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Is parthood identity?

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Abstract
According to a well known, yet controversial metaphysical thesis, Composition is Identity. Recently, Kris McDaniel has articulated and defended a related—and arguably more controversial—thesis, one he calls Parthood is Identity (PI). Roughly the view has it that a whole is, strictly and literally, identical to *each of its parts* considered *individually*. At first sight, the view seems rather implausible. However, McDaniel's formulation and defense are worthy of a serious discussion. In this paper I put forth such a discussion. The result is what we should have expected all along: PI is not only implausible, but arguably false.

Keywords Mereology · Identity · Location · Temporary Identity

1 Introduction

According to a well known, yet controversial metaphysical thesis, Composition is Identity (CAI).[1] Roughly, the view has it that a whole is, strictly and literally, identical to its parts considered *collectively*. Recently, Kris McDaniel[2] has articulated and defended a related—and arguably more controversial—thesis, one he calls **Parthood is Identity** (PI). Roughly the view has it that a whole is, strictly and literally, identical to *each of its parts* considered *individually*. At first sight, the view seems rather implausible.[3] However, McDaniel's formulation and defense are worthy of a serious discussion. The present paper provides such a discussion. McDaniel argues that PI (i) offers an analysis of parthood, in that it *explains* its logical behavior, and (ii) is able to withstand crucial objections. I challenge both of these claims. First, I argue that what explains the logic of parthood is *not* PI (Sect. 6 and "Appendix"). This under-

[1] There is in fact a renewed interest in CAI. The interested reader can start from Baxter and Cotnoir (2014) and references therein.
[2] See McDaniel (2014).
[3] McDaniel himself concedes that. See McDaniel (2014: p. 14).

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mines one of the reasons—as a matter of fact, the most signficant reason according to McDaniel—to endorse PI in the first place. Second, I argue that PI faces a formidable objection that McDaniel does not consider. This objection is so forceful as to warrant the rejection of PI. Or so I contend (Sect. 7).

Before plunging into any details, we should recognize that McDaniel's defense of PI is *conditional*. In particular it explicitly depends on four assumptions, which I shall label *endurantism*, *spacetime fundamentality*, *adverbialism*, and *temporary identity*.[4] Here is a rough formulation:

> **Endurantism**. Objects persist by enduring.[5]
> **Spacetime Fundamentality**. Spacetime is a fundamental entity and instants of time and spatial regions—if there are any—are just spacetime regions of a particular sort.
> **Adverbialism**. Enduring objects instantiate properties *at* different spacetime regions. In particular, given adverbialism, an enduring object instantiates-at region r a property P.
> **Temporary Identity**. The relation of numerical identity always obtains relative to an index.

A few comments are in order. First of all, **Adverbialism** is not strictly speaking necessary. That is to say, what is necessary is that spacetime regions *mediate* the instantiation of properties by enduring objects. Adverbialism is not the only choice here. One might plump for **Relativization** instead. To appreciate the difference, consider a relation R that the absolutist[6] will construe as one that is two place, and instantiated by x and y *simpliciter*. In this case, my official notation will be:[7]

$$x R_r y \tag{1}$$

Relativists and Adverbialists will give different readings of (1). Relativists will read (1) as claiming that x, y, r instantiate *simpliciter* the 3-place relation R, whereas adverbialists will read (1) as claiming that x and y instantiate-at-r the 2-place relation R. This difference arguably plays a crucial role in the metaphysics of persistence,[8] but, as far as I can see, it is not crucial for defending PI. I will simply follow McDaniel in endorsing **Adverbialism**. Second, and importantly, given **Spacetime Fundamentalism**, the index mentioned in **Temporary Identity** is reasonably taken to be a spacetime region. This is in fact what McDaniel does. Thus, in line with (1), I will write $x =_r y$ for "x is identical-at-r to y", and $x \sqsubset_r y$ for "x is part-at-r of y". Finally, the somewhat orthodox set-theoretic construction of spacetime is endorsed. Spacetime is "constructed out" of a set of elements called spatiotemporal *points*. Points are supposed to be mere-

[4] See McDaniel (2014, pp. 15–16).

[5] I assume that the reader is familiar with the metaphysics of persistence.

[6] I take *absolutism* to be, roughly, the thesis that objects instantiate properties *absolutely*, i.e. without any mediation of spacetime regions—be this mediation constructed as relativization or adverbial modification. The terminology is due to Sider (2007), who uses it specifically for the parthood relation.

[7] I will use $r_1, \ldots r_n$ as special terms—both variables and constants—for spacetime regions.

