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A generalized definition of Bell's local causality

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Abstract This paper aims to implement Bell's notion of local causality into a framework, called local physical theory, which is general enough to integrate both probabilistic and spatiotemporal concepts and also classical and quantum theories. Bell's original idea of local causality will then arise as the classical case of our definition. First, we investigate what is needed for a local physical theory to be locally causal. Then we compare local causality with Reichenbach's common cause principle and relate both to the Bell inequalities. We find a nice parallelism: both local causality and the common cause principle are more general notions than captured by the Bell inequalities. Namely, Bell inequalities cannot be derived neither from local causality nor from a common cause unless the local physical theory is classical or the common cause is commuting, respectively.

Keywords Local causality · Bell inequality · Common cause

1 Introduction

Local causality is the principle that causal processes cannot propagate faster than the speed of light. This does not mean that in a physical theory subject to this principle no correlation between spatially separated events can exist; a correlation can well be brought about by a common cause in the past of the events in question. However, since all causal processes propagate within the light cone, fixing the past of an event in a

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detailed enough manner, the state of this event will be fixed once and for all, and no other spatially separated event can contribute to it any more.

In a nutshell, this is the idea which becomes primary focus in John Bell's (2004) seminal papers initiating a whole research program in the foundations of quantum theory. In these papers Bell translated the intuitive idea of local causality into a probabilistic language opening the door to treat the principle in a theoretical setting and to test its experimental validity *via* the Bell inequalities derived from the principle. The logical scheme of this translation was the following: if physical events are localized in the spacetime in a certain independent way, then they are to satisfy certain probabilistic independences. Be this manual as intuitive as it is, to apply it in a formally correct way one had to wait until the advent of a mathematically well-defined and physically well-motivated formalism which was able to integrate spatiotemporal and probabilistic concepts. Without such a framework one could not account for the (otherwise intuitive) inference from relations between spacetime regions to probabilistic independences between, say, random variables. The most elaborate formalism offering such a general framework is quantum field theory, or its algebraic-axiomatic form, algebraic quantum field theory (AQFT).

Thus, it comes as no surprise that AQFT has soon become an important medium to pursue research on the Bell inequalities (Summers and Werner 1987a, b; Summers and Werner 1988; Halvorson 2007); relativistic causality (Butterfield 1995, 2007; Earman and Valente 2014); or the closely related (see below) common cause principle (Rédei 1997; Rédei and Summers 2002; Hofer-Szabó and Vecsernyés 2012a, 2013a). In this paper we follow the route pioneered by the algebraists, but we do not go as far as AQFT. Our aim is simply to establish a *minimal framework* which is needed to formulate Bell's notion of local causality in a strict fashion. Thus we will borrow only a part of AQFT to represent something which we will call a *local physical theory*. A local physical theory is a formal structure integrating the two most important components of a general physical theory: spacetime structure and algebraic-probabilistic structure. Our secondary aim in this paper is to clarify the relation of Bell's local causality to such other important notions as local primitive causality, common cause principle and the Bell inequalities.

There is a renewed interest in a deeper conceptual and formal understanding of Bell's notion of local causality. Travis Norsen illuminating paper on local causality (Norsen 2011) or its relation to Jarrett's completeness criterion (Norsen 2009); the paper of Seevinck and Uffink (2011) aiming at providing a 'sharp and clean' formulation of local causality; or Henson's (2013) paper on the relation between separability and the Bell inequalities are all examples of this inquiry. On the other hand, there is far from being a consensus on what the sound notion of local causality in the algebraic approach should amount to. Just to mention a few, Brunetti et al. (2003) claim that the notion of locality actually consists in two components: localizability in spacetime and the so-called kinematic independence. Summers (1990, 2009) specifies independence in a number of different ways. Ruetsche (2011) regards relativistic covariance as the single manifestation of local causality, whereas Rédei (2014) takes the position that only a whole hierarchy of conditions expresses local causality. Our research runs parallel in some respect to these investigations and we will comment on the points of contact underway.

