

# The received view on quantum non-individuality: formal and metaphysical analysis

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**Abstract** The Received View on quantum non-individuality (RV) is, roughly speaking, the view according to which quantum objects are *not individuals*. It seems clear that the RV finds its standard expression nowadays through the use of the formal apparatuses of non-reflexive logics, mainly quasi-set theory. In such logics, the relation of identity is restricted, so that it does not apply for terms denoting quantum particles; this “lack of identity” formally characterizes their non-individuality. We face then a dilemma: on the one hand, identity seems too important to be given up, on the other hand the RV seems to require that identity be given up. In this paper we shall discuss how the specific characterization of the RV through non-reflexive logics came to be framed. We examine some of the main objections to this version of the RV and argue that they are misguided under this specific “non-reflexive” understanding of the RV. Finally, we shall also argue that this non-reflexive view is not the only option for a metaphysical articulation of the RV: less radical approaches to identity and logic are open. In particular, some of these alternative approaches to the RV we present may be immune to most of the criticisms presented against the non-reflexive approach.

**Keywords** Non-individuality · Non-reflexive logics · Identity · Quantum mechanics

## 1 The problem

The fate of identity and individuality in quantum mechanics is a story of ups and downs. During the first decades of the theory, beginning with the very creation of the

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‘new’ quantum mechanics, identity of quantum particles was declared ‘lost’ by some of the founding fathers of the theory who took the time to reflect upon it—and some of them spent quite a long time reflecting upon it; see (French and Krause 2006, Chap. 3). Quantum particles, according to such pioneer reflections, are no longer *individuals*, mainly due to Permutation Symmetry and the odd quantum statistics. This view was called *The Received View* on quantum non-individuality: we ‘received’ it from the founding fathers, even if only in such an informal and hand waving formulation. By that time, the idea that quantum particles lost identity seemed to be just another ground breaking consequence of quantum mechanics, along with other more well-known assaults on common sense.

That assault on common sense was further developed by Mary Hesse, Heinz Post, and Paul Teller, among others. Beginning with Hesse, and later with Teller, following her, the very idea of non-individuality was explained by the now well-known bank account metaphor. Assuming that Lucy has £10 in her account, she cannot go to the agency, point to some specific pounds and claim: “those are mine £10”. Quantum particles would be like those pounds in a bank account: you can have a quantity of them, but there is nothing to any one of them that makes exactly those pounds the ones in your account. Post followed the lead of Schrödinger (one of the founding fathers that was crucial to shape the Received View, as we shall see soon) and distinguished *form* from *substance*; as a result of quantum statistics, it is form that should be given prominence on metaphysical matters (for further references and discussion on these issues, see French and Krause 2006, pp. 143–144).

That story, if it were left as it is, would mean a victory of quantum mechanics over one of our most deeply rooted common notions: identity. It could be described as a revolution in our conceptual apparatus; just like General Relativity Theory helped to reshape our notions of space and time, for instance. However, after the first impetus of the quantum revolution, philosophers and even some philosophically minded physicists would not sell identity so fast. After all, identity is considered too important and basic to be ‘lost’. As we all know, everything is identical to itself and to nothing else, so how could that be denied? The behavior of quantum particles was further studied and their statistics was seen as compatible with quantum particles having identity (see French and Krause 2006, Chap. 4). The founding fathers and their later followers went perhaps too fast in declaring that quantum particles had lost their identity.

So, as it is generally found in the literature, the problem concerning the identity of the particles could be put as the following dilemma: on the one hand, we could claim to be accepting the Received View, and consequently rejecting identity; on the other hand, we could declare identity as too important and unreplaceable, so that it is the Received View that must go. The fact is that both cannot live together. Indeed, the dialectics of the debate seems to go like that: on the one hand, it is argued that the Received View could be true because we can dispense with identity *in the quantum realm* (see French and Krause 2006, Chap. 4); on the other hand, it is argued that the Received View, as a view on the metaphysical nature of quantum particles, must be

false, because we cannot dispense with identity (see for instance [Jantzen 2011](#); [Bueno 2014](#)).<sup>1</sup>

In this paper we analyze this dilemma, trying to throw some light on the discussions surrounding identity in quantum mechanics. Our claim is that this is a real dilemma only for some formulations of the Received View, the ones that take the idea of something ‘losing identity’ in a very specific sense. There are articulations of the metaphysical idea of non-individuals—that is, articulations of possible metaphysics for the Received View—which may live well with identity. In Sect. 2 we present the basics of what we call the *Non-reflexive* approach to the Received View. This is the standard approach to it, and provides for a complete elimination of identity (it takes the idea of ‘losing identity’ rather seriously, while at the same time interpreting identity in a rather substantial sense). In Sect. 3 we discuss some of the main objections to this approach; they are mainly based on the idea that identity is just too important and cannot be given up. Our main line of attack is that those arguments misunderstand the meaning of identity as it is encompassed in the Non-reflexive approach. In Sect. 4 we present alternative approaches to ground the intuition behind something ‘losing identity’. Those approaches are all compatible with the traditional idea of identity as well as with the Received View, so that the dilemma we began with dissolves for them. The arguments against the Non-reflexive approach to the Received View are innocuous to them. We finish the paper in Sect. 5 with optimistic remarks about the future of the debate. The Received View is seen as a metaphysical thesis; it declares that quantum particles are not individuals. This can be formulated in a plurality of ways, most of them still unexplored.

## 2 The non-reflexive approach to the received view

### 2.1 The logical basis of the received view

As a matter of simplicity, let us abbreviate ‘Received View’ by RV from now on. As we mentioned in the introduction, it is usually thought that the RV must be articulated through some kind of restriction or elimination of identity. This restriction consists in taking seriously the idea of a ‘loss of identity’ as it appears in the writings of some of the founding fathers and their followers. In this sense, the RV is seen as advocating that identity must be eliminated when we are talking about quantum particles.

Now, one could worry that this idea makes no sense at all. How can one eliminate identity? Can we go beyond the bank account metaphor and provide a positive metaphysical characterization of non-individuals? In the recent years this idea was formally underpinned by the development of a family of formal systems called *non-*

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<sup>1</sup> Of course, others may see the dilemma as a sign of deeper problems and avoid taking sides on this issue. In particular, one could follow [French \(2011\)](#) and locate the source of the problem in the very concept of ‘objects’; roughly speaking, given that objects lead to such troubles, one would be better getting rid of them. Without objects there is no issue regarding whether they have identity or not. Others, such as [van Fraassen](#), see the dilemma leading to the end of realism: without scientific realism, there is no need to bother about such issues (see [van Fraassen 1991](#), pp. 480–482). Here, we shall stick with the dilemma and discuss some of its sources and alternatives.

*reflexive logics* (NR). In particular, quasi-set theory is perhaps the most well-known among such logics; it is the one that is most commonly invoked to formally represent the metaphysical intuition underlying the RV. In a nutshell, non-reflexive systems of logic are systems that violate the so-called ‘*Reflexive Law of Identity*’ in the form  $\forall x(x = x)$ . In its ‘metaphysical reading’, the Reflexive Law of Identity is known as a version of the ‘Principle of Identity’, roughly stating that *everything is self-identical*. Versions of this law are restricted in systems of non-reflexive logic, and those systems are said to incorporate in a rigorous fashion the idea of entities somehow losing their identity.

The relation between a rigorous articulation of the RV and non-reflexive systems of logic is very close in actual literature; they rarely appear isolated. French and Krause (2006, Chap. 6), for instance, hold that classical logic is severely committed to identity, so that it is inadequate to capture the ontological claims advanced by the Received View. As we shall discuss later, even critics of the view, such as Bueno (2014), seem to collapse the RV with its formulation in terms of non-reflexive logics. French pushes further the relation between RV and NR, going as far as to condition the intelligibility of the RV on the use of quasi-set theory and similar non-reflexive systems. In discussing whether one should worry about the very intelligibility of the idea of non-individuals, he says in (2014, p.36) (there are similar claims in French 2015):

Steps towards the resolution of this issue have now been taken with the formulation of forms of quasi-set theory and associated logical systems capable of accommodating this non-individuality (French and Krause 2006, Chaps. 7 and 8). I emphasize these developments here because without them, this metaphysical position — of quantum particles as non-individuals — might not be treated as a viable ‘horn’ of an underdetermination argument at all.

