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Analog representations and their users

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Abstract Characterizing different kinds of representation is of fundamental importance to cognitive science, and one traditional way of doing so is in terms of the analog–digital distinction. Indeed the distinction is often appealed to in ways both narrow and broad. In this paper I argue that the analog–digital distinction does not apply to representational *schemes* but only to representational *systems*, where a representational system is constituted by a representational scheme and its *user*, and that whether a representational system is analog or non-analog depends on facts about that user. This aspect of the distinction has gone unnoticed, and I argue that the failure to notice it can be an impediment to scientific progress.

Keywords Representation · Analog · Digital

1 Introduction

Characterizing different kinds of representation is of fundamental importance to cognitive science, and one traditional way of doing so is in terms of the analog–digital distinction (the a/d distinction, from here on). Indeed the distinction is often appealed to, in order to explain both specific facts about cognition, and general features of cognitive architecture. For example, some authors have asked whether the brain is best characterized as analog or digital,[1] the dispute between symbolicists and network theorists is sometimes described as one about whether cognition employs analog or

[1] See e.g., Von Neumann (1958).

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digital representation,[2] the debate concerning mental imagery is often described as one concerning analog mental processes,[3] and more recently, several authors have employed the notion of analog representation in characterizing innate representations of number.[4]

An important fact about the a/d distinction is that it is not agreed upon how best to characterize it. According to the received view, analog representations are continuous, while digital representations are discrete. According to the alternative view, analog representations are structurally similar to that which they represent, while digital representations are not. In this paper, I will present two versions of the received view and two versions of the alternative view. I will argue that all four imply that the a/d distinction does not apply to representational *schemes*, but only to representational *systems*, where a representational system is constituted by a representational scheme and its *user*, and that whether a representational system is analog or non-analog depends on facts about how that user employs the representational scheme. In particular, I will argue that all the accounts surveyed imply that a given representational scheme may be employed in such a way as to constitute an analog system, or in such a way as to constitute a non-analog system. Moreover, I will argue that there is good reason to conclude that any account of the distinction will imply the same result. Finally, I will argue by way of example that the failure to appreciate this aspect of the distinction can impede scientific progress, at least in the sense that it can obscure possible solutions to unsolved problems.

I will take no stand on the question whether the received or alternative view is the correct account of the a/d distinction, nor indeed on whether either of the versions of each is better than the other. To be sure, the authors I discuss present their versions as being correct or more appropriate or more useful than other versions, but I will not be concerned with the soundness of those arguments here. Also, I am not going to offer my own, competing account with those described here. Rather, I will argue that all those surveyed imply that representational schemes are neither analog nor non-analog, and that whether a representational system is analog or not depends on facts about the user.

Moreover, all the accounts surveyed here agree that there are systems that are neither analog nor digital. Therefore my conclusion is not equivalent to the claim that whether a representational system is analog or digital depends on facts about how the user employs the representational scheme. A system may be non-analog and also non-digital. It would take further arguments to show that whether a representational system constitutes a digital or non-digital system turns on how its representational scheme is employed by its user, and I will not provide those here. Thus while the accounts I describe define both analog and digital representation, where possible I will discuss only those portions of the accounts that pertain to analog representation.

It is critical of course to begin with a description of the concepts of representational schemes, users, and systems. A representational *scheme* is a set of representations that form a group, such that rules for creating and interpreting them apply to all the members of the group. Representational schemes are employed by someone, or something, and

2 See e.g., Churchland and Churchland (2000).

3 See e.g., Pylyshyn (1984) and Tye (1991).

4 See e.g., Wynn (1992) and Gallistel et al. (2006).

whatever it is that employs a scheme, in a given case, I will call the *user* of that scheme. More specifically, the user creates ("writes") and interprets ("reads") the individual representations that constitute a scheme. A representational *system* is constituted by a scheme and its user.

For example, consider a machine that performs the following simple task: it takes as input any positive integer, and returns that number multiplied by two. Moreover, suppose the machine is implemented using a supply of tennis balls, two buckets, and one person. An individual representation of a number is just that number of tennis balls in one of the buckets. The representational scheme is just the collection of all such representations. We feed the machine a given positive integer as input by placing that number of tennis balls in the first bucket. The person takes one tennis ball from the first bucket and places it, along with another ball from the general supply, into the second bucket. This process is repeated until there are no tennis balls remaining in the first bucket. The output, i.e., double the input, is just the number of tennis balls in the second bucket when the process is complete. Again, the set of individual representations is the scheme. The person manipulating them is the user. The scheme and user together constitute the system.

The user need not and often will not be a human being. For instance, representations in a personal computer are stored bits of data. They are created and interpreted by other parts of the computer (e.g., the CPU). In some cases there may be more than one user. Thus while the CPU is the user in a personal computer, the computer as a whole also has a user—most likely a human being. Thus it can be useful to distinguish between the *primary user*, and the *secondary user*.[5] In the case of the personal computer, some internal part of the machine is the primary user, and the human is the secondary user.[6]

There is another sense in which a scheme may have more than one user. The part of the system that creates representations may not be the same part that interprets them. Consider for instance a communications network, in which representations are sent from a transmitter to a receiver. Here the transmitter creates representations, and the receiver interprets them. In such a case, it may be useful to describe the transmitter as the *writing user* and the receiver as the *reading user*. I will normally just refer to "the user" of a scheme, except where it is necessary to distinguish between for example the writing user and reading user.