In Sect. 2 we fix our mathematical framework, called local physical theory and list some important relativistic causality principles. In Sect. 3 we formulate Bell's notion of local causality in a local physical theory in a general way including also the noncommutative case. In Sect. 4 we compare local causality with the common cause principle and relate both to the Bell inequalities. We conclude the paper in Sect. 5.

This paper is the philosopher-friendly version of our more detailed and more technical work (Hofer-Szabó and Vecsernyés 2015). Many points (such as local causality in a non-atomic local physical theory; local causality in stochastic dynamics; its complex relation to other locality and causality concepts, etc.) which are treated in a more elementary way here obtain a more detailed mathematical analysis there. We will not refer to these results point-by-point in the paper.

2 What is a local physical theory?

First we set the framework, called local physical theory, within which probabilistic and spatiotemporal notions can be treated in an integrated way. Before introducing it in a full-fledged form, let us briefly sketch the motivating idea behind. In a local physical theory the observable quantities can be localized within bounded spacetime regions. Hence, a local physical theory associates spacetime regions to local observable algebras generated by local observable events or quantities (represented by σ-algebra elements or by self-adjoint elements of a C^*-algebra). The association is regulated by the following two physically motivated rules. First, if an observable can be localized in a spacetime region, then it can also be localized in a bigger region containing the former. Second, observables localized in spatially separated regions are co-possible—as physicists put it, they do not disturb one another. These requirements can be expressed in the following mathematically sound way:

Definition 1 A *local physical theory* is a net $\{\mathcal{A}(V), V \in \mathcal{K}\}$ associating algebras of events to spacetime regions which satisfies *isotony* and *microcausality* defined as follows (Haag 1992):

1. *Isotony.* Let $\mathcal{M}$ be a globally hyperbolic spacetime and let $\mathcal{K}$ be a covering collection of bounded, globally hyperbolic subspacetime regions of $\mathcal{M}$ such that $(\mathcal{K}, \subseteq)$ is a directed poset under inclusion $\subseteq$. The net of local observables is given by the isotone map $\mathcal{K} \ni V \mapsto \mathcal{A}(V)$ to unital C^*-algebras, that is $V_1 \subseteq V_2$ implies that $\mathcal{A}(V_1)$ is a unital C^*-subalgebra of $\mathcal{A}(V_2)$. The *quasilocal algebra* $\mathcal{A}$ is defined to be the inductive limit C^*-algebra of the net $\{\mathcal{A}(V), V \in \mathcal{K}\}$ of local C^*-algebras.
2. *Microcausality* (also called as *Einstein causality*) is the requirement that $\mathcal{A}(V')' \cap \mathcal{A} \supseteq \mathcal{A}(V), V \in \mathcal{K}$, where primes denote spacelike complement and algebra commutant, respectively.

If the quasilocal algebra $\mathcal{A}$ of the local physical theory is commutative, we speak about a *local classical theory*; if it is noncommutative, we speak about a *local quantum theory*. For local classical theories microcausality fulfills trivially.[1]

[1] We note that our definition of a local physical theory does not embrace models beyond the Tsirelson bound. In order to incorporate also such models (Popescu–Rohrlich box) one should generalize the net of

Up to now, there was no mention of probabilities. Probabilities of events as observable quantities are given by states on the quasilocal observable algebra of the theory. A *state* $\phi\colon \mathcal{A} \to \mathbf{C}$ in a local physical theory is a normalized positive linear functional on $\mathcal{A}$. A state assigns *expectation values* to observable quantities of the theory. If the observable quantity is an *event*, namely, a projection in a local observable algebra, then its expectation value is the *probability* of the event. This notion of events comes from the translation of *classical events* represented by elements of a σ-algebra (Ω, Σ) into *projections* in the commutative function algebra on Ω, namely, translation of a measurable set into its characteristic function. In this translation a state on the commutative function algebra will define a probability measure on the σ-algebra. Since generic C^*-algebras do not contain (enough) projections, one usually considers special C^*-algebras, namely von Neumann algebras. The canonical way of this replacement uses the GNS representation $\pi_\phi\colon \mathcal{A} \to \mathcal{B}(\mathcal{H}_\phi)$ corresponding to the state ϕ, which maps the net of C^*-algebras into a net of C^*-subalgebras of $\mathcal{B}(\mathcal{H}_\phi)$. Closing these subalgebras in the weak topology one arrives at a net of local von Neumann observable algebras: $\mathcal{N}(V) := \pi_\phi(\mathcal{A}(V))''$, $V \in \mathcal{K}$. Contrary to generic C^*-subalgebras, von Neumann algebras are rich in, moreover, generated by their projections. The net $\{\mathcal{N}(V),\, V \in \mathcal{K}\}$ of local von Neumann algebras also obeys isotony and microcausality, hence one can also refer to a net $\{\mathcal{N}(V),\, V \in \mathcal{K}\}$ of local *von Neumann algebras* as a local physical theory. Although, the local σ-algebras of classical observable events provided by the projections of the local abelian von Neumann algebras are not the most general σ-algebras, still they provide us a rich enough set of examples for classical theories.

