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Towards a pluralist theory of singular thought

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Abstract This paper investigates the question of how to correctly capture the scope of singular thinking. The first part of the paper identifies a scope problem for the dominant view of singular thought maintaining that, in order for a thinker to have a singular thought about an object o, the thinker has to bear a special epistemic relation to o. The scope problem has it is that this view cannot make sense of the singularity of our thoughts about objects to which we do not or cannot bear any special epistemic relation. The paper focuses on a specific instance of the scope problem by addressing the case of thoughts about the natural numbers. Various possible solutions to the scope problem within the dominant framework are assessed and rejected. The second part of the paper develops a new theory of singular thought which hinges on the contention that the constraints that need to be met in order to think singularly vary depending on the kind of object we are thinking about. This idea is developed in detail by discussing the difference between the somewhat standard case of thoughts about spatio-temporal medium-sized inanimate objects and the case of thoughts about the natural numbers. It is contended that this new Pluralist theory of singular thought can successfully solve the scope problem.

Keywords Singular thought · Epistemic constraint · Natural numbers · Pluralism · Semantic instrumentalism · Mental representation · Scope problem

1 Singular thought and the scope problem

One of the central aims of philosophy of mind is to understand the nature of mental representation. One way of pursuing this aim is to address the questions of what it is for

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a thinker to entertain a *singular* thought about an object, and how to distinguish singular thoughts from *descriptive* thoughts. The dominant view in contemporary philosophy of mind has it that the singularity of our thoughts depends on the satisfaction of a semantic constraint and an epistemic constraint. Here is a minimal statement of the two constraints:

Semantic constraint: a thinker T entertains a singular thought about an object *o* only if the thought can be expressed by a sentence featuring a directly referential singular term *t* for *o*.

Epistemic constraint: T entertains a singular thought about *o* only if T bears some *special* epistemic relation to *o*.

Any theory that proposes and elaborates on these two constraints will here be regarded as an articulation of what I shall dub the *Semantic–Epistemic View* of singular thought (SEV).

The semantic constraint relies on the attack that authors such as Donnellan (1966), Kaplan (1989), Kripke (1980), and Marcus (1961) moved against the so-called Frege–Russell descriptivist theory of meaning. Such a constraint is often specified by appealing to the notion of a *singular proposition*. So, it is often claimed that T entertains a singular thought about *o* only if T entertains, e.g. believes, a singular proposition about *o* (see e.g. Hawthorne and Manley 2012; Recanati 2012; Salmon 1986). The notion of a singular proposition admits itself of many articulations (see Glick forthcoming for a presentation of the various options available). However, such different articulations need not concern us here, for this paper focuses on the epistemic constraint and its role in the characterisation of singular thought.[1]

The epistemic constraint originates in Bertrand Russell's distinction between *knowledge by acquaintance* and *knowledge by description* (see Russell 1959). Russell held that knowledge by description is propositional knowledge, viz. knowledge of truths. By contrast, to be acquainted with something is to bear a "direct cognitive relation to [an] object" (Russell 1959: p. 209), that does not involve "the intermediary of any process of inference or any knowledge of truths" (1959: p. 46). So, to put it roughly, in order for T to satisfy the epistemic constraint, T has to bear an acquaintance relation to the object of thought.

As is well-known, Russell advanced this distinction within a peculiar epistemological and metaphysical framework that results in the view that we are only acquainted with our own mental states, sense-data, universals, and possibly ourselves. In the last 40 years or so, however, the epistemic constraint has been made independent of Russell's overall philosophical commitments.[2] The nowadays-typical examples of singular thoughts accepted by supporters of (SEV) are perceptually-based demonstrative thoughts about medium-sized ordinary objects, such as kettles, trees, persons, and the

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[1] An interesting question is whether, and if so why, one of the two constraints has some priority over the other. This question deserves an extensive examination which is better postponed to another separate investigation.

[2] Russell maintains that sense-data are non-mental external particulars, but it is unclear what his views on the relation between sense-data and *ordinary* objects are. I thank an anonymous referee for urging me to acknowledge this point.

like. So, for instance, when T entertains the thought expressed by the sentence 'That's a kettle' on the basis of T's perceptual links with a particular object, T thinks a singular thought about *that* kettle. Other commonly accepted ways of bearing a special epistemic relation to an object which enable us to satisfy the epistemic constraint are memory, introspection, and testimonial chains of communication.[3]

Throughout the years, (SEV) has been endorsed in various ways by authors such as Bach (1987), Devitt (2014), Donnellan (1977), Evans (1982), Recanati (2012), Salmon (1986), Sawyer (2012). Its dominance is witnessed by the fact that many (e.g. Crane 2011; Hawthorne and Manley 2012; Jeshion 2010)[4] regard it as the view to beat, as it were.

To forestall misunderstandings, let me clarify that I am using the label 'epistemic' to cover both *purely causal* and *mixed* (i.e. causal and non-causal) theories of singular thought. The distinction between these two kinds of theory will be fully explained below. However, as a matter of terminology, let me notice that Devitt (2014: p. 475) follows Hawthorne and Manley (2012: chapter 3) in using 'epistemic' to refer to mixed versions of (SEV)—such as the one developed by Evans (1982)—only. However, I see no harm in taking the causal relations required by the theories of singular thought devised by authors such as Devitt, Donnellan, and Russell to be *epistemically rewarding* (see Recanati 2012) in the minimal sense of providing us with information (and misinformation) about the object we are causally related to. I take this minimal sense of epistemic reward to be the unifying trait behind the significantly different articulations of the epistemic constraint which I will examine below.

Undoubtedly, we have lots of thoughts about spatiotemporal medium-sized objects, be they animate or inanimate. It seems that (SEV) can deliver a simple and compelling explanation of the singularity of some of such thoughts. Thus, we should not be surprised by its prominence. Yet, we also have thoughts about objects to which, *prima facie*, we do not bear any special epistemic relation. For instance, we have thoughts about fictional objects such as Vlad the Impaler,[5] abstract objects such as the number three, non-existent objects such as Vulcan. Since it seems that we are not—perhaps cannot be—epistemically related to these objects in the special way required by the epistemic constraint, if we accept (SEV) we are bound to conclude that we do not—perhaps cannot—have singular thoughts about such objects.

However, this conclusion cannot be embraced blithely. Both advocates and detractors of (SEV) have acknowledged the possibility of having singular thoughts about these objects (see e.g. Azzouni 2009; Burge 2007; Crane 2011; Hawthorne and Manley 2012; Evans 1982; Recanati 2012). The reasons for such an acknowledgement might be various: one can appeal to strong intuitions about a set of clear cases

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[3] I am using 'special' instead of 'direct' since I take testimonial chains of communication to be *mediated* ways of being epistemically related to objects.

[4] These authors challenge (SEV) in different ways. Some of them think that the semantic constraint is both necessary and sufficient to think singular thoughts (so-called *semantic instrumentalists* such as Hawthorne and Manley 2012). Others replace the epistemic constraint with different constraints (e.g. Crane 2011 and Jeshion 2010). More on this below.

[5] Vlad the Impaler is a character of David Foster Wallace's *The Broom of the System*.

and generalise from them; or else, one can appreciate the impact that language has on thought by endorsing Kaplan's observation that language "broadens the horizons of thought" (Kaplan 1989: p. 603) and enables us to go beyond our epistemic limitations.

Be that as it may, the point that is of import here is to emphasise the aforementioned widely shared reluctance to give up on the possibility of singular thoughts about fictional, abstract, and non-existent objects. For once we acknowledge so, we are bound to ask: if (SEV) provides the right account of the singularity of some of our thoughts, how is it that we can have singular thoughts about such objects?

The foregoing question raises, in a crude way, what I shall call the *scope problem* of singular thought. Here is the problem in the form of a loose paradox:

(1) In order for T to have a singular thought about o, T has to bear a special epistemic relation to o.
(2) There are objects o_1, o_2, o_3, to which T does not (cannot) bear any special epistemic relation.
(3) Therefore, T does not (cannot) have singular thoughts about o_1, o_2, o_3.
(4) But T does (can) have singular thoughts about o_1, o_2, o_3.

The scope problem challenges the extensional adequacy of (SEV). Since it seems safe to require of any theory of singular thought that it be extensionally adequate, the urgency of a proper treatment of the scope problem should not be underestimated.

This paper offers an investigation into the scope problem and proposes a new solution to it. To begin with, let me clarify the focus of the present investigation. The scope problem can be—at least in principle—raised in a variety of ways, in that there are different kinds of object to which we do not bear any special epistemic relation, or so it seems.[6] However, it is not possible to responsibly address all the possible different versions of the scope problem in the space of a paper. For this reason, I will be taking up a specific instance of the scope problem which restricts (2)–(4) to the case of thoughts about the natural numbers (henceforth I will omit the "natural" qualification). Nevertheless, I trust that the solution to the specific version of the scope problem I deal with here will look sufficiently compelling as to warrant its generalisation to other instances of the scope problem, thereby eventually offering a systematic approach to it.