That is, without *non-reflexive logics* the RV is simply not a viable *metaphysical* option! If non-reflexive systems fail, then we are left only with the option of treating those entities as individuals (the other horn of the metaphysical underdetermination mentioned in the quote). Indeed, the conclusion seems to follow because without such non-reflexive articulation we would be left at best with the bank account metaphor, which could then be dismissed as way too crude to represent a serious metaphysical contender. So, as it is clear, the connection between non-reflexive logic and a metaphysics of non-individuals is a very close one in this case: if the approach through the formal system fails, the metaphysical counterpart is in trouble too.

As a result of the previous discussion, which is typical of the literature, it seems clear that the approach through non-reflexive logics is simply considered as the formal approach to the RV. So, for the sake of argument in this section, let us consider it as clear that one can, at least, articulate the metaphysical view of the RV according to the strictures of the NR systems. To understand how that formal-*cum*-metaphysical development works, we must take a closer look at non-reflexive systems and how they attempt to capture the informal idea of something ‘losing identity’. We shall make only a rather brief sketch of it, even though we concede that there is much of interest in a longer and more detailed development of the history of the RV–NR marriage.

## 2.2 Non-reflexivity and its development

Perhaps we could say that NR systems, as applied to problems on the identity of quantum particles, began with Newton C. A. da Costa in (2008, pp. 138–141) (first edition of 1979). da Costa's main goal in this work was to show that the so-called Principle of Identity could be restricted, and took quantum mechanics as a main motivation. It is curious that da Costa took Schrödinger's following passage as his *motto*:

I beg to emphasize this and I beg you to believe it: it is not a question of our being able to ascertain the identity in some instances and not being able to do so in others. It is beyond doubt that the question of 'sameness', of identity, really and truly has no meaning (Schrödinger 1996, pp. 121–122).

The use of that quotation as a motivation for non-reflexive systems is a bit curious, because in the wider context of the quote Schrödinger is clearly discussing the problem of *trans-temporal identity*. That is, he is concerned with whether one can determine that a particle at an instant of time  $t_1$  is the same as a particle appearing at a later instant  $t_2$ . Of course, this is not exactly the issue when we deal with the idea of (non-) individuality: in the latter case, we are mostly concerned with synchronic identity, or, generally put, what makes one entity count as one. So, it could be urged that the motivation for the development of non-reflexive systems was misguided. However, this first 'hermeneutic' concern may be easily dispelled by taking into account the fact that Schrödinger, in many other places, is also concerned with synchronic identity, with similar conclusions about its place in quantum mechanics. That happens, for instance, when he is discussing the effects of quantum statistics over the identity of particles (as for example in Schrödinger 1998, and for further discussion, see French and Krause French and Krause 2006, pp. 119–127).

Anyway, according to da Costa there is a simple way to formally capture the idea that identity does not make sense for quantum particles. We may use a two-sorted first-order language with two kinds of terms; let us call them M-terms and m-terms. Let us say that the first kind intentionally refers to middle-seized objects, as we find them in everyday experience, and for which identity claims are thought to make sense. The second kind of terms refers to quantum entities in the intended interpretation. There is also an identity sign in the language, and the trick to capture the idea that identity does not make sense for m-terms consists in restricting the *definition of formula*, so that expressions involving identity like " $t_i = t_j$ " are formulas if and only if both  $t_i$  and  $t_j$  are terms of the first kind (M-terms). So, inside this two-sorted language we cannot write formulas in which identity is involved for terms of the second kind, and it is in this sense that identity *does not make sense* for quantum entities. The resulting system was baptized by da Costa as *Schrödinger logic*.

Schrödinger logics were later on further developed by Décio Krause as higher-order systems (higher-order Schrödinger logics) and the main idea was also employed in the development of quasi-set theory (see French and Krause 2006, Chap. 7 and 8 for details on such systems; Krause's developments of higher-order Schrödinger logics and quasi set theory go back to the late 1980s, resulting in his Krause 1990). What is crucial in Krause's development of those early years is an attempt to incorporate to

Schrödinger logics another main feature of the intuition behind the Received View: the idea that quantum particles are indiscernible but not identical.

Recall, da Costa only attempted to restrict identity, with no explicit intention to articulate the RV. Krause's work now incorporates the indiscernibility present in quantum statistics as a primitive relation. So, in the resulting system it would be possible to take seriously Heinz Post's (1963) claim that indiscernibility should be introduced in the quantum formalism *right at the start* (see French and Krause 2006, Chap. 6 and the references therein). Notice that here two main trends are combined: on the one hand, there is the formal counterpart to the claim that identity does not make sense. On the other hand, there is a formal counterpart to the claim that quantum particles are indiscernible and not identical.

Recall that perhaps the most basic intuition behind quantum non-individuality, the one inherited from the founding fathers, concerns the 'loss of identity' of quantum systems originating from their indiscernibility (again, see the discussions in French and Krause 2006, Chap. 3). So, it is reasonable that quantum indiscernibility does not collapse in identity. After Krause's developments, NR systems formalize this idea, so that quantum particles are indiscernible but not identical, *and this is mainly due to the lack of identity for such systems*. So, Krause's development have already added a spoonful of metaphysics to the formal systems devised by da Costa: so far, they can be seen as incorporating the claim that quantum particles lost their identity in the specific sense of not having any feature (property, quality) that would allow them to be distinguished from other quantum particles of the same kind. This is perhaps what is behind Weyl's (1931) famous claim that you cannot demand an alibi from electrons; they simply do not have a specific property granting them 'identity'.<sup>2</sup>

That kind of metaphysical interpretation can also be seen as working for quasi-set theory. Basically, quasi-set theory is a set theory with two kinds of ur-elements, the *m*-atoms and the *M*-atoms, but with no identity sign as primitive symbol. *M*-atoms represent the usual classical objects, for which identity makes sense, while *m*-atoms represent quantum particles for which identity does not make sense. This time, the idea that identity does not make sense is encapsulated in the definition of identity, which is restricted to classical objects and quasi-sets. Collections of objects are identical when they have the same elements, and *M*-atoms are identical when they belong to the same collections. So, in the end, identity is not defined for *m*-atoms, and it does not make sense for them; one cannot write  $x = y$  when at least one of  $x$  and  $y$  are *m*-atoms (again, see French and Krause 2006, Chap. 7 for details).

Indiscernibility ' $\equiv$ ' is introduced as having the postulates of an equivalence relation. It does not coincide with identity: indiscernibility makes perfect sense for *m*-atoms, while identity is not even defined for *m*-atoms. Also, for *M*-atoms identity and indiscernibility coincide (we shall discuss this issue later). So, as we have been saying, quasi-set theory and higher-order Schrödinger logics, in the second stage of their development, incorporate two features of quantum particles: identity does not apply and indiscernibility holds for all of them.

<sup>2</sup> Of course, this interpretation of Weyl is controversial, as Muller and Saunders (2008) have already pointed out; but we take here the standard interpretation according to which Weyl is seen as claiming that fermions are indiscernible.