It is important to note here the distinction between artificial and naturally occurring representational systems. Artificial systems are, obviously, designed with specific purposes, and the ways in which representations are created and read in such systems are part of the design. Thus it would be unusual for a given scheme to be employed in such a way as to constitute an analog system and also in such a way as to constitute a non-analog system. Nevertheless, I'll argue below that it is possible, given all the accounts of the a/d distinction that I'll survey. Since naturally occurring schemes are not intentionally designed, it may not be as unusual for them to be simultaneously employed by multiple users in multiple ways, and thus to partly constitute multiple

5 If necessary it is also possible to talk of the *tertiary user*, and so on.

6 In this case of course, the primary user and the secondary user do not create and interpret the same representations. The primary user creates and interprets representations in the machine language. The human user creates and interprets representations in the computer's input–output language.

representational systems, some of which may be analog and some non-analog. My claim in the last section of the paper is that this possibility has gone unnoticed, and that this has been something of an impediment to scientific progress.

The rest of the paper will have three main sections. In the first I will describe two versions of the received view, those provided by Goodman (1968) and Haugeland (1998). In the second I will describe two versions of the alternative view, those provided by Maley (2011) and Montemayor and Balci (2007). In all four cases, I will argue that the account described implies that representational schemes are neither analog nor non-analog, that the distinction only applies to representational systems, and that a given scheme may partly constitute both analog and non-analog systems, and that whether a system is analog or non-analog depends on facts about how the user employs the scheme. I will conclude, moreover, that there is good reason to suppose that no matter how the distinction is drawn, the same result will follow. In the third section I will argue by way of one example that the failure to appreciate this fact can impede scientific progress, at least in the sense that it can obscure possible solutions to unsolved problems.

2 The received view

The received view of the a/d distinction holds that analog representation is continuous, while digital representation is discrete. It is not difficult to find appeals to this view in a variety of contexts.[7] Both Goodman (1968) and Haugeland (1998) have offered detailed philosophical accounts of the distinction that at heart rely on the notions of continuity and discreteness. I will begin with Goodman's view. In particular, I will argue that Goodman's view implies that there are no analog representations. This is an intolerable result, as we know there are such representations. I will argue further though, that when the account is remedied, it implies that the a/d distinction applies to systems and not schemes, and that whether a system is analog or not depends on how the representations are employed by their user.[8]

2.1 Goodman

Goodman's view is that a representational scheme[9] is analog just in case it is dense and *finitely undifferentiated* both syntactically and semantically. A scheme is syntactically

[7] The view grows out of the history of the development of computing technology. Thus, Newell (1983) writes, "When computers were first developed in the 1940s they were divided into two large families. Analog computers represented quantity by continuous physical variables, such as current or voltages…Digital computers represented quantities by discrete states…"(p. 195). See also Von Neumann (1958).

[8] The accounts offered by Haugeland (1998), Maley (2011), and Montemayor and Balci (2007) do not face this same problem of implying that there are no analog systems. Thus in those cases I will argue directly that the accounts as they are imply that whether a system is analog or not depends on facts about its user. Therefore the discussion of Goodman's account is considerably longer than each of the others.

[9] Goodman does not draw the distinction between schemes and systems in the way I am. But it will be part of the burden of my argument that his account of the a/d distinction implicitly assumes the notion of a user. I'm attributing the distinction as I use it to him, so as to make that argument clearer.

dense if and only if between any two characters (representational types) there is a third, and it is semantically dense if and only if between any two represented objects there is a third.[10] A scheme is syntactically finitely differentiated if and only if, for any two characters K and K', and any inscription (representational token) m, it must be theoretically possible to determine either that m does not belong to K or that m does not belong to K'. Similarly, a scheme is semantically finitely differentiated if and only if, for any two characters K and K', and any object h, it must be theoretically possible to determine either that h is not denoted by K or that h is not denoted by K'.[11] It is important to note, and Goodman is quite clear, that he does not confound what is theoretically possible with what is possible in practice.[12] I will return to this distinction below, for it forms the basis of my argument that Goodman's account implies that there are no analog representations.

Consider for example an air pressure gauge that has a circular display face with no markings on it, and a single pointer that moves smoothly around the face as the pressure increases. It should fit most intuitions that such a gauge is analog. Indeed it is paradigmatically so. And according to Goodman, it fits the four criteria he describes. Between any two positions of the pointer, there is a third,[13] so the representations are syntactically dense. Between any two states of affairs represented by the system, there is a third, so the representations are semantically dense. And according to Goodman, the representations are neither syntactically nor semantically finitely differentiated. I am going to argue however, that this scheme is in fact finitely differentiated, and so despite what Goodman claims is not on his account analog. Moreover, I will argue that on his account there are no analog representations.

Recall that on the syntactic side finite differentiation means that for any two characters and any inscription, it must be theoretically possible to determine either that the inscription does not belong to one of the characters, or that it does not belong to the other. For instance, Goodman notes that a scheme in which straight marks differ in character if they differ in length "by even the smallest fraction of an inch" would be a scheme in which finite differentiation of character is theoretically impossible. The reason is because, "no matter how precisely the length of any mark is measured, there will always be two (indeed, infinitely many) characters, corresponding to different rational numbers, such that the measurement will fail to determine that the mark does not belong to them" (1968, p. 135). But here Goodman appears to have made a quantifier-alternation error.

The requirement of syntactic finite differentiation, as Goodman states it, is as follows:

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[10] Goodman at times suggests continuity and not density, but the distinction will not be relevant to my argument.

[11] Goodman's account of the a/d distinction grows out of an account of notational systems for art, which according to Goodman serve to "[mark] off performances that belong to the work from those that do not" (1968, p. 128). There are further requirements that Goodman identifies for notational systems.

[12] For present purposes, it will suffice that a technique for differentiating characters and inscriptions is theoretically possible if it is imaginable, even if it does not exist.

