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Charting the landscape of interpretation, theory rivalry, and underdetermination in quantum mechanics

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Abstract
When we speak about different interpretations of quantum mechanics it is suggested that there is one single quantum theory that can be interpreted in different ways. However, after an explicit characterization of what it is to interpret quantum mechanics, the right diagnosis is that we have a case of predictively equivalent rival theories. I extract some lessons regarding the resulting underdetermination of theory choice. Issues about theoretical identity, theoretical and methodological pluralism, and the prospects for a realist stance towards quantum theory can be properly addressed once we recognize that interpretations of quantum mechanics are rival theories.

Keywords Quantum mechanics · Empirical equivalence · Underdetermination · Pluralism · Theory identity

1 Introduction

The interpretation of quantum mechanics is a central subject in the philosophy of physics. Issues like the measurement problem and the constraints imposed by Bell's theorem, among other queries, constitute important challenges regarding how quantum mechanics must be understood. These interpretive deeds are usually presented, explicitly or implicitly, as a matter of semantics. That is, there is supposed to be a theory—quantum mechanics—whose meaning is far from clear, so physicists and philosophers of physics address the task of making sense of it. This endeavor has resulted in several rival interpretations: Copenhagen, many worlds, modal, consistent histories, etc.

These competing attempts to assign a clear meaning to quantum physics, the story goes, have brought along an underdetermination situation. Since, in general, the different interpretations do not result in predictive divergences, empirical evidence cannot

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be the basis for a choice.[1] In turn, this underdetermination constitutes an obstacle for a realist attitude towards quantum mechanics. In short, in order to solve the issue of the unclear meaning of the theory, several interpretive proposals have been introduced, and the variety of interpretations leads to an underdetermination scenario that constitutes a threat for the quantum realist.

Now, since the question of interpretation is usually understood as a matter of semantics, the underdetermination at stake is thus taken as metaphysical. That is, if the question of interpretation is semantic in the sense explained, the underdetermined choice is not between different theories, but between the different extra-empirical posits that can be attached to one and the same theory. This picture provides some relief: at least we are clear regarding theory choice, and the scope of underdetermination does not go beyond epistemological and ontological heuristic discussions about that theory.[2]

There are diverging opinions, though. Some authors state that at least certain interpretations, especially spontaneous collapse and Bohmian mechanics, are rival theories with respect to standard quantum mechanics (e.g., Healey 1989). Some others take it that the whole scenario of different interpretations involves different quantum theories (e.g., Dieks 2017a). Unfortunately, the grounds for these diverging stances are rather vague, or left unaddressed. Thus, apart from the basic perplexities that prompt the task of interpreting quantum mechanics, further confusion results from the disparate opinions about whether the underdetermination affects the semantics of one theory, or a choice between rival theories.

Hereby I attempt to dissipate this confusion showing that the interpretation of quantum mechanics does not involve different heuristic stances regarding one single theory, but contending theories. This clarification is important for several reasons. First, to draw a detailed cartography of underdetermination in quantum mechanics is an interesting subject in and by itself. Apart from addressing particular issues like the measurement problem and non-locality, we also want an accurate description of the big picture: we want to see both the trees and the forest in the landscape of quantum puzzles. Second, the right cartography allows us to understand why the problem of interpretation in quantum mechanics is substantially different from, and more fundamental than, the interpretation of other theories in physics. Third, with a precise cartography we can appropriately address important related queries. Since the interpretation of quantum mechanics involves a variety of predictively equivalent competing theories, then we have a case of underdetermination of theory choice. In turn, this characterization allows us to tackle the situation in the light of the available conceptual strategies to deal with the general problem of empirical equivalence and underdetermination. That is, issues about theoretical identity, theoretical and methodological pluralism, and the

[1] The spontaneous collapse interpretations (Ghirardi 2016) predictively diverge, and the implementation of suitable experimental tests is an ongoing project. For a review and discussion of possible experimental tests of the divergences (see Bassi et al. 2013).

[2] Alberto Cordero, for example, defends a diagnosis of this type: “the underdetermination at hand is clearly one of *limited scope*. The robust physical and structural commonalities between the competing theories are as numerous as they are widespread over the total explanatory narratives […]. So, although the case makes for an intense ontological debate, its corrosive power on belief seems confined to just some aspects of the full narrative” (Cordero 2001, pp. 307–308).

prospects for the realist, can be better and more clearly addressed if we first realize that we have predictively equivalent rival quantum theories, rather than merely rival interpretations of a single theory.

Whether underdetermination in quantum mechanics concerns the semantics of a single theory or involves rival theories, crucially depends on a clear and precise account of what are we really doing when we interpret quantum mechanics. In Sect. 2 I briefly review three recent such accounts, and I show that according to all of them the outcome we get from interpreting quantum mechanics is a variety of rival theories.

In Sect. 3 I chart the landscape of theory rivalry in more detail. I argue that Bohmian mechanics is also a rival quantum theory, but for different reasons. Bohm presented his theory in terms of an independent formalism that does not need to be interpreted in the sense that the standard formalism does. Furthermore, since, rightly understood, von Neumann's so called "impossibility proof" shows that in deterministic hidden variable theories beables cannot be represented by Hermitian operators, Bohm's theory cannot be reasonably formulated on the template of the standard formalism.

In Sect. 4 I evaluate the resulting scenario. The diagnosis that we have a case of empirical equivalence and underdetermnation of theory choice gives us the opportunity to ponder about related issues, both from the point of view of the general problem and from a more local quantum stance. I draw some lessons regarding criteria for theoretical equivalence and theory rivalry, concerning pluralism in quantum theorizing and methodology, and about the prospects for the quantum realist. In Sect. 5 I summarize and conclude.

2 Interpretation and theory rivalry

We often read and hear about different interpretations of quantum mechanics. However, the question of what are we really doing when we interpret quantum mechanics is seldom addressed. I will review three recent answers to this question, highlighting a lesson we can draw from each of them. Although there is a sense in which the talk about different interpretations of quantum mechanics is justified, this way of speaking shrouds that the result of the interpretive task is a physical theory. In short, what we usually call quantum mechanics is not a physical theory, but its interpretation conveys one.

2.1 Interpreting quantum mechanics

Ruetsche's (2011) account is based on a description of what it is to interpret a physical theory in general. In the first stage of interpretation, "the interpreter characterizes the structures by which the theory would represent physical reality" (Ruetsche 2011, p. 8). In turn, this structure specifying phase involves three sub-steps. First, a description of the theory's state-space. In classical mechanics, for example, the state-space is given by points in phase-space represented by ordered pairs (q, p) of position and momentum values. The second sub-step is a specification of the observables of the theory. In classical mechanics this is done by means of functions from elements in the state-space to $\mathbb{R}$, such as $f(q, p) = p^2/2m$ for kinetic energy. The state-space and the observables conform the kinematics of the theory.[3] The third sub-step in the structure specifying stage is given by a description of the theory's dynamics, i.e., by an account of the evolution of states and observables. In classical mechanics the dynamics is specified by trajectories in phase-space determined by the Hamilton equations and the Hamiltonian function $H(p, q)$.

In Ruetsche's second interpretive stage the semantics of the theory is specified. That is, the models that satisfy the structure are identified, models which in turn describe the worlds that the theory portrays as physically possible. In what she calls a *pristine scenario*, this task can be performed by the nomic content of the structure. That is, the physical models can be directly determined by the laws in the structure, with no need of extra metaphysical-epistemological postulates, or of other external conceptual sources. In short, in a pristine scenario the structure of the theory determines its own interpretation.

Once the semantic stage is done, a physical theory gets interpreted. Ruetsche states that a test to determine whether a structure is interpreted and conveys a theory is given by its (in)ability to provide "an account of which propositions attributing determinate values to magnitudes recognized by the theory are true of a system represented by a state of the theory" (2011, p. 9)—let us call this *Ruetsche's test*.

In the case of quantum mechanics, though, the scenario is far from being pristine. According to Ruetsche, the interpretation can be straightforwardly developed only up to structure specification, but the semantic stage cannot be accomplished solely in nomic terms. The interpretation of quantum mechanics is then a problematic task that must take recourse to conceptual (physical–mathematical, epistemological, metaphysical) elements that are alien to the structure.

Ruetsche dubs "ordinary quantum mechanics" (OQM) to the structure that can be straightforwardly identified—and which all interpretations assume as their common core. OQM is given by the following postulates:

($P1_R$) Observables of a physical system correspond to the Hermitian operators acting on a Hilbert space $\mathcal{H}$.[4,5]

($P2_R$) The possible states of the system are in one-to-one correspondence with the density operators on $\mathcal{H}$. The expectation value of an observable A pertaining to a system in the state W is $\mathrm{Tr}(WA)$.

[3] Ruetsche's use of the term "kinematics" is very broad. It refers not only to spatiotemporal properties and relations, but to all observables, including those that are usually considered as dynamic.

[4] A more general formulation of the observables postulate is given in terms of positive operator valued measurements (POVMs) (see Busch et al. 1995, Chap. 1; Peres 1993, pp. 283–284). Interestingly, POVMs allow that some non-Hermitian operators represent observables as well (see Roberts 2018). However, for the purposes of this article it is simpler to retain the formulation in terms of Hermitian operators—as we will see, one of my arguments below directly refers to the representation of observables by this type of operators.

[5] Von Neumann's view in his axiomatization of quantum theory was that the representation is bijective. However, in the 1950s came out discussions about superselection rules and superselection sectors (see Giulini 2016 for a historical and conceptual overview). Fortunately, this complex issue is not relevant for the arguments presented here.

($P3_R$) With H the Hamiltonian of an isolated system with initial state $W_{(0)}$, and the evolution operator $U = e^{-(i/\hbar)Ht}$, the state of the system at time t is given by $W_{(t)} = UW_{(0)}U^{-1}$.

