Synthese (2015) 192:881–922
DOI 10.1007/s11229-014-0498-y

Two papers on existential graphs by Charles Peirce

Ahti-Veikko Pietarinen

Received: 13 February 2014 / Accepted: 20 May 2014 / Published online: 2 July 2014

The following two articles comprise two sets of Charles Peirce's manuscripts, "Recent Developments of Existential Graphs and their Consequences for Logic" (MS 498, MS 499, MS 490 & S-36, 1906) and "Assurance through Reasoning" (MS 669 & MS 670, 1911), written for the National Academy of Sciences meetings in 1906 and 1911. The papers are deposited at Houghton Library, Harvard University. Only some parts of MS 470 have been published before, and in somewhat defective form. Although "Assurance" follows "Recent Developments" chronologically, given the expository style of the former it is recommended to be read before "Recent Developments". As the title indicates, in the latter Peirce goes on to describe his latest discoveries concerning the method and the logic of existential graphs. The transcription reproduces all significant deletions that appear in the original sheets. Editorial comments and additions are given in brackets. **[Alt.:]** means the beginning of an alternative sequence. **[Del.:]** means the beginning of sections that have been crossed out. A couple of paragraph skips have been added to improve readability. In all other respects the transcriptions are diplomatic.

CHAPTER 1

Charles Peirce: Recent Developments of Existential Graphs and their Consequences for Logic (MS 498, 499, 470, S-36, 1906)

The following text consists of three of Peirce's manuscripts, all written for the National Academy of Sciences spring 1906 meeting. The first is "On Existential Graphs as an Instrument of Logical Research" (MS 498, with the deleted title "On that Method

A.-V. Pietarinen (✉)
Department of Philosophy, History, Culture and Art Studies, University of Helsinki,
P.O. Box 3, 00014 Helsinki, Finland
e-mail: ahti-veikko.pietarinen@helsinki.fi

of investigating logical Stechiology which avails itself of Existential Graphs"). The second, "On the System of Existential Graphs, considered as an Instrument for the Investigation of Logic" (MS 499), is an alternative and probably slightly later draft of MS 498. The third, and perhaps the most remarkable one is MS 490, a paper prepared and read for the National Academy of Sciences, Washington DC, 16–18 April 1906, bearing the title of the present selection.

Peirce's presentation is mentioned in *Report of the National Academy of Sciences for the Year 1906*, Senate Document No. 308, 59th Congress, 2d Session, Washington: Government Printing Office, 1907, p. 15 (P 01140). The conference report published in the *Nation* describes it as "a long paper by Mr. C. S. Peirce, on the Method of Existential Graphs, by means of which he showed that this system gives a sort of diagram of the mind in reasoning, and also that there is, strictly speaking, but one way in which the different logical elements of any concept or judgment are combined; namely, by each being indefinite or indeterminate in some respect in which another element renders it determinate" (*The Nation*, April 26, Vol. 82, p. 342, CN 3.269, P 01132). Judging from the content and the timing that occurs in the manuscript marginal, MS 490 was Peirce's actual presentation. At around 35 min into the presentation, Peirce concludes that "the system of existential graphs is a rough and generalized diagram of the Mind", and that the system "recognizes but one mode of combination of ideas, that by which two indeterminate propositions mutually determine each other in a measure". MS 499, in turn, may have served as preparatory remarks setting ground for and describing the "two great logical puzzles", the relationship between logic and cognition and the nature of the proposition, to which EGs were then argued to throw light upon. Altogether the reading time for the both two texts would have been at least 60 min, depending on whether the numerous graphs were prepared in advance on the blackboard or projected, or whether Peirce had to draw ("scribe") them during the talk.