That being said, the specific scope problem I will be focusing on can be presented as follows:

(5) In order for T to have a singular thought about o, T has to bear a special epistemic relation to o.
(6) T does not (cannot) bear any special epistemic relation to numbers.

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[6] A couple of clarifications are in order. First, the number of specific instances of the scope problem might vary depending on one's views about the nature of objects. For instance, one might hold that fictional objects are abstract objects. Secondly, in order for the scope problem to get off the ground, it is sufficient to single out *one* kind of object to which we do not bear the special epistemic relation imposed by the epistemic constraint. In the following, I identify and focus on an instance of the scope problem which enables us to steer clear of complications arising from metaphysics.

(7) Therefore, T does not (cannot) have singular thoughts about numbers.

(8) But T does (can) have singular thoughts about numbers.

I will proceed as follows. In Sects. 2, 2.1, 2.2, 2.3 and 2.4 I focus on claim (6) and argue that attempts at rejecting it fail. In light of this, the main result of Sects. 2, 2.1, 2.2, 2.3 and 2.4 is that some of the most popular versions of (SEV) are unable to solve the scope problem. In Sect. 3 I start looking at the solution I will end up recommending, which consists in rejecting (5). In Sect. 3.1, I briefly register some dissatisfaction with the best-known rejection of (5) that goes under the name of *semantic instrumentalism*. At the end of Sect. 3.1 I take stock and add a further clarification regarding the scope problem: I observe that, in light of the discussion pursued in Sects. 2, 2.1, 2.2, 2.3, 2.4, 3 and 3.1, the prospects of solving the scope problem by rejecting (8) are unattractive. In Sects. 3.2, 4, 4.1 and 4.2 I articulate my own solution to the scope problem. In a nutshell, I propose a new way of rejecting (5) which rests on the contention that there are different constraints governing singular thought about different kinds of object. I develop this idea in detail by discussing the difference between the somewhat standard case of thoughts about spatiotemporal medium-sized inanimate objects and the case of numbers. I dub my view *Pluralism* about singular thought.

2 Rejecting claim (6): meeting the epistemic constraint

Let us look closely at the formulation of the scope problem. Claim (5) is accepted *ex hypothesi*, since it expresses, in a fairly general fashion, the epistemic constraint imposed by (SEV). For the time being, we can rest content with such a generality—for we will go over various specifications of the epistemic constraint below—and move on to claim (6). In the next subsection I present a rough-and-ready argument in its favour.

2.1 A rough-and-ready argument in favour of (6)

The rough-and-ready argument in favour of (6) (RR for short) goes like this:

(9) To be in a position to satisfy the epistemic constraint, we must bear a causal connection to numbers.

(10) To bear a causal connection to an object, there must be (i) contiguity relations between causes and effects, and (ii) energy transfer between objects.

(11) Numbers are non-spatiotemporally located and unchanging objects.

(12) If numbers are non-spatiotemporally located and unchanging, then we cannot be causally connected to them. (From 10 and 11).

(13) We cannot bear a causal connection to numbers (from 11, 12 and *modus ponens*).

(14) We are not in a position to satisfy the epistemic constraint (from 9, 13, and *modus tollens*).

Let us briefly motivate the three assumptions of (RR). Premise (9) articulates the epistemic constraint on singular thought by taking the relevant special epistemic relation to be causal. The causal interpretation of the epistemic constraint is advocated by many

defenders of (SEV), such as Bach (1987), Devitt (2014), Donnellan (1977), Recanati (2012), and Salmon (1986).[7]

Premise (10) states what we might reasonably take to be necessary conditions on efficient causation. More specifically, (i) is accepted by different accounts of causation since Hume; (ii) relies on Newton's third law of motion. So, I take (10) to be warranted.

Premise (11) rests on two further assumptions: (a) numbers are *objects*; (b) *abstract* objects differ from concrete ones in that the former are not spatiotemporally located and unchanging while the latter are. Let me offer some remarks about the initial plausibility of (a) and (b).

Let us take (a) first. One might point out that by taking numbers to be (abstract) *objects* I am ruling out the immanent realist option to the effect that numbers are part of the natural world by being properties of physical objects. The immanent realist could block (RR) by rejecting (11), in that she holds that numbers are not objects. However, a moment of reflection shows that rejecting (RR) in this way does not help the supporter of (SEV): singular thinking is—at least in its basic manifestations—thinking about *objects*, while immanent realism exactly denies that numbers are objects, for they are conceived of as properties. So, *at least* within the project pursued in this paper, assumption (a) is plausible.

To forestall misunderstandings, though, let me stress that I do not foreclose the possibility of there to be singular thoughts about properties. This is indeed a very interesting question, one which has received little attention in the contemporary literature. So, I think that any contribution shedding light on singular thinking about properties, and whether (and if so how) it differs from singular thinking about objects should be welcomed in the debate. However, this is not the project pursued by this paper. Rather, the paper operates under the assumption that the central cases of singular thought are object-directed and seeks to understand how to solve the scope problem conditional on such an assumption. The assumption that object-directed thoughts are the central cases to focus on is ubiquitous in this debate. If we have any pre-theoretical take on what makes a thought singular as opposed to descriptive, it has to do with the fact that we can single out objects somehow independently of the properties they have and we take them to have (I will offer a thorough articulation of this intuition below). Moreover, the almost universally acknowledged examples of singular thought are demonstrative thoughts about concrete medium-sized inanimate objects. So, the project I am after is that of solving the scope problem when formulated under the assumption that thoughts about numbers are thoughts about them *qua* objects. Other philosophers who attempt at vindicating the singularity of our thoughts about numbers (e.g. Azzouni 2009; Evans 1982) or abstract objects more generally (Recanati 2012), articulate their views by taking the relevant thoughts to be object-directed. If such an

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[7] A clarification is in order. Some authors draw a distinction between epistemically special *causal* relations and epistemically special *contextual* relations. Recanati (2012) subsumes them under the label of 'epistemically rewarding relations'. Examples of contextual relations are the relations we bear to our egocentric spatial and temporal locations, and indexical relations of self-identity. However, authors availing themselves of this distinction do not explain how a contextual relation manages to be epistemically rewarding without being causal. Moreover, it strikes me as implausible to maintain that we bear a sort of indexical or egocentric relation to numbers (nor do authors accepting the causal/contextual distinction think so). For these reasons, I'll frame my discussion here in terms of causal acquaintance relations only.

aim would prove to be unattainable, we should turn to address the scope problem by articulating the idea of property-directed singular thoughts in great detail.[8]

As to assumption (b), let me notice that it is customary to define *abstracta* as non-spatiotemporally located and unchanging objects. Moreover, doing so is noncommittal as to their ontological status. That is to say, claiming that such objects are non-located and unchanging does not, per se, establish their existence or non-existence. In order to say something about the latter issue, one should commit oneself to additional substantive views which either prove or disprove the compatibility between the existence of an object and its not being subject to efficient causation. Thus, we can safely claim that this minimal way of distinguishing between *concreta* and *abstracta* is independent of one's own nominalist or platonist proclivities. This establishes the *prima facie* plausibility of (11).

Having said that, it must be noticed that a supporter of (SEV) might solve the scope problem by rejecting premise (6) in two ways: either she shows that we can bear causal relations to numbers, thereby blocking (RR) before step (12); or she shows that there is a plausible non-causal reading of the epistemic constraint such that we can satisfy it while thinking about numbers, thereby rejecting the first premise of (RR). In next two subsections, I argue that both approaches fail. In Sect. 2.4 I consider a way of mitigating the impact of the scope problem which does not consist in a straight rejection of (6).

2.2 Vindicating a causal version of (SEV)

In a series of important contributions, Maddy (1980; 1990) has offered a naturalised version of arithmetic platonism whose driving thought is encapsulated in two theses: first, sets are spatiotemporally located and can therefore be perceived; secondly, numbers are sets. In more recent writings, Maddy has somewhat revised her position, which has also received powerful criticism (see Panza and Sereni 2013, ch. 4, section 4 for an overview). At all events, I contend that even if both claims defended by Maddy were correct, her view could not enable supporters of (SEV) to solve scope problem.

To begin with, let me stress that supporters of (SEV) should not merely block (RR). They have to block it in a way that ensures the possibility of thinking singularly about numbers. Mindful of this observation, let us assume—for the sake of argument— that Maddy's claims are correct. Surely, this enables us to reject premise (11) of (RR). Moreover, Maddy's account also establishes the possibility of having singular thoughts about sets, for we can perceive them. However, on Maddy's view, numbers are not given to us directly, immediately through perception. Rather, they are given to

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us *qua* those sets satisfying such-and-such properties, i.e. the properties specified by Dedekind–Peano axioms. And yet, thinking of an object *qua* satisfier of a property is what has been traditionally taken to be the mark of descriptive thinking (see e.g. Bach 1987; Recanati 2012, I will get back to this distinction below). Thus, even granting the controversial theses that we can perceive sets and that numbers are sets, this view can at most enable us to vindicate the possibility of having singular thoughts about sets. Thoughts about numbers will turn out to be descriptive. Hence, the attempt at harnessing Maddy's naturalistic account of arithmetic to solve the scope problem within a causal interpretation of the epistemic constraint fails.