In this account, $P1_R$ and $P2_R$ configure the kinematics of OQM, whereas $P3_R$ constitutes its dynamics. That is, OQM provides the elements needed in the structure specifying stage. However, its nomic content is not enough to determine models. Although different initial conditions can be introduced in order to solve the Schrödinger equation—so that different possible dynamical evolutions and probabilistic measurement outcomes can be calculated—we do not get physical models yet. This point can be illustrated with Ruetsche's test: OQM is not able to provide an account of which propositions that assign determinate values to the magnitudes in the structure are true of a system represented by a state of the theory. In a word, the structure in OQM underdetermines its own interpretation.

Hence, since the structure in OQM is not enough to determine what worlds are physically possible, it is only a *template* for a theory, and elements must be added and/or amended in order to turn it into one. That is, an interpretation of OQM constitutes a quantum theory, so that rival interpretations constitute rival quantum theories. Ruetsche (2011, Sect. 2.2) states that choices about whether supplementing states with added variables, about introducing collapse-ridden dynamics, about whether a quantum system can possess a determinate magnitude value that its state cannot predict with certainty, about whether quantum reality furnishes one or many worlds, etc., are all choices that result in different quantum theories, all of them built out of OQM, but conveying different classes of physically possible worlds.

Muller (2015) presents a similar account. He states that there is a core formalism, which he dubs minimal quantum mechanics (QM_0), that constitutes "a mathematical recipe to calculate probability distributions over measurement outcomes" (Muller 2015, p. 125). Muller's QM_0 is given by the following postulates[6]:

($P1_M$) Associate a Hilbert space $\mathcal{H}$ to a quantum system S, and a vector $|\psi\rangle \in \mathcal{H}$, or a density operator $W \in \mathcal{H}$, to S at any time t.
($P2_M$) The time evolution of $|\psi\rangle$ is given by $|\psi_t\rangle = U|\psi_0\rangle$, and the time evolution of W is given by $W_{(t)} = UW_{(0)}U^{-1}$, where $U = e^{-(i/\hbar)Ht}$ is the evolution operator, and H is the Hamiltonian operator.
($P3_M$) Observables are represented by Hermitian operators on $\mathcal{H}$.
($P4_M$) Given a system S, the expectation value for an observable represented by the operator A is given by $\langle\psi_t|A|\psi_t\rangle$, or by $\mathrm{Tr}(WA)$.

Considering that QM_0 is unable to provide a clear account of superpositions, for example, Muller states that the physical meaning of the term *state* in the context of this formalism is minute. He claims that a vector or a density operator in Hilbert space should be understood as nothing but a mathematical tool that allows to calculate probability distributions for measurement outcomes. If we further consider that the meaning of a quantum state varies dramatically from interpretation to interpretation (a localized particle and its pilot-wave, the branching structure of the multiverse, systems

6 For brevity and simplicity, I have abridged Muller's more technical formulation of the postulates. He does not include density operators as representatives of systems, nor the trace rule, I have added them.

of beliefs of an agent, etc.), Muller's claim is strengthened. He also makes a similar point concerning *observables*. Considering the unclear status of superpositions, he claims that observables, as represented by Hermitian operators, cannot be understood as *properties*. Thus, in the context of QM_0, the physical meaning of observables is also very weak: they are nothing but mathematical tools to calculate probabilities of measurement outcomes. As we will see below, I share this conclusion, but on the basis of a different argument.

Given the scarce physical meaning of QM_0, Muller draws a conclusion that is similar to Ruetsche's. The formalism does not a constitute a physical theory, for there is

> not a word in QM_0 about physical states, physical properties and physical relations [...]. QM_0 says little if anything about physical reality outside the laboratory, let alone about the microphysical world. This is unacceptable. (Muller 2015, p. 125)

If one includes explanation, understanding, a picture of physical reality, etc., as basic ingredients in a scientific theory, QM_0 is not a theory. Thus, to interpret quantum mechanics is to obtain a quantum theory out of QM_0 by adding postulates, and by enriching and/or changing its physical vocabulary. To undertake this interpretive activity is necessary and urgent because although QM_0 is a powerful device to make predictions, it is unable to accomplish some minimal goals expected from a physical theory.

Yet another similar account can be found in two works by Wallace (2008, 2016). He claims that, contrary to what is normally affirmed in textbooks, the formalism we may call orthodox quantum mechanics must not include the eigenstate-eigenvalue link nor the projection postulate. These two postulates are controversial interpretive principles, and they are not needed in the practical application of the formalism. Thus, Wallace's orthodox quantum mechanics neatly corresponds to Ruetsche's OQM and to Muller's QM_0. Now, regarding the epistemic import of this basic formalism, he coincides in that it does not constitute a physical theory, for orthodox quantum mechanics

> provides a very effective *algorithm* to predict macroscopic phenomena (including the results of measurements which purportedly record microscopic phenomena) but that [...] does not provide a satisfactorily formulated *theory* which explains the success of this algorithm. (2008, pp. 16–17)

From Wallace's formulation and evaluation of orthodox quantum mechanics we can draw the same conclusion once again: to interpret quantum mechanics is an attempt to obtain a physical theory out of the template formalism.[7]

[7] Notice that the added postulates may be of an epistemological or metaphysical nature. That is, it is possible to obtain a quantum theory out of *OQM*-QM_0 without adding any further physical–mathematical postulates, but adding epistemological-metaphysical ones—Bohr's and Everett's interpretations are of this sort, for example. Wallace (2008, p. 21) distinguishes between *pure* interpretations (no extra mathematical formalism apart from the template) and *modificatory* ones (which do add to or amend the mathematical machinery in the template). Now, just like modificatory ones, pure interpretations add to the core formalism in order to obtain a quantum theory—the difference is given by the *type* of postulates they add.

2.2 Theory-rivalry

The reviewed accounts allow us to trace a clear distinction between the task of interpretation in the quantum case and other physical theories. Interpretive issues in special and general relativity, and in statistical mechanics and thermodynamics, for example, are less fundamental in the sense that in those cases there is indeed a theory, but whose conceptual loose ends must be heuristically addressed. Although there remain controversial semantic-epistemic-ontological issues, the nomic structures determine their own interpretations and pass Ruetsche's test. This is not the case in quantum mechanics, so the core formalism must be interpreted in a deeper way: in such a way that it conveys a theory in the first place.

The upshot is that we have a case of several predictively equivalent theories, and a resulting scenario of underdetermination of theory choice. We said above that the talk of different and rival interpretations of quantum mechanics suggests that there is a physical theory—quantum mechanics—that can be interpreted in different ways. If so, the competition between the different interpretations is basically a matter of semantics. We have found that this view is incorrect. The activity of interpreting quantum mechanics consists in building a quantum theory out of the template by adding and/or changing postulates, and these additions and changes usually involve controversial metaphysical and epistemological commitments. In order to underscore that the formal template that Ruetsche, Muller and Wallace identify is uninterpreted, let us dub it *neutral quantum mechanics* (NQM).[8]

The reader may complain that this conclusion (and also Ruetsche's, Muller's, and Wallace's accounts) relies on a certain conception of what a scientific theory must be. This is true. In the three reviewed accounts a requirement for physical theories is assumed: that they provide a description of the world. We do not need, though, to be realists to accept this view: as van Fraassen (1980) states, antirealists/empiricists who reasonably reject the verificationist criterion of significance can certainly take the description of the world offered by a theory literally, and yet not to believe that the theory is true.

QBist (Fuchs et al. 2014) and pragmatist (Healey 2012) interpretations of quantum mechanics assume a different conception of what a physical quantum theory must do: the description of how the quantum world is like is not a requirement that the theory must comply with. In a word, in these interpretations quantum theory is about epistemic-pragmatic propositional attitudes (beliefs of subjects) grounded on the probabilities that the Born rule yields. Anyhow, what is important here is that NQM does not count as a quantum theory for any of these stances either. The uninterpreted formalism does not make reference to propositional attitudes, we can only have a theory like these once suitable extra principles are added. Only then the wavefunction and the Born rule can be understood as referring to epistemic states of subjects (QBists) or agents (pragmatists).

To my mind, the only stance from which NQM can indeed be considered a theory is some form of hard-nosed empiricism—e.g., instrumentalism. If a theory does not

[8] "Uninterpreted" here does not mean "devoid of physical content". NQM is indeed connected, albeit in a loose way, to empirical reality through the Born rule. NQM, though not a theory, is already physics, not just mathematics.

have any extra-empirical meaning, and if it is nothing but a formalism that allows to make empirically testable (statistical) predictions, then NQM does qualify as a physical theory, and from this point of view the very task of interpreting the formalism is unnecessary. However, the seemingly unsurmountable difficulties that the project of reducing the meaning of a theory to its empirical content (e.g., the verificationist criterion of meaning) are well known.[9]

Although I support the standard world-descriptive conception of a physical theory, my present goal is not to defend it or to discard alternative views. What is relevant here is simply that for both the standard conception of physical theories and the alternative epistemic conception of QBists and pragmatists, NQM is not a physical theory. The instrumentalist may have no problem in considering this formalism as a theory, but on the pain of falling prey to several semantic and epistemological problems.[10]

In sum, the situation is more complicated than the usual talk suggests: the right diagnosis is that the rivalry between interpretations of quantum mechanics involves different quantum theories. Now, if we take into account that the different theories do not diverge in their testable predictions, we see that the threat of a central problem in the philosophy of science—empirical equivalence and underdetermination of theory choice—is embroiled in the interpretation of quantum mechanics.[11] Before addressing our scenario from this point of view, we can chart the landscape of theory rivalry in a more detailed way.