Manuscripts 498 and 499 have not been published before. Portions of MS 490 were published in CP 4.573–584 with the misleading title "Introduction to Existential Graphs and an Improvement on the Gamma Graphs". That publication, erroneous and seriously incomplete, omits most of the text (38 entire paragraphs of the original text were not included) as well as almost all the graphs (and the five that it reproduced are all erroneous). The available fragments thus deliver neither the crucial philosophical context that Peirce set up in MSS 498 and 499 nor the important and novel details of the logic of EGs. How EGs serve the advances in logic, including the philosophy of logic is, as Peirce remarks, the substance of his long paper. A major innovation he establishes concerns new philosophical uses of his quantified modal graphs by which to answer the two great logical puzzles.

Peirce's 1906 presentation has a good claim of making him the founder of modern philosophical logic. In tackling the two great logical puzzles he comes to establish the philosophical significance of multi-modal logic, quantification in modal contexts, the idea of world-lines ("references"), as well as what later on became known as the 'Peirce's Puzzle' (which was published in his 1906 *Prolegomena*), namely the question of the meaning of indefinites in conditional sentences, and ultimately, the meaning of logical connectives and quantifiers in such contexts.

Some alternative pages have been taken from MS S-36. All of the manuscripts are located at Houghton Library.

[MS 498:] On Existential Graphs as an Instrument of Logical Research. The system of expressing propositions which is called Existential Graphs was invented by me late in the year 1896, as an improvement upon another system published in the *Monist* for January 1897. But it is curious that 14 years previously, I had, but for one easy step, entered upon the system of Existential Graphs, reaching its threshold by a more direct way. The current of my investigations at that time swept me past the portal of this rich treasury of ideas. I must have seen that such a system of expression was possible, but I failed to appreciate its merits. These merits are, indeed, almost certain to escape recognition by anybody who has a commonplace and superficial notion of the scientific value of a system of symbols.

Just as very few writers upon the history of the differential calculus have put their fingers upon the point of paramount superiority of Leibniz's notation over Newton's, and most of them seem to think the difference between them is merely external; so the majority of students of exact logic seem to look upon the distinction between existential graphs and logical algebra as consisting, except in one particular, in a slight difference in the fashion of dress of the new system, while in that one particular it maims the algebra of one of its most important and beautiful members.

I may instance specially the judgment of a distinguished French logician, to whom, without any authorization from me there had been shown a proof-sheet of an article by me in which I entered upon a comparison of the two algebras, the one of dual relatives and of the other the general algebra of logic with the system of existential graphs to the advantage of this last.[1] He thereupon felt himself called upon to write administrating a castigation of my presumption in setting up my silly invention as a rival of these creations of genius. I had fancied that the circumstance of my being the author of the two algebras would privilege me to speak somewhat lightly of them. However, subsequent reflexion suggested that perhaps he was right.

When a man has given a work to the public and it has been favorably received, the ownership has passed out of his hand, and his relation to it is not essentially different from anybody else's. So I have since endeavored to hold myself in due awe, and to criticize my performances with becoming diffidence.

The great misconception of the majority of non-logicians, and I fear of a good many logicians with them, is that the great purpose of a logical algebra or other system affiliated to a logical algebra is to serve as a *calculus*, that is a contrivance for deducing conclusions from premisses by means of a routine of transformations. If the opinion of the inventor of the systems most in use is of any weight, I wish to declare that I for one never entertained any such ridiculous conception. On the contrary, the structure of those algebras show that quite a contrary purpose presided over their production.

A calculus, in the sense of the definition I just gave, a system of signs, enabling a person by following a routine of rules, to solve any problem of a given kind, in order to fulfill its purpose to perfection, should pass from premiss to conclusion in the smallest number of steps possible; while my design in these algebras I invented was virtually, and in the system of existential graphs was quite definitely and consciously, to dissect the inferential process into as many steps as possible.

[1] [Louis Couturat (1868–1914). See Peirce's letter to Josiah Royce 19 January 1902. The article in question was Peirce's draft entry "Symbolic Logic" for Baldwin's *Dictionary of Philosophy and Psychology*].