One can introduce a number of important locality and causality concepts into the above formalism. Some of them refer only to the observable quantities, i.e. to local observable algebras, some others also involve the states on them. (For a detailed motivation of these concepts see Earman and Valente 2014).

Local primitive causality. $\mathcal{A}(V'') = \mathcal{A}(V)$ holds for any globally hyperbolic bounded subspacetime region $V \in \mathcal{K}$.

Local primitive causality is the requirement that the local algebra associated to a region contains just as many observables as the algebra associated to the causal shadow of the region. Local primitive causality does hold in many AQFTs, but is typically violated in stochastic local physical theories.

A local physical theory satisfying local primitive causality also satisfies local determinism and stochastic Einstein locality:

Local determinism. For any two states ϕ and ϕ' and for any globally hyperbolic spacetime region $V \in \mathcal{K}$, if $\phi|_{\mathcal{A}(V)} = \phi'|_{\mathcal{A}(V)}$ then $\phi|_{\mathcal{A}(V'')} = \phi'|_{\mathcal{A}(V'')}$.
Local determinism is the requirement that fixing the state on a region, the state of any observable in the causal shadow is also fixed.

Footnote 1 continued
local algebras to a net of *order-unit vector spaces*. See (Summers and Werner 1987a) and (Popescu and Rohrlich 1994).

Stochastic Einstein locality. Let $V_A, V_C \in \mathcal{K}$ such that $V_A \subset V_C''$ and $V_C \subset J_-(V_A)$, where $J_-(V_A)$ is the causal past of V_A. If $\phi|_{\mathcal{A}(V_C)} = \phi'|_{\mathcal{A}(V_C)}$ holds for any two states ϕ and ϕ' on $\mathcal{A}$ then $\phi(A) = \phi'(A)$ for any projection $A \in \mathcal{A}(V_A)$.
Stochastic Einstein locality is the requirement that fixing the state on a region, the probability of any event localized in the (future part of) the causal shadow is also fixed.

The next local causality requirement is Haag duality (which can be required only in local quantum theories). A net satisfies Haag duality if

$$\mathcal{A}(V')' \cap \mathcal{A} = \mathcal{A}(V) \tag{1}$$

for all bounded globally hyperbolic subspacetime region $V \in \mathcal{K}$. If a net satisfies Haag duality, then it also satisfies local primitive causality. Note that microcausality alone does not entail local primitive causality. Haag duality is a stronger requirement than microcausality in the sense that the local algebras are "fat" enough to contain *all* observables which commute with the observables localized in their spacelike complement.

Finally, one can go over to the global version of the above local causality concepts (entailed by the local ones):

Primitive causality. Let $\mathcal{K}(\mathcal{C}) \subseteq \mathcal{K}$ be a covering collection of a Cauchy surface $\mathcal{C} \subset \mathcal{M}$ and let $\mathcal{A}(\mathcal{K}(\mathcal{C}))$ be the corresponding algebra. Then $\mathcal{A}(\mathcal{K}(\mathcal{C})) = \mathcal{A}$.

A local physical theory with primitive causality also satisfies

Determinism. If $\phi|_{\mathcal{A}(\mathcal{K}_\mathcal{C})} = \phi'|_{\mathcal{A}(\mathcal{K}_\mathcal{C})}$ for any two states ϕ and ϕ' on $\mathcal{A}$ then $\phi = \phi'$.