3 Bohm's theory

The expression "Bohm's interpretation of quantum mechanics" is usual in the literature. Bohm (1952) himself entitled his two-part seminal paper *A Suggested Interpretation of Quantum Theory in Terms of Hidden Variables*. But phrases like "Bohm's theory" and "Bohmian mechanics" are also common. After the previous section, there seems to be no friction between these two ways of speaking. If different interpretations of quantum mechanics are different and rival theories, these expressions are interchangeable.

There are some diverging voices, though. Healey (1989, p. 24), for example, states that adding further variables to the wavefunction in order to describe the state of a quantum system immediately implies that a different theory other than quantum mechanics is being posed. However, the mere addition of a dynamically evolving

[9] QBists and quantum pragmatists are sometimes accused of being instrumentalists in disguise. The very reference to epistemic states of subjects or agents implies that this accusation is false. The QBist concept of *participatory realism* (Fuchs 2017), and the fact that for the pragmatist the assignment of probabilities to propositions about quantum systems is objectively grounded (Healey 2012), are clear in that these proposals do not amount to an instrumentalist stance. Although they do not require a description of the quantum world from the theory, they nevertheless adopt a *sui generis* realist stance towards quantum mechanics.

[10] The so-called *bare theory* (Albert 1992, pp. 117–125) can be taken as an attempt to get a theory out of the core formalism and nothing else. However, there is general agreement (see, for example, Barrett 1998; Bub et al. 1998; Wallace 2008) that the attempt fails.

[11] Spontaneous collapse proposals are modificatory interpretations of NQM, and therefore quantum theories. However, given their predictive divergence with respect to their rivals, these theories are not subject to underdetermination.

variable to NQM to represent the state of a quantum system is not a good premise to defend the view that Bohm's is an independent theory. Actually, according to the reviewed proposals, the addition of state representing variables constitutes a clear instance of interpreting the quantum template formalism.

In this section I will argue that Bohm's theory should not be taken as an interpretation of NQM, but on a different basis. Bohmian mechanics is presented in terms of a self-standing formalism that does not make essential reference to Hilbert space and that passes Ruetsche's test in nomic terms—thus conveying a physical theory in a pristine way.

3.1 The Bohmian formalism

Reading Bohm's original papers (1952), we find that the theory is not presented on the basis of NQM. Taking the polar expression of the wavefunction $\psi = Re^{iS/\hbar}$, plugging it into the Schrödinger equation, and then separating imaginary and real parts, Bohm obtained the formulas

$$\frac{\partial S}{\partial t} + \sum_{i=1}^{n} \frac{(\nabla_i S)^2}{2m_i} + V - \sum_{i=1}^{n} \left(\frac{\hbar^2}{2m_i}\right) \frac{\nabla_i^2 R}{R} = 0 \qquad (1)$$

$$\frac{\partial R^2}{\partial t} + \sum_{i=1}^{n} \nabla_i \cdot \left(R^2 \frac{\nabla_i S}{m_i}\right) = 0 \qquad (2)$$

He noticed that in the limit $\hbar \to 0$, (1) reduces to the main equation in the Hamilton–Jacobi formulation of classical mechanics, in which the velocities of the particles in an ensemble are given by a guidance equation $\mathbf{v}_i = \nabla S_i/m$. On the other hand, setting $R^2 = P$, Eq. (2) can be written in the following way:

$$\frac{\partial P}{\partial t} + \sum_{i=1}^{n} \nabla_i \cdot (P\mathbf{v}_i) = 0 \qquad (3)$$

Bohm pointed out that in Hamilton–Jacobi mechanics, this equation expresses the conservation of the probability density P for the particles in the ensemble. Thus, when $\hbar \neq 0$, (1) and (3) can be taken as the equations of a quantum theory of motion. Consequently, the guidance equation

$$\mathbf{v}_i = \frac{\nabla S_i}{m} \qquad (4)$$

holds in the quantum case as well.

Furthermore, from (1), a quantum potential

$$U = -\sum_{i=1}^{n} \left(\frac{\hbar^2}{2m_i}\right) \frac{\nabla_i^2 R}{R} \qquad (5)$$

acting on the particles can be read off, which, together with the classical potential V, determines the trajectories. Bohm further stated that the trajectories can be obtained from an equation that is structurally identical to Newton's second law, but that adds a quantum force $-\nabla U$. That is, the trajectories can also be determined by

$$\frac{d\mathbf{p}_i}{dt} = -\nabla_i(V + U) \tag{6}$$

with the condition of restricting the initial momenta according to ∇S_i in (4). Finally, Eq. (3) can also be understood as expressing the conservation of the (assumed) statistical distribution $P = R^2 = |\psi|^2$.

In more recent textbooks (e.g., Holland 1993; Dürr and Teufel 2009), the theory is formulated in terms of the following postulates:

$P1_B$. A system is described by (ψ, Q), where $\psi_{(t)} = Re^{iS/\hbar}$ is the wavefunction in configuration space, $Q_{(t)} = (Q_1, Q_2, \ldots, Q_n)$ is the configuration of the n particles in configuration space, and Q_i is the position of the ith particle.
$P2_B$. The time evolution of ψ is governed by the Schrödinger equation.
$P3_B$. The motion of the particles is governed by the guidance equation $\mathbf{v}_i = \nabla_i S/m$.
$P4_B$. The distribution of the particles in the system is given by $P = R^2 = |\psi|^2$.

At this point we can already see that the formulation of Bohmian mechanics does not result from adding to NQM, but from a simple manipulation of the Schrödinger equation that results in a quasi-Newtonian theory,[12] or from the postulates $P1_B$–$P4_B$. Notice that neither formulation makes reference to Hilbert space, the mathematical structure is defined in configuration space instead. Furthermore, Bohmian mechanics does comply with the epistemic aims that are expected from a theory. In both formulations, the theory saves the phenomena, is not affected by the measurement problem, it provides a concrete description of the quantum world, offers visualizable explanations of quantum phenomena, and so on. In a word, Bohm's theory is presented in terms of an independent formalism (i.e., not on the template of NQM) that does not need to be interpreted in order to convey a physical theory.

This point can be strengthened referring to Ruetsche's account. With respect to the first stage of structure specifying, it is clear that $P1_B$ gives us the state-space, and that $P2_B$ and $P3_B$ give us the dynamics. The specification of the observables (which in Bohm's theory can be understood as properties-beables) requires further comment (see Bohm 1952, p. 172; Holland 1993, Chap. 3). The position $\mathbf{x}_i$ of each particle in a system is given by the configuration $Q_{(t)}$ (not by the position *operator* $\hat{x}$). Concerning the rest of the observables, we can state that the momentum of each particle in the system is given by $\mathbf{p}_i = \nabla_i S$ (not by the momentum *operator* $\hat{p}$), and other properties of the system can be specified as functions of position and momentum (not by the *operators* in NQM). For example, the total energy E is given by Eq. (1).

[12] The theory is quasi-Newtonian in Bohm's original presentation. In later interpretations of the theory, in which there is no pilot wave (the wavefunction is a nomological term) and the second-order equation of motion plays no role (see, for example, Dürr et al. 1996, 1997), the Newtonian flavor is absent. Now, issues about the interpretation of Bohmian mechanics are analogous to issues about interpretation in other physical theories, not analogous to the interpretive issues about *NQM*. For a classification of the different ways to interpret Bohm's theory (see my Acuña 2016).

Let us now consider a single-particle system. If we want to know the determinate value of one of these properties that is possessed by the system at time t, we need to know the particle's position at that instant, but we cannot know that without disturbing the wavefunction. However, we can calculate the expected value of the properties of the system in state ψ using the probability distribution P. Now, since $P = |\psi|^2$, the predicted expected values for the *properties* $\mathbf{x}$, $\mathbf{p}$ and E correspond to the predicted expected values in NQM in terms of the *operators* $\hat{x}$, $\hat{p}$ and H, for:

$$\langle \mathbf{x} \rangle_\psi = \int R^2 \mathbf{x} \mathrm{d}^3\mathrm{x} = \int \psi^* \mathbf{x} \psi \mathrm{d}^3 x = \langle \hat{x} \rangle_\psi$$

$$\langle \mathbf{p} \rangle_\psi = \int R^2 \nabla S \mathrm{d}^3\mathrm{x} = \int \psi^* (-i\hbar\nabla) \psi \mathrm{d}^3 x = \langle \hat{p} \rangle_\psi$$

$$\langle E \rangle_\psi = \int R^2 \left[\frac{(\nabla S)^2}{2m} + U + V \right] \mathrm{d}^3\mathrm{x} = \int \psi^* \left[-\left(\frac{\hbar^2}{2m} \right) \nabla^2 + V \right] \psi \mathrm{d}^3 x = \langle H \rangle_\psi$$

We thus obtain the last ingredient for the kinematics of Bohmian mechanics: the observables, which in this case are plainly beables. Particle positions and momenta are directly determined by the state of the system, and other properties are defined as functions of positions and momentum. Since we cannot simultaneously know ψ and Q, predictions are probabilistic: we calculate expected values using the equilibrium distribution P. However, in spite of this limitation, we have all the elements needed to specify the structure of the theory. On the other hand, since $P = |\psi|^2$, the predictions are the same as in NQM in terms of operators, but in Bohm's theory operators are not representatives of properties.[13] Again, a most relevant point here is that this structure does not rely on the template of NQM.

Now, unlike NQM, such a structure puts us in a pristine scenario, for its nomic content is enough to determine the physical worlds that are possible according to the theory. This can be verified by means of Ruetsche's test: the structure of Bohmian mechanics thus construed certainly provides an account of which propositions attributing determinate values to observables recognized by the theory are true of a system represented by a state of the theory. In this sense, then, the view that Bohm's theory is an interpretation of quantum mechanics is wrong, Bohmian mechanics gets better described as an independent quantum theory.