A person who has no deep comprehension of logic, upon becoming acquainted with Boole's logical algebra, cannot fail to feel that he is in the presence of a novel and great idea; but he has not the logical equipment needed for diagnozing this idea and making out wherein it is so remarkable. He fancies that it is the fact that he can express premisses in this algebra and then, by a quasi-mechanical procedure, can get out the conclusion. He ought not to deem this remarkable. Aristotle did the same thing. Every Jacquard loom, and indeed far simpler machines do the same. But he has a notion that only a conscious person can think and that it is necessary to think in order to reason. This is the crudest of notions of what it is to think. In truth, the kind of "thinking" for which experience seems to show us that, we know not why, consciousness is indispensable, is *self-control*,—whether it be intellectual or moral. Nobody had proved that an automatic engine cannot exercise self-control beyond a specific self-adjustment for which its construction has provided; but nobody has ever succeeded in *designing* such a machine, far less in executing it. But routine operations call for no self-control. And therefore there is nothing in the least surprising or contrary to daily experience in the fact that Boole's algebra works out conclusions correctly. There certainly is something marvellously beautiful in Boole's algebra. Only it is not that. It is something that the mind untrained in logic perceives but cannot distinctly analyze. Falling into this natural mistake, it is supposed that the great problem in inventing an algebra or other system of logical representation is to create something equivalent to a calculating engine. The untrained mind does not see that every machine whatever is a logical machine working out incessantly new conclusions from premisses as long as the steam is kept up.

This same mistake is committed in regard to that greatest of all victories of the human mind, the differential calculus of Leibniz, in which Newton, great mathematician as he was, greater by far than Leibniz as mere mathematician and greater as a philosopher than he usually gets credit for being, had no hand at all. Newton is to be placed along with Fermat among the forerunners of the differential calculus,—or if you please as the beginner of that calculus as a mere calculus in the sense of the definition I just gave; but in the differential calculus as that great instrument of thought about the pseudocontinuity of the theory of functions, Newton had no hand. To return to my point, the routine procedure of the differential calculus, somewhat like the routine methods of the Washington offices of government, is useful in a modest way, but it has not been, as the uninstructed suppose, the chief instrument of modern discoveries in infinitesimal analysis. Those discoveries have employed the routine, because circumstances had made it familiar and therefore convenient, but the great discoveries were not achieved by routine, but by the free and dexterous manipulation of that handy tool, the symbol $\frac{d}{dx}$, for which Newton had no equivalent. It is in a somewhat similar way that existential graphs may be made to do yeoman's service in cleaning up the puzzles of the science of logic. The purpose of this paper is to illustrate the way in which it can be put to use by applying it to two of the subjects under actual discussion among logicians of the present day.

The two puzzles which I select for illustrating the value of existential graphs are, first, that of the relation of a sign to the mind of its utterer and to the mind of its interpreter, and second, the puzzle of the mode of composition of concepts, which is particularly vexed in regard to the structure of a judgment.

The reader will observe that I do not call these logical difficulties *Problems*, but rather *Puzzles*. The reason is that it is their peculiarity that, as long as they are unsolved, we cannot distinctly formulate them as Problems. When that can be done, the puzzle is on the point of being solved if it be not already solved.

I will briefly explain the nature of these two puzzles. To begin with that about the mind, there is a certain opinion in the logical world to the effect that since logic is the science of thought or of the forms of thought, or is the art of reasoning, or something of the kind, and since thought and reasoning are performed by the mind, logic must rest on a psychological basis; or even if scientific psychology cannot properly be taken as its basis, since psychology is in the utmost need of the precepts of logic, and the two sciences cannot logically rest each upon the other, yet still there are certain facts about the mind known to all the world and which cannot really be doubted. Upon what is logic to rest? It cannot furnish support to itself? Evidently, its first propositions must be accepted because they are agreeable to reason. In so far, then, and as regards the foundations of logic, this science must rest upon our knowledge of the mind and our faith in its decisions.