In the rest of the paper a local physical theory obeys only isotony and microcausality by definition without any other locality and causality constraints. Especially, this means that no prescription on states on $\mathcal{A}$ are required. We turn now to Bell's notion of local causality.

3 Bell's notion of local causality in a local physical theory

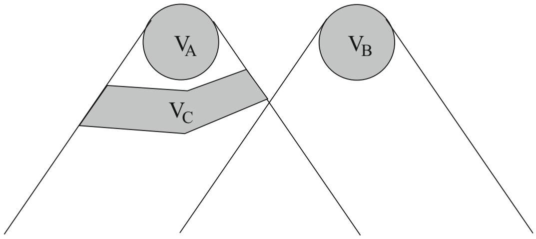
Local causality has been playing a central notion in Bell's influential writings on the foundations of quantum theory. To our knowledge it gets an explicit formulation three times: in Bell (2004). In this latter posthumously published paper "La nouvelle cuisine", for example, local causality is formulated as follows:[2]

> "A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region V_A are unaltered by specification of values of local beables in a space-like separated region V_B, when what happens in the backward light cone of V_A is already sufficiently specified, for example by a full specification of local beables in a space-time region V_C. " (Bell 2004)

(For a reproduction of the figure Bell is attaching to this formulation see Fig. 1 with Bell's caption.) Bell elaborates on his formulation as follows:

[2] For the sake of uniformity we slightly changed Bell's notation and figure.

Fig. 1 Full specification of what happens in V_C makes events in V_B irrelevant for predictions about V_A in a locally causal theory



> "It is important that region V_C completely shields off from V_A the overlap of the backward light cones of V_A and V_B. And it is important that events in V_C be specified completely. Otherwise the traces in region V_B of causes of events in V_A could well supplement whatever else was being used for calculating probabilities about V_A. The hypothesis is that any such information about V_B becomes redundant when V_C is specified completely." (Bell 2004)

The notions featuring in Bell's formulation has been target of intensive discussion in philosophy of science. Here we would like to give only a brief exposé of them.

The notion "beable" is Bell's neologism. (See Norsen 2009, 2011.) "The *be*ables of the theory are those entities in it which are, at least tentatively, to be taken seriously, as corresponding to something real" (Bell 2004). The clarification of the "beables" of a given theory is indispensable in order to define local causality since "there *are* things which do go faster than light. British sovereignty is the classical example. When the Queen dies in London (long may it be delayed) the Prince of Wales, lecturing on modern architecture in Australia, becomes instantaneously King" (p. 236).

Beables are to be local: "*Local* beables are those which are definitely associated with particular space-time regions. The electric and magnetic fields of classical electromagnetism, $\mathbf{E}(t, x)$ and $\mathbf{B}(t, x)$ are again examples." (p. 234). Furthermore, local beables are to "specify completely" region V_C in order to block causal influences arriving at V_A from the common past of V_A and V_B. (For the question of complete *vs*. sufficient specification see (Seevinck and Uffink 2011) and our reply (Hofer-Szabó 2015).

One can "translate" Bell's above terms in the following way. In a classical field theory beables are characterized by sets of field configurations. Taking the *equivalence classes* of those field configurations which have the same field values on a given spacetime region one can generate local σ-algebras. Translating σ-algebras into the language of abelian von Neumann algebras one can capture Bell's notion of "local beables" in the framework of a local physical theory. More generally, one can apply the term "local beables" both for abelian and also for non-abelian local von Neumann algebras, hence treating local classical and quantum theories on an equal footing. We note here that our use of beables and hence our upcoming definition of local causality transcends Bell's intuition and original definition in an important sense. Elements of a noncommutative local algebra can readily be interpreted operationally as measurement outcomes but can hardly be as something ontological. Moreover, nothing in our formalism expresses Bell's explicitly stressed requirement that beables should "correspond to something real". On the one hand we think that at this level of generality, where neither the dynamics nor any other features of the system is specified, we cannot