Notice that Bohm's theory passes the test in terms of beables represented by functions of position and momentum, not by Hermitian operators. Hence, if we wanted to add another postulate to $P1_B$–$P4_B$ to identify the observables, such a postulate

[13] For example, in Bohm's theory ∇S and $\hat{p}$ are neither mathematically nor physically equivalent, but the expected value for the property represented by ∇S calculated in terms of the distribution postulate, and the expected value of a measurement outcome calculated in terms of the operator $\hat{p}$ are numerically equal, as the equations above show. More generally, in Bohm's theory operators are simply bookkeeping devices in measurement outcomes statistics—more about this below. Now, how different regions of the wave function of the measured system "branch out" and get correlated with the eigenvalues of the corresponding operator depends on the specific way in which the measured system and the apparatus interact [this actually all there is to contextuality in Bohm's theory (see Bub 1997, pp. 165–169)]. Thus, in general, "measurements" are perturbative and do not reveal the possessed value of the property before the interaction, so they are better described as *experiments* in Bohm's theory (see Bohm and Hiley 1993, Sect. 6.3).

would not be about Hermitian operators, but, again, about functions of position and momentum.[14] This is not accidental, as we will now see.

3.2 Hidden variables and Hermitian operators

In his seminal *Mathematical Foundations of Quantum Mechanics* (1955), John von Neumann introduced a theorem that was first interpreted as a proof of the impossibility of hidden variable theories. The very existence of Bohm's theory shows that this cannot be correct, of course. In 1966, John Bell showed that von Neumann's theorem is not an impossibility proof, and he further argued that its relevance is rather weak. He stated that the theorem only rules out an uninteresting class of hidden variables theories, for it is supposed to impose a silly constraint on their construction. However, Bub (2010) has recently shown that, rightly understood, von Neumann's theorem establishes that in a viable deterministic hidden variable theory beables cannot be represented by Hermitian operators.

In Section III.1 of his book von Neumann shows that for a state ψ and a quantity $\mathcal{R}$ represented by the Hermitian operator R, the expected value of $\mathcal{R}$ is given by Exp $(\mathcal{R}, \psi) = (R\psi, \psi)$, where $(R\psi, \psi)$ is the inner product between $R\psi$ and ψ.[15] With P_ψ the projector onto ψ, the formula can be written as $\mathrm{Exp}(\mathcal{R}, \psi) = \mathrm{Tr}\big(P_\psi R\big)$. When the system is a mixture of several states $\psi_1, \psi_2, \psi_3, \ldots$, with associated probabilities $\rho_1, \rho_2, \rho_3, \ldots$, the state can be represented by the density operator $W = \sum_i \rho_i P_{\psi_i}$, so we obtain a generalized formula

$$\mathrm{Exp}(\mathcal{R}, W) = \mathrm{Tr}(WR) \tag{7}$$

Given the probabilistic character of Eq. (7), in chapter IV von Neumann ponders if it can be interpreted in classical deterministic way, that is, as a matter of our ignorance. To tackle this issue he introduces two definitions. First, a state W is *dispersion free* if there is no statistical spread in the predictions of measurement outcomes, i.e., such that

$$\big[\mathrm{Exp}(\mathcal{R}, W)\big]^2 = \mathrm{Exp}\Big(\mathcal{R}^2, W\Big)$$

Second, a system is in a *homogeneous* (pure) state W if for any subsystems w_1 and w_2, it holds that

$$\mathrm{Exp}(\mathcal{R}, W) = \mathrm{Exp}(\mathcal{R}, w_1) = \mathrm{Exp}(\mathcal{R}, w_2)$$

By definition, in a deterministic quantum theory that provides an ignorance interpretation of (7), there must be dispersion free states. Furthermore, since from a hidden

[14] Even in a minimalist reading of Bohm's theory in which position is the *only* beable of particles (see Esfeld et al. 2014), the position beable is still given by the configuration of the particles, not by the operator $\hat{x}$ which is merely a bookkeeping device for statistics of experimental outcomes.

[15] In this outline of the theorem, I use von Neumann's own notation.

variables perspective a dispersive state is not completely described, such a system could be divided into dispersion free subsystems. Thus, in a deterministic hidden variables theory a dispersive state cannot be homogeneous.

von Neumann then introduces four "general qualitative assumptions" (1955, p. 295) from which he derives once again Eq. (7):

A′. If the observable $\mathcal{R}$ is by nature non-negative, then $\text{Exp}(\mathcal{R}) \geq 0$.
B′. If $\mathcal{R}, \mathcal{S}, \ldots$ are arbitrary observables and $a, b, \ldots$ are real numbers, then Exp $(a\mathcal{R} + b\mathcal{S} + \cdots) = a\text{Exp}(\mathcal{R}) + b\text{Exp}(\mathcal{S}) + \cdots$.
I. If the observable $\mathcal{R}$ is represented by the Hermitian operator R, then the observable $f(\mathcal{R})$ is represented by the Hermitian operator $f(R)$.
II. If the observables $\mathcal{R}$ and $\mathcal{S}$ are represented by the Hermitian operators R and S, respectively, the observable $\mathcal{R} + S$ is represented by the Hermitian operator $R + S$, regardless of whether R and S commute or not.

I and **II** rely on the principle that observables are represented by Hermitian operators, which is crucial for the relevance of the theorem. Von Neumann explicitly states that "in quantum mechanics [...], the quantities $\mathcal{R}$ correspond one-to-one to the hypermaximal Hermitian operators R" (von Neumann 1955, p. 247).[16] Then, commenting on **II**, he asserts that "this operation depends on the fact that for two Hermitian operators, R, S, the sum $R + S$ is also a Hermitian operator, even if the R, S do not commute" (1955, p. 309).

Using these four assumptions von Neumann derives Eq. (7) (1955, pp. 313–317) and shows that it forbids dispersion free states (320–321), whereas it does admit homogeneous states (321–323). Thus, under assumptions **A′**, **B′**, **I** and **II**, deterministic hidden variable theories are not possible. Since **A′** and **B′** are uncontroversial [**A′** is an obvious requirement, and **B′** is required by the linearity of (7)] von Neumann blames **I** and **II** for the no-go result:

> We have even ascertained that it is impossible that the same physical quantities exist with the same function connections (i.e., that **I**, **II** hold), if other variables (i.e., "hidden parameters") should exist in addition to the wavefunction" (1955, p. 324).[17]

In order to rightly understand the significance of the theorem we must underscore once again that the premises **I** and **II** assume and are justified by the representation

[16] See fn. 5 above.

[17] A quick argument (Bell 1966) to see that von Neumann's assumptions forbid deterministic hidden-variables theories is the following. Let us assume that the values $\lambda \geq n$ of a hidden-variable λ determine dispersion-free states for a fermion. We calculate the expectation value for the beables $\mathcal{S}_x$ and $\mathcal{S}_y$, *represented by the non-commuting operators* σ_x and σ_y, respectively, whose eigenvalues are ± 1. We obtain, say, $Exp(\mathcal{S}_x, \psi_{\lambda \geq n}) = 1$ and $Exp(\mathcal{S}_y, \psi_{\lambda \geq n}) = -1$. Now, from **B'** *and* **II**, for the dispersion-free case it must hold that $Exp(\mathcal{S}_x + \mathcal{S}_y, \psi_{\lambda \geq n}) = Exp(\mathcal{S}_x, \psi_{\lambda \geq n}) + Exp(\mathcal{S}_y, \psi_{\lambda \geq n})$, but the eigenvalues of the operator $\sigma_x + \sigma_y$ (*that represents the beable* $\mathcal{S}_x + \mathcal{S}_y$) are $\pm\sqrt{2}$, so we get a contradiction. More generally, if in a hidden variables theory the beables are represented by Hermitian operators, for dispersion-free states the expected value of a beable is an eigenvalue of the corresponding operator. Hence, ($\mathbf{B}' \wedge \mathbf{II}$) implies that for dispersion-free states the expected value for quantities represented by an operator O which is a sum of two non-commuting operators R and S is equal to a sum of eigenvalues of R and S. However, in general, the eigenvalues of O are not equal to that sum.

of observables by Hermitian operators. Now, since the quantum states represented by density operators W are always dispersive and can be pure, we can conclude, following Bub (2010), that

> what von Neumann's proof precludes, then, is the class of hidden variable theories in which (i) dispersion free (deterministic) states are the extremal states, and (ii) the beables of the hidden variable theory correspond to the physical quantities represented by Hermitian operators of quantum mechanics. (Bub 2010, p. 1340)[18]

Bell (1966) claims that although $\mathbf{B'}$ holds for the dispersive case, for dispersion-free states, when the operators in the linear combination do not commute, additivity is an unjustified assumption that renders the theorem uninteresting. However, he did not notice that the representation of beables in terms of Hermitian operators is crucial for the no-go result, and he thus missed the main relevance of the theorem. In other words, with Bell, we could still render silly a theory in which expectation values for dispersion-free states are additive for observables represented by operators that are a linear combination of non-commuting operators. But he missed that in order to not be silly in this sense, any deterministic hidden variables theory must reject **II** and with it the principle that beables are represented by Hermitian operators.[19]

As I anticipated in the previous section, the beables in Bohm's theory are not represented by Hermitian operators. In Bub's clarification of the relevance of the theorem we find the foundation of this feature, and we can then extend it to any viable deterministic hidden variable theory: in such theories the properties of a system must be represented in a different way, for their representation in terms of Hermitian operators leads to the no-go result. As von Neumann explicitly stated in the quotation above, hidden parameters cannot be added if **I** and **II** hold.[20] In the case of Bohm's theory, we saw that beables are represented by functions of position and momentum, not by operators. Therefore, **I** and **II** do not hold and the no-go result is naturally avoided.