But another section of the logical world replies that if we consider any one of those first principles of logic, as for instance, the principle of contradiction that nothing at once possesses any character and possesses the negative of that character, it is obvious that this proposition says nothing about how we think, but about how things really are. If we firmly believe this principle to be true, there can be no question of proving it, since proof establishes or refutes solely propositions that have been doubted. If there is no doubt there is no occasion and no room for proof of any kind, and therefore there is no occasion to say anything about how we think. On the other hand, if the principle of contradiction [sic.], its truth can be proved without reference to how we may think. Moreover the very kernel of logic consists in the investigation of the validity of arguments, which depend upon their necessary conformity (whether perfect or not) to real facts. Now thinking one way or another will not alter facts. Logic, it is true, does not inquire into the facts of real and non-significant things. It studies especially that which is true or false, as to whether it be necessarily true or not.

But it is necessary to draw a distinction between *thinking* and *thought*. A sentence printed in a book is true or false. It embodies thought, although it does no thinking. It is subject to logic. It does no thinking in the sense that it is not living. It has not, nor has any logical machine, a sufficiently complicated construction, to permit us to recognize its action as thinking. But though a machine cannot be made or even perhaps designed that will exhibit such freedom of logical transformation as to be called thinking, it is possible to describe in a general way a mechanical construction which would merit the name of a thinking machine. Logic itself has nothing to do with the process of thinking. It only compares the premisses with the conclusions and it therefore has no concern with any character of the mind that might not be shared by unconscious machines or by external signs.

The answer I am reporting now goes on to show, what will hardly be disputed, that all deliberative meditation, or thinking proper, takes the form of a dialogue. The person divides himself into two parties which endeavour to persuade each other. From this and sundry other strong reasons, it appears that all cognitive thought is of the nature of a sign or communication from an uttering mind to an interpreting mind. Now in order

that the logician may recognize as germane to his studies all that really can concern them in any essential way, while guarding himself against being diverted from his line of inquiry, it is best that he should recognize that the object of his study is the form of construction and forms of functioning of signs in general, including mental signs but not confined to them. With the matter or contents of ~~human~~ consciousness, except in regard to forms of thought which are equally forms of other signs he will have nothing at all to do. To this answer the first school of logic makes reply both simply and forcibly. Thus: You wish logic to be considered as a section of the science of the physiology of signs. What then is a sign? It is a medium of communication. Communication between? Between two *minds*. So then the mind has to be taken into account.

I will add to this one remark. Two minds can communicate only by becoming in so far one mind. How is this possible? I have thus sketched the first of the two logical puzzles.

The other puzzle can be much more briefly explained. During the last decade and more no question perhaps has more seriously occupied the hesitations of logicians than that of the nature of the judgment. Without going into this further than to say that the main doubt is whether it is the composite nature of a proposition as built up of subject and predicate, with or without a copula, which is its essential characteristic or whether it is the act called assertion, whatever be the essence of this act. It is certainly a difficult question. But I find that upon clearing away from it, what it would be wrong to call the rubbish or *debris* of the examinations which have been made, but matter which certainly obscures its main outlines, I find that the solid core of all this discussion is the question of how concepts can be compounded. Suppose two concepts, A and B, to be combined. What unites them? There must be some cement, and this must itself be a concept C. So then, the compound concept is not AB but ACB. Hereupon, obviously arises the question how C is combined with A or with B. The difficulty is obvious, and one might well be tempted to suspect that compound concepts were impossible, if we had not the most manifest evidence of their existence.

Here, then, are the two puzzles of logic upon which I am going to try what light can be shed by the System of Existential Graphs. They are the puzzle of the relation of signs to minds, and of their communication from one mind to another, and the puzzle of the composition of concepts and the nature of the judgment, or, as we of the antipsychological school say, of the proposition.

I must now describe the system of Existential Graphs; and I must beg your close attention, promising to be as concise as that favor requires of me.