[8] It does, in particular, when it comes to endurantist responses to the *argument from temporary intrinsics*.

ologically simple. Spacetime regions are *non empty sets of points*, and *any* non-empty set of points counts as a region.[9]

2 The region-instantiation principle

Enduring objects are required to instantiate properties and relations at spacetime regions. Which regions? In general, for monadic properties what we can extract from McDaniel's discussion is that enduring objects have properties at regions they *occupy*. In effect, the *occupation* (O) relation plays a central role in McDaniel's formulation and defense of PI, as it does in what I will say henceforth. The following principle, the **Region-Instantiation-Principle** or RIP, is meant to capture the requirement at issue:

$$P_r(x) \rightarrow x O r \tag{2}$$

Informally, RIP claims that if the enduring object x has the property P at r, then it occupies r. Note that, in (2), the O relation is given a special treatment, so to speak, insofar as it is not instantiated at regions. This should not be accepted lightheartedly. However, as far as I can see, this is a problem—if at all—for any endurantist who endorses **Spacetime Fundamentality**, and not just the ones that also endorse PI. Thus, I will consider the O relation special in this respect. Accordingly, I will not write it with any subscript.

What about n-place relations? McDaniel focuses on 2-place relations. Arbitrariness considerations lead him to hold that two place relations are instantiated at the *union* of the regions that the *relata* occupy. We can then safely assume the following general RIP-principle:

$$R_{r_1 \cup \ldots \cup r_n}(x_1, \ldots x_n) \rightarrow x_1 + \cdots + x_n O r_1 \cup \ldots \cup r_n \tag{3}$$

where $x_1 + \cdots + x_n$ stands for the mereological fusion of $x_1, \ldots, x_n$ standardly defined.[10] Suppose now that x occupies r_x, y occupies r_y, and $r_x \subset r_y$. Let R be a relation that holds between x and y. We will then have that $x R_{r_x \cup r_y} y \equiv x R_{r_y} y$. The particular case we will be interested in is parthood. McDaniel writes:

> Suppose the relation (…) alluded to above is the relation of parthood. In this case, intuitively r_x must be a subregion of r_y (…) If r_x is a subregion of r_y, then the union of r_x and r_y just is r_y (…) In general, every region at which something is part of a whole is a region occupied by that whole (McDaniel 2014: 20, notations changed).

9 As a matter of fact, set-theory will play a crucial role. In view of this, I will use set-theoretic notions right from the start.

10 McDaniel argues that PI does not entail mereological universalism, so we are not guaranteed that such a fusion exists. It would be safer to replace (3) with the following: $R_{r_1 \cup \ldots \cup r_n}(x_1, \ldots x_n) \rightarrow x_1 O r_1 \wedge \ldots x_n O r_n$. This difference will not play any role in what follows.

3 Formal reformulation

Informally, PI is the view according to which:

> (i) a whole is identical to *each* of its parts considered *individually* rather than collectively, and (ii) parthood can be analyzed in terms of identity (McDaniel 2014: 14).

As for (i), McDaniel promptly adds:

> More generally, whenever x is part of y at some r, x is identical with y at r (McDaniel 2014: 21, notations changed).

For us, this translates into:

$$x \sqsubset_r y \rightarrow x =_r y \tag{4}$$

As for (ii), McDaniel considers different possibilities and then (almost)[11] settles for the following:[12]

> x is, at r, a part of $y =_{df}$ x is, at r, identical with y and there is some region r' such that (i) x is, at r', identical with x; (ii) x is not, at r', identical with y; and (iii) r' is a proper subregion of r (McDaniel 2014: 21, notations changed).

That is:

$$\begin{aligned} x \sqsubset_r y \equiv_{df} x =_r y \wedge \\ \exists r'(x =_{r'} x \wedge x \neq_{r'} y \wedge r' \subset r) \end{aligned} \tag{5}$$

A few remarks are in order. First, (4) follows from (5). Second, in line with the set-theoretic construction of spacetime, I have cashed out the *proper subregion* relation in terms of *proper subsethood*. This does not betray McDaniel's proposal. As a matter of fact, he would endorse this set-theoretic rendition himself.[13] Finally, according to McDaniel, (5) provides an *analysis* of parthood, and this is a considerable theoretical gain. It is so significant in fact, as to warrant a serious consideration of PI.

Before moving on to the arguments in favor of PI, we should briefly pause to discuss some logical properties of the identity relation. Anyone who endorses **Temporary Identity** has to say something unorthodox about these logical properties. Identity is an equivalence relation. There seems to be no particular difficulty in providing an adverbialist counterpart of Reflexivity, Symmetry and Transitivity. I will just briefly mention the latter. Unsurprisingly, Transitivity boils down to:

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[11] I claim that he "almost" settles for (6) for he claims that he can't prove asymmetry of parthood from it, but only anti-symmetry. He then considers yet another proposal that delivers asymmetry. See McDaniel (2014: p. 25). We need not enter into these details here. As a matter of fact, neither did McDaniel. He proves both Irreflexivity and Transitivity of parthood. Asymmetry follows from those.