[13] Note that here and elsewhere Goodman assumes that space–time is infinitely divisible. I take it this in fact remains an open question, but it will not affect my argument, so I grant it for present purposes.

$$\forall K \forall K' \forall m \exists \mu ((K \neq K') \supset (V(\mu, m, \overline{K}) \vee V(\mu, m, \overline{K'}))) \quad \text{(FD)}$$

where K and K' are characters, m is a mark, μ is a measurement technique, and $V(\mu, m, \overline{K})$ is read as "μ verifies that m is not in (is not a token of) K". In English it reads: for any two characters and any mark, there is a measurement technique that is such that, if the two characters are not identical, the measurement technique will verify either that the mark does not belong to one of the characters, or that the mark does not belong to the other character.

The failure of this requirement—its negation— is as follows:

$$\exists K \exists K' \exists m \forall \mu \sim ((K \neq K') \supset (V(\mu, m, \overline{K}) \vee V(\mu, m, \overline{K'}))) \quad \text{(NFD)}$$

It reads: there are two characters and a mark that are such that for any measurement technique, it is not the case that if the two characters are not identical, the measurement technique will verify either that the mark does not belong to one of the characters, or that the mark does not belong to the other character.

Goodman's concern with the set of straight marks is as follows[14]:

$$\exists m \forall \mu \exists K \exists K' \sim ((K \neq K') \supset (V(\mu, m, \overline{K}) \vee V(\mu, m, \overline{K'}))) \quad \text{(G)}$$

This reads: there is a mark such that for any measurement technique there are two characters that are such that, it is not the case that if the two characters are not identical, the measurement technique will verify either that the mark does not belong to one of the characters, or that the mark does not belong to the other character.

Notice that (G) is the result of reversing the order of the two innermost and two outermost quantifiers in (NFD). But because this reversal involves moving the universal quantifier on μ from the inside of the existential quantifiers on K and K' to the outside of them, it changes the meaning.[15] Goodman's worry concerns a case in which a mark and a measurement technique are held constant. But since any given measurement technique will be precise to within some margin of error, there will always be characters that can be chosen that are such that the measurement technique cannot verify either that the mark does not belong to one character, or that the mark does not belong to the other.

Finite differentiation, however, requires that two characters and a mark be held constant, but not the measurement technique. A failure of finite differentiation would be a case in which given two characters and a mark, no theoretically possible measurement technique would be able to determine either that the mark did not belong to one of the characters, or that it did not belong to the other. But given two characters and a mark, there will always be theoretically possible measurement techniques able to determine

[14] The way Goodman states this worry (quoted above) in fact employs a universal quantifier in the left-most position. But such a formulation will imply the one in-text, with the existential quantifier. The latter makes clear that the problem is an alternation of the universal quantifier on μ with the existential quantifiers on K and K'.

[15] More specifically, although (G) is true of the set of straight marks, (G) does not imply (NFD), and (NFD) is not true of the set. In other words, although (G) is true of it, the set of straight marks is finitely differentiated.

which of the characters the mark does not belong to. These techniques simply involve looking at the marks under ever-higher magnification, so as to measure their length with ever-greater accuracy. In short, despite what Goodman claims, the set of straight marks he envisions is in fact syntactically finitely differentiated, on his account of syntactic finite differentiation.

Now consider again the air pressure gauge. This representational scheme also is syntactically finitely differentiated. For given any two characters (pointer position types) and any inscription (a pointer position token) there are theoretically possible techniques for determining either that the inscription does not belong to one of the characters or that it does not belong to the other. Those techniques again simply involve looking at the gauge under ever-higher magnification. Indeed, it would appear that on Goodman's account, there cannot be any schemes that are not syntactically finitely differentiated, since that would require that no theoretical measurement technique could determine that a given mark did not belong to one character or to another. But given two characters and a mark, it is not difficult to imagine theoretical—even if non-actual—techniques that will make that determination.

Given that analog schemes require a failure of syntactic finite differentiation, and given that on Goodman's account there are no such schemes, it follows that on his account there are no analog representations. But this is clearly false. As I noted above, the air pressure gauge is paradigmatically analog, and there are many other examples. Thus, Goodman's account is unacceptable, at least as is. I will now argue that it can be remedied, but that when it is it implies that representational schemes, aside from their users, are neither analog nor non-analog. The remedied account implies that systems of representations—schemes and their users—are analog or not, and that when employed by different users a given representational scheme may constitute either an analog or non-analog system.

Consider again the set of straight marks Goodman describes. It is syntactically finitely differentiated, because given two characters and a mark, there are theoretically possible measurements that will determine that the mark is not a token of one of the characters, or that it is not a token of the other. However, in the case of some given characters, and some given marks, it will not be true that there are measurement techniques that are possible in practice that will determine that the mark is not a token of one of the characters or that it is not a token of the other. In general, it is in relying on the notion of theoretical possibility that Goodman renders it the case that there are no representational schemes that are not finitely differentiated, and thus that there are no analog representations. If on the other hand the notion of what is possible in practice is part of the account, then there will be representational schemes that are not finitely differentiated, and there will be analog representations.[16]

[16] In fact, I think the notion of possibility in practice is more in the spirit of Goodman's account anyway. After all, his account of the a/d distinction grows out of an account of the distinction between notational and non-notational systems, where notational systems are those systems that allow for unique identification of artwork. His project is thus at root pragmatic, even though Haugeland accuses him of "betray[ing] a mathematician's distaste for the nitty-gritty of *practical* devices" (1998, p. 80). It therefore seems at odds with his project that some of the criteria he sets for notational systems are wholly theoretical.

If what matters in terms of finite differentiation is what is possible in practice though, then the notion of a user is implicit in the account. For what can be determined in practice depends on who or what is creating and interpreting the representations, and what technology is available to them. Return once more to the set of straight marks. Given two characters and a mark, some users will have acute enough measurement techniques (which may just include normal vision) and some will not, to determine either that the mark does not belong to one character or that it does not belong to the other. For the former user the set will be finitely differentiated, and for the latter it will not. But if a representational scheme is finitely differentiated for some users but not for others, and if whether a scheme of representations is analog or not depends on whether or not it is finitely differentiated, then whether a scheme is analog or not depends on its user. Or in other words, the a/d distinction does not apply to representational schemes, but to representational systems.