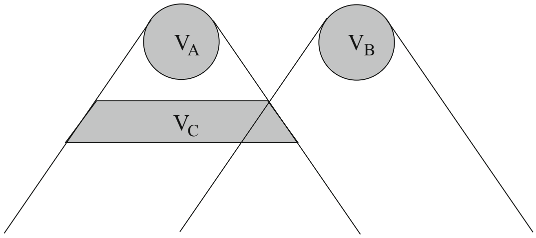


Fig. 2 A region V_C satisfying Requirements (i)–(iii)

determine which elements of the local algebras should be regarded as local beables. On the other hand, we acknowledge that an ontological theory of noncommuting observables would be highly desirable, if not indispensable, to better understand the meaning of noncommutative local causality, especially where it differs (see below) from Bell's original ideas based on classical beables.

Finally, "complete specification" can be translated into this framework as a probability measure having support on the local equivalence class of a single specified configuration. In the abelian von Neumann language this corresponds to a *pure state* on the local von Neumann algebra in question with value 1 on the projection corresponding to the local equivalence class of the specified configuration. If the local algebras of the net are *atomic* (which, by the way, is not the case in a general AQFT), this state can be generated by conditioning on an arbitrary *atomic* event in the local algebra expressing a complete specification of the "beables" of the region in question.[3] With these notions in hand we can formulate Bell's notion of local causality in local physical theories as follows:

Definition 2 A local physical theory represented by a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ of von Neumann algebras is called *(Bell) locally causal*, if for any pair $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ of projections supported in spacelike separated regions $V_A, V_B \in \mathcal{K}$ and for every locally normal and faithful state ϕ establishing a correlation, $\phi(AB) \neq \phi(A)\phi(B)$, between A and B, and for any spacetime region V_C such that

(i) $V_C \subset J_-(V_A)$,
(ii) $V_A \subset V_C''$,
(iii) $J_-(V_A) \cap J_-(V_B) \cap \big(J_+(V_C) \setminus V_C\big) = \emptyset$,

(see Fig. 2) and for any atomic event C_k of $\mathcal{A}(V_C)$ ($k \in K$), the following holds:

$$\frac{\phi(C_k ABC_k)}{\phi(C_k)} = \frac{\phi(C_k AC_k)}{\phi(C_k)} \frac{\phi(C_k BC_k)}{\phi(C_k)} \tag{2}$$

Remark 1. Again we stress that Definition 2 captures local causality only for local physical theories with *atomic* local von Neumann algebras.

[3] For a similar approach to local causality using σ-algebras see (Henson 2013); for a general definition of local causality *via* completely positive maps and for a comparison of the two approaches see our (Hofer-Szabó and Vecsernyés 2015).

2. In case of classical theories a locally faithful state ϕ determines a locally nonzero probability measure p by $p(A) := \phi(A) > 0$, $A \in \mathcal{P}(\mathcal{N}(V))$. By means of this (2) can be written in the following 'symmetric' form:

$$p(AB|C_k) = p(A|C_k)p(B|C_k) \tag{3}$$

or equivalent in the 'asymmetric' form:

$$p(A|BC_k) = p(A|C_k) \tag{4}$$

sometimes used in the literature (for example in (Bell 2004)).

3. The role of Requirement (iii) in the definition is to ensure that "V_C shields off from V_A the overlap of the backward light cones of V_A and V_B". Namely, a spacetime region *above* V_C in the common past of the correlating events may contain stochastic events which, though completely specified by the region V_C, still, being stochastic, could establish a correlation between A and B in a classical stochastic theory (Norsen 2011; Seevinck and Uffink 2011). If V_C is a piece of a Cauchy surface Requirement (iii) coincides with Requirement (iv):

   (iv) $J_-(V_A) \cap J_-(V_B) \cap V_C = \emptyset$

   visualized in Fig. 1. However, for algebras corresponding to coverings of Cauchy surfaces Requirement (iii) is weaker than Requirement (iv) since it allows for regions penetrating into the top part of the common past. For local classical theories Requirement (iii) is enough, but for local quantum theories Requirement (iv) should be used.

Of course the main question is how to ensure that a local physical theory is locally causal. Generally the question is difficult to answer; here we simply mention a sufficient condition in case of *atomic* local algebras:

1. A local *classical* theory is locally causal if the local von Neumann algebras are *atomic* and satisfy *local primitive causality*.