In this system, every sign, and every complete part of a sign, that is, everything which if it stood alone would be a sign, is of a nature to be fully interpreted in a proposition. We shall see how that perfectly provides for arguments and for names of all kinds. But it leaves signals of command,—such as the infantry-officer's "Ground arms",—as well as pieces of concerted music, though they communicate not merely sentiments, but also esthetic forms,—it leaves all these unrepresented. That is an imperfection of this system. The signs which it leaves unrepresented are signs of feeling presented as such and signs of energy presented as such. The signs it represents are signs presented as cognitive, that is as conformed to a real object. By *real*, I always mean that which is such as it is whatever you or I or any generation of men may opine or otherwise

think that it is. There must not be any confusion between reality and exteriority that is real which is as it is no matter what one may think *about it*. The external is that which is as it is whatever one may think about *anything*. No doubt there are grades of reality, meaning that objects of signs may yield with more or less resistance to opinion or other representation. According to the definition absolute resistance is essential to reality. But an approach to reality, something that is not in the slightest of the nature of pretense is found wherever an object of thought is sufficiently obstinate to enable us to say, it has *not* these characters, but it *does* have these. There is already a lesson in logic. Namely that one may lay down the very best of definitions, going to the very heart of things; and yet there will be, as it were, a little living mouse of a quasi-exception which will find or make a hole to get in when all seemed hermetically closed. This mouse will not be a mere pest to be got rid of and forgotten. It will be a fellow being to be remembered and to be appraised.

Every sign of this system is perfectly interpretable by a proposition, that is, is necessarily either true or false, and every proposition whatever is capable of being represented according to the conventions of this system. Every expression of a proposition according to the conventions of this system is called either a *graph*, or a *graph-instance*. It is necessary to recognize the facile distinction between a graph and a graph-instance. A graph-instance is a *token*, that is, is an existent individual object, which signifies a proposition. It can never be duplicated. Although I duplicate it, and the duplicate will be a graph-instance of the same signification in all respects, but it will not be that individual graph-instance of which it is the precise copy. I *scribe*, that is, write or draw, a sign meaning Tully was Cicero. I duplicate it precisely. The new sign will be substantially the same. It will only differ so much as is necessary to make it a second scribing of precisely the same type. But it will not be the same graph-instance. A *graph*, on the other hand, is a type. If two graph-instances are precisely interpretable by the same proposition. [sic.] In order to show you that you are already perfectly familiar with this distinction between a *Type* and a *Token*, let me point out that upon any ordinary English page the word "the" will be found to occur about 20 times. In these twenty occurrences, it is always one and the same word "the". It is said that Shakespeare uses 16,000 English words. Of course in this country "the" is a unit. But a common fashion of expressing the length of a magazine article or of a book introduced by the late Dr Edward Eggleston is by the number of words in it. In this count "the" will figure us about 20 on each page. Now in the sense in which "the" is *one* word only, no matter how many times it may occur, it is a *Type*, a form or habit. In the sense in which it is 20 words on each page, of each copy, or in an edition of ten thousand copies of a book of 300 pages is sixty million words, each of these is a *Token*. An *instance* of a graph is a Token. The distinction between a *Graph* and a *Graph-instance* has a certain importance. [end]

[MS 499:] On the System of Existential Graphs, considered as an Instrument for the Investigation of Logic. The majority of those writers who place a high value upon symbolic logic treat it as if its value consisted in its mathematical power as a calculus. In my article on the subject in Baldwin's Dictionary I have given my reasons for thinking, on the contrary, that [if] it had to be so appraised, it could not be rated as much higher than puerile. Peano's system is no calculus; it is

