T5 does not. These results are established in Appendix 3. When it comes to the hard problem, in contrast, T1–T8 are all in the same boat: each of them falls prey to the hard problem. This follows from the fact that each of them falls prey to the disjunction problem or the distality problem.

These results are summarized in Table 2. A "Yes" in a cell indicates that the theory in question can handle the problem in question; a "No" in a cell means that the theory cannot.

What about the two hundred forty hybrids of two or more of T1–T8 noted in Sect. 3.7? It turns out that *none* of them can handle the hard problem either. We show this in Appendix 4. Some of them can handle the disjunction problem, and some can handle the distality problem, but none can handle both.

We are not the first to note that various ppc theories have trouble with distality. For example, consider this passage from Artiga and Sebastián (forthcoming):

> Consider the Fusiform Face Area (FFA), which is usually thought to represent faces…. Suppose we discover that a certain neural network R in the FFA selectively fires with significant intensity when there is a face and also that, given that R is active, the entity that is more likely to be present is a face. One might think these observations suffice for establishing the fact that the brain state represents *face* according to SGIT. Unfortunately, it is unclear that SGIT can deliver this result. Consider, for instance, the set of neuronal structures in the thalamus that are active whenever there is a face in front of the subject. If R has the highest statistical dependence with faces, it will also normally have the highest statistical dependence with these neuronal states in early vision. Thus, SGIT would entail that this activity in FFA represents neuronal activa-

Table 2 Which problems can T1–T8 handle?

	Disjunction problem	Distality problem	Hard problem
T1	No	No	No
T2	Yes	No	No
T3/T4	Yes	No	No
T5	No	Yes	No
T6	Yes	No	No
T7	Yes	No	No
T8	Yes	No	No

tion in another part of the brain. This is of course an extremely counterintuitive result. Indeed, even if there was some principled way of excluding other brain states from being represented, other inadequate contents such as *face-looking thing* could probably not be avoided. (Artiga and Sebastián forthcoming, p. 8, italics original)

Here "SGIT" is short for "Scientifically Guided Informational Theories." T6 is an example of such a theory, and so is T8 when understood in terms of DOC_{RM}.[27] In our terminology, and focusing on T6, Artiga and Sebastián's worry is that T6 outputs the mistaken result that b's content is the proposition C_p, which describes neuronal activation in the brain, and isn't X, which describes a face.

There's an important difference, though, between Artiga and Sebastián's discussion and ours. Consider the the following claims from the passage just quoted:

If R has the highest statistical dependence with faces, it will also normally have the highest statistical dependence with these neuronal states in early vision.
Indeed, even if there was some principled way of excluding other brain states from being represented, other inadequate contents such as *face-looking thing* could probably not be avoided.

These claims are *prima facie* plausible, but Artiga and Sebastián provide no argument in support of them. The claims are simply asserted. In contrast, we prove in Appendix 3 that there are no probability distributions such that (i) (DIST2) and (DIST3) hold and (ii) given the assumption that (DIST1) holds, T6 outputs the result that b's content is X and isn't C_p or C_d.[28]

One final note is in order. The fact that a given theory solves the disjunction problem or the distality problem doesn't mean that there are *realistic* probability distributions—probabilitiy distributions in line with the relevant frequencies in the actual world—of the sort in question. Consider T5, for example, and the fact that it can handle the distality problem. It could be that no realistic probability distribution is such that (i) (DIST2) and (DIST3) hold and (ii) given the assumption that (DIST1) holds, T5 outputs the result that b's content is X and isn't C_p or C_d. Our adequacy

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[27] Why are the theories in question called "informational"? See Artiga and Sebastián (forthcoming, p. 3, n. 4) for an explanation. In Sect. 5.2, we discuss two senses of "mutual information" in the literature, and how they relate to ppc theories and the hard problem.

[28] Artiga and Sebastián consider a theory that we have yet to address. They call it "INFO." It can be put like this:

INFO: For any b and X, b's content is X if and only if (i) $Pr(B|X) > Pr(B|Y)$ for any Y distinct from X and (ii) $Pr(X|B) > Pr(X|B^*)$ for any B^* distinct from B.

This is like T6 except that it also requires that the probability of X given B be greater than the probability of X given any other proposition B^* to the effect that the organism in question has a token of belief type b* at the time in question. It turns out, though, that our proof that T6 falls prey to the distality problem carries over to INFO. The problem is that any probability distribution on which (DIST2) and (DIST3) hold is such that $Pr(B|X) < Pr(B|C_p)$.

conditions are very weak, which is why failing to meet them is a death blow to a theory, whereas meeting them is a minor victory.[29]

5 Three potential responses to the hard problem

It would be premature to give up hope at this point, and conclude that no ppc theory can handle the hard problem. There are potential responses to consider.

5.1 Weaken T1–T8

In Sect. 3.7, we noted 240 hybrid ppc theories that were formed by using two or more of T1–T8. Each of those hybrid theories is like T1–T8 in that it gives a necessary and sufficient condition for b's meaning *X*, and each is like T1–T8 in that it falls prey to the hard problem. What about ppc theories that are like T1–T8 except that they give only a sufficient condition for b's meaning *X* or give just a necessary condition for b's meaning *X*? Let "Ti$_S$" be Ti (for any i = 1, 2, …, 8) when weakened so as to give only a sufficient condition for b's meaning *X*, and let "Ti$_N$" be Ti (for any i = 1, 2, …, 8) when weakened so as to give only a necessary condition for b's meaning *X*. Can any of theses weaker theories handle the hard problem?

The situation is perfectly uniform when it comes to T1$_S$–T8$_S$: none of them can handle the disjunction problem or the distality problem; hence none of them can handle the hard problem. The reason why is straightforward. None of T1$_S$–T8$_S$ gives a necessary condition for b's meaning *X*, and thus none of them can rule out any candidate proposition as b's content. For example, although there might be cases where T5$_S$ outputs the result b's content is *X*, T5$_S$ is unable to output the result b's content is *not* say, $X \lor Y$.[30]

It might seem that the situation is similar with respect to T1$_N$–T8$_N$. For, it might seem that because none of T1$_N$–T8$_N$ gives a sufficient condition for b's meaning *X*, none of them can "rule in" any candidate proposition as b's content. However, consider (DISJ1) and (DIST1). The former says that b's content is *X*, $X \lor Y$, or *Y*, while the latter says that b's content is *X*, C_p, or C_d. Take some case where (DIST2) and

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[29] An anonymous referee suggests, in effect, that if *X* causes different retinal states on different occasions, and each such retinal state always causes *B*, then *X*'s probability given *B* is greater than the probabilities of the various retinal-state descriptions (taken individually) given *B*, and that because of this, b's content is *X* and isn't some retinal-state description. This idea is captured by T5, but we have two comments. First, T5 falls prey to the disjunction problem. Second, even if *X*'s probability given *B* is higher than the probabilities of the various retinal-state descriptions given *B*, it might be that *X*'s probability given *B* is *not* greater than the probability of the *disjunction* of the various retinal-state descriptions. See Roche and Sober (forthcoming, third to last paragraph in section 5) for further discussion.