Consider once more the air pressure gauge. My conclusion is that Goodman's account implies that the gauge is neither analog nor non-analog. Rather it partly constitutes an analog system when employed by a user for whom the representations are not finitely differentiated, and a non-analog system when employed by a user for whom the representations are finitely differentiated. This should seem counterintuitive. Indeed, I said above that the gauge is paradigmatically analog. I said that because we normally envision a human user, with normal eyesight, using the gauge under normal lighting conditions. Indeed, the gauge was designed for just such a user. And for such a user under such conditions, there will be position types and position tokens that that user cannot differentiate. When employed by such a user, the system constituted by that user and the gauge is analog. Still, the gauge could be employed by a being with supernatural eyesight. In that case, the system constituted by both the scheme and user would be non-analog. Indeed, we could imagine the gauge having been designed for such users. In that case, the gauge might intuitively seem non-analog.

In short, Goodman's account implies that there are no analog representations, and is therefore unacceptable. It can however, be remedied by supplanting a notion of possibility in practice for the notion of theoretically possible that is present is Goodman's description of finite differentiation. When that is done, the account implies that representational schemes are neither analog nor non-analog, and that whether a system is analog or not depends on facts about how the user employs the representational scheme. Specifically, it depends on whether the representations are finitely differentiated, given the manner in which the user employs them. I will now argue that Haugeland's account implies this same result.

2.2 Haugeland

Haugeland (1998) offers a general account of the distinction between analog and digital "devices," where he is "noncommittal" about what qualifies as a device. However, as will become clear, it is obvious that the notion of a user is implicit in his account. The burden of my argument therefore, will be to show that on his account whether a device (*system* in my terminology) is analog or not depends on facts about how the user employs the representational scheme, and indeed, that a given scheme may partly constitute an analog system when employed in some ways and a non-analog system when employed in other ways. Moreover, while the present concern is with analog representations, the part of Haugeland's account that pertains to analog devices is built in contrast with the part of his account that pertains to digital devices, so I will first describe that part of the account.

Haugeland claims that digital devices are defined by the following four features:

(i) a set of types,
(ii) a set of feasible procedures for writing and reading tokens of those types, and
(iii) a specification of suitable operating conditions, such that
(iv) under those conditions, the procedures for the write–read cycle are positive and reliable. (1998, p. 78)

Since Haugeland's account of digital devices is based in part on how representations are written and read, and since a user is whoever or whatever reads and writes the representations, it is clear that Haugeland's account implies the notion of a user. The same will be true of his account of analog devices. His idea of a positive procedure is one that "can succeed absolutely and without qualification." A reliable procedure is one that "under suitable conditions, can be counted on to succeed virtually every time" (1998, p. 77). He explains that "Parking the car in the garage (in the normal manner) if getting it all the way in is all it takes to succeed [and] cutting boards six feet, plus or minus an inch," (1998, p. 77) are examples of positive procedures. So too are "raising the dead, writing poetry, winning at roulette, or counting small piles of poker chips" (1998, p. 84). Some of these will be reliable, others not. Standard examples of digital devices then, include "Arabic numerals, abacuses, electrical switches, musical notation, poker chips, and (digital) computers" (1998, p. 75). Indeed, at least some of these are paradigmatic examples of digital devices.

Haugeland thinks of analog devices as also employing write–read cycles (1998, p. 83) but he thinks of these procedures as "approximation" procedures that are defined by margins of error, such that:

(v) the smaller the margin of error, the harder it is to stay within it,
(vi) available procedures can (reliably) stay within a pretty small margin,
(vii) there is no limit to how small a margin better (future, more expensive) procedures may be able to stay within, but
(viii) the margin can never be zero—perfect procedures are impossible. (1998, p. 83)

Since Haugeland allows that analog devices employ representational types and write–read cycles, it follows that analog devices also have the features (i)–(iv), but with a modification on (iv). In the case of analog devices, it should read as follows:

(vi$_a$) under those conditions, the procedures for the write–read cycle are approximate and reliable.

Here the notion of "approximate" is the idea that the device will employ a margin of error, and (v)–(viii) explain the relationship between a margin of error and the reliability of a write–read cycle. In general, the smaller the margin of error, the less reliable the cycle. Examples of approximation procedures, Haugeland explains, include "all ordinary (and extraordinary) procedures for parking the car right in the center of the garage, cutting six-foot boards, measuring out three tablespoons of blue sand, and copying photographs" (84).[17] Examples of analog devices "include slide rules, scale models, rheostats, photographs, linear amplifiers, string models of railroad networks, loudspeakers, and electronic analog computers" (1998, p. 82). Again, at least some of these are paradigmatic examples of analog devices.

However, consider again Haugeland's claim that cutting a board to six feet, plus or minus an inch is a positive procedure. This claim does not sit well with the general idea that approximation procedures involve margins of error while positive procedures do not, since "plus or minus an inch" is a margin of error. Moreover, Haugeland writes that whether a procedure "*can* succeed [absolutely and without qualification]… will depend on the technology and resources available" (1998, p. 77). That is, whether a procedure is positive or not will depend on the available technology and resources. But this suggests that the difference between positive procedures and approximation procedures is not that only the latter involve margins of error, but that the latter involve relatively narrow margins of error while the former involve relatively broad margins of error. The difference between cutting a board to six feet plus or minus an inch and cutting a board to exactly six feet, after all, is that the typical carpenter using typical tools is able to cut a board to the former specifications but not to the latter.[18]

If this is correct though, then it is not difficult to show that whether a write–read procedure is an approximation procedure or a positive procedure depends on facts about who or what does the writing and reading, and thus, that whether the representational system based on that procedure is analog or not depends on facts about its user. For given some margins of error, some users will be able to stay within them and others will not. Keeping with the example, as noted above the typical carpenter using typical tools is able to cut a board to six feet plus or minus an inch. But she is not able to cut a board to six feet plus or minus one one-hundredth of an inch. In contrast, technology companies brag about their ability to cut plastic and metal to specifications within just a few microns. Cutting a board to six feet plus or minus one one-hundredth of an inch is an approximation procedure for a typical carpenter, and any representational system based on a carpenter cutting boards to within one one-hundredth of an inch is an analog system. But the same procedure will be a positive procedure for a company that can cut boards to within microns, and any representational system based on such a company cutting boards to within one one-hundredth of an inch will be non-analog.