2.3 Vindicating a "mixed" version of (SEV)

I turn now to consider the *mixed*, i.e. causal and non-causal, version of (SEV) offered by Evans in his posthumous book *Varieties of Reference* (1982).

Evans' theory of singular thought hinges on what he calls *Russell's Principle*:

> In order to be thinking about an object or to make a judgement about an object, one must *know which* object is in question—one must *know which* object it is that one is thinking about.

> [Evans 1982: p. 65]

Russell's Principle can be satisfied in a variety of ways which need not involve causation. Let me explain. Evans contends that, for any object, there is an account of what makes it different from other objects at a given time *t*. This is what Evans calls an *Idea* of an object.[9] The second key move in Evans' proposal is the distinction between *fundamental* and *non-fundamental* Ideas. To use Hawthorne and Manely's apt characterisation (2012: p. 75), a fundamental Idea of an object is "a concept which encodes a property that is had uniquely by that object at that time and which explains what makes that object different from all other objects at that time". Evans mentions the case of numbers and maintains that having fundamental Ideas of numbers amounts to "being able to differentiate one number from another by their position in an infinite ordering" (Evans 1982: pp. 106–107). This, crucially, does not require having a causal link to numbers. Moreover, Evans says that we often employ such fundamental Ideas of objects in our thinking about them and that "this is especially clear with abstract objects" (*ibid.*, fn. 30). Since thinking about an object by deploying a fundamental Idea of it is one way of satisfying Russell's Principle, when T thinks of the number three via the position it occupies in the relevant infinite ordering, viz. that of the natural numbers, and ascribes to it, say, the property of being prime, T has a singular thought about the number three.

Here we have, in kernel form, Evans' version of (SEV).[10] If correct, this view would enable the supporter of (SEV) to reject (RR) in this way: the argument does

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[9] In order to have an Idea of an object, T should satisfy the *Generality Constraint* (1982: p. 104): "If a subject can be credited with the thought that *a* is F, then he must have the conceptual resources for entertaining the thought that *a* is G, for every property of being G of which he has a conception".

[10] I am focusing here on how Evans interprets the epistemic constraint imposed by (SEV). I grant that his view satisfies the semantic constraint.

not get off the ground since its first assumption, namely a causal interpretation of the epistemic constraint, is dispensed with by Evans' version of (SEV). On reflection, however, Evans' vindication of (SEV) does not succeed, for a well-known argument against reductive arithmetic platonism due to Paul Benacerraf can be mobilised to show that we cannot satisfy Russell's Principle in the way suggested by Evans.

In his 1965 seminal paper "What Numbers Could Not Be", Benacerraf identifies the *adequacy constraint* that any account of numbers should meet in order for it to be formally adequate. The adequacy constraint requires both that any such account provide a model of Dedekind–Peano axioms thereby entailing the truth of the standard theorems of arithmetic, and that it explain cardinality in terms of counting. Then, Benacerraf argues for the existence of multiple and equally adequate set theoretical reductions of numbers to sets by noticing that different set theoretic progressions, e.g. Zermelo's and Von Neumann's, satisfy the adequacy constraint.[11] To put things in an Evansian fashion. If T has a fundamental Idea of the number three, T thinks of it as the next to the next to the next to the least element of the infinite ordering satisfying the adequacy constraint. Yet, this property can be ascribed to both $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$ and $\{\{\{\varnothing\}\}\}$, for both Zermelo's and Von Neumann's progressions satisfy the adequacy constraint. So, it turns out that the number three is both $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$ and $\{\{\{\varnothing\}\}\}$. Clearly, this cannot be the case, since these are different objects. Therefore, we are unable to discriminate which object the number three is on the basis of the fundamental Idea of it. This is tantamount to saying that Evans' non-causal version of (SEV) fails to vindicate the possibility of entertaining singular thoughts about numbers.[12]

2.4 Mitigating the impact of the scope problem

So far I have considered two (SEV)-based strategies for rejecting claim (6) of the scope problem by undermining the main argument, i.e. (RR), in its favour. However, the supporter of (SEV) might adopt a different strategy: instead of flatly rejecting (6), she might acknowledge the existence of the problem but tone down, as it were, its negative impact on (SEV). This strategy can be elicited from the recent version of (SEV) advocated by François Recanati. Recanati's proposal hinges on three core ideas:

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[11] From this result, Benacerraf draws two conclusions: there is no non-arbitrary reason to choose one reduction over the other and, for this reason, numbers do not exist. The point I want to make is independent of both conclusions, though.

[12] A critic could argue that I haven't spelled out fundamental Ideas of numbers correctly. For instance, one could reject the claim that numbers must be identified on the basis of their arithmetical properties only and avoid the Benacerraf Problem. One way to do so would be to accept the neo-logicist claim that the essence of numbers is given by what Frege called *Hume's Principle* (see Wright 1983). Hume's Principle says that the cardinal number belonging to the concept F is identical to the cardinal number belonging to the concept G iff the entities falling under F are in 1-1 correspondence with the entities falling under G. Neo-logicists maintain that this principle works as an abstraction operator which produces numbers as its values. By adding Hume's Principle to second-order logic, we can derive the Dedekind–Peano axioms, thereby accounting for arithmetic to in purely logical terms. I cannot address this option in depth here, but let me notice that controversies about the status of Hume's Principle are such that, if our account of the possibility of having singular thoughts about numbers had to rely on such controversial principle, it would certainly lack the required generality to make such an account theoretically attractive.

(i) T cannot entertain a singular thought about an object *o* without possessing and exercising a *mental file* of *o*. A mental file is a nondescriptive mode of presentation whose role is to enable the collection and storage of information about *o*.

(ii) Mental files refer (or are supposed to refer) to objects, but their reference is not determined by the information stored into the file; rather, reference is secured via *epistemically rewarding relations* to these objects which, in Recanati's view, are various kinds of (causal) acquaintance relations.

(iii) Acquaintance is a *de jure*, i.e. normative constraint on singular thought.

Even though the basic function of mental files is that of storing information about objects via epistemically rewarding relations to them, Recanati mentions the possibility that mental files serve *derived functions* where the acquaintance norm is neither *de facto* satisfied, nor exploited.[13] This possibility rests on the following empirical hypothesis:

> The object tracking system which exists in perception is used throughout cognition—even in high-level cognition, e.g. in thought about abstract objects. The units of the system—the files—acquire new functions when they are recruited in this way to do extra jobs in thought.
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> [Recanati 2013: p. 212]

Recanati explains the role played by the acquaintance norm in this case as follows:

> […] The derived functions correspond to new jobs assigned to the files, but the referential function of the prototypical files is primary because *the files are what they are in virtue of belonging to the object tracking system* (where they have a referential function), and the new roles correspond to uses of *these files* to do new things thanks to, inter alia, the interaction with the linguistic system.
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> [Recanati 2013: pp. 212–213]

If Recanati's evolutionary hypothesis were correct, a supporter of (SEV) could mitigate the consequences of the scope problem on (SEV) as follows. Even if we cannot meet the epistemic constraint when we think of numbers, there's some sense in which the mental representations we deploy while thinking of them are importantly related to the mental representations we token while thinking about spatiotemporally located objects. Insofar as (SEV) delivers a plausible account of the latter and there is a robust cognitive-evolutionary connection between them and the former, we should not worry too much about the scope problem, or so the thought goes.

In order for such a mitigating strategy to pan out, we should ask: Is Recanati's hypothesis confirmed by empirical data?

To begin with, it must be stressed that advocates of mental files appeal to a flurry of psychological data supporting the psychological legitimacy of mental files. Mental files are claimed to be importantly related to what psychologists call *object files*, which are representations of the mid-level visual system stored in working memory invoked to explain how we visually track objects. Experiments in developmental psychology support the contention that such a visual tracking takes place without relying

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[13] The acquaintance norm is exploited just in case, even if T is not acquainted with the object, T presupposes the acquaintance norm and makes "*as if* [T] were using the file normally (i.e. according to the norm) to refer to an object of acquaintance" (Recanati 2013: p. 209).

on the properties possessed (or taken to be possessed) by the tracked objects. This has led supporters of mental files (e.g. Jeshion 2010; Recanati 2012; see Murez and Smortchkova 2014 for an overview) to stress the similarity between object files and mental files and take such a similarity to provide empirical support for the view that singular thinking is thinking through mental files.