But if there is no representative connection between beables and Hermitian operators, one may ask how can it be that in Bohmian mechanics measurement outcomes respect the expectation value rule Eq. (7). The answer was anticipated in the previous section. We saw that the value for physical quantities possessed by a system are specified by functions of position and momentum, such as Eq. (1) in the case of total energy. But if we consider the equilibrium distribution $P = |\psi|^2$ and the Bohmian account of measurements, the quantum mechanical trace rule can be derived. Thus, rather than a basic postulate of the theory, the quantum mechanical expectation value rule Eq. (7) is a deductive consequence of the structure of Bohmian mechanics, and

[18] Jammer (1974, p. 274, fn. 45) proposes an evaluation of the theorem that is close to Bub's.

[19] From Bell's quick argument (see fn. 17) we saw that in consistent deterministic hidden variables theories it must be the case that $\neg(\mathbf{B'} \wedge \mathbf{II})$, but $\neg\mathbf{B'}$ is incompatible with the trace rule, so avoidance of the no-go result requires that $\neg\mathbf{II}$.

[20] Bub (2010) and Dieks (2017b), state that considering that passage, it seems that von Neumann understood that his theorem is not an impossibility proof of hidden variables theories, but a proof that deterministic hidden variables theories in Hilbert space *which adopt the representation of properties by operators* (*i.e.*, which adopt **I** and **II**) are not possible.

it is not about values of properties possessed by systems. As Dürr & Teufel put it, in Bohm's theory "the operator observables of quantum mechanics are book-keeping devices for […] wavefunction statistics" (2009, p. 228).[21]

3.3 Bohm's theory, not Bohm's interpretation

If we consider Bohm's original formulation, and also its presentation in terms of postulates $P1_B$–$P4_B$, we realize that the theory is not built on the template of NQM. This questions the stance that Bohm's theory is an interpretation of that formalism. This statement gets strengthened when we consider Bub's clarification of von Neumann's theorem. If we wanted to add a postulate specifying the beables of the theory, we would add a postulate about functions of position and momentum, not about Hermitian operators—so that Eq. (7) would be a theorem in the theory, not a basic postulate. Furthermore, neither Bohm's original formulation nor the postulates $P1_B$–$P4_B$ make reference to Hilbert space. In Bohm's theory, vectors, density operators and Hermitian operators in Hilbert space are mere mathematical devices that allow us to calculate statistical expectation values of measurement outcomes, they are not representatives of states or beables. Bohmian states and properties are defined in configuration space by the wavefunction, the configuration of the particles and suitable functions, so there is no reason to include vectors, density operators and Hermitian operators in Hilbert space in its basic formalism.

It could be replied that there is a way in which we can formulate Bohm's theory as an interpretation of NQM. We could retain Muller's $P2_M$–$P4_M$,[22] replace $P1_M$ by a postulate asserting that states of systems are represented by vector or density operators in Hilbert space (specified by the wavefunction in configuration space) and by the configuration of the particles, and then add $P3_B$ [the guidance Eq. (4)] and $P4_B$ (the distribution postulate)—this is actually how Muller presents Bohmian mechanics. But this does not work. Bohm's theory has well-defined beables, but they do not appear in this list of postulates. Furthermore, and oddly, despite the absence of the beables, we find constraints for measurement outcomes among the postulates in terms of Hermitian operators ($P3_M$ and $P4_M$). Using Ruetsche's terminology, with the structure presented in terms of these postulates the observables (beables) are not identified, even though they are well-defined. But then if we opted for adding postulate(s) about the Bohmian beables, then the trace rule (7) in $P4_M$ would be a theorem, not a postulate. Thus, a forced presentation of Bohm's theory on the template of NQM simply does not

[21] A complete treatment of how the trace rule follows from the Bohmian account of measurements and quantum equilibrium can be found in Holland (1993, Chap. 8). For the role that operators play in the statistics of measurement outcomes in the context of Bohmian mechanics—in which the account of observables in terms of POVMs is especially clarifying—see Daumer et al. (1997); Dürr and Teufel (2009, Chap. 12), and, especially Dürr et al. (2004).

[22] After our discussion of von Neumann's theorem, we know that Hermitian operators cannot represent properties in Bohm's theory, so the meaning of $P3_M$ and $P4_M$ should not go beyond a constraint on measurement outcomes (more on this below).

capture its physical meaning. Thus, we conclude once again that Bohm's theory is not an interpretation of NQM, but a theory in its own right.[23]

4 Underdetermination

We can now return to our cartographic task. What we call interpretations of quantum mechanics are actually rival theories. Bohm's proposal is yet another rival theory, but it does not result from an interpretation of the template formalism in Hilbert space. Now, all these theories (excepting spontaneous collapse) are predictively equivalent. This means that in quantum physics we face a case of underdetermination of theory choice. There are several rival quantum theories, but given their predictive equivalence, empirical evidence is not able to determine a choice. As it is well known, empirical equivalence and underdetermination involves a list of problems in the philosophy of science. Thus, the diagnosed situation in quantum physics offers a twofold opportunity. Its analysis can yield important lessons with respect to the issue of empirical equivalence and underdetermination in general, and we can draw important morals about the situation in quantum physics.

4.1 Theoretical equivalence

Theoretical equivalence is an issue that is essentially connected to predictive equivalence. If two theories turn out to be not only empirically, but also theoretically equivalent, they are not rivals but different formulations of a single theory. Logical positivists dispensed with underdetermination in this way. Given the verificationist criterion of meaning, empirical equivalence is a sufficient condition for theoretical equivalence, so the choice that is (unproblematically) underdetermined by the evidence concerns only a particular formulation of the same theory.

The unsurmountable problems that the verificationist criterion faces are well known, so from a post-positivism standpoint further conditions apart from empirical equivalence must be met for theoretical equivalence to be the case. Some philosophers of science state that such conditions can be expressed in terms of formal intertranslability relations between theoretical structures (e.g., Glymour 1970). Their view, roughly, is that theoretical equivalence holds (if and) only if a certain formal-mathematical relation holds between two empirically equivalent theoretical structures. Furthermore, this approach assumes that a universal and unique such criterion can be formulated.

Other authors argue that this approach cannot succeed.[24] For example, Coffey (2014) argues that theoretical equivalence between theories is not a distinctive question, but simply an instance of a more general issue: the interpretation of physical theories. From a post-positivism point of view, the content of a theory goes beyond

[23] Valentini and Westman (2005) have explored the idea of dropping the quantum equilibrium postulate in Bohmian mechanics and then derive it dynamically. By formulating the theory without a quantum equilibrium postulate it would be even clearer that Bohm's theory is not an interpretation of NQM. However, the issue of the foundations of the quantum distribution in Bohm's theory is a contentious one—whether quantum equilibrium is a postulate or not is a controversial subject.

[24] For a general overview of this controversy (see Weatherall 2018).

its empirical consequences, so that in order to determine what a theory says about the world, its theoretical baggage must be interpreted. Hence,

> two theoretical formulations are theoretically equivalent exactly if they say the same thing about what the world is like, where that content goes well beyond their observable or empirical claims. Theoretical equivalence is a function of interpretation. It's a relation between completely interpreted formulations. (Coffey 2014, pp. 834–835)

In this view, theoretical equivalence is an issue that is subordinated to the general question of interpretation of physical theories. Notice that this stance does not imply that formal relations between theoretical structures are irrelevant regarding theoretical equivalence. Rather, the point is that if a certain formal relation is relevant, it is so given an interpretive background.

We can use the situation in quantum mechanics to probe these arguments. There is indeed a formal relation between the interpretations of quantum mechanics that, *prima facie*, seems to be a candidate for a criterion of theoretical equivalence: the Stone-von Neumann theorem. This theorem states that all quantizations of a classical Hamiltonian theory that display the canonical commutation relations (CCRs) are unitarily equivalent.[25] A few definitions and deductive relations, which I take from Ruetsche (2011, pp. 24–30), give us an outline of the theorem's meaning. A Hilbert space $\mathcal{H}$ and a collection of operators $\{O_i\}$ is *unitarily equivalent* to another pair $\left(\mathcal{H}', \left\{O_i'\right\}\right)$ iff there exists a one-to-one, linear, invertible, norm-preserving transformation $U : \mathcal{H} \rightarrow \mathcal{H}'$ such that $U^{-1}O_i'U = O_i$ for all i. Let $\mathcal{M}(\mathcal{H})$ be the set of observables operators (which form an operator algebra) in $\mathcal{H}$, and $\mathcal{S}(\mathcal{H})$ the set of density operators representing states in $\mathcal{H}$, so that $(\mathcal{M}, \mathcal{S})$ determines the kinematics of a quantum theoretical structure. Let P_i and P_i' be the canonical operators that generate the algebras $\mathcal{M}(\mathcal{H})$ and $\mathcal{M}'\left(\mathcal{H}'\right)$, respectively. The Stone-von Neumann theorem establishes that if P_i and P_i' satisfy the CCRs, then $(\mathcal{M}, \mathcal{S})$ and $\left(\mathcal{M}', \mathcal{S}'\right)$ are unitarily equivalent. On the other hand, $(\mathcal{M}, \mathcal{S})$ and $\left(\mathcal{M}', \mathcal{S}'\right)$ are *kinematically equivalent* iff there exists a bijection $i_s : \mathcal{S} \rightarrow \mathcal{S}'$ and an algebraic-structure preserving bijection $i_o : \mathcal{M} \rightarrow \mathcal{M}'$ such that for all density operator $W \in \mathcal{S}$ and for all Hermitian operator $A \in \mathcal{M}$, it holds that $W(A) = [i_s(W)](i_o(A))$. Consider now the dynamics given by a set $\mathcal{D}(\mathcal{H}) = \{d_t\}$ of flows in a state-space, so that $d_t(W)$ is the state into which W evolves according to d_t during time t. We have that $(\mathcal{M}, \mathcal{S}, \mathcal{D})$ and $\left(\mathcal{M}', \mathcal{S}', \mathcal{D}'\right)$ are *dynamically equivalent* iff they are kinematically equivalent and there is a bijection $i_d : \mathcal{D} \rightarrow \mathcal{D}'$, such that for all $W \in \mathcal{S}$, for all $A \in \mathcal{M}$, and for all $d_t \in \mathcal{D}$, it holds that $d_t(W)(A) = i_d(d_t)[i_s(W)](i_o(A))$. Finally, if $(\mathcal{M}, \mathcal{S}, \mathcal{D})$ and $\left(\mathcal{M}', \mathcal{S}', \mathcal{D}'\right)$ are unitarily equivalent, they are kinematically and dynamically equivalent.