If that were left as it is, we would have a specific way of encapsulating the idea of ‘loss of identity’. Following the main remarks by the founding fathers, according to which indiscernibility implied somehow the lack of identity, NR systems would encompass a way of talking about entities which are indiscernible but not identical. One way to metaphysically dress the intended interpretation of these formal systems is to take it as implying that quantum indiscernibility does not imply identity, violating thus the famous *Principle of the Identity of Indiscernibles*. In this sense, they would have lost their identity in the sense that there is simply no *property* granting the particles’ identity; that is, no particle has as its individuating feature a property of its own. This would be a violation of so-called *bundle theories of individuality*. However, even though this is an option—and we shall discuss it in Sect. 4—this is not the only way the relation between the RV and NR systems can be developed.

This is the time to add another spoonful of metaphysics to the formalism. The tie between the limitations on the identity sign in the language and a metaphysical background took a distinct direction with (French and Krause 2006, Chap. 1). In their discussions on the many forms a *principle of individuality* may take, they classify—following Post (1963) again—a broad spectrum of such principles under the label *Transcendental Individuality* (henceforth, TI). In a nutshell, TI principles are those that attribute individuality to individuals by ‘something’ that goes beyond the individual’s qualities. So, TI principles help us keeping the concepts of individuality and discernibility apart: an individual is that particular individual it happens to be due to something over and above its properties. This seems recommendable because *two* individuals may share every property while still being *two* individuals, and that is important in the quantum domain due to quantum indiscernibility.

Candidates for such TI are principles such as *bare particulars* and Lockean *substrata* on the one hand, and also *primitive thisness* and *haecceities* on the other. As we know, bare particulars and substrata are usually thought of as an extra ingredient of the individual, so that it is usual to find them associated with so-called *constituent ontologies*. On the other hand, primitive thisnesses and haecceities are usually thought of as “non-qualitative” properties, properties that are not representing a quality (so that they do not count as a qualitative difference between two numerically distinct individuals), but are still understood as a property possessed by each individual (in the sense that each individual has its own thisness that accounts for its individuation). In general, such principles need not be associated with a constituent ontology (see Lowe 2003 for more on individuation principles and their differences). French and Krause (2006, p. 5, pp. 13–14, p. 140) suggest that the representation of the principles comprised under TI, regardless of their differences, can be reasonably achieved in terms of self-identity of the kind  $a = a$ . An individuation principle of the TI family may be formally understood as that relation which every individual has with itself, and only with itself, so that it is encapsulated by the reflexive law of identity  $a = a$  for every individual  $a$ .

Now, after such a small digression, we are ready to present the second link between the non-reflexive formalism and a metaphysics of non-individuals: *individuals*, according to French and Krause’s interpretation of TI principles, *are those things that have a transcendental individuality*, which on its turn happens to be expressed formally by the reflexive law of identity. In metaphysical terms, the RV would be dressed according to the following idea: to fail to be an individual something must not have a TI. In

formal terms: non-individuals are represented formally as those entities for which the reflexive law of identity fail.

So, there are at least two distinct approaches to individuality that seem to be associated with NR systems and which may be employed to capture the idea of a non-individual. On the first alternative, non-individuality and loss of identity are understood as indiscernibility without numerical identity, leading to a failure of the PII. On the second interpretation, non-individuality and the failure of identity are understood as a failure of a TI principle of individuality. We shall take the second interpretation as standard for the discussion which follows. This is due to the fact that, as we shall argue in Sect. 4, the failure of PII is indeed compatible with the acceptance of a universally applicable symbol of identity in some formal languages. So, NR would go farther than required in this first metaphysical interpretation. Anyway, this is still a live option.

The problem we have to face, then, is the following: how can the non-reflexive law of identity ever fail? As we have already mentioned while discussing da Costa's strategy to capture the idea that identity is meaningless, that failure must not be spelled in terms of negation, that is, as  $a \neq a$  for some  $a$ , but rather in the *non-application* of the relation of identity for some items (the non-individuals, of course!). Hence, non-reflexive logics are just the tool to deal with those items! There we have it. Metaphysics and formalism complement each other; *non-individuals*, according to TI principles, *are the entities not bearing a transcendental individuality*, and their formal representation is achieved in non-reflexive systems of logic. On the other hand, those items may also be indiscernible without being identical (given that identity does not apply), so an alternative metaphysics in which PII is violated is also available.<sup>3</sup>

In this section we attempted to make clear the connection between the formalism of non-reflexive logics and a metaphysics of non-individuals, as it is generally formulated. It seems to us that it is the second, anti-TI, formulation that French has in mind when claiming that non-reflexive systems help us making the RV a sensible position. Notice that once individuality is framed in terms of TI principles, it gets a linguistic representation in terms of the reflexive law of identity, and consequently, in every system where this law holds universally there are only individuals. Furthermore, it is impossible to deny this law by using negation; one must understand the failure/restriction of the law of identity as a kind of 'reduction of the scope of application' rather than strict violation through denial. Notice also that this choice of the relation between identity and TI makes it virtually impossible to keep identity in a language and use it to deal with non-individuals. But isn't this kind of strategy too strict, given the relevance of identity for us? That is the point the dilemma in the introduction raises to us. We shall have to face it now. Keeping in mind that identity here has a heavy metaphysical weight (it encompasses TI principles of individuality), we believe that it is easier to soften the blow of such criticisms.

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<sup>3</sup> To make sure, French and Krause (2006, p.150) do not seem to allow such a second option. They claim that when particles are regarded as non-individuals, the status of PII simply does not come into question; that is, it doesn't even fail, it simply does not apply. This happens because those entities have no identity to begin with. But notice that this assumes right from the start that non-individuality is cashed in terms of failure of identity according to our second option here, that is, TI individuality. Our point, as we shall argue later, is that one may cash non-individuality alternatively, in terms of entities for which the PII fails.



### 3 Identity lovers against NR logics

#### 3.1 The basic distinctions

As we have discussed in the previous section, by characterizing individuality through the bearing of some form of TI and by pointing out that this TI is formally captured by the reflexive law of identity, French and Krause managed to provide for a reasonable relation between *the metaphysical* characterization of (non-)individuality and the corresponding role of *identity relation in a formal system*. Interpreted with such a metaphysical weight, the formal relation of identity has obviously to be restricted in logical systems that are meant to deal with non-individuals. That is, if the reflexive law of identity is seen as attributing a kind of TI for everything, then if we are going to deal with non-individuals we will have to restrict identity.

To put the issues that will follow in a clearer fashion, it will be very helpful to set the terminology that the non-reflexive approach to the RV adopts, following the development of NR systems (which on its turn, aims at representing aspects of quantum mechanics). As a result of the development of non-reflexive systems and its metaphysical interpretation, we have the following three ingredients, *which are kept apart* for non-individuals:

- Cardinality* The cardinal of a collection is either taken as primitive or else defined (at least for the finite case; see [Arenhart 2012](#)). Whichever path is chosen, the number of elements in a collection is determined *independently of the sign of identity* in the system.
- Individuality* Individuality is closely related with identity, as explained in the previous section. The identity sign introduces individuality in a metaphysically robust sense (TI) (see also [French 2015](#), Sect. 5).
- Indiscernibility* Indiscernibility is a primitive relation that does not collapse neither on identity nor on cardinality. That is, indiscernible items may not count as one and need not be identical or different (see [French and Krause 2006](#), Chap. 7).

The claim then is that this conceptual framework is able to provide for a rigorous basis to ground the development of the metaphysics of the Received View. Notice that much of the conceptual work that is usually taken to be performed requiring identity (such as determining a cardinality) is now taken as being performed by an independent specific concept. This perception will be crucial for the discussion that follows.