Haugeland himself gives an example that seems to make this clear. He writes,

> It is common digital electronics practice to build pulse detectors that flip 'high' on signals over about two and a half volts, flipping 'low' on smaller signals… What saves the day for engineers is that pulse *generators* produce only signals very close to zero and five volts respectively, and the whole apparatus can be

[17] Of course, Haugeland developed his account well before the mass availability of "digital" photography.

[18] This is not quite precise enough. If space–time is not infinitely divisible, then it is possible for a typical carpenter using typical tools to cut a board to exactly six feet, but it is not possible for her to know that she has done so.

> well shielded against static, so the detectors never actually get confused. (1998, pp. 79–80)

Because the pulse detectors respond in one way to anything below two and half volts and respond in another way to anything above two and a half volts, the pulse generators need not generate exactly zero or exactly five volts. Instead, they are allowed a margin of error of two and a half volts, and they have no difficulty staying within those margins. That relatively wide margin of error, given the abilities of both the writing and reading users (the generators and detectors, respectively) is what makes this a positive procedure, and any representational system based on it non-analog. Were the abilities of the users different, it might be an approximation procedure and an analog system of representation (or more specifically, were the system designed with other users, with different abilities, the procedure might well be an approximation procedure and the system might well be analog).

Someone might object here, however, that Hagueland in fact intends something different by "margin of error" in the digital case than he does in the analog case. Therefore, my argument above equivocates on "margin of error" and the conclusion does not follow. The objection runs that in the digital case, there are allowable differences among representational tokens that are to be interpreted as being of the same type, and a margin of error is the boundary within which any token will be interpreted as being of a certain type. By keeping the margins wide relative to users' abilities (e..g, in Haugeland's example in the above quotation) the system ensures reliability. In the analog case, the objection continues, there are no allowable differences in tokens that are to be interpreted as being of the same type, or as Haugeland puts it, "every difference makes a difference" (1998, p. 83). A margin of error, therefore, marks the amount of deviation there may be between the content in response to which a representation was created, and the content that representation will be interpreted as carrying (e.g., if voltage represents number, with xV representing x, 4V will be interpreted as representing 4, but may have been created in response to 3 or 5). By keeping margins narrow relative to the amount of allowable error (given the proper functioning and use of the device as a whole), reliability is again ensured.

Again, it is clear that Haugeland's account implies the notion of a user—the account is aimed at devices, not just representational schemes, and includes the idea that tokens must be written and read. So the burden here is to show that whether a device (system) is analog or not depends on facts about how the representations are employed by their user, and indeed, that a given scheme may partly constitute an analog system when employed by some users and a non-analog system when employed by others. And even granting the objection, Hagueland's account implies this. For whether or not there are allowable differences between tokes that are to be typed together is a matter of how the representations are to be employed by their users. More specifically, it is a matter of how the system was designed, and a system may be designed with a set of representations to be used in one way, or in another. Return once more to Haugeland's example of cutting boards to particular lengths. Someone designing a representational system may use boards of various lengths. The representational scheme is just the collection of boards. Consider all the boards that are between $5'11''$ and $6'1''$. One user may type these all together and another may type them to their measured length.

On Haugeland's account, the system including the former user will be non-analog and the system including the latter user will be analog. Even allowing the objection, that is, Hagueland's account implies that whether a system is analog or not depends on facts about how the user employs the representations.

3 The alternative view

Thus I have argued that on both versions of the received view surveyed here, representational schemes are neither analog nor non-analog, and that whether a representational system is analog or not depends on facts about how the user employs the representations. I will now argue that on each of two versions of the alternative view, the same result follows. The alternative view holds that analog representation bears a structural similarity to that which it represents, and digital representation does not. As with the received view, it is not difficult to find examples of this view in the literature.[19] Both Maley (2011) and Montemayor and Balci (2007) have offered detailed accounts in these terms.

3.1 Maley

On Maley's view analog representations are physical quantities that vary linearly with that which they represent. More technically, he writes,

> A representation R of a number [20] Q is analog if and only if:
> (1) There is some property P of R (the representational medium) such that the quantity or amount of P maps onto Q; and
> (2) As Q increases (or decreases) by an amount d, P increases (or decreases) as a linear function of $Q + d$ (or $Q - d$) (2011, p. 8).

Suppose for example we want to use a container of water to represent the number of people in a room, and we do so by pouring one ounce of water into the container for each person in the room. Such representations fit with intuition as analog and according to Maley's account they would be. The amount of water maps onto the number represented, and that amount varies linearly with the number represented. Now consider the familiar base-ten system of Arabic numerals. These are everyone's idea of digital representations, and they are certainly non-analog according to Maley's account (as we'll see in a moment, they are digital on his account). Numbers are not represented by quantities (the size or weight or etc. of the numerals has no bearing on what number they represent). It is true that as the numbers represented grow in size, so too does the number of digits needed to represent those numbers, but this growth is not linear. It takes no more digits to represent the number ninety, for example, than it does to represent the number ten.

[19] See for example Horak (2000).