[30] It might be objected that since b's content can't both be *X* and be $X \lor Y$ (though its content might both entail *X* and entail $X \lor Y$), any case where T5$_S$ outputs the result b's content is *X* is ipso facto a case where T5$_S$ outputs the result b's content isn't $X \lor Y$. The problem, though, is that T5$_S$ itself implies otherwise. For, any case where $Pr(X|B) = 1$ is also a case where $Pr(X \lor Y|B) = 1$.

Table 3 Which problems can $T1_N$–$T8_N$ handle?

	Disjunction problem	Distality problem	Hard problem
$T1_N$	No	No	No
$T2_N$	Yes	No	No
$T3/T4_N$	Yes	No	No
$T5_N$	No	Yes	No
$T6_N$	Yes	No	No
$T7_N$	Yes	No	No
$T8_N$	Yes	No	No

(DIST3) hold, and suppose that one of $T5_N$, for example, rules out each of C_p and C_d as b's content because $\Pr(X|B)$ is greater than each of $\Pr(C_p|B)$ and $\Pr(C_d|B)$. Then given the assumption that (DIST1) holds, it follows by $T5_N$ that b's content is X.

The situation with respect to $T1_N$–$T8_N$ is summarized in Table 3. First, neither $T1_N$ nor $T5_N$ can handle the disjunction problem, but the other theories can. Second, $T5_N$ can handle the distality problem, but none of the other theories can. Third, none of the theories can handle the hard problem. These results are established in Appendix 5.

It's interesting that the pattern for the necessary-condition theories $T1_N$–$T8_N$ (as summarized in Table 3) is *identical* to the pattern for the necessary-and-sufficient-condition theories T1–T8 (as shown in Table 2). We conjecture that the reason for this confluence is to be found in (DISJ1) and (DIST1).

5.2 Move from confirmation to either correlation or mutual information

It's not uncommon for theorists to use the term "correlation" in such a way that a high degree of correlation is just a matter of a high conditional probability. Consider, for example, the following passage from Fodor:

> However, the crude treatment just sketched clearly won't do: it is open to an objection that can be put like this: If there are wild tokenings of *R*, it follows that the nomic dependence of *R* upon *S* is imperfect; some *R*-tokens – the wild ones – are not caused by *S* tokens. Well, but clearly they are caused by *something*; i.e., by something that is, like *S*, sufficient but not necessary for bringing *R*s about. Call this second sort of sufficient condition the tokening of situations of type *T*. Here's the problem: *R* represents the state of affairs with which its tokens are causally correlated. Some representations of type *R* are causally correlated with states of affairs of type *S*; some representations of type *R* are causally correlated with states of affairs of type *T*. So it looks as though what *R* represents is not either *S* or *T*, but rather the disjunction ($S \vee T$): The correlation of *R* with the disjunction is, after all, *better than* its correlation with either of the disjuncts and, ex hypothesi, correlation makes information and information makes representation. If, however, what *R*s represent is not *S* but

($S \lor T$), then tokenings of *R* that are caused by *T aren't, after all, wild token-ings* and our account of misrepresentation has gone West. (Fodor 1984, p. 240, emphasis original)

Dretske (1983, pp. 83–84) also uses "correlation" to refer to a single conditional probability.

Fodor's claims in the above quote make sense, if "high degree of correlation" means high conditional probability. For, switching to our notation, if *X* and *Y* each entails $X \lor Y$ but not vice versa (since *X* and *Y* are mutually exclusive), it follows that $\Pr(X|B) < \Pr(X \lor Y|B)$ and $\Pr(Y|B) < \Pr(X \lor Y|B)$, which means that *B*'s "degree of correlation" with $X \lor Y$ is greater than both its degree of correlation with *X* and its degree of correlation with *Y*.

This usage of "correlation," however, is miles away from standard usage in statistics. Consider:

$$r(H, E) = \frac{\Pr(H \& E) - \Pr(H)\Pr(E)}{\sqrt{\Pr(H)\Pr(\sim H)\Pr(E)\Pr(\sim E)}}$$

This is the Pearson correlation coefficient applied to propositions rather than to quantitative variables.[31] Pearson's $r(H, E)$ has a range of [1, −1], where $r(H, E) > / = / < 0$ if and only if $\Pr(H|E) > / = / < \Pr(H)$. Suppose, for example, that $\Pr(H|E) = 0.990 < 0.999 = \Pr(H)$. Then though $\Pr(H|E)$ is high and thus *E*'s degree of correlation as understood by Fodor is high, $r(H, E)$ is negative and thus it isn't high.

But are there cases where *X* entails $X \lor Y$ but not vice versa, and yet it's not the case that $r(X, B) < r(X \lor Y, B)$? Yes, for there are cases where *X* entails $X \lor Y$ but not vice versa, and yet $\Pr(X|B) > \Pr(X)$ whereas $\Pr(X \lor Y|B) < \Pr(X \lor Y)$.[32] Any such case is a case where $r(X, B) > 0 > r(X \lor Y, B)$.

There are clear respects in which correlation as standardly understood in statistics is similar to confirmation as standardly understood in Bayesian confirmation theory. But, at the same time, there are important differences. Unlike DOC_{DM}, *r* is symmetric in that $r(H, E) = r(E, H)$ in all cases. And unlike DOC_{RM}, *r* is maximal at 1 precisely when *H* and *E* entail each other.[33]

This suggests that the way to solve the hard problem may be to replace Highest-Degree-of-*Confirmation* Theories such as T7 with a Highest-Degree-of-*Correlation* Theory like the following:

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[31] See Kemeny and Oppenheim (1952, p. 314, proof of Theorem 2) for discussion of how to understand correlation in the context of propositions.

[32] We give an example in Appendix 2.

[33] $DOC_{RM}(H, E)$ can be arbitrarily close to 1 (the neutral point for DOC_{RM}) when *H* and *E* entail each other. For, when *H* and *E* entail each other, $DOC_{RM}(H, E) = 1/\Pr(H)$, and this ratio approaches 1 as $\Pr(H)$ approaches 1.

T9: For any b and X, b's content is X if and only if $r(X, B) > r(Y, B)$ for any Y distinct from X.[34]

However, we show in Appendix 6 that T9 falls prey to the distality problem.[35] Hence T9 cannot solve the hard problem.

Perhaps if T9 were modified in terms of some degree-of-correlation measure other than r, the resulting theory would be able to handle the hard problem. We leave that question for the future, and turn now to "mutual information."