[12] As I will point out in Sect. 4, this is better understood as providing an analysis of *proper part*. I stick to "part" here because this is the term McDaniel uses.

[13] This will be important when it comes to the explanation of the logic of parthood. See Sect. 6.

$$x =_r y \wedge y =_r z \rightarrow x =_r z \tag{6}$$

This is not problematic. What is problematic is how friends of **Temporary Identity** will re-phrase the Indiscernibility of Identicals (II):[14]

$$x =_r y \rightarrow \forall P^* \left(P^*_r(x) \leftrightarrow P^*_r(y) \right) \tag{7}$$

where P^* stands for what McDaniel calls a *region-free* property, i.e. a property P that does not have any

> [I]nformation about either a particular region or regions in general built into the property (McDaniel 2014: 17).

The restriction in (7) is crucial, for otherwise we could for example derive from (5) that I am part of my hand at the region that I occupy. And this is absurd (McDaniel 2014: p. 22).[15] Now, you might protest that any restriction of II is problematic. For *any* restriction brings forth a source of *discernibility*. Suppose the scope of the second-order quantifier in (7) is restricted so as to exclude F-properties. Then x and y will be F-discernible. One might insist that discernibility in *any respect* is enough to guarantee that the relation at stake is not identity after all.[16] To give the PI theorist the best fighting chance I will not push this line of argument here.

4 In favor of PI

McDaniel offers various considerations—if not arguments—in favor of PI. In particular, McDaniel (2014: p. 32) considers the following:

> **Intimacy**. PI explains the intimacy between parthood and identity while steering clear of the problems that affect alternative explanations, such as CAI;

[14] In higher-orderese.

[15] The argument would go roughly as follows. Suppose x is part of y at r_y. Then x is identical to y at r_y. Suppose furthermore that II is not restricted. Hence, x at r_y has *all* the properties that y has at r_y. y has, at r_y, the property of *having* x *as a part*. Thus, x has that property as well, that is, x is part of x at r_y. But x and y are identical at r_y, so that I can substitute y for the first occurrence of x and get that y is part of x at r_y. Let x be my hand, y be my body, and r_y be the region occupied by my body. The argument shows that I am part of my hand at the region that I occupy, as claimed in the main text. The endorsement of the restricted version of II undermines the argument. This is because, given (5), *having* x *as a part* is not a *region-free property*. Thus it falls outside the scope of restricted II.

[16] Compare this with the debate over CAI. Hovda (2014) suggests that friends of CAI could answer some of the objections against it by restricting the second order quantifier, so as to exclude the plural predicate "being one of". He then claims:

> It might be argued that the symbol = (…) *does not express genuine identity*, since some instances of the original SID [i.e. substitutivity of identicals] axioms fail (Hovda 2014: 209, italics added).

For a discussion see Calosi (2018).

Explanation of the Logic of Parthood. PI offers an analysis of parthood, and explains why parthood behaves the way it does. In particular, it explains that it is a strict partial order,[17] and that it is extensional;

No Devastating Objection. PI does not suffer from any devastating objection.[18]

I don't have much to say about **Intimacy**. The thought seems to be that theses such as CAI and PI are appealing because they have a direct, straightforward explanation of the intimacy between parthood and composition on the one hand, and identity on the other: they are—so the thought goes—one and the same. One might wonder why intimacy needs to be explained in terms of identity itself—assuming it needs to be explained at all. Hopefully, after all, distinctness does not preclude intimacy. One might for instance think that the relations of "almost exact similarity" and identity are very intimate, and yet they are distinct relations.[19]

I will mainly focus on **Explanation of the Logic of Parthood** and **No Devastating Objection**. These will be the focus of Sects. 6 and 7, respectively. Hence, it is worth spending a few words on McDaniel's defense of such claims.

Let me start with **Explanation of the Logic of Parthood**. According to McDaniel, Irreflexivity, Anti-symmetry, Transitivity, and Extensionality of $\sqsubset$ can be proven from PI, i.e. from (5). This might sound a bit odd. Usually, in mereology, we distinguish between *proper parthood* and *improper parthood*. The former is a *strict (partial) order*: it is irreflexive, asymmetric and transitive. The latter is a *(partial) order*: it is reflexive, anti-symmetric, and transitive. Be that as it may, asymmetry follows from irreflexivity and transitivity—as I pointed out in footnote 11. So we may take (5) to provide an analysis of *proper parthood*. The only minor qualm I have about McDaniel's proofs concerns Transitivity. It is instructive to see the latter in some detail. First of all McDaniel formulates it as follows:[20]

$$x \sqsubset_r y \wedge y \sqsubset_r z \rightarrow x \sqsubset_r z \tag{8}$$

I find this formulation problematic. Remember: the regions at which parthood relations are instantiated are the regions occupied by the wholes. Different wholes might be involved in the formulation of transitivity: how are we supposed to be guaranteed that the regions they occupy are the same? I think this crucially depends on what you take the relation of *occupation* to be. The entire next section will deal with this issue. My suggestion is that it is safer to formulate Transitivity as:[21]

$$x \sqsubset_{r_y} y \wedge y \sqsubset_{r_z} z \rightarrow x \sqsubset_{r_z} z \tag{9}$$

[17] Intended as *proper parthood*, as I mentioned already.