nothing but a pasigraphy; and while it is undoubtedly useful, if the user of it exercises a discrete freedom in introducing additional signs, few systems of any kind have been so wildly overrated, as I intend to show when the second volume of Russell and Whitehead's Principles of Mathematics appears. As for the other systems the aim that was kept in view in the construction of them was directly inconsistent with their having any special value as calculi. I venture to say this because I have thoroughly studied the genesis of Boole's original algebra, not merely in his own writings but in others which occupied his mind; and I cannot well be mistaken as to how it came into being. Too many independent indications point the same way. As to the three modifications of Boole's algebra which are in much use, I invented these myself,—though I was anticipated as regards one of them,—and my dated memoranda show perfectly what my aim was. I was to make the algebras as analytic of reasoning as possible and thus to make them capable of exhibiting every kind of deductive reasoning. But to say that the aim was to make the algebra as analytic of reasonings as possible is to say that the aim was to make every demonstration as long as it possibly could be made without being circuitous. A calculus, on the other hand, is a system of symbols by transforming which according to a certain routine one is enabled to pass from a premiss to a conclusion in a particularly speedy and direct way. I ought, therefore, to have been obvious in advance that an algebra such as I was aiming to construct could not have any particular merit as a calculus.

This brings us to the question of what value, then, are the algebras of logic? To this I shall begin by giving the indirect reply that they have a value of the same sort, as the value of the notations of the differential and integral calculus. Non-mathematicians who have only an elementary acquaintance with the differential calculus usually have an exaggerated idea of its value, considered as a calculus. It can be proved by means of principles capable themselves of demonstration by the methods to be described in this communication that a calculus, or notation to be transformed according to general prescribed rules, laid down once for all,—which will only differ from a logical machine in that the changes are not effected by mechanical force but by quasi-mechanical rules,—must have one or other of two characters, or else a mixture of these. Namely, it must either be artificial, in the sense that its own working does not depend upon the very elements of the problem in hand, in which case it will be entirely inapplicable to any problems except those of a special kind which had to be solved beforehand in their generality in order to *design* the calculus; or else it is natural, by which I mean that it is like a tank of water with powder disposed through it and specially illuminated this tank being considered as a reasoning machine for solving problems of hydrodynamics, that is the powers which determine its operations are identical in form with the conditions of the problem to be attacked. In this case, it may be applicable to a very wide range of problems, but except in special cases, to ascertain what the result of the calculus is will be the very operation of solving the problem; so that there will be no advantage in it, but rather encumbrance. It is a truism to say that in all the great advances in infinitesimal analysis, the routine that constitutes the calculus (as defined by Leibniz) has played an altogether secondary role. The principal service has been Leibniz's notation, $\frac{d}{dx}$, in which Newton had no part whatever. This symbol $\frac{d}{dx}$ has enabled all subsequent mathematicians to form what Leibniz terms a "symbolic concept" of the relation upon which all geometry, or mathematics of continua, turns.

I now proceed to illustrate the ways in which the system of existential graphs is serviceable to the advance of the science of logic. This is to be the substance of this paper.

But I must preface that substance with some remarks which have but a loose connection with it but which are needed in order to make the point of view of that substance intelligible. This substance, as I just said, is to illustrate how existential graphs are serviceable to the science of logic. But this is not strictly accurate language. It could indeed have no precise meaning until the science of logic is defined. Therefore, although I may seem to be faring far afield, I must explain in what sense I speak of a "science",—which is an abridged expression for a heuretic science, or science aiming at the discovery of new truth. Namely, I do not mean by science, as the ancients did, that doctrine which is beyond all doubt. Nor do I use the word in the sense in which Coleridge at the beginning of the XIXst-century defined science as systematized or ordered truth. But I use science in the sense of a business, that is, of a total of real acts exerting reciprocal effects one upon another, and concerned with closely analogous purposes. When I speak of any given heuretic science, I mean the body of doings in Past and Future time, not too remote from the present, of the members of a certain social group. These persons constitute a social group in their acquaintance with, understanding of, and sympathy for one another's doings. And the peculiarity which make it a *scientific* group are, first, that the members are devoted to ascertaining truths of a given kind on account of their speculative interest in the matters, that they have each of them some special facilities or capacities for such research, that they employ approved methods, and that each seeks aid from the results of the others. From this point of view, the question whether a given class of investigations ought to be regarded as belonging to this science or to that is not to be settled by mere logical analysis, but is a question of fact; namely, it is the question whether the men who in our day will undertake in a scientific way investigations of the class in question will naturally mingle with one group or with another group.