It is indeed hard to deny the existence of some continuity between the object tracking system in perception and cognition when we focus on cases where a thinker has a perceptually-based demonstrative thought about an inanimate object.[14] However, empirical research seems to provide evidence for the presence of some noticeable discontinuity between object files and mental representations of numbers (*qua* objects). Carey (2009) has offered a detailed account of how children acquire the latter. She argues at length that neither the ability of representing approximate cardinalities via a physical magnitude, nor the deployment of object files tracking the numerical identity of objects in parallel on the basis of their changes in spatiotemporal location can ground representation of numbers. Rather, the acquisition of numerical concepts[15] witnesses a conceptual discontinuity which is explained by what Carey calls *Quinean Bootstrapping*.

Quinean boostrapping is a two-stage learning process whereby children acquire new concepts. The bootstrapping process can be generally described as follows. At a first stage, a network of symbolic yet meaningless placeholders is constructed. At a second stage, the placeholders get interpreted through modelling processes such as analogical mapping, thought experiments, limiting case analysis, inductive and abductive inferences. In the case of numbers, at the first stage the child learns a portion of the sequence "One, two, three, …" as a list of meaningless lexical items. At the second stage, the meaning is partly supplied by what Carey calls "enriched parallel individuation", viz. an object tracking system which encodes the numerical identity of individual items by initiating object files for each item enriched with quantifiers. Parallel individuation is limited, in that working memory has a limit of three or four items, and so can only provide the meanings of "one", "two", "three", "four" figuring on the placeholder count list. The bootstrapping proceeds with the child noticing the analogy between next in the numeral list and next in a sequence of sets related by one additional individual. This enables the child to make the following inductive step: if 2 is followed by 3 in the numeral sequence, then adding an individual to a set with cardinal value 2 results in a set with cardinal value 3. This step is crucial for the acquisition of the concept of the successor function. When the learning process is complete (at around age $3^{1/2}$), the child bootstraps the ordered list of numerals representing numbers.

Surely, bootstrapping enables children to acquire numerical concepts thanks, at least partly, to those cognitive mechanisms, such as parallel individuation, that are in the same psychological category as mental files. However, the contribution of parallel individuation is limited, for it merely provides a partial interpretation of some of the

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[14] Things get more complicated when we focus on thoughts about animate objects, such as persons. Murez and Smortchkova (2014) argue that, in this case, conceptual and empirical reasons support the introduction of a distinct type of file, what they call a *person file*. I need not go into these details here.

[15] I'm using here "concept" as synonymous with mental representation to conform to Carey's terminology.

placeholders. Moreover, and most importantly, the output of the bootstrapping process is a representational system having a significantly different and greater expressive power than the parallel individuation system's. Carey insists on the latter point by stressing and documenting the conceptual discontinuity between parallel individuation and the representational systems acquired through bootstrapping (see especially the third part of Carey 2009).

The previous discussion indicates that one of the most influential and successful strand of empirical research in psychology speaks against Recanati's empirical hypothesis: the mental representations we deploy to think of numbers are not in the same psychological category as the ones we deploy to think of (e.g. perceived) inanimate objects. The discontinuity argued for on empirical basis by Carey undermines the contention that the same object tracking system is used throughout cognition. Thus, we cannot mitigate the negative impact of the scope problem on (SEV) by claiming that representations of numbers are importantly related to representations of medium-sized spatiotemporal objects.

Summing up. The argument from (5)–(6) to (7) is valid, and (6) cannot be rejected by some of the most popular versions of (SEV). This, to my mind, warrants examination of alternative solutions to the scope problem.[16] More specifically, I turn now to evaluate the prospects for possible rejections of (5).

3 Rejecting claim (5): semantic instrumentalism and *pluralism*

A rejection of (5) amounts to parting with the epistemic constraint on singular thought. As a consequence, rejecting (5) is tantamount to rejecting (SEV) itself. I will firstly and very briefly consider the best-known approach that rejects (5) one can find in the literature, viz. semantic instrumentalism.[17] The nowadays well-known reasons for being dissatisfied with semantic instrumentalism will lead me to explore a new way of rejecting (5).

3.1 Semantic instrumentalism

Semantic Instrumentalism (see e.g. Borg 2007; Harman 1977; Hawthorne and Manley 2012; Kaplan 1989) is the view that by simply manipulating the linguistic apparatus of direct reference we can entertain a singular thought about an object even though we do not bear any epistemic relation to it. All that is required of us to entertain singular thoughts is to exercise our semantic competence. For instance, if T is able to introduce a new singular representation, such as a proper name, into the language

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[16] Supporters of (SEV) might acknowledge this failure yet maintain that, all things considered, this is not enough to give up (SEV). They might insist that (SEV) has further advantages which trump its inability to solve the scope problem. I don't find this reply incredibly convincing: the scope problem is the problem of capturing the correct extension of singular thinking, and I find it hard to downplay the importance of such an issue. At all events, I think that supporters of (SEV) should grant that, *ceteris paribus*, a theory of singular thought that solves the scope problem is preferable to one that does not.

[17] There are other ways of rejecting (5), see e.g. Crane (2011) and Jeshion (2010). Since Crane's and Jeshion's views bear some similarity to the view I will end up defending, I will comment on them below.

via a reference-fixing description and mentally tokens that representation, T thereby has a singular thought about the denotation of the name. Semantic instrumentalism opposes (SEV), in that it denies that T needs to stand in any special epistemic relation to objects in order for them to have singular thoughts about them.

Semantic instrumentalists might block the scope problem by rejecting (5) on the following grounds. Suppose that numbers-naming expressions are directly referential singular terms whose reference is fixed via description. Let us now assume the principle Hawthorne and Manley (2012: p. 38) call *Sufficiency*: "Believing a singular proposition about an object is sufficient for having a singular thought about it". If T believes the proposition *that 3 is prime*, and under the assumption that the numeral '3' is used as a directly referential singular term, T has a singular thought about the number three.[18]

The very permissiveness about the generation of singular thoughts that enables semantic instrumentalism to easily account for the possibility of having singular thoughts about numbers has been regarded as its major weakness. As is well-known, two main objections have been levelled against it. The first objection, due to Donnellan (1977), goes roughly as follows. If semantic instrumentalism were true, then one could gain pieces of empirical knowledge—such as the proposition *that if some unique individual is the first born in the next century, it is Newman 1*—via simple linguistic stipulation. The second objection, due to Evans (1982), is this. Semantic instrumentalism implausibly countenances the generation of singular thoughts at will by pointing out that simple linguistic stipulation cannot alter cognitive mechanisms whereby we represent objects in thought.

It is unclear how semantic instrumentalists respond to these objections. By way of illustration, take the last in-depth defence of semantic instrumentalism offered by Hawthorne and Manley in their 2012 book. They discuss at length Donnellan's objection, but after pointing out that there is something suspicious about the notion of the contingent a priori, they fail to explain what the source of the suspiciousness is. So, it is hard to see what their response to Donnellan's objection is. As for Evans' objection, it is striking that Hawthorne and Manley do not even mention it.[19]

I should emphasise, though, that I do not take Donnellan's and Evans' objections to show that semantic instrumentalism has been conclusively refuted. Rather, I think that it is safe to observe that the debate between advocates and detractors of semantic instrumentalism is stuck in an *impasse* and that no real progress has been made regarding a proper evaluation of its pros and cons. This stalemate, in my view, warrants

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[18] Hawthorne and Manley might actually offer a slightly different argument, in that they accept a second principle they call *Harmony* (2012: p. 38): "Any belief report whose complement clause contains either a singular term or a variable bound from outside by an existential quantifier requires for its truth that the subject believe a singular proposition". Since, my dissatisfaction with semantic instrumentalism will be largely independent of the specific argument they might use to reject (5), these details won't matter.

[19] An anonymous referee suggests that Evans' point might be resisted by saying that any linguistic stipulation would be in some important sense related to some cognitive mechanism. Then, semantic instrumentalism would become the view that our cognitive systems generate singular representations via reference-fixing descriptive representations alone. I cannot examine this hypothesis in detail, but let me notice that those philosophers accepting semantic instrumentalism in virtue of their endorsement of the externalist idea that reference depends on social and causal factors that are external to our mind will not light-heartedly accept that linguistic stipulation is somehow parasitic upon mechanisms internal to the thinker's mind.

investigating the possibility of alternative way of rejecting (5) which is not affected by the worries usually levelled against semantic instrumentalism.

Before turning to the articulation of such an alternative rejection of (5), however, I want to take stock and make, in light of the discussion pursued so far, a general observation about the scope problem. Bear in mind that the scope problem consists of four claims, i.e. (5)–(8). So far, I have dealt with possible rejections of (5) and (6), and I have advertised that my own solution will consist in a new way of rejecting (5). Yet, one might wonder: why not rejecting (8), that is, the claim that we have (or can have) singular thoughts about numbers? One might indeed maintain that, if anything, the discussion pursued in the previous sections shows that the possibility of entertaining singular thoughts about numbers is merely illusory, for our best theories of singular thought cannot account for it. We should rest content with the claim that we can entertain only descriptive thoughts about numbers, or so the thought goes.