In the next subsections we shall explore this relation between identity, TI, and non-reflexive systems. We shall do that by presenting some objections that were advanced against this kind of articulation of the RV.<sup>4</sup> We hope to convince the reader that the objections are far from refuting the non-reflexive view and that when we read identity as contributing to the individuality of an entity, in particular as attributing a form of TI to each entity, restricting identity seems to be a reasonable move to deal with non-individuals. In fact, the dilemma presented in the first section now reads as follows: either we accept that quantum particles have a kind of TI and the Received View fails,

<sup>4</sup> Recall that for most authors this is simply the only articulation of the RV available.

or else they have no TI, and the Received View is right. But now, who would seriously support the adoption of TI for quantum particles? There are not many people willing to advance such a defense, it seems. From now on, when we speak about the ‘*non-reflexive articulation of the RV*’, we mean the specific link forged by French and Krause between the metaphysical aspects of individuality conferred by TI and its formal counterpart captured by non-reflexive formal systems. From that point of view it seems entirely reasonable to have suspicions on identity: it is just too metaphysically loaded!

### 3.2 A problem with the formulation

The first worry about such an articulation comes from questions about the plausibility of its formulation. An objection that could be advanced against this kind of formulation of the RV was raised by Muller (2011, p. 231, fnt. 19). The objection in the particular case of Muller was not directly addressed to the non-reflexive articulation of the RV, but it could be so employed. It goes as follows: if we represent the attribution of some form of TI<sup>5</sup> with the reflexive law of identity, then in any language with identity every entity necessarily has a TI, given that the reflexive law of identity is an axiom. So, it becomes impossible to deny TI for any entity whatever, given that such a denial would imply denying a statement having the status of a logical necessity, which on its turn would lead us to a contradiction. So, it is not reasonable to represent TI by the reflexive law of identity.

This objection would render the efforts of non-reflexive logics useless in order to capture the idea of non-individual entities. The identification between non-individuals and ‘not having identity’, as explained in the previous section, would be hopeless, to say the least. Notice that the appeal of the argument consists in deriving undesirable consequences from the assumption that TI individuality is represented by the reflexive law of identity.

We must show now that the argument, as it is presented, can be resisted. First of all, as Muller remarks, that identification would certainly lead to trouble in languages with identity, in which the reflexive law of identity is an axiom. *In those languages*, due to the reflexive property of identity, we would attribute TI to everything. *In those languages*, denying the reflexive law of identity would amount to denying a logical axiom. However, nothing is being said about languages in which identity is restricted to one kind of entity only (as in Schrödinger logics) or about languages without the identity sign, in which identity may be defined as applying only to some entities, but not to others (as in quasi-set theory). That would render the idea that TI applies overall inapplicable, and for those languages the conclusion of the argument would not work. That is, in non-reflexive systems we deny that identity must apply to everything even when identity is in the language; the axiom of the reflexivity of identity still applies to *all the entities of a given kind*. However, the argument does nothing to prevent that identity does not apply to entities of other kinds.

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<sup>5</sup> Muller deals only with haecceitas, while we follow French and Krause and group under TI other kinds of individuality principles, like bare particulars and primitive thisnesses; that difference needs not concern us now.

This is also connected with a second point. What exactly is meant when we say that we deny a tautology (or a logical law, or a logical necessity)? In denying that an axiom of classical logic is valid in general, don't we have to accept that this 'axiom' is false in at least one interpretation of an alternative system in which the same formula may be expressed? Consider, for instance, intuitionistic logic. In denying the validity of some instances of the law of excluded middle, it is not the case that intuitionists accept its negation in its place. However, they do accept that the law may be false sometimes (mostly when we deal with infinite collections). For another example, consider some paraconsistent logics, like those in da Costa's  $C_n$  hierarchy. In denying the Explosion Law, it is certainly not accepted as an axiom (or as logically valid in the system) the negation of the Explosion Law, but some of its instances must be false in some valuations. So, the argument could go, in denying the universal validity of the reflexive law of identity in non-reflexive logics, are we not committed to accepting that it may be false sometimes?

As we have said in the previous section, in non-reflexive logics we do not accept the *negation* of the reflexive law of identity. Also, we don't have to accept that it must fail in at least some interpretations. Rather, we adopt its *restriction* in the form of its inapplicability. Here, 'inapplicability' is couched in terms of identity not making sense, not being a formula, for some kinds of terms. Recall from our discussion in the previous section that this is the formal counterpart of the idea of something not having TI. So, if this is correct, the link between metaphysics and logic that underlies the non-reflexive formulation of the RV is reasonable, in fact, but it does not go in the same lines as we think it is reasonable to reject some classical principles of logic in any non-classical logic with the same vocabulary.<sup>6</sup>

### 3.3 Problems with the fundamentality of identity

There is a second kind of worry, related with the first, that also appears in the literature (it may be found explicitly in Bueno [Bueno 2014](#), Sect. 4, and it also appears in related discussions by [Jantzen 2011, 2015](#), and by [Dorato and Morganti 2013](#)). It goes like this: identity is something just too fundamental to be dispensed with. It is required for our understanding of most of our conceptual machinery, such as cardinal attribution and even the application of concepts. So, it should not go. However, still according to these claims, keeping identity, at the same time, should not commit us with any metaphysically robust thesis, such as TI forms of individuality. In this sense, there is an agreement with the non-reflexive approach to quantum mechanics: TI should go. The efforts of such identity lovers is to show that identity, as a metaphysically thin notion, unavoidably permeates all over our conceptual machinery, so it cannot be eliminated, but rather should be taken (in general) as a primitive notion.<sup>7</sup>

<sup>6</sup> Furthermore, notice that even if in a system of logic  $L_2$  we deny a formula  $\alpha$  that is a logical law of a system  $L_1$ , we do not necessarily get a contradiction:  $\alpha$  may simply not be logically valid in  $L_2$ . That is, logical validity is always relative to a system.

<sup>7</sup> Primitiveness of identity is emphasized by [Dorato and Morganti \(2013\)](#) as well as by [Bueno \(2014\)](#).

Instead of examining separately each claim advanced for the fundamentality and necessity of identity,<sup>8</sup> we shall advance general considerations to the fact that the non-reflexive approach to quantum mechanics can dispense with the identity sign in some cases, allowing it to be endowed with a very specific function: attribute individuality. When we examine the claims that identity is important and not eliminable, what kind of considerations do we find? We find considerations that it is required for counting objects, for attributing cardinality to collections of objects, to distinguish objects through application of concepts, and even to attribute individuality in a very thin sense of individuality (certainly not a TI form; see the papers referred to in the previous paragraph for all those claims).

What can we say about the alleged involvement of identity in all such “activities”? The best we can do is to recall that NR logics keep most of those tasks that are usually attributed to identity clearly separated. Recall the conceptual distinction between cardinality, individuality and indiscernibility. We have disambiguated identity in NR logics in order to avoid identity in cases where it is not needed. So, for instance, when properties are found that distinguish two entities (electrons from protons, say), we need not employ the concept of identity, but rather the concept of discernibility (the negation of indiscernibility). When we need to talk about individuality or non-individuality, identity comes in, and we are able to separate adequately between individuals and non-individuals. So, the general idea is that NR logics in general and quasi-set theory in particular, have the conceptual resources to do almost everything that was done with identity. . . without identity. Identity is reserved to the individuality attribution role. The result is still a rich conceptual system: structures of Fock spaces employed in quantum mechanics may be ‘constructed’ inside quasi-set theory basing the construction only on non-individuals (see [Domenech et al. 2008](#)).

But what of the claim that identity is required to attribute individuality in a ‘thin sense’? Doesn’t the NR approach put too much weight on identity, committing us with a metaphysically robust notion of individuality? The answer to that worry comes in two parts. As a first stab on the question, it is clear that one could disagree with TI as the most adequate principle of individuality. However, if one agrees with [Lowe \(2003\)](#) that individuality is an explanatory notion, the alternative principle will have to be metaphysically robust anyway (such as a discerning property; see next section). So, even if one does not agree with TI, other principles for individuality may be just as metaphysically robust, it seems, and identity will have to bear some metaphysical weight anyway.