Accepting this understanding, it will be necessary for the present and for a long time to come to regard logic, not as a distinct science, but as only a department of the ~~inquiry into~~ science of the general constitution of signs,—the physiology of signs,—cenoscopic semeiotics. For it we roughly define a sign as a medium of communication, a piece of concerted music is a sign, and so is a word or signal of command. Now logic has no ~~special~~ positive concern with either of these kinds of signs, but it must concern itself with them negatively in defining the kind of signs it does deal with; and it is not likely that in our time there will be anybody to study the general physiology of the non-logical signs except the logician, who is obliged to do so, in some measure.

But at this point we come upon one of the important questions which I have selected as illustrating the utility of the system of existential graphs. Ever since there has been any definite science of logic, there have always been logicians who have held that although logic does not deal with all forms of signs, yet it does apply to any sign that has a certain form of constitution, and that, be our interests ever so restricted, yet in order to comprehend reasoning, it is necessary, in the first instance, entirely to disregard all the psychical ingredients of it, the qualities of consciousness, the effort of attention, and so forth, even if these have to be taken into account later, in order to consider without confusing circumstances, the formal relations of signs. I may be permitted to say, by the way, that this is and always has been my position. But in answer to this, it

is urged, probably by the majority of logicians, counting noses indiscriminately, that a sign cannot function as such unless it be ultimately interpreted by a personal mind; so that if we limit ourselves to *concepts*, or the mental interpretations of signs, we shall therein include every sign that is a sign in actual function, while if we consider signs regardless of the relation of each to a mind, we neglect to consider a most essential characteristic of signs, and thus make room for errors of logic. There is much force in that answer; but under cover of it those who support it introduce considerations which greatly confuse logical questions. Now existential graphs furnish us concepts wherewith to reply to that answer, so as to do full justice to its truth, while entirely eliminating its error; and without the conception furnished by existential graphs I know not how the answer can be fairly and justly met. How the puzzle is at once, by means of existential graphs, I shall presently show you.

Another great puzzle of logic is that of the composition of concepts, or thoughts. It is evident that a thought may be complex. Darwin's Origin of Species is devoted to the presentation of one thought to which every chapter and almost every paragraph of the work is an essential part. That shows, what nobody has ever doubted, that there are thoughts compounded of thoughts. But let A and B be two simple thoughts which can be compounded. But how are they compounded? They are compounded in thought. Very well, then, the composition must then be a third thought, which we may denote by C; so that the compound is not AB but is ACB. Then the question arises, how are A and C compounded, as they certainly are in ACB; and how are C and B compounded? This puzzle has been formulated in all its generality by some great logicians. In that abstract form, however, it has not attracted any great attention, owing to the feebleness of mind of the rank and fit of logicians. But a particular phase or case of this puzzle has been the arena for a good half of the logical controversy of our time, which has turned on the question, What constitutes a judgment or proposition? The common doctrine has always been that just as an argument is a compound of propositions, so a proposition is a compound of names, with or without a copula, *is*. But many logicians answer that such a proposition as "It rains" is not compound, and that it consists simply in taking the idea of rain, assertorically; so that a proposition according to them is merely a name-concept apprehended assertorically. The reply that is made to this answer is that if the proposition "It rains" consisted in taking a simple concept of rain assertorically, then the question, Does it rain? would consist in apprehending the same concept problematically. But in fact, it is very easy to discern two distinct ideas in this question; namely, the familiar idea of rain and the idea of the present common environment of the questioner and person questioned, and the question asked is whether that idea of rain applies veraciously to that environment. To this reply the rejoinder is that according to the reply, the interrogation is as complex as the assertion. But the interrogation is not a proposition or assertion; and thus the reply confesses that the essence of the proposition does not lie in its being compound, but on the contrary upon its being asserted or at least conceived to be asserted. Now assertion does not add a new element of thought. For an assertion is not a thought but a deed. If one goes before a notary and takes one's affidavit to a statement, that is nothing but highly emphatic assertion. The law permits the quaker to "affirm" as fully equivalent. But the law-phrase is quite correct that the assertion before the notary is an *execution* of one's *act* and *deed*. Whether it consists in assuming a responsibility