Now, since the interpretations of quantum mechanics fall under the scope of this theorem—the CCRs hold in all of them—Ruetsche states that unitary equivalence

[25] For the canonical operators P and Q acting on a Hilbert space, the CCRs are $\left[P_i, P_j\right] = \left[Q_i, Q_j\right] = 0$, $\left[P_i, Q_j\right] = -i\hbar\delta_{ij}I$, where I is the identity operator. In the case of spin quantum systems, the Jordan-Wigner theorem states that unitary equivalence holds if the canonical observables satisfy the canonical anticommutation relations (CARs). For a spin system and canonical operators $\sigma_x, \sigma_y, \sigma_z$ acting on a Hilbert space, the CARs are $\left[\sigma_i, \sigma_j\right] = i\sigma_k$, and $(\sigma_x)^2 = \left(\sigma_y\right)^2 = (\sigma_z)^2 = I$.

looks like a candidate for a criterion according to which all the resulting quantum theories are predictively *and theoretically* equivalent:

> Unitarily equivalent ordinary quantum theories agree about which observables are physical, about which states on those observables are possible, about how those states change in time [...], about which data confirm or falsify the statistical predictions they're capable of making. Their agreement is stalwart enough to suggest that *unitarily equivalent ordinary quantum theories are presumptively physically* [theoretically] *equivalent.* (Ruetsche 2011, p. 29)

This judgment can be further strengthened by the fact that unitary equivalence is actually a way to account for the theoretical equivalence between Heisenberg's matrix mechanics and Schrödinger's wave mechanics.

However, in the case of the different theories that result from interpreting NQM, the theorem does not really work as a test for theoretical equivalence. Let us recall that the interpretive problem with quantum theory is that, in and by itself, NQM is not able to make it past the structure-specifying stage, so interpretation is needed to progress to the semantic stage. Hence,

> they're only "presumptively" equivalent because unitarily equivalent theories subject to different semantics could come out physically [theoretically] inequivalent. [...]. Physical equivalence is properly understood as a relation between *fully interpreted physical theories*. (Ruetsche 2011, p. 29).

That is, the different interpretive revisions or elements added to NQM that are required for the template to make it through the semantic phase convey theories which are theoretically inequivalent, even though they are unitarily equivalent. Thus, in the context of quantum mechanics, Coffey's interpretive stance gets vindicated.

We can also use the case of empirical equivalence and underdetermination in quantum mechanics in order to draw a lesson—also confirming Coffey's view—about the general issue of theoretical equivalence. Ruetsche's analysis of the relevance of the Stone-von Neumann theorem is quite coherent with the stance that theoretical equivalence between empirically equivalent theories is an interpretation-dependent matter. The fact that unitary equivalence is useful to illustrate and justify the theoretical equivalence between matrix and wave mechanics, but not when it comes to the theories that result from interpreting NQM, is a clear indication that theoretical equivalence cannot be reduced to an issue about formal relations.[26]

Finally, the interpretation-dependency stance regarding theoretical equivalence reinforces and clarifies that we have a situation of rivalry between empirically equivalent quantum theories, and a resulting scenario of underdetermination. Given the analysis of what is to interpret quantum mechanics in Sect. 2, we can see that the theoretical inequivalence between the quantum theories is grounded on interpretive issues, so that unitary equivalence does not work as a formal criterion for theoretical equivalence. The crucial point for the theory-rivalry diagnosis is that the template of NQM

[26] One may wonder, though, if matrix and wave mechanics can really count as theories. They may turn out to be simply two unitarily equivalent quantizations unable to make it through the semantics phase. That is, they may be no more than templates for a quantum theory, and the equivalence between them holds only up to the structure-specifying stage.

fails Ruetsche's test: it cannot give us an account of the true propositions assigning determinate values of observables to states. Only when interpretation comes in such an account can be provided and we get a theory. But since the interpretive ingredients determine different and incompatible ways to pass the test, rival theories result. The upshot is that we cannot even talk about the identity of quantum theories before NQM is endowed with a full interpretation.

4.2 Pluralism

The predictive equivalence between quantum theories implies that a choice between them is underdetermined by the evidence. However, agnostics aside, both in physics and philosophy of physics there are several parties which defend one or another of these theories, so it is obvious that some philosophers and physicists do make a choice. Such a choice is based on non-empirical considerations. Pragmatic, epistemic, ontological and metaphysical criteria are usually invoked in order to defend one interpretation or another. These non-empirical factors, though subject to controversy, are not arbitrary, which means that, evidential underdetermination notwithstanding, a rationally supported choice between quantum theories can be made.

The interesting point is that there is a wide variety of different and incompatible but rationally based choices. Although non-empirical virtues can be invoked in order to justify the choice of a particular quantum theory, such virtues cannot determine a universal decision. First, the very task of establishing whether this or that theory scores better with respect to a putative non-empirical virtue is problematic. For example, if conceptual economy is a feature that we are willing to invoke in order to make a choice, it is not clear whether the many-worlds or the pragmatist interpretation is the most economic theory—in Wallace's terminology, they are both pure interpretations of NQM.

Secondly, even we take for granted a clear and precise assessment of how the different quantum theories stand with respect to each and every relevant non-empirical virtue, we still face the problem that there is no universally accepted ranking of theoretical virtues. Some may value conceptual economy, but some others may value a clear ontology, or locality, or determinism, and so on. Different assessments of different non-empirical virtues typically result in different choices of quantum theories. In a word, although non-empirical features may be invoked to make a rationally founded choice, such features are not enough to determine a unique and universal choice (see Acuña and Dieks 2014).

This situation may be evaluated as problematic. However, if we consider the main difficulty at stake—that NQM does not provide us with a description of the quantum world—a pluralist stance seems quite right. If we want a quantum theory with a clear meaning, the committed exploration of different ways to make sense of the template formalism is a reasonable and recommendable approach. In other words, the root problem with quantum mechanics is not that we have many theories, but that the core formalism cannot be assigned a clear meaning. In this sense, the plurality of theories results from looking for a solution to the main problem.

This judgment of a virtuous pluralism gets strengthened when one considers that there is a methodological dimension involved (cf. Belousek 2005). Invoking a Kuhnian mantra, we should remind that the practice of science occurs under a certain theoretical framework that determines the methodology for that practice. Thus, the choice of a quantum theory involves methodological issues concerning how to do science. For example, a specific choice of a quantum theory may have consequences on how issues like the classical limit and the role of decoherence are addressed. Furthermore, this methodological dimension of quantum theory choice may be crucial in the development of future physics. We can find examples in theoretical proposals in quantum gravity. The work by Aguirre and Tegmark in quantum cosmology assumes an Everettian interpretation (e.g., Aguirre and Tegmark 2011), Hartle's approach assumes the consistent histories interpretation (e.g., Hartle 1991), and there is also an active group of researchers working on a Bohmian setup (e.g., Tovar Falciano et al. 2015; Pinto-Neto and Fabris 2013). Of course, there is no way to guarantee that a particular interpretive approach has more chances to succeed than others when it comes to theoretical proposals about challenges like quantum gravity. However, my point is that the plurality of quantum theories offers a wide variety of methodological approaches to deal with those challenges, and if a viable theory is to be found, that all possible paths get explored is a reasonable way for physics to proceed.

In this particular sense thus, the situation of empirical equivalence and underdetermination in quantum physics is not, in and by itself, a problem. Once again, the real trouble is that the template formalism does not constitute a theory. The predictive equivalence scenario is actually the result of looking for a solution to that problem, so, in this quantum context, underdetermination is linked to a potentially fruitful pluralism.

From this assessment of the situation in quantum physics we can also learn a lesson for the general problem of predictive equivalence and underdetermination. That we have predictively equivalent theories is not *tout court* a problem, if the situation involves a plurality of methodological approaches for the development of science—as in quantum physics—there is actually a virtuous side in the situation.

4.3 Realism and explanatory power

Things look grimmer when we consider the prospects of a realist stance towards quantum theory. From a general point of view, empirical equivalence and underdetermination of theory choice is taken as a severe difficulty for the realist (see Psillos 1999, Chap. 8). The rationale for this view is that if there are two or more rival theories that are (dis)confirmed by the same (available and/or possible) evidence, there is no way to pick the true one.

Let us now consider how the quantum realist could escape the underdetermination menace (see Acuña and Dieks 2014; Laudan and Leplin 1991). First, in some cases at least, the empirical equivalence can be broken. We can picture two ways to break it. Theories are not able to predict by themselves, they need to be supplied with initial conditions and auxiliary hypotheses, where the later are usually provided by other theories. Now, we can envision that the development of future science may be such that newly available auxiliary hypotheses may lead to divergent predictions when conjoined with the different theories. Another way to obtain a breakdown of the predictive equivalence can be given by the introduction of new measurement technologies. What is observable and measurable in physical theories is theory-laden and a function of experimental techniques, so that new techniques may expand the observational scope of one theory, but not of others. If the empirical equivalence gets removed in one of these ways, then the basic difficulty for the realist gets removed as well.