Some philosophers use the expression "mutual information" to refer to the logarithm, base 2, of $DOC_{RM}(H, E)$:

$$mi(H, E) = \log \left[\frac{\Pr(H|E)}{\Pr(H)} \right]$$

We noted in Sect. 3.4 that T7 and T8 can be understood in terms of DOC_{RM}, and that if they are so understood, then T6, T7, and T8 are all logically equivalent to each other. The same is true if T7 and T8 are understood in terms of mi. This is because DOC_{RM} and mi are ordinally equivalent in that for any H_1, H_2, E_1, and E_2, $DOC_{RM}(H_1, E_1) > / = / < DOC_{RM}(H_2, E_2)$ if and only if $mi(H_1, E_1) > / = / < mi(H_2, E_2)$; see Sect. 3.6 for relevant background. Hence it won't help to modify T7 and T8 in terms of mi.

However, there's another usage of the expression "mutual information" in the literature. In information theory (Cover and Thomas 2006), the expression "mutual information" is standardly used to refer not to mi, but to a *weighted average of mi*:

$$MI(\Gamma_H, \Gamma_E) = \sum\nolimits_{i,j} \Pr(H_i \& E_j) \log \left[\frac{\Pr(H_i|E_j)}{\Pr(H_i)} \right]$$

Note that whereas H and E are propositions, $\Gamma_H = \{H_1, H_2, \ldots, H_n\}$ and $\Gamma_E = \{E_1, E_2, \ldots, E_m\}$ are *partitions of propositions* (i.e., sets of pairwise mutually exclusive and jointly exhaustive propositions).[36] Can T7 and T8 be modified by using MI, and if so, would this help in terms of handling the hard problem?

It isn't clear how best to modify T7 and T8 by using MI, but we have a suggestion. Consider:

T10: For any b and X, b's content is X if and only if there are partitions $\Gamma_1 = \{B, \sim B\}$ and $\Gamma_2 = \{X, Y_1, \ldots, Y_n\}$ such that *(i)* $MI(\Gamma_1, \Gamma_2) > MI(\Gamma_1, \Gamma_3)$ for any Γ_3 distinct from Γ_2 and (ii) $mi(X, B) > mi(Y_i, B)$ for any Y_i in Γ_2.[37]

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[34] Since r is symmetric, it follows that T9 is logically equivalent to T8 when understood in terms of r.

[35] T9 can handle the disjunction problem, but we won't explain why here.

[36] For discussion of how best to interpret MI, see Roche and Shogenji (2018).

[37] Since mi is symmetric, it follows that the second condition on the right-hand side of T10 is logically equivalent to the condition that $mi(B, X) > mi(B, Y_i)$ for any Y_i in Γ_2.

Think of this as working in two steps. First, *MI* narrows down b's content to the members of a particular partition. Call this "the content partition." Second, *mi* induces a further narrowing, to a particular member of the content partition. For example, suppose that the candidate content partitions are $\Gamma_2 = \{X, Y, Z\}$ and $\Gamma_3 = \{X^*, Y^*, Z^*\}$, that $MI(\Gamma_1, \Gamma_2)$ is greater than $MI(\Gamma_1, \Gamma_3)$, and that $mi(X, B)$ is greater than each of $mi(Y, B)$ and $mi(Z, B)$. Given that $MI(\Gamma_1, \Gamma_2)$ is greater than $MI(\Gamma_1, \Gamma_3)$, it follows that the content partition is Γ_2, and so, as none of X^*, Y^*, and Z^* is a member of that partition, b's content isn't X^*, Y^*, or Z^*. Given that Γ_2 is the content partition, and that $mi(X, B)$ is greater than each of $mi(Y, B)$ and $mi(Z, B)$, it then follows that b's content is X and isn't Y or Z.

What are the candidate content partitions in the disjunction problem and the distality problem? It might seem that one of the candidate content partitions in the disjunction problem should include X, $X \vee Y$, and Y, and that, similarly, one of the candidate content partitions in the distality problem should include X, C_p, and C_d. But note that X, $X \vee Y$, and Y aren't pairwise mutually exclusive, and neither are X, C_p, and C_d, so no set with X, $X \vee Y$, and Y as members or with X, C_p, and C_d as members is a partition. We will assume, instead, that the candidate content partitions in the context of the disjunction problem are $\Gamma_2 = \{X, Y, {\sim}X \& {\sim}Y\}$ and $\Gamma_3 = \{X \vee Y, {\sim}X \& {\sim}Y\}$, and that the candidate content partitions in the context of the distality problem are $\Gamma_2 = \{X, {\sim}X\}$, $\Gamma_3 = \{C_p, {\sim}C_p\}$, and $\Gamma_4 = \{C_d, {\sim}C_d\}$.

We show in Appendix 7 that T10 falls prey to the distality problem and thus can't handle the hard problem.[38] Hence, just as it won't help solve the hard problem to move from confirmation to correlation in the sense of *r*, it also won't help to move from confirmation to mutual information in the sense of *MI*.

5.3 Move to a degree-of-confirmation measure other than DOC_DM

T7 and T8, when understood in terms of $\mathrm{DOC_{DM}}$, fall prey to the distality problem, but what if they are modified in terms of some degree-of-confirmation measure other than $\mathrm{DOC_{DM}}$ (and also other than $\mathrm{DOC_{RM}}$)? Does this allow them to handle the distality problem? If so, does this modification also enable them to handle the hard problem?

Before answering this question, it's important to note here that our proofs in Appendix 3 regarding T7 and T8 go well beyond these theories when understood in terms of $\mathrm{DOC_{DM}}$. Our proof that T7 falls prey to the distality problem generalizes to T7 when understood in terms of any degree-of-confirmation measure such that:

Weak Law of Likelihood: For any E, H_1, and H_2, if (i) $\Pr(E|H_1) > \Pr(E|H_2)$ and (ii) $\Pr(E \mid {\sim}H_1) < \Pr(E \mid {\sim}H_2)$, then $\mathrm{DOC}(H_1, E) > \mathrm{DOC}(H_2, E)$.[39]

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[38] It can be shown that T10, like T9, can handle the disjunction problem, but we won't bother with that here.

[39] For discussion of *Weak Law of Likelihood* and related theses, see Roche and Shogenji (2014).

Further, our proof that T8 falls prey to the distality problem generalizes to T8 when understood in terms of any degree-of-confirmation measure such that:

Final Probability Incrementality: For any E_1, E_2, and H, if $\Pr(H|E_1) > \Pr(H|E_2)$, then $\mathrm{DOC}(H, E_1) > \mathrm{DOC}(H, E_2)$.[40]

It won't help, then, to simply understand T7 and T8 in terms of any old degree-of-confirmation measure that is distinct from $\mathrm{DOC}_{\mathrm{DM}}$.

However, there are degree-of-confirmation measures on which *Weak Law of Likelihood* or *Final Probability Incrementality* does not hold. Here is an example:

$$\mathrm{DOC}_{\mathrm{DM}*}(H, E) = \Pr(H \mid E)\Pr(H \mid E) - \Pr(H)\Pr(H)$$

This is a variant of $\mathrm{DOC}_{\mathrm{DM}}$ on which *Final Probability Incrementality* holds but *Weak Law of Likelihood* does not. Now consider:

T11: For any b and X, b's content is X if and only if $\mathrm{DOC}_{\mathrm{DM}*}(X, B) > \mathrm{DOC}_{\mathrm{DM}*}(Y, B)$ for any Y distinct from X.