Proof Due to isotony and local primitive causality $\mathcal{N}(V_A) \subset \mathcal{N}(V_C'') = \mathcal{N}(V_C)$ and hence for any atom C_k of $\mathcal{N}(V_C)$: either (i) $AC_k = 0$ or (ii) $AC_k = C_k$. In case of (i) both sides of (2) are zero, in case of (ii) (2) holds as follows:

$$\frac{\phi(ABC_k)}{\phi(C_k)} = \frac{\phi(BC_k)}{\phi(C_k)} = \frac{\phi(AC_k)}{\phi(C_k)}\frac{\phi(BC_k)}{\phi(C_k)}. \tag{5}$$

2. A local *quantum* theory is locally causal if the local von Neumann algebras are *atomic* and satisfy *local primitive causality*, and if Requirement (iii) in the definition of local causality is replaced by Requirement (iv).

Proof Since region V_C is spatially separated from region V_B, $B \in \mathcal{N}(V_B)$ and an atomic event $C_k \in \mathcal{N}(V_C)$ will commute due to microcausality. Using $C_k A C_k = r\, C_k$

(where $r \in [0, 1]$ depends on both A and C_k) we obtain:

$$\frac{\phi(C_k ABC_k)}{\phi(C_k)} = \frac{\phi(C_k AC_k B)}{\phi(C_k)} = r\frac{\phi(C_k B)}{\phi(C_k)} = \frac{\phi(C_k AC_k)}{\phi(C_k)}\frac{\phi(BC_k)}{\phi(C_k)}. \tag{6}$$

Since local primitive causality, which originates from solving initial value problems of hyperbolic partial differential equations, grasps the property of a local causal dynamics, we can feel reassured that local primitive causality implies Bell's local causality. However, one can also expresses solicitude by looking at Point 2 and asking: how can a local *quantum* theory be locally causal if local causality implies various Bell inequalities, which are known to be violated for certain set of quantum correlations. Does Definition 2 correctly grasp Bell's intuition of local causality? We answer these questions in the next section.

4 Local causality, common cause principle and the Bell inequalities

Local causality is closely related to Reichenbach's (1956) common cause principle. The *common cause principle* (CCP) states that if there is a correlation between two events A and B and there is no direct causal (or logical) connection between the correlating events, then there always exists a common cause C of the correlation. Reichenbach's original classical probabilistic definition of the common cause can readily be generalized to the local physical theory framework. (See Rédei 1997, 1998; Rédei and Summers 2002; Hofer-Szabó and Vecsernyés 2012a, b, 2013a, b; Hofer-Szabó, Rédei and Szabó 2013).

Let $\{\mathcal{N}(V), V \in \mathcal{K}\}$ be a net representing a local physical theory. Let $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ be two events (projections) supported in spacelike separated regions $V_A, V_B \in \mathcal{K}$ which correlate in a locally normal and faithful state ϕ. The common cause of the correlation is an event screening off the correlating events from one another and localized in the past of A and B. But in which past? Here one has (at least) three options. One can localize C either (i) in the *union* $J_-(V_A) \cup J_-(V_B)$ or (ii) in the *intersection* $J_-(V_A) \cap J_-(V_B)$ of the causal past of the regions V_A and V_B; or (iii) more restrictively in $\cap_{x\in V_A\cup V_B} J_-(x)$, that is in the spacetime region which lies in the intersection of causal pasts of *every* point of $V_A \cup V_B$. We will refer to the above three pasts in turn as the *weak past*, *common past*, and *strong past* of A and B, respectively (Rédei and Summers 2002).

Depending on the choice of the past we can define various CCPs in a local physical theory:

Definition 3 A local physical theory represented by a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ is said to satisfy the *(Weak/Strong) CCP*, if for any pair $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ of projections supported in spacelike separated regions $V_A, V_B \in \mathcal{K}$ and for every locally faithful state ϕ establishing a correlation between A and B, there exists a nontrivial common cause system, that is a set of mutually orthogonal projections $\{C_k\}_{k\in K} \subset \mathcal{N}(V_C)$, $V_C \in \mathcal{K}$ summing up to the unit of the algebra, satisfying

$$\frac{\phi(C_k ABC_k)}{\phi(C_k)} = \frac{\phi(C_k AC_k)}{\phi(C_k)} \frac{\phi(C_k BC_k)}{\phi(C_k)}, \qquad \text{for all } k \in K \tag{7}$$

such that the localization region of V_C is in the (weak/strong) common past of V_A and V_B.