In reply to this train of thought, two things must be stressed. First, the option of solving the scope problem by rejecting (8) is not available to those theorists who have actually tried to make sense of the possibility of having singular thoughts about numbers. Secondly, and more importantly, I believe that this way of solving the scope problem is at odds with the motivations offered in favour of both semantic instrumentalism and the most refined versions of (SEV). To see why, it is worth keeping in mind that the main motivation behind semantic instrumentalism is to make room for the possibility of acquaintanceless singular thoughts—notice that semantic instrumentalists call themselves "liberals" (see Hawthorne and Manley 2012). Thus, unless semantic instrumentalists provide substantive reasons to maintain that, within the category of acquaintanceless singular thoughts, those thoughts which are about numbers should never wind up being singular, a rejection of (8) seems to run contrary to the very idea of being a *liberal* about singular thought. Thus, I believe that semantic instrumentalists had better focus on the worries raised against them rather than solving the scope problem by rejecting (8).

As for (SEV), I submit that the rationale behind proposing less radical and ingenious interpretations of the epistemic constraint, such as Evans' and Recanati's, is that we can have singular thoughts about objects with which we are not causally connected. Given this broad motivation, and absent additional and substantive reasons to think that we cannot have singular thoughts about numbers, I think that these sophisticated supporters of (SEV) had better not solve the scope problem by rejecting (8), for otherwise they would come too close to more radical versions of (SEV).

These remarks suggest that solving the scope problem by rejecting (8) would be something of a pyrrhic victory. A better solution is called for. In the remainder of the paper I show how such a solution should look like.

3.2 A new approach to the scope problem: pluralism

To present my new solution to the scope problem, I wish to draw attention to what I take to be an overlooked assumption made by most—to the best of my knowledge,

all—theories of singular thought. Theories of singular thought assume what I shall call *Monism about singular thought* (MONISM for short). To put it roughly:

MONISM: Thinking singularly about an object requires of a thinker *the same thing* no matter what the objects of thought are.

Let me show that MONISM is—at least implicitly—subscribed to by the theories of singular thought canvassed so far. Versions of (SEV) maintain that in order for T to have singular thoughts about a tree, a person, a fictional entity such as Vlad the Impaler, or an abstract object such as the natural number two,T has to meet a given epistemic constraint. Surely, the epistemic constraint can be met in different ways (as claimed by Evans), or it can be taken to play a role in determining the singularity of the thought even if T actually fails to satisfy it (as is argued by Recanati). Either way, supporters of (SEV) claim that the epistemic constraint governs *all instances* of singular thinking. By contrast, semantic instrumentalism maintains that in *all these cases* T need not bear any special epistemic relation to the objects of thought, for entertaining a thought which could be expressed via a sentence featuring a directly referential term for that object— or, to put it differently, entertaining a singular proposition about it—is sufficient for thinking singularly about *all kinds of* object.

My solution to the scope problem is to reject (5) by rejecting MONISM and arguing for a *Pluralist* theory of singular thought. A rough formulation of pluralism about singular thought is as follows:

PLURALISM: Thinking singularly about an object requires of a thinker *different things* depending on the kind of object the thought is about.[20]

This means, roughly, that singular thoughts about different kinds of object are governed by different constraints.

In order for pluralism to be put on the table as a real option, three questions need to be addressed:

A. To explain what it is that makes singular thought-episodes about different kinds of object as varieties of the *same phenomenon*. Call this the *Unity Question*. The Unity Question relies on the fact that we are ready to classify our thoughts about Vlad the Impaler, trees, Barack Obama, and numbers as all being *singular*. Hence, we have to explain why it seems that they share some core features that led us to use a single word (i.e. 'singular'), and a single concept (i.e. SINGULAR), for all of them. To put it differently, the apparent *unity* of singular thought must vindicated.
B. To clarify what the relation between the different kinds of episodes of singular thought is. That is to say, once we have answered the Unity Question, we have to explain how the vindication of the apparent unity of singular thought interacts with the plurality of constraints. Call this the *Architecture Question*.

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[20] To forestall misunderstandings: there is an intuitive sense in which any version of (SEV)—as well as any version of semantic instrumentalism—countenancing the possibility of singular thoughts about at least two different kinds of object, e.g. concrete and abstract, is pluralist. While I do not deny this, let me stress that I am using "monism" and "pluralism" in the somewhat technical sense of there to be just one (monism) or more than one (pluralism) constraint(s) to be met in order to have singular thoughts about different kinds of object.

C. Once the Architecture Question has been solved, we have to give substance to the new theory by specifying different constraints. This requires explaining why a given constraint applies to a kind of singular thought and not to another, as well as complying with intuitive verdicts about the singularity of various thought-episodes. Call this the *Specification Question*.

I will proceed as follows. I will firstly articulate the general structure of the pluralist view by addressing the Unity Question and the Architecture Question. I will then take up the Specification Question and fill in the specific details of the pluralist theory by taking up two cases: the somewhat standard case of thoughts about spatiotemporal medium-sized inanimate objects and the case of thoughts about numbers.[21] This will enable us to solve the specific version of the scope problem at stake.

4 Pluralism exposed

4.1 Unity and architecture

I submit that a good entry point to the Unity Question is a claim that is often made to bring about the distinction between singular thought and descriptive thought. It is commonplace to say that what distinguishes singular thought from descriptive thought is that while latter involves thinking of an object *qua* satisfier of certain properties, the former involves thinking of an object independently of taking it to satisfy certain properties.[22] Clearly, however, insofar as we do not provide a suitable gloss to the idea of thinking of an object in a non-satisfactional way, this claim is hardly explanatory.

As we have seen, the usual epistemic gloss will not do: to think of an object in a non-satisfactional way cannot be identical to think of it via a special epistemic relation, for otherwise we cannot solve the scope problem. Moreover, if we had to understand the distinction in merely semantic terms, by taking non-satisfactional thinking to be defined by the presence of a representation of the object which could be expressed via a directly referential singular term and taking satisfactional thinking to be defined by the

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[21] As the reader might have foreseen, I am distinguishing different kinds of singular thought on the basis of the kinds of object thoughts are about. I acknowledge that this is far from uncontroversial, especially in light of heated disputes in metaphysics about the nature of certain objects (e.g. fictional objects). Yet, I believe that since the main aim of this paper is to outline an as yet unexplored approach to singular thought, and since this aim is pursued by relying on the generally accepted distinction between concrete and abstract objects ("generally accepted" in the sense that many accept the existence of the distinction while, at the same time, debating on how to trace it), we can postpone a full discussion of the identification method of different kinds of singular thought to a separate investigation.

[22] To cite a few authors making this claim: Bach (1987), Goodman (2016), Murez and Smortchkova (2014), Recanati (2012). Something very similar is proposed by Azzouni (2011: p. 59) and Evans (1982: pp. 35–36). Hawthorne and Manley (2012: pp. 17–19) have reservations about the idea that what is opposed to satisfactional thinking is *relational* thinking, where the latter is understood in broadly epistemic-causal terms (as is done by supporters of SEV). However, I am not subscribing here to the satisfactional/relational dichotomy, for the pluralist view precisely denies the identification of singular thinking with relational (i.e. epistemic-causal) thinking. Rather, I am starting to characterise the category of singular thought in a negative way, as it were, by saying that it is constituted by all those thoughts which do not involve thinking of objects *qua* satisfiers of properties. As far as I can see, this claim is not touched by Hawthorne and Manley's concerns.

presence of a representation of the object which could be expressed via a quantified expression, we would be dragged back into the semantic instrumentalism debate's quagmire. For this reason, I suggest exploring the prospects for a *cognitive* gloss of the distinction between satisfactional and non-satisfactional thinking.

By offering a 'cognitive gloss' of the satisfactional/non-satisfactional distinction I intend to pause on and bring out the kind of *cognitive function* that is served by thinking of an object independently of its satisfying (or taking to satisfy) such and such properties. The question of the cognitive function (or role) of singular thought has been brought to the fore by authors such as Tim Crane (2011), Robin Jeshion (2010), Sarah Sawyer (2012) and Ken Taylor (2010). I will proceed as follows: I will present my account and point out similarities and dissimilarities between it and the views advocated by these authors along the way.

To understand what kind of cognitive function is performed by non-satisfactional thinking, I deem helpful to start off with satisfactional thinking. When we think of an object *qua* satisfier of a given property, what we really do is to think of the property, namely the property that is instantiated by at most one object. To illustrate. When we think something like 'The 44th President of the United States is born in America' in a satisfactional way, we think of Barack Obama *qua* the 44th President of the United States. It is a structural/syntactic feature of the mental representation we deploy in such a case, which can be plausibly taken to be a descriptive mode of presentation of the object, that of enabling us to focus *directly* on the property *being the 44th President of the United States* independently of the unique individual satisfying it, and only *indirectly* on the unique individual satisfying it. By contrast, when we think of Barack Obama in a non-satisfactional way, e.g. by thinking 'He's born in America' on the basis of a perceptual link with Barack Obama, we deploy a mental representation, which might be plausibly taken to be a nondescriptive mode of presentation of the object, whose structural/syntactic feature is that of enabling us to aim to refer to that (perceived) particular object.