It turns out, surprisingly, that T11 *can* handle the hard problem. We show this in Appendix 8.

This does not mean that T11 has any real plausibility in the context of ppc theories. $\mathrm{DOC}_{\mathrm{DM}*}$ is a rather strange-looking measure (to say the least). Why *square X*'s posterior and prior probabilities? Why not instead take them to the third power, or the fourth? Further, there are variants of the disjunction problem and the distality problem to consider. Consider:

(DISJ4) $\Pr(B|X) > \Pr(B)$ and $\Pr(B|Y) > \Pr(B)$.

Let's say that a given ppc theory T can handle the disjunction* problem if and only if there is a probability distribution D such that (i) (DISJ2), (DISJ3), and (DISJ4) hold and (ii) given the assumption that (DISJ1) holds, T outputs the result that b's content is X and isn't $X \vee Y$ or Y. If, as it seems, an adequate theory of content should be able to handle the disjunction* problem thus understood, then T11 is inadequate. For, as we show in Appendix 9, T11 falls prey to the disjunction* problem.

However, the fact that at least one Highest-Degree-of-Confirmation Theory can handle the hard problem perhaps provides some hope for ppc theories.

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[40] *Final Probability Incrementality* is logically weaker than the principle that Crupi et al. (2013) call by the same name.

6 Conclusion

The disjunction problem is a problem for *some but not all* of T1–T8 and their 240 hybrids, and likewise with respect to the distality problem. The hard problem, in contrast, is a problem for *every single one* of T1–T8 and their 240 hybrids (Sect. 4). This generalizes to various weakened versions of T1–T8 (Sect. 5.1), to T7 and T8 both when they are modified in terms of Pearson's correlation measure *r* and when they are modified in terms of mutual information *MI* (Sect. 5.2), to T7 when modified in terms of any degree-of-confirmation measure that, like DOC_{DM}, meets *Weak Law of Likelihood* (Sect. 5.3), and to T8 when modified in terms of any degree-of-confirmation measure that, like DOC_{DM}, meets *Final Probability Incrementality* (Sect. 5.3). The hard problem is *recalcitrant*! However, it doesn't bring down every ppc theory in logical space. T11, for instance, can handle it. We don't claim that T11 is the right theory of semantic content, in part because it succumbs to the strengthened version of the disjunction problem described in Sect. 5.3. Rather, our point is that ppc theories have resources that should be scrutinized further, in connection with the hard problem, and with respect to other conditions of adequacy as well.[41] We note, in this regard, that although we have looked at a largish number of candidate ppc theories, we have used a small handful of formal tools to construct those theories; many of those tools are Bayesian. There are other formal frameworks that are worth exploring.[42]

We close with an analogy. Adaptationism is a research program in evolutionary biology that aims to explain the current traits of organisms in terms of natural selection in ancestral populations. It would be absurd to dismiss this research program on the grounds that adaptationism has so far failed to explain this or that trait in a given biological population. We feel the same way about the development of ppc theories of semantic content. This is a research program, and noting defects in this or that ppc theory hardly suffices to show that the whole program is bankrupt. Philosophers should be just as tenacious as scientists!

Appendices

Some of the claims in the appendices below can be readily verified by using elementary probability theory, but others are more difficult. Some are based on various results in the extant literature; here readers can refer to the cited works for details.

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[41] We remind the reader that what we've been saying about "the disjunction problem" and "the distality problem" has, for most of this paper, really been about highly specific "versions" of each. As noted in Sect. 2, other versions are possible, and exploring them is worthwhile.

[42] Here we draw the reader's attention to Roche and Sober (forthcoming), where the Akaike Information Criterion is used to investigate the epistemology of hypotheses that attribute false beliefs.

Still others were verified or found on *Mathematica* with Fitelson's PrSAT (on which see Fitelson 2008). This is true, for example, of (A2.3.1) in Appendix 2.

Appendix 1: DOC$_{RM}$ and T6, T7, and T8

Bayes's theorem implies that:

$$\frac{\Pr(X|B)}{\Pr(X)} = \frac{\Pr(B|X)}{\Pr(B)} \tag{A1.1}$$

$$\frac{\Pr(Y|B)}{\Pr(Y)} = \frac{\Pr(B|Y)}{\Pr(B)} \tag{A1.2}$$

These equalities imply:

$$\frac{\Pr(X|B)}{\Pr(X)} > \frac{\Pr(Y|B)}{\Pr(Y)} \text{ for any } Y \text{ distinct from } X \text{ iff}$$
$$\frac{\Pr(B|X)}{\Pr(B)} > \frac{\Pr(B|Y)}{\Pr(B)} \text{ for any } Y \text{ distinct from } X \text{ iff} \tag{A1.3}$$
$$\Pr(B|X) > \Pr(B|Y) \text{ for any } Y \text{ distinct from } X.$$

Given this, it follows that if T7 and T8 are understood in terms of DOC$_{RM}$, then the right sides of T6, T7, and T8 are all logically equivalent to each other. Hence, if T7 and T8 are understood in terms of DOC$_{RM}$, then T6, T7, and T8 are all logically equivalent to each other.

Appendix 2: How T1–T8 fare in terms of the disjunction problem

T1 and Disjunction

Any probability distribution on which (DISJ2) and (DISJ3) holds is such that $\Pr(X|B) < 1$. It follows that any such distribution is such that T1 outputs the result that b's content isn't X. Hence T1 falls prey to the disjunction problem.

T5 and Disjunction

Any probability distribution on which (DISJ2) and (DISJ3) hold is such that $\Pr(X|B) < \Pr(X \vee Y|B)$ and $\Pr(Y|B) < \Pr(X \vee Y|B)$. It follows that any such distribution is such that given the assumption that (DISJ1) holds, T5 outputs the result that b's content is $X \vee Y$ and isn't X or Y. Hence T5 falls prey to the disjunction problem.