As the second part of the answer, one must consider the possibility of alternative thin notions of individuality. Those are said to be principles of individuality accounting merely for the plurality of entities, with no further metaphysical ingredient ([Dorato and Morganti 2013](#) and [Jantzen 2011, 2015](#) seem to have that in mind). Individuality, then, amounts to a mere fact of cardinality: whenever there are many items, they have identity in a thin sense and this accounts for their individuality. This is also a proposal advanced in another context by [Ladyman \(2007\)](#). What is required is that the concept of individuality does not have an explanatory robust sense, as required

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<sup>8</sup> This task was performed in related ways in [Krause and Arenhart \(2015\)](#) and in [Wehmeier \(2012\)](#).

by Lowe. Mere cardinality is all that matters to establish individuality. However, as we hope is clear from our previous exposition, identity is not required for that kind of approach. We may deal with those cases by employing the concept of cardinality. NR logics are able to account for such cases with the concept of cardinality. In fact, the NR approach even has the advantage of not conflating the concepts of identity, cardinality and individuality!

### 3.4 Problems with cardinality

Cardinality was involved in the discussion above in an essential way. It deserves a separated discussion, and it forms the third kind of worry we shall consider against the non-reflexive approach to the RV. We shall here present two main arguments relating identity and cardinality. The first argument first.

In the first case, the opponents main claim is the following: independently of whether it counts as a ‘thin’ notion of individuality or not, attribution of cardinality to collections is one of such activities that essentially involve identity, so that it is not possible to dispense with identity. Roughly speaking, the objection holds that whenever we have a cardinality, there must also be identity, and vice-versa. Jantzen (2015, p. 3) puts the issue in two parts, as follows:

If ‘numerical distinctness’ is understood in anything like the usual way, then non-individuals cannot be numerically distinct and yet fail to be identifiable. In the standard set-theoretic definition, cardinality essentially involves a notion of mapping or correspondence that is conceptually equivalent to labeling. Specifically, the cardinal associated with a set  $A$  (intuitively, the number of things in  $A$ ) is the smallest ordinal number  $n$  such that there is a bijection from the elements of  $A$  to the elements of  $n$ .

[. . .]

The relation between identity and cardinality is not a metaphysical fact, but rather a semantic one. I do not mean to suggest that there is some metaphysically necessary association between identity and cardinality. Rather, I am claiming that what it means for entities in a collection to be numerically distinct is for the collection to possess a cardinality greater than one. And part of what it *means* for a collection to possess a definite cardinality—on any standard account of cardinality—is for the entities to be identical with themselves and no others in the collection.

The objections go from cardinality to identity. Similar objections are framed in Jantzen (2011), Bueno (2014), with Dorato and Morganti (2013) also jumping from cardinality to identity (more on Dorato and Morganti coming soon). As Jantzen puts it in the first part, whenever we have cardinality we have also the possibility of identification of the items in the collection. In fact, cardinal attribution is closely related to labeling, so that an epistemological feature (identification) is unavoidably introduced.

Now, let us examine those claims. The first claim, as it is put, is just too strong. Set theory, or any theory that attributes cardinals to collections, does not grant identification of the elements labeled through an ordinal. In fact, as a simple counterexample,

consider the set of real numbers. It surely has a cardinal in standard set theory and, if we follow Jantzen and attribute this cardinal through an ordinal (*i.e.*, von Neumann's definition of cardinal), then we are also well-ordering the set of real numbers. No one doubts that such a well-ordering exists in set theories such as ZFC (Zermelo–Fraenkel with the axiom of choice), but we cannot benefit from its existence to *identify* the elements by their position in the well-ordering. That is, we cannot ask for the first element according to the well-ordering, for the second, and so on. As it is known, such a well-ordering is not even expressible in the language of ZFC. So, even in classical set theories the link between cardinality and identifiability is not granted.

Let us now take a closer look at the second claim in the quote of Jantzen. We may understand the argument as attempting to grant that the general applicability of identity is implied by the simple fact that we have a plurality of items, because in having more than one item we must be able to say somehow that they are *different*. Bueno seems to advance such a claim in Bueno (2014, p. 330), and Jantzen (2011) also advances such a claim. Difference is introduced because we must always have a first element, a second element, and so on (more technically: an ordinal is always present when there is a cardinal). In this case, the argument goes from the fact that we have a plurality to the universal validity of identity; identity (in fact, identification for finite collections at least) forces itself in such cases. If this is correct, the argument goes, it does not make sense to deal with formal systems restricting identity, they cannot account for pluralities of many entities without also implying identity. In the terminology we introduced before: cardinality would imply individuality by introducing identity for every item.

Now, to show how we can resist such an argument, we begin by remarking that non-reflexive logics, viewed as formal systems intentionally performing a metaphysical job, keep two important concepts—individuality and cardinality—separated (and this a virtue of these systems, it seems to us). In classical logic with identity we can use the identity sign to attribute a TI kind of individuality and also to account for the plurality of entities in most cases. However, in non-reflexive logics the tasks are separated. Individuality and non-individuality in the TI sense are a matter for identity, while cardinality *is not*. As one can see by the usual formulations of quasi-set theory (see French and Krause 2006, Chap. 7), cardinality is not introduced through the concept of ordinal: it is taken either as a primitive concept, detached from ordinals and identity, or else it is introduced as a defined concept, once again, defined with no use of identity, employing rather indiscernibility and other conceptual resources of quasi-set theory (see Arenhart 2012). So, in this sense, there is no danger that a plurality of entities may imply that identity applies: the concepts that do such jobs are unrelated for non-individuals!

But perhaps this is still not satisfactory. One could hold that the objection's main claim is that *even in systems of non-reflexive logic* one may always introduce identity to the entities forming a plurality. This is aimed at showing the implausibility of such systems. So, perhaps the original claim of the argument was that systems of non-reflexive logic are, after all, reflexive, and being reflexive they cannot account for the separation between cardinality and individuality made above, right? To ground that claim, consider, for instance, the usual example used as motivation for the introduction of cardinality in quasi-set theory by definition: we use radiation to ionize an Helium

atom in a cloud chamber. After applying the procedure a first time, we will see the track of one electron. Repeat the procedure and we will see the track of a new electron. Repeat the procedure and no further electrons will be extracted. So, we can claim that two electrons were extracted, without requiring identity. This procedure is modeled in the definition of cardinality in quasi-set theory, which is developed without using identity (again, see [Arenhart 2012](#)). The problem, as we are mentioning, is that Jantzen and Bueno claim that even this kind of approach fails to avoid identity. By claiming that there is a second electron, they say, we are claiming that there is another electron, that it is *not the same as the first one*. So, identity is back! The very meaning of cardinality implies identity.

This remark, however, is not correct. There are at least three considerations against the idea that the meaning of cardinality introduces identity.

First, even if it were correct, the argument would not prove the general case. Consider situations in which we weigh an amount of a certain kind of particles. We can discover how many of them are there by dividing the total weight by the weight of the kind of particles present. In this case, there is no way to attribute such a label as ‘first’ and ‘second’, and we understand very well what is going on. So, the argument against separating cardinality and identity seems at best to labor on a specific interpretation of one kind of example.

Second, one can even resist the argument by invoking Toraldo di Francia’s notion of *mock individuality* (see [Toraldo di Francia 1985](#)). By judging from the tracks left in the cloud chamber, we are already dealing with the systems in (semi) classical terms, the measurement was already performed and the problematical superposition is temporarily absent. In those cases, we can, if only for a moment, forget the idea that quantum particles are indiscernible and somehow ‘identify’ them through the tracks. However, this is only a mock individuality, attributed through the tracks, and it should not be imported to the metaphysical realm of the particles in the Helium atom, granting them identity. That is why we must confine ourselves to quasi-cardinality, and not to classical cardinals.