[18] Apart from its being admittedly "weird", and apart from the objections against the assumptions in Sect. 1.

[19] I owe this example to Maria Scarpati.

[20] McDaniel (2014: p. 23).

[21] I offer the reformulation in the text to alert the reader about how careful one has to be when providing an adverbialist counterpart of mereological axioms. Some subtelties depend on the intended understanding of the occupation relation. As we will see in due course, this understanding turns out to be crucial for the very fate of PI.

where r_i is the region that is occupied by i. To be fair, substituting (9) for (8) does not undermine McDaniel's proof.[22] As McDaniel rightly points out the proof depends crucially on the fact that the relation of "being a subregion of" is *transitive*. This will take center stage in Sect. 6.

Let us turn to **No Devastating Objection** then. McDaniel considers six potential objections. In Sect. 7 I will set forth an objection that McDaniel does not consider, and that I take to be the strongest one against PI. As a matter of fact, I think it warrants its rejection. In the light of this, I will not discuss any of the objections McDaniel addresses. The only one that needs to be mentioned is Objection 3. This is because McDaniel's reply to Objection 3 will be important for the arguments in Sect. 6 and the "Appendix". There, I will show that, contrary to what McDaniel claims, what really explains the logic of parthood is not PI. Rather, it is the logical behavior of the subsethood relation. And subsethood is exactly what McDaniel uses to dispel Objection 3.
The objection has it that PI employs the notion of proper subregion, and so is circular. The thought here is that the relation of proper subregion is just the relation of proper parthood when restricted to regions of spacetime. McDaniel replies that there are analyses of the proper subregion relation that do not appeal to proper parthood. He considers two possible candidates: set theory and plural logic. This is one of the reasons I used set-theory myself.[23] As I mentioned already, this will be crucial in what follows.

5 Occupation and location

As we saw, the relation of *occupation* plays a crucial role in McDaniel's formulation and defense of PI. Yet, McDaniel does not provide any details on how to understand such a relation. I think it is crucial to consider such details. This will become most clear in Sect. 7. There, I will argue that, depending on different understandings of the occupation relation, PI faces different problems. Thus, in this section I spell out the necessary details, using *formal theories of location.*

Formal theories of location attempt to give a precise characterization of different locative notions that capture different ways an object can be located at—or , in other words, *occupy*—a spacetime region. An elegant, and widely accepted,[24] way of doing that is to start with a primitive notion of *exact location* @. The following is the somewhat orthodox gloss on @:

> [A]n entity x is exactly located at a region y if and only if x has (or has-at-y) exactly the same shape and size as y and stands (or stands-at y) in all the same spatial or spatiotemporal relations to other entities as does y (Gilmore 2018: 6).

We can then define other locative notions in terms of @ and standard set theory. It is customary to define *Weak Location* ($@_{\circ}$), *Entire Location*($@_{<}$), and *Pervasive Location* ($@_{>}$), as (10), (11), and (12) respectively:

22 McDaniel (2014, pp. 23–24). The proof contains a few typos that are easily fixed.

23 I will return to plural logic, albeit briefly, in the "Appendix".

24 See e.g. Casati and Varzi (1999), Hudson (2001), Sattig (2006) , Hawthorne (2008) and Donnelly (2010). For alternatives see Parsons (2007), and Eagle (2010). The differences between these theories do not matter here.

$$x@_{\circ}r \equiv \exists r' \left(x@r' \wedge r' \cap r \neq \emptyset\right) \tag{10}$$

$$x@_{<}r \equiv \exists r' \left(x@r' \wedge r' \subseteq r\right) \tag{11}$$

$$x@_{>}r \equiv \exists r' \left(x@r' \wedge r \subseteq r'\right) \tag{12}$$

In plain English, something is weakly located at a region r iff it is exactly located at a region that intersects r, something is entirely located at a region r iff it is exactly located at a sub-region (i.e. a subset) of r, and something is pervasively located at a region r iff it is exactly located at a super-region (i.e an extension) of r. As an illustration, suppose I stick my arm out of my office into the corridor. I count as weakly located both at my office and in the corridor. I am entirely located in Switzerland, in Europe, in the northern hemisphere and so on. I am pervasively located where my hands are, where my spleen is, and so on. The question facing us is then: what is *occupation*? Is it Exact, Weak, Entire or Pervasive Location?