T2, T3/T4, T6, T7, T8 and Disjunction

Consider the following probability distribution:

X	Y	B	Pr
T	T	T	0
T	T	F	0
T	F	T	$\frac{10}{57}$
T	F	F	0
F	T	T	$\frac{3}{31}$
F	T	F	$\frac{16}{29}$
F	F	T	$\frac{10}{57}$
F	F	F	$\frac{32}{51243}$

It follows on this distribution that

$$1 > \Pr(X \vee Y) \approx 0.824 > \Pr(X \& Y) = 0 \tag{A2.3.1}$$

$$\Pr(B\&X) \approx 0.175 > 0 \text{ and } \Pr(B\&Y) \approx 0.097 > 0 \tag{A2.3.2}$$

Hence both (DISJ2) and (DISJ3) hold. It also follows that:

$$\Pr(B|X) = 1 > \Pr(B|X \vee Y) \approx 0.330 > 0.149 \approx \Pr(B|Y) \tag{A2.3.3}$$

$$\Pr(X|B) \approx 0.392 > 0.175 \approx \Pr(X) \tag{A2.3.4}$$

$$\Pr(X \vee Y|B) \approx 0.608 < 0.824 \approx \Pr(X \vee Y) \tag{A2.3.5}$$

$$\Pr(Y|B) \approx 0.216 < 0.649 \approx \Pr(Y) \tag{A2.3.6}$$

Given (A2.3.3) and the assumption that (DISJ1) holds, each of T2 and T6 outputs the result that b's content is X and isn't $X \vee Y$ or Y. Given (A2.3.4), (A2.3.5), (A2.3.6), and the assumption that (DISJ1) holds, each of T3/T4, T7, and T8 outputs the result that b's content is X and isn't $X \vee Y$ or Y. Hence each of T2, T3/T4, T6, T7, and T8 can handle the disjunction problem.[43]

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[43] There's nothing special about the above probability distribution. There are lots of other probability distributions on which (A2.3.1)–(A2.3.6) all hold, and each of them would have sufficed for our purposes. This point carries over to the probability distribution given below in Appendix 3.

Appendix 3: How T1–T8 fare in terms of the distality problem

T1 and Distality

Any probability distribution on which (DIST2) and (DIST3) hold is such that $\Pr(X|B)=1$ only if $\Pr(C_p|B)=1$. Hence any probability distribution on which (DIST2) and (DIST3) hold is such that T1 outputs the result that b's content is X only if it also outputs the result that b's content is C_p. Hence T1 falls prey to the distality problem.

T2, T6, T8 and Distality

Any probability distribution on which (DIST2) and (DIST3) holds is such that:

$$\Pr(C_p|X) > \Pr(C_p) \tag{A3.2.1}$$

$$\Pr(B|C_p) > \Pr(B) \tag{A3.2.2}$$

$$\Pr(B|C_p\&X) = \Pr(B|C_p) \tag{A3.2.3}$$

$$\Pr(B| \sim C_p\&X) = \Pr(B| \sim C_p) \tag{A3.2.4}$$

(A3.2.3) and (A3.2.4) imply (see Shogenji 2003):

$$\Pr(B|X) - \Pr(B) = \frac{\left[\Pr(C_p|X) - \Pr(C_p)\right]\left[\Pr(B|C_p) - \Pr(B)\right]}{1 - \Pr(C_p)} \tag{A3.2.5}$$

They further imply:

$$\Pr(C_p|X) < 1 \tag{A3.2.6}$$

Given (A3.2.5) and (A3.2.6), and given (A3.2.1) and (A3.2.2), it follows that:

$$\Pr(B|X) - \Pr(B) < \Pr(B|C_p) - \Pr(B) \tag{A3.2.7}$$

This inequality implies:

$$\Pr(B|X) < \Pr(B|C_p) \tag{A3.2.8}$$

By similar reasoning, it can be shown that:

$$\Pr(B|C_d) - \Pr(B) < \Pr(B|C_p) - \Pr(B) \tag{A3.2.9}$$

$$\Pr(B|C_d) < \Pr(B|C_p) \tag{A3.2.10}$$

Given (A3.2.8), T2 outputs the result that b's content isn't *X*. Given (A3.2.8), (A3.2.10), and the assumption that (DIST1) holds, T6 outputs the result that X's content is C_p and isn't *X* or C_d. Given (A3.2.7), (A3.2.9), and the assumption that (DIST1) holds, T8 outputs the result that X's content is C_p and isn't *X* or C_d. Hence each of T2, T6, and T8 falls prey to the distality problem.

T3/T4 and Distality

Any probability distribution on which (DIST2) holds is such that:

$$\Pr(B|C_p) > \Pr(B) \tag{A3.3.1}$$

$$\Pr(B|X) > \Pr(B) \tag{A3.3.2}$$

$$\Pr(B|C_d) > \Pr(B) \tag{A3.3.3}$$

Given (A3.3.2), T3/T4 outputs the result that *b*'s content is *X*. But given (A3.3.1) and (A3.3.3), T3/T4 also outputs both the result that *b*'s content is C_p and the result that *b*'s content is C_d. Hence T3/T4 falls prey to the distality problem.

T5 and Distality

Consider the following probability distribution:

C_d	X	C_p	B	Pr	C_d	X	C_p	B	Pr
T	T	T	T	$\frac{72}{565}$	F	T	T	T	$\frac{51}{2260}$
T	T	T	F	$\frac{96}{113113}$	F	T	T	F	$\frac{17}{113113}$
T	T	F	T	$\frac{1}{113}$	F	T	F	T	$\frac{17}{10848}$
T	T	F	F	$\frac{96}{1243}$	F	T	F	F	$\frac{17}{1243}$
T	F	T	T	0	F	F	T	T	0
T	F	T	F	0	F	F	T	F	0
T	F	F	T	$\frac{1}{67}$	F	F	F	T	$\frac{19395421}{313141920}$
T	F	F	F	$\frac{96}{737}$	F	F	F	F	$\frac{19395521}{35880845}$

It follows on this distribution that:

$$\Pr(B|C_p) \approx 0.993 > 0.237 \approx \Pr(B) \tag{A3.4.1}$$

$$\Pr(B|X) \approx 0.636 > 0.237 \approx \Pr(B) \tag{A3.4.2}$$

$$\Pr(B|C_d) \approx 0.421 > 0.237 \approx \Pr(B) \tag{A3.4.3}$$

$$\Pr(C_p|X) \approx 0.598 > 0.151 \approx \Pr(C_p) \tag{A3.4.4}$$

$$\Pr(C_p|C_d) \approx 0.357 > 0.151 \approx \Pr(C_p) \tag{A3.4.5}$$

$$\Pr(X|C_d) \approx 0.595 > 0.252 \approx \Pr(X) \tag{A3.4.6}$$

$$\Pr(B|C_p\&X) = \Pr(B|C_p) \approx 0.993 \tag{A3.4.7}$$

$$\Pr(B| \sim C_p\&X) = \Pr(B| \sim C_p) \approx 0.103 \tag{A3.4.8}$$

$$\Pr(B|C_p\&C_d) = \Pr(B|C_p) \approx 0.993 \tag{A3.4.9}$$

$$\Pr(B| \sim C_p\&C_d) = \Pr(B| \sim C_p) \approx 0.103 \tag{A3.4.10}$$

$$\Pr(B|X\&C_d) = \Pr(B|X) \approx 0.636 \tag{A3.4.11}$$

$$\Pr(B| \sim X\&C_d) = \Pr(B| \sim X) \approx 0.103 \tag{A3.4.12}$$

$$\Pr(C_p|X\&C_d) = \Pr(C_p|X) \approx 0.598 \tag{A3.4.13}$$

$$\Pr(C_p| \sim X\&C_d) = \Pr(C_p| \sim X) = 0 \tag{A3.4.14}$$

Hence (DIST2) and (DIST3) both hold. It also follows that:

$$\Pr(X|B) \approx 0.676 > 0.637 \approx \Pr(C_d|B) > 0.632 \approx \Pr(C_p|B) \tag{A3.4.15}$$

Given the assumption that (DIST1) holds, T5 outputs the result that b's content is X and isn't C_p or C_d. Hence T5 can handle the distality problem.