As a third point concerning cardinality and identity, we could not only argue that the association between cardinality and identity is not as Bueno and Jantzen claim, but also advance a clear understanding of this relation. The meaning of “cardinal” is established by its explicit definition. And there are many definitions of cardinality. When we advance a definition of cardinal inside a formal system, it is important to distinguish between what is going on in our object language and how we explain that in the metalanguage. Inside the language of quasi-set theory we can model the attribution of cardinality without invoking identity. We understand how the formulas are put together and how they end up defining a concept we call cardinality. Now, there is also an heuristic explanation in a metalanguage, which is our natural language. In our description in the metalanguage we benefit from asymmetries to ‘label’ things (the use of ‘mock individuality’ devices, as mentioned in the previous paragraphs). For instance, we label a ‘first’ and a ‘second’ electron track. In this case, natural language only superficially introduces identity to the particles; for serious metaphysical work we employ the definition developed in non-reflexive logics.

This approach is related to the idea that one should (or at least could) regiment natural language for serious metaphysical purposes whenever there is the danger that

natural language may cause confusion. We could underpin this point in an even clearer and more general fashion as follows. Suppose we would like to support the claim that the concept of ‘cardinal’ has meaning in the absence of identity; suppose we are looking for an explanation of it. We must provide for an *explicatum* for our *explicandum* (in this case, the concept of cardinal). According to the traditional explanation of what attributing a cardinal to a collection means (as formalized in ZFC, say), that concept is such that cardinality implies identity for the members of collections having a cardinal. Jantzen (2015, p. 3), for one, claims that it is part of the meaning of cardinal that the entities composing the plurality should have identity, in any account of the term. The non-reflexive analysis of cardinality, on the other hand, introduces cardinality without requiring identity. Does that imply that it is wrong or that it does not capture the same concept? Not really. We must recall that we are dealing with a very different kind of entity, one that violates the claims of our common sense intuitions in many respects. New entities require the framing of new concepts, that is the typical procedure of science. Think about the concept of property in quantum mechanics, for instance; it is the origin of many intricate problems, most of them due to our unallowed association of the meaning of property attribution for everyday objects. The same goes for the concept of cardinal: it must be reframed to account for the peculiarities of non-individuals.

This ‘reforming’ approach to concepts is not a novelty in philosophy of science. The explication of a concept may clearly depart from common usage for the benefit of clarity and fruitfulness. This is a clear lesson from the discussions about explication, for instance, developed by Carnap and other Logical Empiricists (see the historical discussion related with the concept of theory in Lutz (2012, Sect. 5.1), and also Dutilh Novaes and Reck 2015). So, even if our common use of ‘cardinality’ seems to require identity—which is also doubtful, but let us grant it for the sake of argument—it is not a problem that we have one explanation for this concept which is not in complete agreement with common use; even so we can claim we have provided for a useful clarification of this concept. For the concept of a cardinal we do have one such formalization accounting for cardinality without identity, and it is provided for using the apparatus of non-reflexive systems. It is not in agreement with common usage, perhaps, but it works for a rigorous enlightening of the concept. So, what else is required to grant the possibility of dissociation of identity and cardinality?

Let us now briefly consider a second kind of argument relating identity and cardinality; it was advanced by Dorato and Morganti (2013, Sect. 6). It may be understood as a kind of ‘inference to the best explanation’, and goes as follows: given that most accounts of quantum individuality present one difficulty or another, we may attribute those particles primitive identity. Primitive identity should be preferred not only on the grounds of its simplicity and conservativeness with philosophical entrenched beliefs (Dorato and Morganti 2013, p. 605), but also because “one could maintain that the “presence” of  $n$  particles at the formal level has a direct ontological counterpart, so that it can be concluded that those particles are  $n$  individuals” (Dorato and Morganti 2013, p. 606). That is, cardinality leads to primitive individuality.

How is that for an argument favoring identity? There are two main remarks to be made. First, concerning the quotation, notice that the best one may infer from the fact that there are  $n$  particles is that there are  $n$  items; when non-individuals are an option, the inference about the metaphysical nature of the entities may be relevant



questions if one assumes individuality beforehand. But (and here comes the second part), perhaps primitive individuality is the most simple alternative; it would rule over non-individuality on such pragmatic grounds. Notice first that this is no argument against the coherence or possibility of the RV as framed until now, but it only concerns the preferability of primitive individuality over the RV.

How can one evaluate such preferability claims? It seems that the conclusion that primitive individuality is preferable is not so straightforward. On the grounds of simplicity, it seems to fail by being closely related with problems concerning haecceities: the states of a system before and after a permutation of such particles are not the same, due to the identities of the items involved. However, this goes clearly against quantum mechanics, and any attempt to avoid the conclusion would introduce further complications that would count against the so-called simplicity of the approach. So, on this aspect, it is not as problem-free as Dorato and Morganti seem to think. On the grounds of conservativeness with traditional philosophical entrenched beliefs, the effectiveness of the argument will depend on how much weight one puts on such traditional beliefs and what one is willing to sacrifice in order to keep traditional beliefs. In the face of a scientific revolution, it seems that conservativeness is not the first virtue to be safeguarded. For those embracing metaphysical naturalism, or a form of it, then, conservativeness will not be a priority.

So, it seems that the argument by Dorato and Morganti do not establish that primitive identity should be granted on the face of cardinality. Cardinality facts may be assumed without implying necessarily identity.

It seems to us those are the main arguments challenging the NR approach to the RV. In particular, as we have seen, by denying that individuality and cardinality can be framed without identity, opponents seem to conflate some of the typical meanings of identity that the NR approach urges us to keep separated. By the framing of a formal system that distinguishes between the many concepts that go under the name ‘identity’, one is able to achieve some enlightenment on their use in quantum mechanics and on the role identity may play in some cases. As we have seen, ‘mock individuality’ takes care of cases in which we think identity is being introduced, and for metaphysical purposes there is a venerable tradition on conceptual regimentation according to which a rigorous explanation of a concept is allowed to depart from common usage. If one is willing to hold that some concepts must only be understood and explained with the use of identity, then a non-reflexive therapy may help us disentangle identity from other important concepts.

Now, even if that defense of a non-reflexive approach is successful, there is still room for further options. As we mentioned before, it is disputable whether identity really needs to be clothed with such a heavy metaphysical dress. Individuality by itself does not require TI. So, one may still agree with the RV while disagreeing with its non-reflexive articulation. How could that be?

#### **4 Non-individuals everywhere**

Our point in the previous section was that the objections on the same lines of those by Bueno and Jantzen are only addressed to a very specific shaping of the RV: the non-

reflexive foundation of the RV. Also, as we argued in the previous section, it is not at all clear that these objections work, and even if they did, it is not necessary that the RV as a view on the nature of quantum particles fails: it would be a particular rendering of the RV that would fail, or perhaps, more specifically, a particular rendering of the RV related with a particular formal system. Can't we think about alternative metaphysical renderings of non-individuals? Of course we can, in particular, we may frame the RV in such a way as to be safe from the above kind of objections, directed against the specific kind of interpretation of the role of identity advanced by French and Krause.

Let us begin by taking a closer look at the idea involved in the standard characterization of a non-individual by French and Krause, and the one most people have in mind when objecting to non-individuals. Given that individuals are particular objects  $a$  satisfying the condition  $a = a$ , non-individuals are the particular objects not satisfying this condition, that is, they lack self-identity. This is more than saying something about numerical identity, recall, this is saying that they don't have some form of TI. Of course, as we have been calling attention, this is due to the specific interpretation of identity given by French and Krause, relating identity and TI. Now, following similar lines, we may provide for alternative approaches for the metaphysical understanding of non-individuals which are not related in that particular way with a metaphysically robust interpretation of identity.

The gate is open for many such alternatives when we recognize the fact that in the metaphysical literature the very idea of something acting as a 'principle of individuality' may be characterized in a plurality of ways. Allowing for such 'failures of application' of a principle to a particular object should then also be viewed as a way in which such a view can be termed a metaphysics with non-individuals. That is, one must acknowledge that the particular objects of a given scientific theory may be classified as individuals or as non-individuals, so that a failure of the application of some previously chosen principle of individuality does not always count against the validity of that same principle, but may be seen as attesting to the fact that we are facing a non-individual. Before approaching such alternatives, one doubt may still be left hanging in the air: when a principle of individuality fails, doesn't that mean that it is false or incorrect? Not always, as we shall briefly point out.