Different passages from McDaniel suggest different answers. As an example, recall the passage I already quoted:

> Suppose the relation (…) alluded to above is the relation of parthood. in this case, [1] intuitively r_x must be a subregion of r_y (…) If r_x is a subregion of r_y, then the union of r_x and r_y just is r_y (…) In general, every region at which something is part of a whole is a region occupied by that whole. A similar and shorter argument can be given for the claim that, [2] if x is identical with y at r, then both x and y occupy r (McDaniel 2014: 20–21 notation changed, numbers inserted).

Consider claim [1] above. The line of argument seems to be the following. Suppose x is part of y. In this case, the region r_x occupied by x is *intuitively a subregion* of the region r_y occupied by r_y. This is true if occupation is exact location. But it is not true if occupation is weak location or entire location. x is weakly and entirely located at any region that has r_y as a subset. By contrast, consider claim [2]. Claim [2] is true if occupation is weak location or entire location, but false if occupation is exact location or pervasive location, for x is neither exactly, nor pervasively located at r_y. This is problematic. Without an unambiguous characterization of the crucial notion of *occupation*, how is one suppose to seriously evaluate PI?[25]

There is an argument to the point that "occupation" should be exact location. For reasons that will be obvious, I shall call it the *arbitrariness* argument. In a nutshell, the argument has it that an enduring object has a *unique* exact location at an instant—*modulo* time-travel complications that I shall set aside—whereas it has many different weak, entire and pervasive locations. As a result, if occupation is weak, entire, or pervasive location it would be arbitrary to single out one among the many as the one that mediates the instantiation of a property.

[25] The following is perhaps a charitable reading. Beside pervasive location, any other location relation would do for McDaniel's purposes. As a matter of fact, I will argue that no matter what locational relation is used PI faces insurmountable difficulties. This will become most clear in Sect. 7. Thanks to two anonymous referees for this journal for suggesting this reading.

Let me be a little more precise. Define an instant of time t as any set of all spacetime points that are absolutely simultaneous with one another.[26] Consider me. At any instant of my existence I have a unique exact location that is a proper subregion (i.e. a subset) of the instant in question. Focus on one such instant t, and let r be my unique exact location at t. Clearly, there is nothing arbitrary about r inasmuch as it is the *unique* subregion of t at which I am exactly located. By contrast, how many weak locations do I have at t? Infinitely many, for every subregion of r counts as one of my weak locations. As a matter of fact, every subregion of r also counts as a pervasive location of mine. Furthermore, every super-region of r, i.e. every region that has r as a subregion, counts as my entire location. Thus, I also have infinite entire locations. The thought is that if occupation were weak, pervasive, or entire location it would be arbitrary to say that I have a property P at one of those regions and not at others. This does not happen with exact location. An example might help. Suppose that, at t, I am in my office, and I weigh 73 kg. Which one is the relevant region at which I weigh 73 Kg? If occupation is weak location, I weigh 73 Kg at my office, where my heart is exactly located, in Switzerland, where my spleen is exactly located and so on—for these are all weak locations of mine. The same goes, *mutatis-mutandis*, if occupation is entire or pervasive location. By contrast, if occupation is exact location the only region at which I weigh 73 Kg is the "me-shaped" region of my office I exactly fit into—at t. This—absent any other considerations—seems to provide a reason to hold that occupation is exact location.[27]

Be that as it may, McDaniel might find *the arbitrariness argument* less than persuasive, especially in the light of some of the things he writes. Thus, I am going to consider *all* the further options.[28] In Sect. 7 I will argue that, on the one hand, if occupation is either weak or entire location then PI is committed to highly implausible claims—if not straightforwardly incompatible with highly plausible principles. On the other, I will contend, if occupation is exact location then PI runs afoul of II, even in its restricted version (7). Either way, this spells trouble for PI. Before that, I focus on the alleged explanation of the logic of parthood, that PI is supposed to deliver.

6 PI and the logic of parthood

McDaniel claims that PI *explains* the logic of parthood inasmuch as PI entails that $\sqsubset$ is *irreflexive*, *asymmetric*,[29] *transitive*, and *extensional*. Transitivity is important. McDaniel is explicit in claiming that *it is PI* that explains that $\sqsubset$ is transitive, because $\sqsubset$ inherits transitivity from transitivity of $=$:

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[26] Clearly, I am not considering relativistic complications.

[27] As an example take Gibson (2006). They explicitly defend both **Endurantism** and **Spacetime Fundamentality**. Unsurprisingly, they endorse the view that enduring objects have properties at their exact locations—albeit with no arguments.

[28] McDaniel might respond that *occupation* is none of the relations above. But then he owes us a clear, fully-fledged account of what *occupation* is. I will not consider pervasive location in what follows for it seems to be ruled out by several remarks of McDaniel's. I think this is right. Pervasive location strikes me as the worst way to qualify what occupation is.