T7 and Distality

Any probability distribution on which (DIST2) holds is such that:

$$\Pr(B|C_p) > \Pr(B) \tag{A3.5.1}$$

$$\Pr(C_p|X) > \Pr(C_p) \tag{A3.5.2}$$

These inequalities imply:

$$\Pr(C_p|B) > \Pr(C_p) \tag{A3.5.3}$$

$$\Pr(X|C_p) > \Pr(X) \tag{A3.5.4}$$

Any probability distribution on which (DIST3) holds is such that:

$$\Pr(B|C_p \& X) = \Pr(B|C_p) \tag{A3.5.5}$$

$$\Pr(B|\sim C_p \& X) = \Pr(B|\sim C_p) \tag{A3.5.6}$$

These equalities imply:

$$\Pr(X|C_p \& B) = \Pr(X|C_p) \tag{A3.5.7}$$

$$\Pr(X|\sim C_p \& B) = \Pr(X|\sim C_p) \tag{A3.5.8}$$

(A3.5.3), (A3.5.4), (A3.5.7), and (A3.5.8) together imply (see Roche and Shogenji 2014):

$$\Pr(B|C_p) > \Pr(B|X) \tag{A3.5.9}$$

$$\Pr(B|\sim C_p) < \Pr(B|\sim X) \tag{A3.5.10}$$

(A3.5.9) and (A3.5.10) together imply (see Roche and Shogenji 2014):

$$\Pr(C_p|B) - \Pr(C_p) > \Pr(X|B) - \Pr(X) \tag{A3.5.11}$$

By similar reasoning, it can be shown that:

$$\Pr(C_p|B) - \Pr(C_p) > \Pr(C_d|B) - \Pr(C_d) \tag{A3.5.12}$$

Given the assumption that (DIST1) holds, T7 outputs the result that b's content is C_p and isn't X or C_d. Hence T7 falls prey to the distality problem.

Appendix 4: How hybrid theories fare in terms of the hard problem

Conjunctive theories based on one or more of T2, T6, T7, and T8

We show in Appendix 3 that any probability distribution on which (DIST2) and (DIST3) hold is such that:

$$\Pr(B|X) - \Pr(B) < \Pr(B|C_p) - \Pr(B) \tag{A3.2.7}$$

$$\Pr(B|X) < \Pr(B|C_p) \tag{A3.2.8}$$

$$\Pr(B|C_d) - \Pr(B) < \Pr(B|C_p) - \Pr(B) \tag{A3.2.9}$$

$$\Pr(B|C_d) < \Pr(B|C_p) \tag{A3.2.10}$$

$$\Pr(C_p|B) - \Pr(C_p) > \Pr(X|B) - \Pr(X) \tag{A3.5.11}$$

$$\Pr(C_p|B) - \Pr(C_p) > \Pr(C_d|B) - \Pr(C_d) \tag{A3.5.12}$$

It follows that for any conjunctive theory T based on one or more of T2, T6, T7, and T8, there are no probability distributions such that (i) (DIST2) and (DIST3) hold and (ii) given the assumption that (DIST1) holds, T outputs the result that b's content is X and isn't C_p or C_d. Hence no conjunctive theory based on one or more of T2, T6, T7, and T8 can handle the distality problem. Hence no such theory can handle the hard problem.

Conjunctive theories based on T5

We show in Appendix 2 that any probability distribution on which (DISJ2) and (DISJ3) hold is such that $\Pr(X|B) < \Pr(X \lor Y|B)$ and $\Pr(Y|B) < \Pr(X \lor Y|B)$. It follows that for any conjunctive theory T based on T5, there are no probability distributions such that (i) (DISJ2) and (DISJ3) hold and (ii) given the assumption that (DISJ1) holds, T outputs the result that b's content is X and isn't $X \lor Y$ or Y. Hence no conjunctive theory based on T5 can handle the disjunction problem. Hence no such theory can handle the hard problem.

The T1&T3/T4 Conjunctive Theory

There is only one conjunctive theory left to consider: T1&T3/T4. We note in Appendix 2 that any probability distribution on which (DISJ2) and (DISJ3) hold is such that $\Pr(X|B) < 1$. It follows that there are no probability distributions such that (i) (DISJ2) and (DISJ3) hold and (ii) T1&T3/T4 outputs the result that b's content is X. Hence T1&T3/T4 falls prey to the disjunction problem. Hence it falls prey to the hard problem.

Disjunctive theories based on one or more of T6, T7, and T8

We show in Appendix 3 that any probability distribution on which (DIST2) and (DIST3) hold is such that:

$$\Pr(B|X) - \Pr(B) < \Pr(B|C_p) - \Pr(B) \tag{A3.2.7}$$

$$\Pr(B|X) < \Pr(B|C_p) \tag{A3.2.8}$$

$$\Pr(B|C_d) - \Pr(B) < \Pr(B|C_p) - \Pr(B) \tag{A3.2.9}$$

$$\Pr(B|C_d) < \Pr(B|C_p) \tag{A3.2.10}$$

$$\Pr(C_p|B) - \Pr(C_p) > \Pr(X|B) - \Pr(X) \tag{A3.5.11}$$

$$\Pr(C_p|B) - \Pr(C_p) > \Pr(C_d|B) - \Pr(C_d) \tag{A3.5.12}$$

It follows that for any disjunctive theory T based on one or more of T6, T7, and T8, there are no probability distributions such that (i) (DIST2) and (DIST3) hold and (ii) given the assumption that (DIST1) holds, T outputs the result that b's content isn't C_p. Hence no disjunctive theory based on one or more of T6, T7, and T8 can handle the distality problem. Hence no such theory can handle the hard problem.

Disjunctive theories based on T5

We show in Appendix 2 that any probability distribution on which (DISJ2) and (DISJ3) hold is such that $\Pr(X|B) < \Pr(X \vee Y|B)$ and $\Pr(Y|B) < \Pr(X \vee Y|B)$. It follows that for any disjunctive theory T based on T5, there are no probability distributions such that (i) (DISJ2) and (DISJ3) hold and (ii) T outputs the result that b's content isn't $X \vee Y$. Hence no disjunctive theory based on T5 can handle the disjunction problem. Hence no such theory can handle the hard problem.

Disjunctive theories based on T3/T4

Any probability distribution on which (DIST2) holds is such that $\Pr(B|C_p) > \Pr(B)$. It follows that for any disjunctive theory T based on T3/T4, there are no probability distributions such that (i) (DIST2) and (DISJ3) hold and (ii) T outputs the result that b's content isn't C_p. Hence no such theory can handle the Ditality Problem. Hence no disjunctive theory based on T3/T4 can handle the hard problem.