As a first point, principles of individuality were supposed to deal *with every particular item*, and when an alleged counterexample to the principle appeared, the principle was thought of as failing to be a reasonable principle for individuality. As we mentioned, this is due to the fact that the ontological category of non-individuals was not acknowledged as a legitimate one, and every particular object was thought to be an individual. The problem, then, was "only" to provide for the correct principle of individuality. However, when the category of non-individuals is recognized, there is conceptual space for the rise of situations in which a principle of individuality may fail to apply and still not be strictly false (we shall discuss such cases soon). As a second relevant point, this idea (*i.e.* that whenever a principle of individuality does not apply implies that it must be an incorrect principle) is a consequence of the fact that most of those theories identify the concepts of 'particular' with 'individual object'. We do not have the space to discuss the issue here with full details, but as Lowe (1998, Chap. 3) has argued, the category of 'particular' may encompass distinct subcategories, such as objects, quasi-objects, individuals, non-individuals, and so on. That is, being a partic-

ular is more general than being an object or an individual, which leaves space for some items to be particulars and still not be individuals. So, allowing that particulars may come in at least two kinds, individuals and non-individuals, we may clarify the field and allow for particulars which fail to satisfy the principle of individuality, whatever it is. Of course, these considerations are not proofs that there are non-individuals or something like that, but they are enough to grant that the idea of a restricted principle of individuality (a principle that does not apply to every particular) makes sense.

However, that possibility does not mean we may save any principle of individuality whatever from being incorrect by providing for non-individuals in quantum mechanics. Assuming, as we are, that quantum mechanics may be seen as describing some kind of non-individuals, some restrictions are imposed by quantum mechanics itself if a metaphysical theory is to underpin the RV (and this provides for the first kind of constraint on a scientifically informed metaphysics; see for instance [Arenhart \(2012\)](#)). As two minimal requirements for a principle of non-individuality compatible with quantum mechanics, a theory of individuals/non-individuals must have the following two features:

*Indiscernibility* the theory must account for an item's indiscernibility with other items of the same kind.

*Cardinality* the theory must allow for us to account for the precise number of non-individuals in every situation without the resources of the principle of individuality.

As we mentioned, to begin with, it is not every theory of individuals that comes equipped with an accompanying theory of non-individuals. Many of the traditional theories implicitly assumed the view that every particular is an individual. So, failure of application of the principle of individuality amounts to failure of the theory as a whole. In the second place, even if we suppress that implicit premise, we may still judge an account of non-individuality for quantum mechanics by its own merits and as to how it meets the two requirements above. Those requests are not satisfied by the traditional theory of bundles, for instance. According to this view, an object is a bundle of properties (when this is taken as a kind of composition principle), and its uniqueness is granted by a strong version of the Principle of the Identity of Indiscernibles: no two things have every property in common. As is well-known, when the individuality of particles depends on the same sets of properties (not relations, which we shall discuss later) quantum particles render this principle false (see [French and Krause 2006](#), Chap. 4). So, a straightforward adherence to this view would lead to adoption of a false principle.

Now, there are some ways to overcome this trouble and render the PII inapplicable, for instance, instead of false. One of such alternatives is provided by French and Krause themselves. It goes as follows. Suppose we adopt spatial position as a principle of individuation (which amounts to STI: space-time individuation). This principle works well for classical mechanics, for instance, where a principle of impenetrability (IA) holds for classical particles: no two particles have the same spatio-temporal location. This vindicates a form of PII which quantifies over properties for classical particles, with spatio-temporal properties included in the scope of the quantifiers: no two entities have the same properties. However, for quantum particles IA does not hold. What to

do then? Instead of claiming that PII *fails* for quantum particles, one may go on and claim that it simply does not apply. As French and Krause (2006, p. 153) put it:

one could respond by arguing that to be an individual is to be understood in terms of what we have called Space-Time Individuality, and insisting on the metaphysical importance of IA. In that case, if (IA) does not hold, then the entities concerned are simply not individuals and PII is not violated, rather, it simply fails to apply. This would amount to a further version of the Received View.

The general idea is simple: on this account, IA is behind the STI granting individuals' their individuality. When it fails, as it does in quantum mechanics, we simply don't have individuals anymore. The PII applies to individuals (and given IA, it holds for them when quantification over spatial properties is allowed). For non-individuals it simply does not apply due to the fact that quantum particles violate IA. There we have the first lines of an alternative version of RV, one which does not require that identity should be interpreted as playing a metaphysically robust role. The relation of identity is deflated to state merely that there is one thing (when the relation obtains) or two things (when it doesn't).

But if that kind of move is allowed (and, following French and Krause, we think it is), the main strategy behind French and Krause's idea may be taken further. We don't need to adhere to STI and wait for the failure of IA to grant some non-individuals in our metaphysics. One possibility is to follow the suggestion by Rodriguez-Pereyra (2004), according to whom the bundle theory is compatible with distinct but indiscernible bundles of properties. If we allow that PII is the principle conferring individuality to particulars and that, as quantum mechanics seems to establish, PII does not work in the case of bosons at least, then this version of the bundle theory may have both individuals and non-individuals.<sup>9</sup> It would go like this: (i) when unity is granted by PII, there we have an individual, *intrinsically discerned* from everything else by its properties; (ii) when PII fails, so that a plurality of indiscernible bundles is found, there we have non-individuals: no intrinsic feature plays the role of individuating a particular entity. In this second case, the cardinality of the indiscernible bundles may be taken as a brute fact or else be grounded on the fact that property-instances have well determined cardinality conditions. Once again, identity is not required to play a substantial role for non-individuals here (for individuals it does collapse on indiscernibility, so it has more content than usual). In this case, even though PII may be false, its failure does not invalidate the whole approach to particulars, it just advocates in favor of the claim that we are facing non-individuals.

For a distinct yet related approach, consider the so-called weak discernibility defenses of objects in quantum mechanics (see Muller and Saunders 2008 and Muller 2011). We shall neither review the whole subtleties of the literature concerned with this issue here nor comment on the most audacious purposes of the weak discernibility strategy, but the main idea is that we may allow relations in the scope of PII and provide for an account according to which it doesn't fail in quantum mechanics (at least not

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<sup>9</sup> This may not be how Rodriguez-Pereyra sees the facts, of course.

for fermions). Some relations which are symmetric and irreflexive are said to *weakly discern* objects: it is not possible for an object to have such a relation with itself, and this relation, being symmetric, seems to allow for quantum permutation symmetry. Weak discernibility should be compared with “absolute discernibility”: when there is a property that object *a* has and *b* doesn’t, then *a* and *b* are *absolutely discernible*. Quantum objects are absolutely *indiscernible*, but, according to Muller and Saunders, are weakly discernible, so that allowing weakly discerning relations in the scope of PII saves it in quantum mechanics: no two particles share every property and relation. We shall not discuss whether weak discernibility is legitimate discernibility; what concerns us now is that Muller and Saunders (2008, p. 504) define individuals as *absolutely discernible* objects. Given that quantum objects are absolutely indiscernible they are not individuals according to Muller and Saunders’ definition (see the related discussion on discernibility and individuality in Caulton and Butterfield 2012). Now, given the approach of Muller and Saunders themselves, weak discernibility implies that quantum objects are indeed particular objects, in fact, *non-individual* objects (even though they do not use this terminology). So, again, this is clearly an approach to non-individuality which does not require a substantial interpretation of identity. This approach differs from the previous one, which allowed for the failure of PII. If one has doubts about the power of some kinds of relations to discern entities, then one can still adhere to the previous paragraph’s approach; on the other hand, if one considers that relations do discern and may be employed to save PII, then this approach seems to fit better. Here non-individuals obey a weaker form of PII; in the previous paragraph, non-individuals do not obey such principle. Notice, however, that both approaches assume that individuality is conferred by a property unique to the individual, understood in general as an *intrinsic property*.