The first way to break the predictive equivalence does not look too promising in the case of quantum mechanics. Given unitary equivalence, it is unlikely that new auxiliary hypotheses may lead to diverging predictions. However, the second way is, at least in principle, possible, although restricted to the rivalry between Bohmian mechanics and the rest of the theories. In quantum tunneling experiments all theories can predict an average dwelling time within the barrier. Unlike other quantum theories, though, in Bohmian mechanics the concept of trajectory is well-defined. Thus, the tunneling time of reflected particles and the tunneling time of transmitted particles (which averaged over give the dwelling time) can be discerned and calculated.[27]

Thus, for the Bohmian, different readings of a suitable clock represent the time of flight of reflected and transmitted particles. But for theories in which the notion of trajectory is absent, those readings are physically meaningless, so the clock only reads average dwelling times. Now, if the Bohmian predictions for time of flight for reflected and transmitted particles are correct, the opponent theorists would reasonably reply "so what? Those measurements are meaningless, Bohm's theory is not better confirmed than mine". However, if the Bohmian predictions are wrong, there would be evidence against Bohm's theory that is harmless for the other theories.[28]

Given the state of the art, though, experiments sensitive enough as to measure the reflected and transmitted times are not possible. Furthermore, the described imagined scenario would put only Bohmian mechanics out of the game. Anyhow, what is interesting is that the breakdown of empirical equivalence is at least conceivable. The different theoretical structures in the different quantum theories differ as to what is real. Thus, what is observable (in principle) can vary from one theory to another—average tunneling times gives us an example. It is not inconceivable that with theoretical and experimental ingenuity, plus the development of experimental technology, an evidential tiebreak may result. Actually, the very possibility of this scenario gives some epistemological relief to the realist stance: at least it is not a non-starter when empirical equivalence is the case.

A second way in which realism can find its way is by a breakdown of the evidential tie, despite empirical equivalence. This way to break underdetermination requires non-entailed empirical evidence. At least two forms of this type of evidence can be identified. First, suppose theories T and T' are predictively equivalent. When first formulated, both theories are consistent with the rest of accepted knowledge. Assume now

[27] There are technical issues involved in the calculation of arrival times, but the conceptual distinction in Bohm's theory is actually rather straightforward. See Muga and Leavens (2000) for a comprehensive treatment of this issue. See also Das and Dürr (2019) for a recent proposal of a feasible experimental test of the predictions of arrival times in Bohm's theory.

[28] See Cushing (1995). C.R. Leavens has published extensively on arrival times Bohm's theory, see for example (Leavens and Aers 1993; McKinnon and Leavens 1995; Leavens 1996, 1998).

that later development of science is such that a new theory N—which is incompatible with T' but consistent with T—gets evidentially accepted. The empirical evidence in favor of N is then evidence against T', whereas T remains as confirmed as before the introduction of N (cf. Boyd 1970; Acuña and Dieks 2014). Second, consider T and T' once again. This time a later theory U, which encompasses T (as a special case, for example) but not T', is evidentially accepted. Assume that U is confirmed by observational statements e that cannot be derived from T nor from T'. Hence, assuming the principle that evidence confirming a hypothesis is also evidence for the statements that follow from that hypothesis, e counts as evidence for T, but not for T' (see Laudan and Leplin 1991; Acuña and Dieks 2014).

Coming back to real physics, it is at least conceivable that future development of science may bring a theory like N or like U that breaks the evidential tie between quantum theories. If we recall the potentially fruitful methodological pluralism mentioned above, a scenario of a quantum gravity theory like U, i.e., that encompasses one quantum theory but not the others, is at least possible. Once again, the mere possibility of this situation offers some epistemological relief for the quantum realist.[29]

Apart from the hope for a breakdown of the evidential tie along the lines just described, the realist can also adopt the strategy of focusing her epistemic commitment on the uninterpreted core structure. That is, the realist may take NQM as true, but adopting a quietist or agnostic attitude with respect to the interpretive elements with which the template is turned into different theories. This strategy thus avoids the challenge posed by underdetermination, so it is interesting to assess it as a viable form of quantum realism.

Scientific realism can be described in terms of two theses. First, the realist states that the empirical success of a theory is explained because it latches onto an objective aspect of the world. Second, the realist states that the extra-empirical content of a theory correctly represents unobservable features of the world, so that extra-empirical beliefs motivated by the content of the theory get justified by its empirical success. In short, the realist states *i)* that theories are empirically successful because they are (partially, approximately) true, and *ii)* that the terms in empirically successful theories that denote unobservable entities have an objective reference in the world.

Saatsi (2017) proposes a minimal realism stance towards quantum mechanics. His view is, in a sense, negative. The recommendation for the realist is to adopt a realist attitude, but not to commit to the parts of the theory that fall under the scope of

[29] Some may think this scheme is applicable in the case of relativistic versions of Bohm's theory. The intuition would be that since Bohmian quantum field theory is not Lorentz-covariant it is incompatible with special relativity—so that we could discard the former. But this is too quick. First, it can be argued that the fact that Bohmian field theory is not Lorentz-covariant does not imply an incompatibility with special relativity, even if a preferred foliation that reflects the non-locality of the theory is added to Minkowski spacetime structure (see Maudlin 2008). Besides, Maudlin (2014) has convincingly argued that what Bell's theorem proves is that any empirically viable quantum theory must be non-local, regardless of whether hidden variables are included or not, so that most quantum theories should ultimately add some extra structure to Minkowski spacetime to provide an account of non-locality after all. Secondly, there is not one official Bohmian relativistic field theory, but a variety of different approaches, where some of them are actually Lorentz-covariant at a fundamental level (see, e.g., Dürr et al. 1999, 2014; Lienert et al. 2017).

underdetermination.[30] After the discussion in Sect. 2, we can add a positive dimension to Saatsi's proposal. That is, we can clearly and precisely identify the target of the minimal realist attitude in quantum physics, namely, NQM. Empirical evidence confirms this template formal structure, which is not subject of underdetermination. In this sense, our remarks about the Stone-von Neumann theorem are supportive of the weak realism stance. Since it is a proven fact that all theories observing the CCRs are unitarily equivalent, and therefore kinematically and dynamically equivalent, the realist attitude towards the formal template is firmly based. If it is formally guaranteed that the core formalism is, so to speak, *contained* in all quantum theories, then the underdetermination that affects the latter does not affect the former.[31]

Now, after our discussion in Sect. 3 we must carefully reconsider the physical meaning of the common mathematical structure. As we saw, Bohmian mechanics is a theory that does not result from interpreting NQM, and in which Hermitian operators do not represent the beables. Thus, if we use the Stone-von Neumann theorem in order to identify a mathematical structure that is shared by all quantum theories—including Bohm's—then the physical meaning of the set of Hermitian operators $\mathcal{M}(\mathcal{H})$ and $\mathcal{M}'(\mathcal{H}')$ in our sketch of the theorem above is only that they represent *measurement outcomes, not physical properties of systems*. If we label them "observables", we must use the term in a strictly phenomenological way, not as describing or representing the ontology of quantum systems.[32] Furthermore, something similar holds for the term *system* in NQM: its reference is not specified by the formalism, so *the state of a system* is nothing but a map that assigns values to observables. What type of entity the expression "a system in a certain state" refers to, if any, NQM does not say—this is a blank that interpretations of the template formalism come to fill.

Considering these remarks we can formulate the physical meaning of the shared uninterpreted formalism of NQM in precise terms. In such a structure, states, Hermitian operators and the trace rule refer only to measurement outcomes, and we cannot establish a connection with properties of quantum entities. Consequently, *the physical meaning of NQM is purely phenomenological, not ontological*. Then, from the two

30 The underdetermination Saatsi has in mind seems to be of the metaphysical type, not a result of empirical equivalence between rival theories.

31 The Stone-von Neumann theorem and the Jordan-Wigner theorem guarantee that different quantizations with finite degrees of freedom are unitarily equivalent. Different quantizations with infinite degrees of freedom, which Ruetsche dubs QM_∞—as in quantum field theory and quantum mechanics in the thermodynamical limit—are not necessarily unitarily equivalent. Thus, our minimal realism maneuver does not work for QM_∞. The alternatives that the realist can explore in this context are treated in Ruetsche (2011, 2015).

32 Muller (2015, p. 121) characterizes the meaning of Hermitian operators in *NQM* in a similar way. If the eigenstate-eigenvalue link is added to the template, for a superposed state on the spectral decomposition basis of a Hermitian operator it cannot be said that the system has a definite value for the observable represented by the operator. This is why Muller states that Hermitian operators cannot be taken to represent properties in *NQM*. I think that this is not enough to support the conclusion. We could simply state that Hermitian operators do represent observables of systems (even in the sense of properties) in *NQM*, but that *if we assume the eigenstate-eigenvalue link* states which are superpositions of eigenstates do not have a definite value for the corresponding property. The eigenstate-eigenvalue link does not oblige us to deny that Hermitian operators represent properties in *NQM*. However, if we want to take *NQM* as the mathematical structure that is shared by all quantum theories, then in order to make sense of the meaning of Hermitian operators in Bohm's theory we must indeed interpret them in a strictly phenomenological way. The QBist interpretation requires this restricted and phenomenological conception of Hermitian operators as well.

theses that constitute scientific realism, in Saatsi's proposal only *i)* can be taken on board. Entity realism is certainly challenged by the underdetermination scenario. The uninterpreted core does not introduce a quantum ontology, it only has a phenomenological meaning, so what entities the theory refers to is interpretation-dependent. Hence, entity realism is discarded as a justifiable attitude towards quantum mechanics given the landscape of underdetermination. But if entity realism is unaffordable, how can we give some physical content to minimal quantum realism?