T1 ∨ T2

There is only one disjunctive theory left to consider: T1 ∨ T2. We note in Appendix 3 that any probability distribution on which (DIST2) and (DIST3) hold is such that:

$$\Pr(X|B) = 1 \text{ only if } \Pr(C_p|B) = 1$$
$$\Pr(B|X) < \Pr(B|C_p)$$

It follows that there are no probability distributions such that (i) (DIST2) and (DISJ3) hold and (ii) T1 ∨ T2 outputs the result that b's content is X and isn't C_p. Hence T1 ∨ T2 falls prey to the distality problem. Hence it falls prey to the hard problem.

Appendix 5: How T1$_N$–T8$_N$ fare in terms of the hard problem

T1$_N$ and the disjunction problem

We note in Appendix 2 that any probability distribution on which (DISJ2) and (DISJ3) holds is such that $\Pr(X|B) < 1$. It follows that any such distribution is such that T1$_N$ outputs the result that b's content isn't X. Hence T1$_N$ falls prey to the disjunction problem.

T2$_N$, T3/T4$_N$, T6$_N$, T7$_N$, and T8$_N$ and the disjunction problem

We show in Appendix 2 section that there are probability distributions on which (DISJ2) and (DISJ3) hold and:

$$\Pr(B|X) = 1 > \Pr(B|X \vee Y) \approx 0.330 > 0.149 \approx \Pr(B|Y) \qquad \text{(A2.3.3)}$$

$$\Pr(X|B) \approx 0.392 > 0.175 \approx \Pr(X) \qquad \text{(A2.3.4)}$$

$$\Pr(X \vee Y|B) \approx 0.608 < 0.824 \approx \Pr(X \vee Y) \qquad \text{(A2.3.5)}$$

$$\Pr(Y|B) \approx 0.216 < 0.649 \approx \Pr(Y) \qquad \text{(A2.3.6)}$$

Given (A2.3.3), and given the assumption that (DISJ1) holds, T2$_N$, T6$_N$, and T8$_N$ all output the result that b's content is X. Hence each of T2$_N$, T6$_N$, and T8$_N$ can handle the disjunction problem. Given (A2.3.4), (A2.3.5), and (A2.3.6), and given the assumption that (DISJ1) holds, T3/T4$_N$ and T7$_N$ both output the result that b's content is X. Hence T3/T4$_N$ and T7$_N$ can handle the disjunction problem.

T5$_N$ and the disjunction problem

We note in Appendix 2 that when (DISJ2) and (DISJ3) hold, it follows that $\Pr(X|B) < \Pr(X \vee Y|B)$. Hence when (DISJ2) and (DISJ3) hold, T5$_N$ rules out X as b's content. Hence T5$_N$ falls prey to the disjunction problem.

T1$_N$ and the distality problem

We note in Appendix 3 that when (DIST2) and (DIST3) hold, it follows that $\Pr(X|B) = 1$ only if $\Pr(C_p|B) = 1$. Hence when (DIST2) and (DIST3) hold, T1$_N$ rules out C_p as b's content only if it also rules out X as b's content. Hence T1$_N$ falls prey to the distality problem.

T2$_N$, T3/T4$_N$, T6$_N$, T7$_N$, and T8$_N$ and the distality problem

We show in Appendix 3 that when (DIST2) and (DIST3) hold, it follows that:

$$\Pr(B|X) < \Pr(B|C_p) \tag{A3.2.8}$$

$$\Pr(B|C_p) > \Pr(B) \tag{A3.3.1}$$

$$\Pr(C_p|B) - \Pr(C_p) > \Pr(X|B) - \Pr(X) \tag{A3.5.11}$$

(A3.2.8) implies that $\Pr(B|X)$ isn't equal to 1, that $\Pr(B|X)$ isn't greater than $\Pr(B|C_p)$, and that $\Pr(B|X) - \Pr(B)$ isn't greater than $\Pr(B|C_p) - \Pr(B)$. Hence when (DIST2) and (DIST3) hold, $T2_N$, $T6_N$, and $T8_N$ all rule out X as b's content. Hence each of $T2_N$, $T6_N$, and $T8_N$ falls prey to the distality problem. Given that (A3.3.1) holds when (DIST2) and (DIST3) hold, it follows that when (DIST2) and (DIST3) hold, $T3/T4_N$ doesn't rule out C_p as b's content. Hence $T3/T4_N$ falls prey to the distality problem. Given that (A3.5.11) holds when (DIST2) and (DIST3) hold, it follows that when (DIST2) and (DIST3) hold, $T7_N$ rules out X as b's content. Hence $T7_N$ falls prey to the distality problem.

$T5_N$ and the distality problem

We show in Appendix 3 that there are probability distributions on which (DIST2) and (DIST3) hold and:

$$\Pr(X|B) \approx 0.676 > 0.637 \approx \Pr(C_d|B) > 0.632 \approx \Pr(C_p|B) \tag{A3.4.15}$$

Given (A3.4.15), $T5_N$ rules out each of C_p and C_d as b's content. Given this, and given the assumption that (DIST1) holds, it follows by $T5_N$ that b's content is X. Hence $T5_N$ can handle the distality problem.

Appendix 6: How T9 fares in terms of the hard problem

First, note that:

$$r(C_p, B) > r(X, B) \text{ iff}$$

$$\frac{\Pr(C_p \& B) - \Pr(C_p)\Pr(B)}{\sqrt{\Pr(C_p)\Pr(\sim C_p)\Pr(B)\Pr(\sim B)}} > \frac{\Pr(X \& B) - \Pr(X)\Pr(B)}{\sqrt{\Pr(X)\Pr(\sim X)\Pr(B)\Pr(\sim B)}} \text{ iff}$$

$$\frac{\left[\Pr(C_p \& B) - \Pr(C_p)\Pr(B)\right]^2}{\Pr(C_p)\Pr(\sim C_p)\Pr(B)\Pr(\sim B)} > \frac{[\Pr(X \& B) - \Pr(X)\Pr(B)]^2}{\Pr(X)\Pr(\sim X)\Pr(B)\Pr(\sim B)} \text{ iff}$$

$$\frac{\left[\Pr(C_p|B) - \Pr(C_p)\right]^2}{\Pr(C_p)\Pr(\sim C_p)} > \frac{[\Pr(X|B) - \Pr(X)]^2}{\Pr(X)\Pr(\sim X)} \text{ iff}$$

$$\left[\frac{\Pr(C_p|B)}{\Pr(C_p)} - 1\right]\left[\frac{\Pr(C_p|B) - \Pr(C_p)}{1 - \Pr(C_p)}\right] > \left[\frac{\Pr(X|B)}{\Pr(X)} - 1\right]\left[\frac{\Pr(X|B) - \Pr(X)}{1 - \Pr(X)}\right] \tag{A6.1}$$