It may be instructive to discuss, even if rather briefly, the idea that weak discernibility may be employed to ground a metaphysics of non-individuals. In his discussions on metaphysical underdetermination, from which we quoted before, French saw weak discernibility as providing further support for the *individuals* side (see for instance French 2011, Sect. 9). In fact, as French took it, the approach could be coupled with a *thin notion* of individuality and be seen as granting some form of ‘thin’ individuality in quantum mechanics. As we have seen, this is not how Muller and Saunders see it, however. They approach individuality rather as a *thick* concept, somehow endorsing Lowe’s (2003) claim that individuality plays an explanatory role (of course, Muller and Saunders do not state the issue that way). There is a sense, then, according to which weak discernibility contributes rather to the Received View. Of course, French would still have his metaphysical underdetermination in place, but we need not follow him in interpreting weak discernibility as lending support specifically to the individuality horn of the underdetermination.

That brings to the center of the stage questions about the proper role of individuality in science and how should it be better understood. Questions such as “are ‘thin individuals’ legitimate?” and “is individuality an explanatory notion?” are central to this debate and its further development. Depending on how we address those issues, much could be gained in other relevant parts of the philosophy of science. Consider, for instance, the approach by Ladyman in (2007). In the context of an ontic structural

realist approach, Ladyman contends that when we only take relations into account (and for the ontic structural realist relations are all we must take into account), what we have in quantum mechanics is, at best, weak discernibility. Fermions are weakly discernible, bosons are not. Anyway, Ladyman claims that a cardinal can be reasonably introduced in both contexts. That is, we may always know how many items there are in a structure: for fermions, weak discernibility grounds the claims of cardinality; for bosons, those claims are primitive, ungrounded. This cardinality grants their identity and their individuality. It seems to us that instead of conflating the concepts of identity, individuality and cardinality, it would be clearer to accept that such objects do not have identity and individuality conditions, so that mere cardinality is not identified with individuality. In this case, at least some versions of non-eliminative ontic structural realists, as Ladyman's, could also be seen as much more naturally providing support for a metaphysics of non-individual objects.

As a final possible theory on individuals and non-individuals, let us consider Bueno's approach to individuality in [Bueno \(2014\)](#). According to Bueno, an individual is a particular object that is discernible from every other individual and is re-identifiable through time. Now, even if we accept Muller and Saunders's proposal of weak discernibility (which is not really considered by some to be an approach to *discernibility*, recall), this would not be enough to render quantum particles individuals on Bueno's account: as is well-known, quantum particles are not re-identifiable through time. So, adopting this approach to individuals would grant that quantum particles, by failing one of the conditions, are not individuals. Once again, notice that this approach to individuality puts no pressure on synchronic identity to play a double duty on individuation and numerical difference. So, again, this is compatible, as [Bueno \(2014\)](#) claims, with a deflationist account of identity.

So, in the end, the moral is this: non-individuality may be understood in a plurality of ways. This should come as no surprise, given that individuality is also understood in many ways. Of course, just as there is dispute over an appropriate principle of individuality, there is also dispute over an appropriate understanding on non-individuality, but that fact *per se* should not trouble us; as philosophers, we should be already used to it. What is relevant is that a particular system of logic, such as non-reflexive logic, is not constitutive of the meaning of non-individual as a metaphysical concept. There are perfectly respectable ways to render the notion metaphysically clear. In some sense, this is analogous to Lowe's contention that the notion of 'object' should be understood metaphysically, not related with typical Quinean definition through linguistic concepts such as being the value of a variable and other related syntactical concepts (see [Lowe \(1998, Chap. 3\)](#); it is important to notice that [Muller and Saunders \(2008\)](#) do adopt a typical Quinean framework in their work). In this sense, objections against the formal system are not the end of the accompanying metaphysical position, and vice-versa.

## 5 Conclusion

We have examined the RV in quantum mechanics from a formal and a metaphysical point of view. As we have seen, traditionally the association between both approaches

is thought of as being very close: as the quote of French in the introduction shows, *the* articulation of the RV is thought of as being made in terms of non-reflexive logics. This articulation has framed the general lines in which the discussion has developed in current literature, with friends of non-individuals mostly adhering to non-reflexive logics and the foes of non-individuals criticizing the view by attacking such systems.

Here we have discussed how this traditional articulation was developed so that logic and metaphysics contribute to our understanding of the failure of identity. Two main steps may be distinguished: the creation of Schrödinger logics by da Costa and of quasi-set theory by Krause on the one hand, and the metaphysical interpretation of the formalism by French and Krause on the other (with an earlier proposal by Krause also taken into account in our exposition). This provides for the most well-known articulation of the RV nowadays. We have argued that this articulation takes into account a very specific metaphysical view of non-individuals, a view according to which non-individuals are cached in metaphysical terms by a lack of the TI kind of individuality. It is precisely in this vein that non-reflexive systems of logic are said to capture the intuitive idea of something failing to be an individual: identity fails in the formal system, which is a sign that TI fails for the corresponding entities.

Criticism addressed to such an approach does not seem to take into account the specifically metaphysical interpretation of identity that is peculiar to this approach. The general claim seems to be that identity, even if understood in a deflated sense, cannot be abandoned mainly due to the necessity of identity to define and talk about cardinality and individuality. Our claim has been that non-reflexive logics solve those difficulties by having the conceptual resources to do everything identity was supposed to do without having to invoke identity. That is, identity is used in multiple senses, some of them may be captured by distinct concepts, such as cardinality and indiscernibility. This is a virtue of such systems, it seems, given the disambiguation of identity and the clear metaphysical picture that emerges from this construction.

Clear and formally well established as it is, though, the idea of a non-individual needs not to be developed exclusively along those lines: as we have argued, the concept of a non-individual may be framed in alternative ways. We could put it thus: the NR approach to the RV is not forced on us by Physics. Of course, this could follow from the already well known *Underdetermination of Metaphysics by the Physics* (see [French and Krause 2006](#), Chap. 4); what we have contributed is to show that the ‘non-individuals horn’ produces its own underdetermination. As we mentioned in passing, this should be expected, given that a strict naturalism on metaphysics has little hope to offer definitive answers to those questions. It seems that our options include the possibility to address those questions with a more robust metaphysical methodology or else to embrace a form of quietism about such issues.

Anyway, however one is going to address such methodological questions, by sticking to the metaphysical problem of individuality, we were able to discern answers to the question of what is an individual that were also compatible with the existence of non-individuals. In particular, those approaches to non-individuality did not require any kind of commitment with identity representing TI. That is, some things may form collections with cardinality greater than one (be a plurality), be indiscernible from each other, but still not be individuals in some metaphysically robust sense of individuality. Consequently, in these alternative accounts of non-individuality the failure

of individuality does not amount to a failure of identity, given that identity, taken by itself, is not the individuality-conferring device in these interpretations (rather, intrinsic properties or other individuating devices act as the individuating principle). In this sense, there is no need to necessarily associate non-reflexive systems of logic with these approaches to non-individuality.

Realizing that there is a broad spectrum of theories of non-individuality allows us, in particular, to make clear that most arguments against the RV are generally framed against the standard, non-reflexive approach to the RV. As we hope to have shown, these arguments are not compelling, and further, they have no impact on alternative approaches to the metaphysical concept of a non-individual. Thus, the RV is in fact a rich subject awaiting for further development and articulation. Quantum mechanics provides the clues and may still be seen as illuminating some foundational work in metaphysics. It is up to us to develop and study those approaches.

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