A common cause is called *nontrivial* if $C_k \not\leq X$ with $X = A, A^\perp, B$ or $B^\perp$ for some $k \in K$. If $\{C_k\}_{k \in K}$ commutes with both A and B, then we call it a *commuting* common cause system, otherwise a *noncommuting* one, and the appropriate CCP a *Commutative/Noncommutative CCP*.

The status of these six different notions of the CCP has been thoroughly scrutinized in a special local quantum theory, namely AQFT (See Rédei (1997, 1998); Rédei and Summers (2002); Hofer-Szabó and Vecsernyés (2012a, 2013a).) Now, what is the relationship between the various CCPs and Bell's local causality? The following list of *prima facie* similarities and differences may help to explicate this relationship:

Similarities:

1. Both local causality and the CCPs are *properties of a local physical theory* represented by a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$.
2. The core mathematical requirement of both principles is the *screening-off condition* (2) or equivalently (7).
3. The *Bell inequalities* can be derived from both principles. (But see below.)

Differences:

1. In case of local causality the screening-off condition (2) is required for *every* atomic event (satisfying certain localization conditions). In case of the CCP for every correlation *only a single* subset of events is postulated satisfying the screening-off condition (7).
2. In case of local causality the screening-off condition is required only for *atomic* events expressing the complete specification of the shielding-off region blocking any causal information from the past. In case of the CCPs these atomic screener-offs of the algebra $\mathcal{A}(V_C)$ are called trivial, since they screen any correlation off irrespectively to the state. What one is typically looking for are *nontrivial* common causes.[4]
3. In case of local causality screener-offs are localized *'asymmetrically'* in the past of V_A; in case of the CCP they are localized *'symmetrically'* in either the weak, common or strong past of V_A and V_B.

Let us come back to Point 3 of the Similarities, that is to the relation of local causality and the CCPs to the Bell inequalities. In (Hofer-Szabó and Vecsernyés 2013b, Proposition 2) we have proven a proposition which clarifies the relation between the CCPs and the Bell inequalities. It asserts that the Bell inequalities can be derived from the existence of a (local, non-conspiratorial joint) common cause system for a set of correlations *if* common causes are understood as *commuting* common causes. However, if

[4] Finding a common cause for a correlation does not mean to provide the most detailed description for the physical situation; it simply means that at this coarser level of description correlations can be causally accounted for. For an opposing view see Uffink (1999) and Henson (2005).

we also allow for *non*commuting common causes, the Bell inequalities can be derived only for *another* state which is *not* identical to the original one. And indeed in (Hofer-Szabó and Vecsernyés 2013a, b) a noncommuting common cause was constructed for a set of correlations violating the Clauser–Horne inequality. Moreover, this common cause was localized in the *strong* past of the correlating events.[5]

Now, an analogous proposition holds for the relation between local causality and the Bell inequalities. We assert here only the proposition without the proof since the proof is step-by-step the same as that of the proposition mentioned above.

Proposition 1 *Let* $\{\mathcal{N}(V), V \in \mathcal{K}\}$ *be a locally causal local physical theory with atomic (type I) local von Neumann algebras. Let* $A_1, A_2 \in \mathcal{A}(V_A)$ *and* $B_1, B_2 \in \mathcal{A}(V_B)$ *be four projections localized in spacelike separated spacetime regions* V_A *and* V_B*, respectively, which pairwise correlate in the locally faithful state* ϕ *that is*

$$\phi(A_m B_n) \neq \phi(A_m)\phi(B_n) \tag{8}$$

for any $m, n = 1, 2$*. Let furthermore* $\{C_k\}_{k\in K} \subset \mathcal{N}(V_C), V_C \in \mathcal{K}$ *be a maximal partition of the unit, where the set* $\{C_k\}_{k\in K}$ *contains mutually orthogonal atomic projections satisfying Requirements (i)–(iii) in Definition 2 of local causality. Then the Clauser–Horne inequality*