[20] Maley holds that the a/d distinction applies only to representations of number. Though they do not argue for it, most authors disagree, assuming that representations with all manner of content may be either analog or digital. Maley gives historical reasons for his view, but I will not address the topic here.

My argument will require discussion of Maley's account of digital representations as well. He writes that,

> . . . we can. . . precisely characterize digital representation as
> (1) A series of digits, each of which is a numeral in a specific place in the series; and
> (2) A base, which determines the value of each digit as a function of its place, as well as the possible number of numerals that can be used for each digit. (2011, pp. 9–10)

As I noted above, this characterization captures our normal usage of the Arabic numerals, as of course it should. That's no surprise either, since Maley builds his account specifically around the Arabic numerals. Now imagine the same base-ten set of representations, but using cups of water instead of the Arabic numerals. Suppose for example we let a cup with one ounce of water replace the numeral '1', a cup with two ounces replace the numeral '2', and so on. An array of three cups in which the furthest left cup had two ounces of water, the middle cup had seven ounces, and the right cup had five ounces, for example, would represent the number two-hundred-seventy-five. These representations are also digital on Maley's account. That makes perfect sense, since they differ from the Arabic numerals only in that cups of water are used as numerals, in place of the familiar symbols.

However, imagine the same cups of water, but with the following modification: the temperature of the water in the cups varies, such that when the number represented is n the temperature is n °C. On Maley's account these representations are both digital and analog. Numbers are represented by series of digits together with a base, but also there is a property of the medium that maps onto the number represented and that varies linearly with it. I said above that on all the accounts discussed here, the a/d distinction is not exhaustive, but it is mutually exclusive. No account is intended to allow for representations to be both analog and digital, and indeed, no account should.[21] What has gone wrong here?

The source of the problem is that Maley's characterizations of both analog and digital representation depend on the simple possession, by representational schemes, of certain kinds of properties. But there is no demand that those properties play any role in the creation or interpretation of individual representations. According to the account, if a representational scheme possesses a property of the medium that varies linearly with that which is represented then the representations are analog, regardless of whether that property is relevant to the interpretation of individual representations. Similarly, according to the account, if a scheme is a series of digits with a base then the representations are digital, regardless of whether that feature of the representations plays any role in their interpretation.

Of course, one would normally assume that if a representational scheme possesses such features as Maley describes then those features determine the content of individual representations. Indeed, Maley seems to have made that assumption. But the example

[21] Again, on my view a representational scheme is neither analog nor non-analog, but may with some users constitute an analog system and with others constitute a non-analog system. But no system will be both analog and non-analog.

shows that that need not be the case. Moreover, the features that determine content are the features that users rely on in creating representations with particular contents, and interpreting representations as having particular contents. That is, the assumption implies the notion of a user, and that whether a system of representations is analog or digital rests on facts about how the user employs the representations. Specifically, it rests on which properties of the representations the user considers when creating and interpreting them.

Again, how the system is designed will determine whether it is analog or non-analog. My point is that a given scheme may be employed by both analog and non-analog systems. The cups of water might have been designed so as to be interpreted in terms of the base ten nature of the representations, or they might have been designed so as to be interpreted in terms of the temperature of the water in the cups. the former system would be digital, the latter analog. Maley's account could include explicitly the idea that the features of the representations that render them analog or digital be the very same features that determine content. That would be do build the notion of a user, and the idea that the a/d distinction applies to representational systems (and not merely schemes), explicitly into the account.

3.2 Montemayor and Balci

Montemayor and Balci (2007) think of analog representations as representations that possess *metric structure*, as opposed to representations that possess *linguistic structure*. Whereas the latter are composed of atomic constituents and get their content from the content and structure of those atomic components, the former either have no atomic components or have atomic components that play no role in the content of the whole. And whereas the latter may bear no resemblance to what they represent, the former do. Specifically, Montemayor and Balci define analog representations as follows:

> A representation *R* is magnitude-based [22] if and only if the rules of composition governing *R* do not require the existence of atomic constituents. Computations on these representations
>
> (a) Produce no *syntactic* compositions and decompositions and
> (b) Must bear a *causal* or *structural* isomorphism to what they represent: they preserve metric structure.[23] (2007, pp. 55–56)

Thus, compare the Arabic numerals as normally used to represent the positive integers, and amounts of sand to represent the same. The numerals include compound representations that are composed of atomic parts, whose content and structure determine the content of the whole. For example, '345' represents the number it does in virtue of the content of the individual component representations '3', '4', and '5', and in virtue of their arrangement (were those components rearranged, the whole would have a different content). So the numerals fail to meet Montemayor and Balci's first

[22] Note that Montemayor and Balci use 'analog', 'metric', and 'magnitude–based' interchangeably.

[23] Indeed, Montemayor and Balci call this the "resemblance constraint".

requirement for analog representation. Moreover, there is no causal or structural isomorphism that preserves the metric structure of the numbers represented. In particular, while the binary asymmetric relation 'greater-than' holds among the integers, no such relation holds among the numerals. Hence, they fail to meet Montemayor and Balci's second requirement as well.

In contrast, suppose we allow *n* ounces of sand to represent the number *n*. Here we have representations that meet both requirements. A pile of sand has no syntactic structure, so the first requirement is met. Moreover, the asymmetric binary relation 'heavier-than' will hold between two piles of sand *a* and *b*, representing the numbers *i* and *j* respectively, just in case 'greater-than' holds between *i* and *j*. So the second requirement is met as well.

Consider though, how a user might determine what number a given pile of sand represents. One way of doing so would be to weigh the sand. In this case it makes no difference whether or not the pile was formed by combining individual one-ounce piles. That is, used in this way there is no syntactic structure to the pile; no syntactic structuring is needed. But imagine instead that in order to determine the content of a representation, the user must count the number of one-ounce piles of sand. In that case the representations do have syntactic structure, at least in the sense that they must be composed of atomic components (each component being a one-ounce pile of sand). But then the representations turn out not to be analog on Montemayor and Balci's account, since computations on them require atomic constituents.