[29] Despite what McDaniel himself writes.

> In short […] the transitivity of the parthood relation falls out of the transitivity of identity (McDaniel 2014: 23).

However, as he recognizes immediately after—McDaniel (2014: p. 24)—one *needs* transitivity of $\subset$ in order to get the transitivity of $\sqsubset$. This is crucial.

If parthood is analyzed in terms of subsethood and occupation *alone*—call this view **Parthood is Sub-Occupation**—it turns out that proper parthood is a strict partial order:[30]

$$x \sqsubset_{r_y} y \equiv_{df} xOr_x \wedge yOr_y \wedge r_x \subset r_y \tag{13}$$

(See the Appendix for the proofs). Things are different for *extensionality*. In the case of *extensionality* PI *is* doing the work.[31]

In effect, **Parthood is Sub-Occupation** alone does not entail extensionality. In the "Appendix", I show that **Parthood is Sub-Occupation** entails *extensionality* only *together* with **No-Colocation**. And, as I will point out, **No-Colocation** is plausible iff *occupation* is *exact location*. Note that this is a result that should please some endurantists.

The argument here is that it is *not* PI that explains the logic of parthood. Rather it is the logical behavior of $\subset$. To appreciate this, note that transitivity of $\subset$ is both *necessary* and *sufficient* to derive the transitivity of $\sqsubset$. This lends credit to the claim that PI plays no necessary role in deriving transitivity of $\sqsubset$, and thus it does not play any role in explaining its logical behavior. I will leave the proofs for the "Appendix". Here I just want to point out that for those who were willing to follow McDaniel in providing an analysis of parthood in terms of subsethood and occupation,[32] **Parthood is Sub-Occupation** is still a viable option. They might even argue that **Parthood is Sub-Occupation** sheds some light on *what parthood is.*

7 RIP PI

I already raised several worries about PI. Can we hammer the proverbial final nail in its coffin? Consider the following argument. Take my left hand. Suppose I paint it *red*, completely *red*.[33] It also has the property of being *uniformly colored*. Now, my body is not uniformly colored. In fact it is multi-colored—for I did not paint my entire body. It follows from PI and II that my hand is multi-colored at the region I occupy.

30 The same can be done using plural logic, and natural axioms for the "is one of" relation. See the "Appendix".

31 McDaniel might reply that the result about *extensionality* is good enough. This seems an interesting line of argument. However, two things are worth noting. First, transitivity is indeed crucial. To appreciate this, consider that *any* formalization of classical mereology contains *transitivity* as an axiom—see Cotnoir and Varzi (Forthcoming). Second, *extensionality* is much more controversial than *transitivity*, especially for endurantists. McDaniel himself acknowledges this much, when he writes: "Unlike transitivity, extensionality is fairly controversial" (McDaniel 2014: p. 27). In the light of the foregoing, it would be advisable to endorse a putative explanation of the logic of parthood that delivers *transitivity* rather than *extensionality* as a theorem. Thanks to an anonymous referee for pushing this point.

32 To lay my cards on the table, I am not one of them.

33 The reason I am using *red* will be clear later on.

This is not a straightforward contradiction, but it strikes me as problematic—especially for someone who, like McDaniel, cherishes intuitions.[34] Intuitively, there is no region[35] at which my hand is multi-colored.

Let me make the point more vivid. Take any asymmetric and irreflexive relation R holding—at r_w—between a whole w and one of its proper parts p, e.g. "being bigger". Construct from R a property of the whole—at r_w, such as "being bigger than p". It follows from the argument above that p is bigger than itself at the region r_w that w occupies. But, the argument insists, p is not bigger than itself at *any* region. In other words, nothing is never bigger than itself -anywhere!

As I pointed out, this is no straightforward contradiction. It is just highly counter-intuitive, one might reply. But we can strengthen the argument. I will now phrase this strengthened argument in terms of *entire location*, for I find it more persuasive. However, note that *weak location* would do just as well. So, suppose O is $@_<$. Arbitrariness considerations push endurantists towards the following principle, to the point that if x has a property P at one of its entire locations r that belongs to an instant t, it has P at every entire location belonging to t:

$$P_r(x) \wedge r \in t \rightarrow \big(\forall r_1 \in t(x @_< r_1 \rightarrow P_{r_1}(x)\big) \tag{14}$$