In light of this, we can claim that singular thinking serves the following cognitive function:

Object-directed aim function: *A singular thought about o enables T to aim to refer to a particular o.*

Three quick clarifications about the "aim to refer" metaphor are in order. First, the aim metaphor should be understood as singling out a *cognitive* relation between T and the object of thought. So, this relation should not be identified with some special epistemic relation, such as acquaintance, T bears to *o*. Secondly, T can have an object-directed aim towards a plurality of objects, that is, T can aim to refer to three objects in particular and keep them distinct from any other salient object. However, for the sake of simplicity, I will ignore these cases. Thirdly, I take the "aim to refer to a particular object" to be a primitive cognitive relation between T and *o* that should not be reduced to the mental state of intending to refer to an object.

Crane (2011) has been the first to insist on the centrality of the object-directed aim function. He seems to hold (see e.g. Crane 2011: p. 23) that this is the only cognitive function served by singular as opposed to descriptive thinking. Yet, on closer inspection, singular thinking serves another cognitive function.

I submit that a second feature of singular thinking is that it provides us with representations that aim to track objects through actual and possible changes to their features without requiring us to possess uniquely identifying information and in spite of any misinformation we have about them.[23] To put this intuitive idea in more theoretically substantive terms, let us say that singular thought enables us to represent objects without taking them to fall under a sortal concept. A sortal concept is a concept of a property which determines criteria of application, identity and persistence for objects that possess the property. The sortal independency at stake here should be understood as follows: T can aim to refer to a particular o even if T does not know that there is a given kind k such that o falls under k. That is to say, T need not know which kind o belongs to in order to aim to refer to it. The sortal independency of singular thought has been acknowledged by Dickie (2014), Goodman (2012) and Evans (1982: p. 178) in relation to demonstrative thoughts, and it can be reconciled with empirical research in psychology (see Murez and Smortchkova 2014).

To see why this feature of singular thinking is cognitively relevant, it must be noticed that representing an object as falling under a sortal might generate dissimilarities amongst thinkers disagreeing about how the relevant sortal concept determines the criteria of identity and persistence for the target objects. To illustrate this idea, let me borrow an example from Murez and Smortchkova (2014: p. 643). If a philosopher defends the view that two persons dwell inside a body with a bihemispheric brain, she will apply the sortal PERSON differently from us. However, Murez and Smortchkova argue, when such a philosopher tracks somebody having a body with a bihemispheric brain perceptually, she does track it as a single medium-sized animate object. This seems to speak in favour of the idea that perceptually-based thoughts about persons are sortal-independent.

In my view, the possibility of aiming to refer to an object in a sortal-independent way ensures the possibility of representing an object in a *cognitively impenetrable* way.[24] The cognitive impenetrability of the representation is to be understood as its independence from one's beliefs about the nature of the object that are encapsulated in the introduction, deployment and definition of the sortal concept under which the object is taken to fall. So, the second function served by singular thinking can be put as follows:

Cognitive impenetrability function: *A singular thought about o enables T to aim to refer to a particular o in a cognitively impenetrable way.*

The object-directed aim function and the cognitive impenetrability function jointly pinpoint what we might take to be the *singular thought-role*. Thus, the view I recommend is that the job of singular thoughts is that of allowing us to aim to refer to a particular object in a cognitively impenetrable way.

Let us now avail ourselves of property talk and maintain that a given property P plays the singular thought-role if, and only if: when T deploys a representation of o in thought, T aims to refer to that particular o in a cognitively impenetrable way. In order to develop the idea of there to be different constraints on different kinds of singular

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[23] I am grateful to an anonymous referee for suggesting this intuitive formulation.

[24] I am borrowing this expression from Murez and Smortchkova (2014: p. 643).

thought without ending up with a mere disjunctive deflation of singular thought, let us say that there are different properties that play the *same* singular thought-role depending on the kind of object the thought is about. I shall maintain that the property of being a singular thought is a single higher-level property whose instantiations across different kinds of thought-episodes are determined by a class of numerically distinct lower-level properties. Singular thoughts about different kinds of object fall under a unique category, viz. the category of singular thoughts, in that these different thoughts are such that they possess the same higher-level property, even though the singular thought-role can be played by different lower-level properties, depending on the kind of object the thought is about.

This broadly functionalist view enables us to see how the idea of there to be different constraints on singular thoughts about different kinds of object could chime with the contention that there is something which unifies different instances of singular thinking.[25] For this reason, I regard the Architecture Question and the Unity Question as answered.

I have now presented the scaffolding of the pluralist view. In the next section I fill in the details of this view by specifying the different constraints governing singular thoughts about numbers and singular thoughts about spatiotemporal medium-sized objects.

4.2 Specifying constraints

I begin with focusing on thoughts about numbers. It must be kept in mind that we are looking for a constraint which enables us to vindicate the possibility of having singular thoughts about numbers without countenancing too libertine a generation of such thoughts. This observation, which is one of the upshots of the critical examination of (SEV) and semantic instrumentalism pursued in the first part of the paper, suggests that not all cases in which we mentally token a representation of a number give rise to singular thoughts about them. This suggestion relies in turn on the fact that we mentally token representations of numbers while being engaged in different kinds of *cognitive project*. We can indeed token representations of numbers while counting the number of apples on the table, mentally performing simple computational operations, thinking hard about the nature of arithmetic, wondering about which numbers are beautiful,

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[25] An anonymous referee worries that the appeal to functionalism might not carry the required explanatory weight, in that functionalism could be apt to explain notions on which we have a pre-theoretical take whereas singular thought is a theoretical construct. It is off the agenda of the present discussion to put forward a sustained defence of the functionalist framework. So, I shall confine myself to the following two observations. First, various philosophers have developed functionalist-like accounts of (to some extent) theoretically loaded notions, such as truth (Lynch), theoretical terms (Lewis), and properties (Shoemaker). So, I'm in good company in thinking that the functionalist approach can be extended to more theoretically loaded notions. Secondly, while I agree that we have less pre-theoretical purchase on singular thought than on, say, feeling pain, the phenomenon of singular thinking is not a mere theoretical construct—as opposed, for instance, to the related notion of a singular proposition. It seems to me clear that we have an intuitive take on what thoughts—and other representational vehicles such as utterances—are about, and in explaining the object-directed aim function and the cognitive impenetrability function I have also invoked some intuitive considerations. In light of this, I think that my appeal to a broadly functionalist framework to develop a pluralist approach to singular thought is not misplaced.

and so on. So, the question is: what is the feature that distinguishes those cognitive projects in which we think singularly about numbers from those in which we do not?

In what follows, I defend the contention that thought-episodes about numbers playing the singular thought-role are those episodes occurring when T is engaged in a given cognitive project for which it is *instrumentally indispensable* to mentally token a *given* representation of numbers. So, the property playing the singular thought-role in the case of episodes of thought about numbers is the property of being instrumentally indispensable for a given cognitive project: if a given mental representation tokened by T while engaged in a given cognitive project satisfies this property, then the resulting thought-episode will be singular. To put the same idea by using the constraint talk: the constraint T has to satisfy in order to entertain a singular thought about numbers is that of mentally tokening a mental representation for the number in question while pursuing a given cognitive project for which it is *instrumentally indispensable* to mentally token *that* type of representation. Let me articulate and defend this idea by taking the notion of instrumental indispensability first.

Following David Enoch (2011: chapter 3), who expands on Colyvan (2001), I will maintain that something is instrumentally indispensable for a project just in case it cannot be eliminated without defeating whatever reason we had to find the project attractive in the first place. To illustrate this idea, let me begin with Enoch's own proposal. Enoch contends that a belief in the existence of (irreducibly) normative truths is instrumentally indispensable for the project of deliberating about what to do. That is to say, without believing the existence of irreducibly normative truths, we would no longer have reasons to be engaged in the deliberative project. Enoch's argument, in a nutshell, is that given the phenomenological features of deliberation, such as the fact that you think that deliberative questions have answers, that while the decision is up to you which decision the one it makes most sense for you to take is not up to you, and so on, it seems that we cannot but accept the existence of such truths.

I hasten to acknowledge that it is unclear what the value of instrumental indispensability is unless we clarify the role that such a notion plays in the overall argument on offer. Some remarks on this point are in order. To begin with, notice that Enoch's aim is to provide an indispensability argument for the existence of irreducibly normative truths by showing that these truths are instrumentally indispensable within the intrinsically indispensable deliberative project. An intrinsically indispensable project is such that it is not a rational option for us to opt out of it. This allows Enoch to claim that the indispensability of irreducibly normative truths for the intrinsically indispensable deliberative project provides us with a justification for the belief that there are irreducibly normative truths.[26]

Be that as it may, the aim of the present paper is not to justify ontological commitment in the existence of metaphysically suspicious entities, be they irreducibly normative facts or numbers. The project I am after is eminently that of getting clear about the extension of singular thinking by investigating what conditions should be satisfied in order for thought-episodes about numbers to count as singular. This project is independent of the question whether numbers exist or not; as has emerged previously,

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[26] Obviously, whether Enoch's indispensability argument succeeds is not at issue here.