Once again, the representations themselves are neither analog nor non-analog. Rather, if employed in one way then the system they partly compose is analog, and if employed in another way the system they partly compose is non-analog. If the user relies on the number of one-ounce piles of sand, the system is not analog. If the user relies on the total weight of the sand, then the system is analog. Thus, whether the system is analog or not turns on facts about the user. Specifically, it turns on facts about how the user interprets the content of the representations. And again, we would normally expect a given scheme to be designed to be employed in a particular manner. That will determine whether the resulting system is analog or non-analog. But still, a given set of representations may be part of analog system or part of a non-analog system, depending on how they are used.

4 One example

So far I have argued that each of the views presented implies that representational schemes are neither analog nor non-analog, and that whether or not a representational system is analog depends on facts about how the user employs the representations. To be sure, there are other extant accounts.[24] But I believe that the foregoing discussion suggests that any account of the a/d distinction will imply the same result. That is because any account of the distinction must ultimately rest on some salient feature of

[24] See e.g., Lewis (1971), Blachowicz (1997), Frigerio et al. (2013), and Schonbein (2014). See Dretske (1981) for a non-standard account, and for a closely related discussion see Cummins (1996) and Cummins et al. (2001).

the representations, which determines how they are to be interpreted or how computations on them will be performed. But interpretation implies an interpreter—a user—and different users may employ different salient features. And so a given scheme may be employed in different systems, where what distinguishes the systems are differences among users and specifically what salient features of the scheme the users rely on in creating and interpreting them. Thus for example, a system might be designed such that the user employs the base-ten structuring of an array of water-filled cups, while another may be designed such that the user employs the temperature of the water in the very same cups. Or, one system may be designed such that the user exploits the differences in length of straight lines that differ by millimeters, while another may be designed such that the user need not exploit such small differences in the same lines.

I will use the rest of this section to discuss, by way of one example, part of the significance of this result. In general, I will argue that the failure to notice that the a/d distinction applies to representational systems and not representational schemes can impede scientific progress, at least in the sense that it can obscure possible solutions to unsolved problems. The example comes from the literature on the acquisition of number concepts.

In particular, recent experimental evidence suggests that human infants possess an innate cognitive mechanism that represents cardinality and can perform various arithmetic operations. Much of the discussion surrounding this system has focused on whether and in what way it may play a role in the acquisition of mature natural number concepts. Many authors have argued though, that this system cannot be the sole source of natural number concepts, because it is "approximate", or indeed because it is "analog", while natural number concepts are "precise" or "digital". Some authors have argued that this system must work in concert with other systems in generating natural number concepts, while others have argued that human beings must possess innate natural number concepts. I will argue, however, that these views overlook a way in which the representations that partly constitute this this system could be the sole source of natural number concepts, and they do so because they fail to recognize that these representations by themselves are neither analog nor non-analog, but rather partly constitute an analog system, given how they are employed by their user.

Recent decades have seen a massing of evidence (from a variety of looking-time methods)[25] that infants are able to compare the cardinalities of sets of objects, and to compute sums and differences. However, these abilities appear to be dependent on the relative cardinalities of the sets. For example, although six-month-olds distinguish between sets containing eight and sixteen objects, they fail to distinguish between sets containing eight and twelve objects.[26] Infants' abilities to compute sums and differences also appear dependent on the relative sizes of the cardinalities involved.[27] This suggests that infants' numerical abilities conform to Weber's Law, which states that whether or not a subject can discriminate two stimuli depends not on the absolute values of the stimuli, but on the ratio between the two.

[25] See Carey and Spelke (1996) for a useful discussion of looking-time methods and results.

[26] See Xu and Spelke (2000).

[27] See McCrink and Wynn (2004).

In order to account for these abilities, many authors posit an innate system of *mental magnitudes*. Sometimes termed an *accumulator*, the model was originally proposed by Meck and Church (1983) to explain numerical competencies in rats. Meck and Church originally described the system as having three parts: a *pacemaker*, a *switch*, and several *basins*. The pacemaker creates pulses at a (somewhat) constant rate. When the switch is closed, these pulses are transferred into a basin, where they are stored. The switch may close and open again at periodic intervals, thus increasing in steady increments the pulses that are passed into the basin. If n objects are observed, n increments may be passed from the pacemaker to the basin.

There are several features of this system that are important to mention. First, the representations are distinguished by their size, and in particular they grow in size in proportion to the number represented. Second, the increments that compose the representations are inherently variable in size, such that any two increments are only roughly equivalent in size. Because the increments compose to form completed representations, the result is that two representations of the same number may differ in size. Third, because representations of larger numbers are composed of more increments than representations of smaller numbers, they are subject to more variability than representations of smaller numbers. More exactly, the standard deviation of the sizes of magnitudes representing a given number is a linear function of the size of that number. This feature of the system is known as *scalar variability*. An effect of scalar variability is that, at a certain point, the range of magnitudes which may denote a given number becomes large enough that there is overlap with the ranges of magnitudes that may denote other nearby numbers. Thus two representations with the same magnitude may differ in content.[28]

An accumulator can be used to compare sets of objects by cardinality, by filling one basin for each set, and comparing the levels of fullness of the basins. It can be used for addition, by combining the contents of two or more basins. It can also be used for subtraction, multiplication, division, and so on. Notice, moreover, that the model provides an especially powerful explanation of why infants' numerical abilities are subject to Weber's Law. In particular, because the system exhibits scalar variability, it cannot reliably distinguish between nearby cardinalities, where what counts as "nearby" is proportional to the size of the given cardinalities.