Recall my previous example: I am 73 Kg at my exact location.[36] What (14) claims is that, under the assumption that O is $@_<$, I am 73 Kg in Switzerland, in the northern hemisphere, and so on. Furthermore, endurantists should hold that enduring objects *have incompatible properties at different regions*. Let P_1 and P_2 be two such incompatible properties. Then:[37]

$$P_{1_{r_1}}(x) \wedge P_{2_{r_2}}(x) \rightarrow r_1 \neq r_2 \tag{15}$$

Now the argument is that PI, together with II, is inconsistent with (14) and (15). To appreciate that, recall the example of my uniformly colored hand. My hand is uniformly colored at the region it occupies. Thus, by (14), it is also uniformly colored at the region I occupy. I am not, however, uniformly colored at the region I occupy. By PI, my hand is identical to me at the region I occupy, and therefore, by II, it follows that my hand is not uniformly colored at the region I occupy. Clearly, being uniformly colored and being multi-colored are incompatible properties. Thus my hand instantiates incompatible properties at the very same region, the region I occupy. And this is incompatible with (15). I myself find (14) and (15) more plausible than PI. Or

[34] McDaniel (2014: p. 13).

[35] At the given instant.

[36] Note that any exact location of x is also an entire location of x. To appreciate this, go back to the definition of entire location. Informally, an entire location is a *proper* or *improper* subset of an exact location. Given that any set is an improper subset of itself, it follows that every exact location of x is also an entire location of x.

[37] Note that an endurantist might want to endorse a stronger principle—modulo time-travel considerations, namely: $P_{1_{r_1}}(x) \wedge P_{2_{r_2}}(x) \wedge r_1 \in t_1 \wedge r_2 \in t_2 \rightarrow t_1 \neq t_2$. The principle is stronger for it entails (15), whereas the converse does not hold.

at least, I would find them more plausible if I were an endurantist. How should the defender of PI respond to such an argument? I shall consider two different ways.

First, the defender of PI could claim that *being multi-colored* or *uniformly colored* are covertly *region encoding* properties. Thus they fall outside the scope of II.[38] The challenge now consists in finding genuine *region-free* properties that make II a substantive principle. As a matter of fact, I used color-properties because McDaniel himself suggests these are *region-free* properties.

Second, the defender of PI could insist that the argument simply shows that *occupation* is neither *entire*, nor *weak* location. In fact, she will go on, *occupation* should be *exact location*. We also have *the arbitrariness argument* of Sect. 5 to back-up that claim! Modulo time-travel considerations, one might want to endorse the view that an object has a unique exact location at a given instant, as I already pointed out. But then, the argument goes, (14)—in its current formulation—is incompatible with RIP. For RIP entails that things have properties only at their exact locations. By contrast, if we replace $@_<$ in (14) with @ we get:

$$P_r(x) \wedge r \in t \rightarrow \big(\forall r_1 \in t(x@r_1 \rightarrow P_{r_1}(x)\big) \quad (16)$$

which is trivially *not* problematic. Under the assumption that things have a unique exact location at any instant of their existence, (16) boils down to $P_r(x) \rightarrow P_r(x)$, given that for every $r_1 \in t$, $r = r_1$. In either case, we can resist the argument.

The point is basically the following. Given that things have properties at their exact locations, and that my hand is not exactly located at the region where I am exactly located, my hand *does not have any property* at the region where I am exactly located. In particular, it is not multi-colored there, so that the problem vanishes.

But this seals the fate of PI, it seems. For the very same argument establishes that II is incompatible with RIP. Suppose $x \sqsubset_r y$. By PI, $x =_r y$. Then by II, x has the very same region-free properties that y has at r. But it follows from RIP that x is exactly located at r, which is false. In fact, proper parts are never exactly co-located with the wholes they are proper parts of.[39] We may phrase the argument as a dilemma: either *occupation* is *entire* or *weak location*, but then PI is inconsistent with extremely plausible principles—principles that are, as a matter of fact, much more plausible than PI itself; or *occupation* is *exact location*, but then PI is incompatible—given RIP—with II, even in its restricted formulation. But if so, it is unclear that we are really talking about identity anymore—even temporary identity.

38 This is as akin to McDaniel's move against Objection 2.

39 To be fair, this might be the case in mereological scenarios that violate *Weak Supplementation*. As an illustration, consider Fine's theory of *qua-objects*, developed in Fine (1982). According to Fine, every basic object such as, say, Lear, qualifies as the only proper part of its incarnations, say King Lear, or Lear-*qua*-King. *Weak Supplementation* is violated because it prohibits composite objects with a single proper part. In this un-supplemented scenario it is plausible to claim that a proper part, namely Lear, is co-located with the whole it is a proper part of, namely Lear-*qua*-King. Thanks to an anonymous referee here.

8 Conclusion

On the one hand, the reasons McDaniel offers in favor of PI are hardly compelling. Or so I have argued. On the other, I think the view faces substantial problems. This should come as a relief. We can truly say: we are not our hands, we are not our spleens.

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Appendix

In this Appendix, I show that **Parthood is Sub-Occupation** "explains" the (minimal) logic of parthood, inasmuch as it entails that proper parthood is a strict partial order.