Epistemic structural realism (ESR) is a natural option. ESR was originally introduced by Worrall (1989) as a way to cope with the pessimistic meta-induction argument (Laudan 1981). Empirically successful theories have been replaced by successors that portray radically different ontologies, so realist commitments towards the ontology of the superseded theory get refuted. The inductive part of the argument states that our currently accepted theories are likely to be superseded by theories with different ontologies, so a realist commitment towards the ontology of our currently accepted theories is unjustified. However, Worrall argues, if we take not the ontology, but the mathematical structures of empirically successful theories as the target of the realist attitude, the pessimistic meta-induction is not a threat. Despite radical ontology change, superseded and superseding theories share a mathematical structure: usually the core mathematical structure of the superseded theory is recovered as a special case of the mathematical structure of the superseding one. Thus, a realist attitude towards the conserved mathematical structure is not challenged by the pessimistic argument, and it offers a realist explanation of the success of past and present theories: they succeed because their mathematical structures latch on an objective aspect of the world. As it is clear, this form of realism is about the truth of the mathematical structure of a theory, not about its ontological reference.

We can apply this argument in our case: regardless of the theory that may come to supersede quantum mechanics, the mathematical formalism given by NQM will have to be recovered, one way or another (reduction, limiting case, etc.), by the new theory. But apart from the pessimistic meta-induction—and this is especially relevant here—the structural-realist, following Saatsi's advice, can find motivation in the avoidance of the threat of empirical equivalence and underdetermination. Given unitary equivalence and the Stone-von Neumann theorem, there is a mathematical structure that all quantum theories share. Therefore, if we take this structure as the target of the structural realist attitude, empirical equivalence and underdetermination of theory choice do not erode this form of realism.

As we mentioned, this form of realism is rather weak. First, it has no ontic dimension. The structure about which an underdetermination-free realist attitude can be adopted is not a physical structure, NQM is not about quantum things, it is just about the phenomena. Therefore, ontic structural realism (Ladyman and Ross 2007; French and Ladyman 2011) is not an available stance for our minimal quantum realist: to assign any specific ontic reference to the formal template is an interpretive maneuver that falls prey to underdetermination.[33] Thus, a statement of the quantum realist attitude is simply something like 'the empirical success of any quantum theory is

[33] French and Ladyman (2003) defend ontic structural realism in quantum mechanics referring to issues about (non-)individuality in the context of indistinguishable particles. However, this argument does not work on the basis of the uninterpreted formalism. *NQM* is not even enough to formulate issues about particle

grounded on the (partial, approximate) truth of NQM'. Most remarkably, Worrall himself makes a passing comment about a structural realist stance of this sort:

> The view would simply be that quantum mechanics does seem to have latched on to the real structure of the universe, that all sorts of phenomena exhibited by microsystems really do depend on the system's quantum state, which really does evolve and change in the way quantum mechanics describes. (Worrall 1989, p. 123)

Now, on the bad side, that the uninterpreted formalism has no ontological import seems to imply that we cannot find explanatory power in it. We could refer to the nomological statistical model of explanation (Hempel 1965), but this account implicitly assumes that the nomological aspect grasps causal factors, or at least that the nomic structure provides a pristine interpretation, but this does not hold in the case of the quantum template formalism. If by "physical explanation" we understand an account of quantum phenomena in terms of causal processes and underlying mechanisms, then NQM certainly does not have any explanatory power.

However, there seems to be a promising note for the minimal structural quantum realist to assign some form of explanatory import to the uninterpreted formalism. In 1919, Einstein introduced a distinction between theories of principle and constructive theories. Constructive theories "attempt to build a picture of the more complex phenomena in terms of a relatively simple scheme from which they start out" (Einstein 1954, p. 227), where the scheme refers to a model in terms of basic entities that allows causal and mechanistic explanations. Einstein's paradigm of a constructive theory is statistical mechanics. On the other hand, in theories of principles "the elements which form their basis are […] general characteristics of natural processes, principles that give rise to mathematically formulated criteria which the separate processes or the theoretical representation of them have to satisfy" (*ibid*.). That is, these theories introduce some very general constraints that all physical processes must obey, without making reference to underlying mechanisms. Einstein's paradigmatic example of a theory of principle is thermodynamics.

This distinction can be stated also in explanatory terms. Constructive theories provide bottom-up explanations, whereas theories of principle explain in a top-down fashion. Flores (1999) reformulates Einstein's dichotomy in terms of a functional distinction between *theories of framework* and *interaction theories*. The function of a framework-theory is to constrain other (interaction) theories by means of the general rules it imposes. These theories "provide the framework on which other theories are built" (Flores 1999, p. 129). On the other hand, interaction theories "describe specific physical processes *within* the constraints imposed by the principles […] of a framework theory" (*ibid.*). Flores further states that the type of explanations that theories of framework provide correspond to the unificationist top-down model introduced by Kitcher (1989), whereas the explanations that interaction theories allow correspond to the causal bottom-up model defended by Salmon (1989).

Footnote 33 continued
indistinguishability in a conceptually well-defined way. Furthermore, in Bohm's theory particles are always distinguishable, so the arguments presented by French and Ladyman assume an interpretive stance, thus falling prey to underdetermination. For a critical assessment of French and Ladyman (2003) (see Morganti 2004).

More recently, van Camp (2011) argues that apart from unificationist explanations, theories of framework can also explain insofar as they can "establish the conceptual framework necessary for a theoretic structure with empirical meaning, by providing the preconditions for the understanding and explanation of phenomena that fall under the theory" (van Camp 2011, p. 30). In a word, a theory of framework can provide explanations by constituting, in a loosely Kantian sense, the scaffolding of intelligibility for a realm of phenomena.

My suggestion for the minimal quantum realist—I am not introducing a fully-fledged argument here—is to explore the possibility of taking NQM as a framework in Flores' sense. If this is possible, some explanatory import could be found in it. For example, NQM could be taken as setting up a theoretical framework that constraints several interaction theories about different types of microphysical entities. More concretely, the suggestion would be that theories of nucleons, theories of atoms, theories of condensed matter, etc., are all interaction theories that respect the rules set by NQM. Hence, the uninterpreted formalism could be read as providing explanation in terms of theoretical unification.

However, this suggestion of a framework reading has a limited scope, for NQM is not a fully-fledged framework. As we saw above, it is not able to pass Ruetsche's test, so it cannot provide us with intelligible models of physical phenomena—even when we fill it with interaction theories. Without a clear account of the physical meaning of superpositions, for example, NQM is unable to provide a scaffolding of intelligibility of quantum phenomena, so the second type of explanation that van Camp identifies seems to be beyond the reach of the quantum formalism. Rather than as a *theory* of framework, NQM, could perhaps be taken as a *truncated* theoretical framework.

We end up thus with a very humble stance that realists can adopt. Besides, there still remains a general challenge for structural realism that affects the whole proposal, not only its application to quantum mechanics. If the basic claim is that a physical theory is empirically successful because its mathematical structure latches on some structural objective feature of the world, then the notion of a mathematical structure must be assigned some metaphysical meaning—in the quotation above Worrall states that for the structural realist quantum mechanics latches on to *the real structure of the universe*. Without a metaphysical basis for the structures, epistemic structural realism collapses into constructive empiricism (van Fraassen 1980). Now, if the mathematical structure is literally taken as the structural aspect of the world onto which the theory latches, then a (rather questionable) form of mathematical Platonism results. But if the metaphysical status of the structure is not strictly mathematical, it is unclear what that status can be—the theory itself is not able to answer this question, and we already know from the pessimistic meta-induction that it cannot be given in terms of an ontology of physical objects.[34]

However, even if we are skeptics about the prospects for the quantum structural realist, the maneuver of reading NQM as a framework in Einstein's and Flores' sense, if successful, may allow us to find some unificatory explanatory power in the uninterpreted formalism that the anti-realist could value. The concepts of scientific explanation and scientific understanding are not private property of the realist. Anti-

[34] For a discussion of this issue (see Bueno 2011).

realists that agree in that explanation and understanding have an essential value will be interested in finding quantum explanations and understanding that are not affected by underdetermination. In this sense, conceiving NQM as a framework may yield some epistemic gains for the quantum anti-realist as well.[35]

5 Concluding summary

An explicit characterization of the activity of interpreting quantum mechanics shows that we have several predictively equivalent quantum theories. Most of them arise from adding or amending to the postulates in the basic template formalism of NQM. Bohm's theory, though, does not result from interpreting that template, it constitutes a theory in its own right. This chart of the landscape of quantum theories allows us to extract some lessons. First, theoretical identity, both in general and in the quantum case, is an essentially interpretation-dependent issue, not resolvable solely in terms of formal relations between empirically equivalent theories—such as the Stone-von Neumann theorem in the case of quantum mechanics. Second, the variety of quantum theories involves a positive aspect: theoretical and methodological pluralism may constitute a way to progress in quantum physics, and it may also lead to a way out of the underdetermination scenario. Finally, the Stone-von Neumann theorem allows us to identify an uninterpreted formalism with respect to which an epistemic structural realist attitude, unaffected by underdetermination, may be adopted; and which is also a candidate to be a source of underdetermination-free explanatory power that also anti-realists could value.

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[35] This proposal seems to be coherent with Bueno's (1999, 2011) *structural empiricism*—a sort of structuralist version of van Fraassen's anti-realist constructive empiricism.

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