In explaining both of these features of the system, authors have appealed to both the received and alternative accounts of analog representation. For example, Wynn (1992) appeals to shared structure between representation and content in describing how the system explains infants' numerical abilities. She writes,

> In the accumulator mechanism, numerosity is inherently embodied in the structure of the representations... the relationships between the representations exactly reproduce the relationships between the quantities they represent. For example, four is one more than three, and the representation for four (the magnitude of fullness of the accumulator) is one more increment than the representation for three (p. 219).

[28] Other models posit a system in which representations compress logarithmically as the number represented grows. See for example, Dehaene et al. (2008).

Gallistel et al. (2006) appeal explicitly to the difference between analog and digital representation to describe how the system explains why infants' numerical abilities are subject to Weber's Law. They write,

> When a device such as an analog computer represents [numbers] by different voltage levels, noise in the voltages leads to confusions between nearby numbers. If, by contrast, a device represents [numbers] by [discrete] symbols, as digital computers and written number systems do, then one does not expect to see the kind of variability seen [in the experiments described]. For example, the bit-pattern symbol for 15 is 01111 while for 16 it is 10000. Although the numbers are adjacent, the discrete binary symbols for them differ in all five bits. Jitter in the bits (uncertainty about whether a given bit was 0 or 1) would make 14 (01110), 13 (01101), 11 (01011), and 7 (00111) all equally and maximally likely to be confused with 15, because the confusion arises in each case from the misreading of one bit. These dispersed numbers should be confused with 15 much more often than is the adjacent 16. Similarly, a scribe copying a handwritten English text is presumably more likely to confuse seven and eleven than to confuse seven and eight (p. 252).

However, authors also appeal to the analog and approximate nature of the system in arguing that it cannot be the sole source of natural number concepts. For example, Spelke (2003) writes,

> [The system of mental magnitudes] represents... numbers of objects or events as sets with cardinal values, and it allows for numerical comparison across sets. This system, however, fails to represent sets exactly... and therefore it fails to capture the numerical operations of adding or subtracting one (p. 299).

Similarly, Margolis and Laurence (2008) write that mental magnitudes "are by their nature approximate and hence incapable of expressing a difference of exactly one" (p. 935). These and other authors conclude therefore, that the system either works in concert with other innate systems to produce natural number concepts, or indeed that human beings must innately possess natural number concepts.

It is important to notice though, that the arithmetic calculations the mechanism is able to perform require positing some part or parts of the overall system that serve to, for example, compare levels of fullness of basins. Meck and Church (1983) did indeed posit a "comparator" in addition to other functional parts of the system. Given the present terminology, the comparator is the system's *reading user*. The reason the system seems approximate or analog, is because the comparator only "looks at" completed representations, which are subject to scalar variability, such that comparing them will not allow discrimination between nearby cardinalities. However, there is nothing approximate in the number of increments that compose to form a given magnitude. Again, if n objects are observed, n increments may be passed from the pacemaker to the basin, and there is nothing approximate about n, whatever n is. What this means is that if there were some way for the comparator to "look at" the number of increments that were composed to form a magnitude, rather than only the completed magnitude itself, it would be able to discriminate between nearby cardinalities. This would be a different system, employing the same representations, and it would not be approximate or analog (on at least some accounts of the a/d distinction). And thus, the major premise in the argument for why the accumulator cannot be the sole source of natural number concepts would not be true. Or more exactly, what would emerge is a way in which the representations that partly constitute the accumulator may also partly constitute another system, and this other system may indeed be the sole source of natural number concepts.

Elsewhere I have argued[29] that learning the number words may allow for this very adjustment to take place. In particular, children first come to learn the number words as a meaningless string of symbols, and only later do they associate them with exact cardinalities.[30] This process takes considerable time, and proceeds in recognizable steps. Children first learn the meaning of “one”, then “two”, then “three”, and finally all the rest at once. The hypothesis I propose is that this process is one whereby the child learns to use the number words to mark in memory the number of increments that were composed to form a mental magnitude. The idea is that the child then has a record in memory of the precise number of objects or events that were observed, and can compare that to other similar records, rather than just being able to compare the completed magnitudes. This would in essence provide the child with a precise representation of the number of objects or events that were observed, since the number of increments composing a magnitude is precise. These representations then, would be natural number concepts, and the sole innate source for them would be the same representations that partly constitute the accumulator.

I do not intend to present and defend this hypothesis in detail here. I intend only to note that by recognizing the difference between comparing completed mental magnitudes and comparing the number of increments in mental magnitudes, we reveal an otherwise overlooked hypothesis, according to which mental magnitudes are after all the sole source of natural number concepts. Or to put the point another way, by failing to recognize that mental magnitudes must be interpreted, and that describing them as approximate or analog assumes that they are interpreted in a specific way, contributors to the literature have failed to appreciate possible hypotheses for their role in the acquisition of natural number concepts. In short, the failure to recognize that representations have users, and that how those users interpret representations can determine whether the systems constituted by both representations and users are analog or not, approximate or not, and so on, has been something of an impediment to scientific progress—at least insofar as one considers exploring the space of possible hypotheses a component of progress.

5 Conclusion

I have considered four accounts of the a/d distinction, and I have argued that all four imply that representational schemes are neither analog nor non-analog, that representational systems—which are constituted by schemes and their users—are, and that a given scheme may partly constitute both analog and non-analog systems, depending

29 Katz (2013).

30 See Spelke (2003) for a description of the process.

on facts about how the users of those systems employ the representations. Again, there are other accounts of the distinction, but it seems likely that any account will imply this same result. The reason is because any account of the distinction will invariably rest on some salient feature of the representations, which determines how they are to be interpreted. But interpretation implies an interpreter—a user—and different users may exploit different salient features of representations. I have also argued, by way of the literature on number concept acquisition, that a failure to appreciate this result can impede scientific progress, at least insofar as it can obscure possible solutions to unsolved problems.

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