We show in Appendix 3 that any probability distribution on which (DIST2) and (DIST3) hold is such that:

$$\Pr(B|C_p) > \Pr(B|X) \tag{A3.5.9}$$

$$\Pr(B|\sim C_p) < \Pr(B|\sim X) \tag{A3.5.10}$$

These inequalities imply (see Roche and Shogenji 2014):

$$\frac{\Pr(C_p|B)}{\Pr(C_p)} - 1 > \frac{\Pr(X|B)}{\Pr(X)} - 1 \tag{A6.2}$$

$$\frac{\Pr(C_p|B) - \Pr(C_p)}{1 - \Pr(C_p)} > \frac{\Pr(X|B) - \Pr(X)}{1 - \Pr(X)} \tag{A6.3}$$

It follows that any probability distribution on which (DIST2) and (DIST3) hold is such that:

$$r(C_p, B) > r(X, B) \tag{A6.4}$$

By similar reasoning, it can be shown that any probability distribution on which (DIST2) and (DIST3) hold is such that:

$$r(C_p, B) > r(C_d, B) \tag{A6.5}$$

It follows that any probability distribution on which (DIST2) and (DIST3) hold is such that given the assumption that (DIST1) holds, T9 outputs the result that b's content is C_p and isn't X or C_d. Hence T9 falls prey to the distality problem.[44] Hence it falls prey to the hard problem.

Appendix 7: How T10 fares with respect to the hard problem

We noted in Appendix 3 that any probability distribution on which (DIST3) holds is such that:

$$\Pr(B|C_p \& X) = \Pr(B|C_p) \tag{A3.5.5}$$

$$\Pr(B|\sim C_p \& X) = \Pr(B|\sim C_p) \tag{A3.5.6}$$

It follows from these equalities that:

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[44] What about the disjunction problem? The probability distribution given in Appendix 2 is such that (DISJ2) and (DISJ3) hold. It's also such that $r(X, B) \approx 0.512 > -0.510 \approx r(X \vee Y, B)$ and $r(X, B) \approx 0.512 > -0.815 \approx r(Y, B)$. Given the assumption that (DISJ1) holds, T9 outputs the result that b's content is X and isn't $X \vee Y$ or Y. Hence T9 can handle the disjunction problem.

$$\Pr(\sim B|C_p\&X) = \Pr(\sim B|C_p) \tag{A7.1}$$

$$\Pr(\sim B|\sim C_p\&X) = \Pr(\sim B|\sim C_p) \tag{A7.2}$$

$$\Pr(B|C_p\&\sim X) = \Pr(B|C_p) \tag{A7.3}$$

$$\Pr\left(B|\sim C_p\&\sim X\right) = \Pr\left(B|\sim C_p\right) \tag{A7.4}$$

$$\Pr(\sim B|C_p\&\sim X) = \Pr(\sim B|C_p) \tag{A7.5}$$

$$\Pr(\sim B|\sim C_p\&\sim X) = \Pr(\sim B|\sim C_p) \tag{A7.6}$$

This means that each member of $\Gamma_3 = \{C_p, \sim C_p\}$ screens-off each member of $\Gamma_2 = \{X, \sim X\}$ from each member of $\Gamma_1 = \{B, \sim B\}$. Given this, it follows by the so-called "Data Processing Inequality" (Cover and Thomas 2006, Ch. 2) that:

$$MI(\Gamma_1, \Gamma_2 \leq MI(\Gamma_1, \Gamma_3) \tag{A7.7}$$

But then there are no probability distributions such that (i) (DIST2) and (DIST3) hold and (ii) given the assumption that (DIST1) holds, T10 outputs the result that b's content is X and isn't C_p or C_d. Hence T10 falls prey to the distality problem. Hence T10 falls prey to the hard problem.

Appendix 8: How T11 fares with respect to the hard problem

The probability distribution given in Appendix 2 section is such that (DISJ2) holds, (DISJ3) holds, and:

$$\text{DOC}_{\text{DM*}}(X, B) \approx 0.123 > -0.309 \approx \text{DOC}_{\text{DM*}}(X \vee Y, B) \tag{A8.1}$$

$$\text{DOC}_{\text{DM*}}(X, B) \approx 0.123 > -0.374 \approx \text{DOC}_{\text{DM*}}(Y, B) \tag{A8.2}$$

The probability distribution given in Appendix 3 section is such that (DIST2) holds, (DIST3) holds, and:

$$\text{DOC}_{\text{DM*}}(X, B) \approx 0.393 > 0.377 \approx \text{DOC}_{\text{DM*}}(C_p, B) \tag{A8.3}$$

$$\text{DOC}_{\text{DM*}}(X, B) \approx 0.393 > 0.277 \approx \text{DOC}_{\text{DM*}}(C_d, B) \tag{A8.4}$$

Given (A8.1), (A8.2), and the assumption that (DISJ1) holds, T11 outputs the result that b's content is X and isn't $X \vee Y$ or Y. Hence T11 can handle the disjunction problem. Given (A8.3), (A8.4), and the assumption that (DIST1) holds, T11 outputs the result that b's content is X and isn't C_p or C_d. Hence T11 can handle the distality problem. Hence T11 can handle the hard problem.

Appendix 9: How T11 fares with respect to the disjunction* problem

First, note that:

$$
\begin{aligned}
\mathrm{DOC}_{\mathrm{DM*}}(X \vee Y, B) &= [\Pr(X \vee Y|B)]^2 - [\Pr(X \vee Y)]^2 \\
&= [\Pr(X|B) + \Pr(Y|B)]^2 - [\Pr(X) + \Pr(Y)]^2 \\
&= \Pr(X|B)\Pr(X|B) + \Pr(Y|B)\Pr(Y|B) \\
&\quad + (2)\Pr(X|B)\Pr(Y|B) - \Pr(X)\Pr(X) \qquad (A9.1) \\
&\quad - \Pr(Y)\Pr(Y) - (2)\Pr(X)\Pr(Y) \\
&= \mathrm{DOC}_{\mathrm{DM*}}(X, B) + \mathrm{DOC}_{\mathrm{DM*}}(Y, B) \\
&\quad + (2)[\Pr(X|B)\Pr(Y|B) - \Pr(X)\Pr(Y)]
\end{aligned}
$$

Any probability distribution on which (DISJ2), (DISJ3), and (DISJ4) hold is such that $\mathrm{DOC}_{\mathrm{DM*}}(X, B) > 0$, $\mathrm{DOC}_{\mathrm{DM*}}(Y, B) > 0$, and:

$$
(2)[\Pr(X|B)\Pr(Y|B) - \Pr(X)\Pr(Y)] > 0 \qquad (A9.2)
$$

It follows that any probability distribution on which (DISJ2), (DISJ3), and (DISJ4) hold is such that given the assumption that (DISJ1) holds, T11 outputs the result that b's content is $X \vee Y$ and isn't X or Y. Hence T11 falls prey to the disjunction* problem.

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