As I read the arguments, they just prove what I claimed in Sect. 6, namely that what explains the logic of parthood is not PI: rather, it is the logical behavior of $\subset$. However, anyone that has reductive inclinations might take **Parthood is Sub-Occupation** more seriously, and claim that it provides a viable analysis of the notion of parthood.

Parthood is Sub-Occupation (PiSO): $x \sqsubset_{r_y} y \equiv_{df} x O r_x \wedge y O r_y \wedge r_x \subset r_y$

In what follows, $\subset$ is what we sometimes refer to as *proper subsethood*. In terms of $\in$, this is defined as follows: $x \subset y \equiv \forall z(z \in x \rightarrow z \in y) \wedge x \neq y$. $\subset$ is a strict partial order: it is irreflexive and transitive, and thus asymmetric. It is also extensional. Once again, the proofs are trivial. Given **PiSO**, $\sqsubset$ inherits (some of) its logical properties from $\subset$.[40]

Parthood is Irreflexive: $\neg x \sqsubset_{r_x} x$.

Proof By reductio. Suppose $x \sqsubset_{r_x} x$. Then by **PiSO**, $x O r_x$ and $r_x \subset r_x$. Contradiction. □

Parthood is Transitive: $x \sqsubset_{r_y} y \wedge y \sqsubset_{r_z} z \rightarrow x \sqsubset_{r_z} z$.

Proof Assume $x \sqsubset_{r_y} y$ and $y \sqsubset_{r_z} z$. Then, by **PiSO**, (i) $x O r_x$, (ii) $y O r_y$, (iii) $z O r_z$, (iv)$r_x \subset r_y$, and (v) $r_y \subset r_z$. By transitivity of $\subset$, (vi) $r_x \subset r_z$. Thus, $x O r_x \wedge z O r_z \wedge r_x \subset r_z$. This is equivalent to $x \sqsubset_{r_z} z$. □

It also follows that $\sqsubset$ is asymmetric. Thus, it is a strict partial order. Now we could define *improper partood* as:

Improper Parthood: $x \sqsubseteq_{r_y} y \equiv x \sqsubset_{r_y} y \vee x =_{r_y} y \equiv x O r_x \wedge y O r_y \wedge r_x \subseteq r_y$

Similar arguments establish that $\sqsubseteq$ is a *partial order*, i.e. it is *reflexive*, *anti-symmetric*, and *transitive*. It just inherits (some of) its logical properties from $\subseteq$.

[40] We could have used plural logic rather than set-theory. In that case, $\sqsubset$ would have inherited its logical properties from the plural logical predicate "being one of", $\prec$. As a case in point, we could require $\prec$ to be transitive: $\forall x(x \prec xx \rightarrow x \prec yy) \wedge \forall y(y \prec yy \rightarrow y \prec zz) \rightarrow \forall x(xx \prec xx \rightarrow x \prec zz)$, where double signs such as xx stand for plural variables.

Two things are worth noting about these rather boring proofs. The first is that they do not depend on the choice of O. That is, O might be @, or $@_<$, or $@_\circ$, and they would still go through. The second is that **PiSO** alone is not enough to derive *extensionality*. Let me spend a few more words on this. In general, I take *extensionality* to be the following:[41]

Extensionality: $\forall z \left(z \sqsubset_{r_x} x \leftrightarrow z \sqsubset_{r_y} y\right) \rightarrow x =_{r_x \cup r_y} y$

To see that **PiSO** alone does not yield *extensionality*, think of the classic case of the statue and the clay. It follows by **PiSO** that the region occupied by the statue is identical with the region occupied by the clay.[42] But nothing forces that the statue *is* the clay. In fact, we need some sort of **No-Colocation** principle in order to ensure that. The following one will do:

No-Colocation: $x O r_x \wedge y O r_x \rightarrow x =_{r_x} y$

We can now prove that **PiSO** and **No-Colocation** *do* entail extensionality.

Proof Assume $\forall z(z \sqsubset_{r_x} x \leftrightarrow z \sqsubset_{r_y} y)$. From **PiSO**, (i) $x O r_x$, (ii) $y O r_y$, and (iii) $r_z \subset r_x \leftrightarrow r_z \subset r_y$. Therefore, (iv) $r_x = r_y$, by extensionality of $\subset$. By **No-Colocation**, $x =_{r_x} y$.[43] □

No-Colocation is a substantive principle. Foes of extensionality are likely to reject it. Also note that **No-Colocation** is plausible iff O is @. If O is $@_\circ$ or $@_<$, it is simply false. If I were an endurantist I would welcome these results!

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[41] McDaniel (2014: p. 26) offers a different formulation. My take here parallels my take on Transitivity in Sect. 4.

[42] By extensionality of $\subset$. I take it that identity for sets is not *temporary identity*.

[43] Or $x =_{r_y} y$ for what matters.

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