I maintain that the existence of an object is not necessary for having singular thoughts about it. Thus, insofar as I can see, I need not impose an intrisicality constraint on the set of relevant projects. As a consequence, I will proceed by discussing some examples of cognitive projects to illustrate how the instrumental indispensability of a given mental representation of a number ensures the singularity of those thought-episodes involving that mental representation.

As is emphasised, among others, by Azzouni (2009) and Burge (2007), there are two abilities that are deeply connected with numeration: perceptual recognition of small quantities of objects and pure computation. In such cases, we associate small numerals to small groups of objects through perception and we use numerals in thought to mentally perform simple additions, subtractions and multiplications.[27] While in the former project numbers behave as properties, in the latter they behave as objects. The claim I set out to defend now is that it is instrumentally indispensable to token the mental analogous of numerals while being engaged in the project of mentally performing simple pure computations.

To begin with, let us reflect on the reasons we have to find such a project attractive. Philosophers (Azzouni 2009 and Burge 2007) and cognitive scientists (Dehaene 1997; Zhang and Norman 1995) alike stress that, while mentally performing simple additions, subtractions or multiplications, we exhibit some notation facility: there are computational tricks due to our base 10 notation which enable us to perform such computations immediately and (relatively) effortlessly. Complex number notation systems using a place-value coding are of particular interest here.[28] In such systems, numbers are represented by being decomposed into multiples of powers, and "the quantity that a digit represents varies depending on the place it occupies in the number" (Dehaene 1997: p. 85). For instance, in the Hindu-Arabic numeral '13' the power is denoted by the position (second from the right), while the quantity is represented by the symbol 2. Dehaene (1997: pp. 85–88) emphasises two advantages of such a system for the pure computation project. First, it makes magnitude relations between numbers completely transparent. To illustrate. Since successive places in the string of the notation system represent successive powers of the base, and since we can effortlessly obtain the quantity of a number by multiplying the digit by the corresponding power of the base, we can easily see that 70 is bigger than 7. This is not the case, for instance, with the Greek notation system, in which 7 is represented by 'ζ' and 70 by 'o'. Secondly, place-value

[27] I will be interchangeably using expressions such as 'using numerals in thought', 'mental tokening of numeral-like representations', 'tokening the mental analogous of numerals'. I use them as follows: Numerals occur in sentences which express thoughts. If the sentence contains a numeral which aims to refer to a number n, in order for a thinker to entertain the relevant thought expressed by the sentence she has to deploy a mental representation which also aims to refer to n. So, the mental representation in question is a syntactic constituent of the thought just like the numeral is a syntactic constituent of the sentence.

[28] A wealth of research in cognitive science supports the hypothesis that humans represent the same numbers in different forms. The prominent triple-code model (see Dehaene 1997; Dehaene et al. 1999; Spelke and Tsivkin 2001) has it that we represent numbers in an analog magnitude code which is used for approximate calculation, in an Arabic visual number code which is used for parity judgements and multi-digit operations, and in a verbal code which is used for pure computational operations. Clearly, the analog magnitude system is not the one relevant here, in that it produces a merely approximate representation of numbers. So, the cognitive project of mental computation involves representations of numbers belonging to the Arabic visual number code and the verbal code.

notations make multiplication very easy, in that such an operation requires memorising a table of products from 2×2 up to 9×9 only. These two features of numeral-like representations are integral to the use and development of simple algorithms to perform pure computational tasks.[29]

These data suggest that an important—I dare to say the *main*—reason we have to engage in the project of mental pure computation by tokening numeral-like representations is that is that it is immediate, non-inferential and therefore not very cognitively costly.

Let us now try to envisage what it would take to carry out the same project without mentally tokening numeral-like representations of numbers. It seems that we should mentally token more complex representations, the obvious candidates being definite descriptions which aim to refer to numbers by identifying their position in the natural number structure. Suppose now that a thinker has to mentally compute $2+3$: the thinker should token the mental analogous of something like 'the next to the next to the least element of the sequence of objects having such and such (i.e. arithmetical) properties' instead of '2', and 'the next to the next to the next to the least element of the sequence of objects having such and such (i.e. arithmetical) properties' instead of '3'. Surely, computing in this way is cognitively costly: first, it requires deploying a sort of successor notation which does not use a place-value coding, thereby losing the advantage of using simple and rapid algorithms based on notational tricks. Secondly, even those thinkers who master these representations must deploy high-level cognitive resources to have a cognitive fix on the number the relevant description aims at picking out.[30] For instance, the mental tokening of these representations would require us to think of the number two as, roughly, *the object which comes after one*, thereby requiring the deployment of a prior conceptualisation of two via recursion. Thus, it seems that without using numeral-like representations of numbers we would lose the main reason we had to find the project attractive in the first place, viz. the fact that it didn't require too much cognitive effort. For this reason, tokening numeral-like representations is instrumentally indispensable for the project of mentally executing simple and pure computations.

The previous discussion establishes that tokening numeral-like representations of numbers is instrumentally indispensable for the project of executing simple mental computations.[31] The next step is to argue for the thesis that the numerical thinking involved in the pure computation projects at stake is singular. To do so, I have to show that mentally tokening numeral-like representations of numbers while being

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[29] The registered advantages hold for other place-value representational systems.

[30] The same applies, perhaps to a greater extent, to more computationally complex mathematical formulas and non-arithmetical descriptions such as 'The number of gin and tonics my friend Hichem has had tonight'.

[31] An anonymous referee has pointed out that there is an analog of simple pure computations that could be carried out by taking numbers to behave as properties. I do not want to dispute this claim, for the point I am arguing for is compatible with it. To wit, my point is that when we focus on the cognitive project of mentally performing simple computations *in which numbers behave as objects*, it is instrumentally indispensable to token numeral-like representations of numbers. This is what enables us to regard the object-directed thoughts involved in such a project as singular, as opposed to descriptive.

engaged in this project enables us to aim to refer to a particular object in a cognitively impenetrable way.

To see whether the object-directed aim function is preserved, let us look again at a simple mental pure computation project, viz. the project of getting the total amount of two quantities via the operation of addition. Let '2' and '3' be the mental representations for the two quantities. It seems that if T did not aim to refer to the two distinct particular objects, we could neither explain the successfulness of the computing project had T concluded that the total amount of the two quantities is equal to five, nor could we successfully correct T (nor could T rationally accept such a correction) had T concluded that the total amount is equal to four. This is evidence for the claim that numeral-like representations used in projects for which they are instrumentally indispensable enable T to aim to refer to particular numbers.

Secondly, there are reasons to think that the mental pure computation project is executed in a sortal-independent way, thereby guaranteeing that when engaged in such a project, thinkers aim to refer to particular numbers in a cognitively impenetrable way. To illustrate, take a nominalist structuralist, that is, somebody who believes that numbers do not exist. Clearly, she will never apply the sortal concept NUMBER to any object, she will not take numbers-naming expressions to refer to objects, and she will regard arithmetical statements as general claims about the natural number structure. So, a nominalist structuralist would apply the sortal concept NUMBER differently from a platonist. Within the platonist field itself, advocates and detractors of neo-logicism will also disagree about how to introduce NUMBER. Famously, neo-logicists believe that NUMBER is introduced via Hume's Principle. However, platonists who do not defend neo-logicism and nominalists notice that when introduced this way, NUMBER is unable to give a criterion of application for the natural numbers, as is witnessed by the so-called *Julius Caesar problem*.[32]

Be that as it may, while engaged in the mental pure computation project described above, platonists and nominalists of various stripes execute such a project in the same psychologically immediate, direct and cognitively non-demanding way. This supports the contention that the sortal concept NUMBER is not deployed while carrying out the computation project, thereby guaranteeing that these representations actually serve the cognitive impenetrability function.[33]

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[32] The problem, in a nutshell, is that Hume's Principle tells us whether two objects described in mathematical terms are identical or not but it does not tell us which objects can be described in mathematical terms. Thus, Hume's Principle does not provide us with the resources to establish whether Julius Caesar is a number or not. Neo-logicists offer a solution to the Julius Caesar problem, but their solution is contested by some non-logicist platonists and nominalists. For a nice overview of the debate, see section 6 of MacBride (2003).

[33] Let me emphasise that what I said so far is compatible with the claim that we have acquired numerical concepts through the bootstrapping process described by Carey. Surely, this process requires a certain cognitive effort, and it might seem to involve the acquisition of some descriptive knowledge about numbers. Yet, this does not mean that while deploying these concepts in the computing project thinkers have to access and deploy that descriptive knowledge. This consideration bears close resemblance to a point made in relation to descriptive names by authors defending different theories of singular thought, such as Azzouni (2011), Hawthorne and Manley (2012) and Recanati (2012). They all acknowledge that we can think of Jack the Ripper singularly even if all we have is descriptive information about him.