$$-1 \leqslant \phi_{\{C_k\}}(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0. \tag{9}$$

holds for the state $\phi_{\{C_k\}}(X) := \sum_k \phi(C_k X C_k)$*. If* $\{C_k\}$ *commutes with* A_1*,* A_2*,* B_1 *and* B_2*, then the Clauser–Horne inequality holds for the original state* ϕ*:*

$$-1 \leqslant \phi(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0. \tag{10}$$

The moral of Proposition 1 is the same as in the case of the CCPs: the Bell inequalities can be derived in a locally causal local physical theory only for a modified state $\phi_{\{C_k\}}$; it can be derived for the original state ϕ *if* the set of atomic projections $\{C_k\}$ localized in V_C commutes with A_1, A_2, B_1 and B_2. What is needed for this to be the case?

In local *classical* theories any element taken from any local algebra will commute, therefore the Bell inequalities will hold in local classical theories. In locally causal local *quantum* theories, commutativity of $\{C_k\}$ and the correlating events is not guaranteed. If V_C is spatially separated from V_B (due to Requirement (iv) in Definition 2), then $\{C_k\}$ will commute with B_1 and B_2 and hence (2) will be satisfied. However, for noncommuting A_1 and A_2 one cannot pick a maximal partition $\{C_k\}$ commuting with both projections, and therefore the theorem of total probability, $\sum_k \phi(C_k A_m C_k) = \phi(A_m)$, will not hold for the original state ϕ at least for one of the projections A_1 and A_2 (it will hold only for the state $\phi_{\{C_k\}}$). This fact blocks the derivation of Bell inequalities for the original state ϕ. (For the details see (Hofer-Szabó and Vecsernyés

[5] For an argument to use noncommuting common causes in causal explanation of quantum correlations see (Hofer-Szabó and Vecsernyés 2013a, b). For a criticism of noncommuting common causal explanation see Cavalcanti and Lal (2014) and for an answer to this see our (Hofer-Szabó and Vecsernyés 2015).

2013b, p. 410)). In short, the Bell inequalities can be derived in a locally causal local quantum theory only if all the projections commute.

At this point we would like to note that the violation of the theorem of total probability is a straightforward consequence of our approach transcending Bell's classical beables. It has the conceptually challenging consequence that we cannot reconstruct the original state ϕ from the states conditioned on the atomic projections. Similarly, in the noncommutative case violation of the theorem of total probability makes the standard interpretation of the common causal explanation as a "finer description of the same physical situation" impossible. (See our (Hofer-Szabó and Vecsernyés 2013b); Cavalcanti and Lal (2014) and our response (Hofer-Szabó and Vecsernyés 2015).)

Coming back to the question posed at the end of the previous Section, namely how a local *quantum* theory can be locally causal in the face of the Bell inequalities, we already know the answer: the Bell inequalities can be derived from local causality if the 'beables' of the local theory are represented by *commutative* local algebras. This fact is completely analogous to the relation shown in (Hofer-Szabó and Vecsernyés 2013b), namely that the Bell inequalities can be derived from a (local, non-conspiratorial, joint) common cause system if it is a *commuting* common cause system. Thus, the violation of the Bell inequalities for certain quantum correlations is compatible with locally causal local quantum theories but not with locally causal local classical theories. Local causality is a more general notion than captured by the Bell inequalities.

5 Conclusions

In this paper we have shown the following:

(i) Bell's notion of local causality presupposes a clear-cut framework in which probabilistic and spatiotemporal entities can be related. This aim can be reached by introducing the notion of a *local physical theory* represented by an isotone net of algebras.
(ii) We have implemented Bell's notion of *local causality* in this general framework and shown sufficient conditions on which local physical theories will be locally causal.
(iii) Finally, we pointed out some important similarities and differences between local causality and the CCPs and showed that in a locally causal local quantum theory one cannot derive the Bell inequalities from local causality just as one cannot derive them from noncommuting common causes.

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