The foregoing discussion establishes that while engaged in the computing project described above, the instrumentally indispensable mental representations deployed by T play the singular thought-role, thereby ensuring the singularity of the target thoughts.

Let us take stock. I have offered a principled way to distinguish between mental tokenings of numbers' representations which give rise or fail to give rise to singular thought-episodes.[34] To repeat it, the constraint that has to be met in order for T to have a singular thought about a number is this: T has to mentally token a representation of the number in question while pursuing a cognitive project for which it is instrumentally indispensable to mentally token *that* representation. This constraint makes room for the possibility that some of our thought-episodes about numbers turn out to be singular, yet it does not overgenerate singular thoughts about them. It is not the case any deployment of any mental representation of a number is such that the correspondent thought is singular, for there are cognitive projects when we deploy, say, the mental analogous of numerals without such deployment being instrumentally indispensable for the project we are carrying out. Let me explain.

Suppose that we are engaged in the project of figuring out which numbers are beautiful and which are not because we love doing so, and suppose that I think that even numbers up to fifty are beautiful. Surely, I can go piecemeal and mentally token all the relevant numerals; I can also think of the relevant numbers through their reference-fixing descriptions, or I could simply think quantificationally something like 'All even numbers up to fifty are beautiful', without losing the main reason—however strange it is—I had to find this project to be attractive in the first place. So, while we engage in the project of figuring out which numbers are beautiful, there is no representation of numbers that is instrumentally indispensable for carrying out the project.

The foregoing shows that the constraint regulating the generation of singular thoughts about numbers allows for the possibility of thinking singularly about numbers without countenancing a voluntaristic and cognition-independent generation of singular thoughts about numbers.[35] So, the constraint on offer avoids the worries raised against (SEV) and semantic instrumentalism.

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[34] Two clarifications are in order. First, one might wonder how the instrumental indispensability constraint interacts with the semantic constraint I have presented at the beginning of this paper. I want to remain neutral on the question whether the purely cognitive characterisation of the singular thought-role and the semantic constraint are *both* necessary to capture the singularity of our thoughts. I wish to stress that if one thinks that they both are (or must be), the instrumental indispensability constraint is compatible with different ways of showing that thoughts about numbers playing the singular thought-role also have singular truth-conditions. To illustrate. If one is a platonist, a standard truth-theoretic semantics will guarantee the semantic singularity of these thoughts (as well as of the correspondent sentences). By contrast, if one is a nominalist, one could show that the relevant representations satisfy the semantic constraint via a pretence-theoretic semantics. Secondly, let me stress the mental representations tokened in the computation project are not object-dependent, in that the existence of numbers is not required to carry out the computing project (see also Azzouni 2009 on this), nor can they be identified with mental files if we stick to Recanati's acquaintance-based type-individuation of mental files.

[35] The view I am articulating might be taken to bear some similarities to Robin Jeshion's *cognitivist* theory of singular thought. Jeshion's idea, in a nutshell, is that a thinker entertains a singular thought about an object just in case the thinker opens a mental file for the object; to open a mental file for the object, the object must be *significant* to the thinker's plans, projects, affective states, motivations, (see Jeshion 2010). Even though a comparison between my view and Jeshion's should be postponed to another separate investigation,

The final step in my argument in favour of the pluralist theory consists in showing that the instrumental indispensability constraint governs singular thoughts about numbers but does not govern singular thoughts about at least one kind of object which differs from the kind under which numbers fall. This is needed since in order to support the pluralist view that the property of being a singular thought is multiply realised, I should make the case for the claim that different constraints govern singular thought-episodes about different kinds of object. To accomplish this task, I shall focus on singular thoughts about spatiotemporal medium-sized inanimate objects.

Take, for instance, a demonstrative thought expressed by the sentence 'That's a kettle' entertained by T on the basis of a perceptual link with the kettle, and let us assume—somewhat plausibly—that T entertains a singular thought about *that* kettle. If this thought is singular, then the relevant representation deployed in thought is such that it enables T to aim to refer to that particular kettle. On closer inspection, what enables T to aim to refer to that particular object rather than other objects that are in T's perceptual field are, at least partly, the perceptual interactions between T and the kettle. So, it seems plausible to hold that without such a perceptual contact, T could not have tokened a mental representation directed to *that* kettle. In light of this, the best explanation of why demonstrative thoughts are singular involves perception. Since perception is an epistemic relation between T and *o*, it is therefore plausible to maintain that in order for T to entertain a demonstrative singular thought about *o*, T must bear some epistemic relation to *o*.

To generalise the lesson drawn from the previous example: it is plausible to hold that T has to meet an epistemic constraint in order to think singularly about spatiotemporal medium-sized inanimate objects. One might suspect, however, that the instrumental indispensability constraint and the epistemic constraint are related in such a way as to support the view that singular thought about spatiotemporal medium-sized inanimate objects is also governed by the instrumental indispensability constraint. One might indeed argue that singular thought about medium-sized spatiotemporal objects is also instrumentally indispensable for certain kinds of project. Yet, since such an instrumental indispensability in these cases only provides a partial explanation for how we think singularly about such objects, a fuller explanation of our singular thought about medium-sized spatiotemporal objects seemingly requires an epistemic account of what enables it.[36] If this line of reasoning were correct, we would have established that instrumental indispensability plays a role also in the case of singular thoughts about medium-sized spatiotemporal objects, thereby undermining the distinctively pluralist claim that there be genuinely different constraints governing singular thought about different kinds of object.

Footnote 35 continued

let me stress two important differences. First, Jeshion takes the cognitive significance constraint to govern all kinds of singular thoughts, whereas my pluralist view holds that thoughts about numbers are subject to the instrumental indispensability constraint while thoughts about spatiotemporal inanimate medium-sized objects are subject to an epistemic constraint (more on this below). Secondly, instrumental indispensability for a cognitive project and significance for an individual's plans, motivations and so on appear to be two rather different properties.

[36] Thanks to an anonymous referee for suggesting this line of reasoning.

On closer inspection, however, this line of reasoning does not succeed. In order for the foregoing argument to undermine the pluralist view on offer, it must be the case that: first, singular thought about medium-sized spatiotemporal objects must be instrumentally indispensable for some *cognitive* project, and not for any kind project. Secondly, constraints on singular thought should be regarded as imposing necessary conditions on such kinds of thought. Thus, if both the instrumental indispensability constraint and the epistemic constraint have to play a role in the account of singular thoughts about medium-sized spatiotemporal object, it must be the case that the instrumental indispensability of mentally tokening a, say, demonstrative-like representation of *o* for a given cognitive project and the existence of an epistemic link to *o* are *both* necessary conditions for thinking singularly about *o*. And yet, I don't think that this is plausible, in that we happen to have—and it is certainly possible to envisage that we have—demonstrative thoughts about objects even though we're not pursuing any additional cognitive project. Suppose that I'm just walking on the street and I happen to think of that tree in front of me that it's brown and I keep on walking. I find it hard to come up with a sufficiently specific cognitive project this thought is an instrumentally indispensable part of. To put the point more generally: it seems that we can think singularly about a spatiotemporal object even though we are not engaged in any other cognitive project. Thus, instrumental indispensability cannot be taken to be a necessary condition on the singularity of demonstrative thought. This, to my mind, vindicates the pluralist claim that genuinely different constraints govern thought-episodes about spatiotemporal medium-sized inanimate objects and thought-episodes about numbers.

5 Summary and conclusion

Summing up. The paper has addressed a specific version of what I have called the scope problem of singular thought. The scope problem has proved to be resilient to solutions proposed within the dominant semantic–epistemic view of singular thought. This, together with the current stalemate about the tenability of semantic instrumentalism, has warranted exploration of a new approach to the scope problem. The pluralist theory I have proposed in the second part of this paper provides a new solution to the specific version of the scope problem: the problem does not arise for we can reject the contention that in order for T to have a singular thought about a number, T has to bear a special epistemic relation to it. Instead, the pluralist view maintains that there are different constraints that need to be met in order for us to think singularly about numbers and, for instance, spatiotemporal medium-sized inanimate objects. Meeting these different constraints ensures that the same twofold cognitive function of singular thinking, that is, enabling T to aim to refer to a particular *o* in a cognitively impenetrable way, is served in the two cases.

Let me close by venturing the hypothesis that the pluralist theory here outlined could help us dealing with other instances of the scope problem involving fictional and nonexistent entities. A pluralist treatment of these cases will have to be deferred to further works, so I shall content myself with the following: if the arguments of this paper are sound, we should take seriously the possibility of further developing the

pluralist theory here offered in order to correctly capture the whole scope of singular thinking.[37]

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