or in something else is a side-question. The point is that it is no *thought* but is an *act* related to the thought. To this rejoinder a somewhat weak surrejoinder is often made, which though weak is not uninstructive. The surrejoinder, such as it is, is this:

> A man appears before a notary and says, I wish to make what I have written on this paper my act and deed, I will swear to it and be responsible for it. The notary takes the paper and finds nothing written on it but these words—hard red crystal. He says you cannot make yourself responsible nor so much as wish to make yourself responsible for a crystal of any kind.

To make yourself responsible means that if on a certain defined occasion a certain definitely described phenomenon does not emerge then you confer upon somebody a right which he otherwise would not have. This bet need not be definitely expressed, for if it is indefinite you at least give everybody the right to call you untrustworthy. But you must say, whether on the occasion of a hard thing being red, it will be found to be a crystal or whether upon the occasion of a crystal's being red it will be found to be hard, or what conjunction of two possible events it is to which you wish to make yourself responsible. One can only be responsible for future conjunctions of definitely described kinds.

I have said enough to give some idea of what this question of the constitution of the judgment is. It is still unsettled and is the most prominent perhaps among unsettled questions in the logical literature of recent years. I wish in this communication to exhibit to you the unexpected solution not merely of this problem but of the more general problem of the composition of concepts to which the system of existential graphs leads.

I must warn the Academy beforehand that I do not propose today to enter into the demonstration of the truth of the solution suggested by existential graphs, of the two problems in regard to the relation of signs to the mind and of the composition of concepts. The reasons I exclude this part of the discussion are, first, that it would render my paper tedious since the proof presents no very striking idea, or other great novelty, and second, that if I should go into the tedious development it is unlikely that any one member would carry it away from an oral statement without dropping out some point which is essential to its cogency; and at any rate it would be unintelligible to the great majority. In case the paper should be printed, I will append the proof for the benefit of those who may desire to examine it.

I have now sufficiently indicated two great logical puzzles. I do not call them problems because it is the nature of logical difficulties that until they are solved we cannot distinctly state what the problem is. Even after they are solved, it is often no easy matter to say what the problem was. The puzzle of the measure of force is one of many instances of this that I might adduce. These two puzzles relate to the mode of composition of concepts in general and to the nature of the proposition in particular, and to the relation of concepts and signs to the mind.

These puzzles being indicated, I must describe the system of existential graphs, and then go on to show how this system throws a light upon these puzzles.

But in order that my description of existential graphs may be quite clear, I must ask you to observe certain essential relations of signs, from which is deduced a classifi-

cation of signs, and a nomenclature which I shall be obliged to employ in describing existential graphs.

I may remark that in no science, not even in comparative anatomy, are questions of classification so vital as they are in phaneroscopy and semeiotics. My classification of signs is not yet fully matured. I have been at work upon it, or at least have kept it in mind since 1863, but still confidently expect important improvements in it. If I live to complete it, it will be the contribution to exact logic that has cost me the most labor, and it will be recognized by exact logicians as a very positive and indisputable contribution to exact logic even if I should leave it in its present imperfect state. There remain many hundreds of difficult questions yet to be considered, though the majority of them have received an examination which cannot justly be called careless or summary. Lest it should be suspected that I exaggerate in saying that there are many hundred questions I will say that the exact number, so far, is 205 billion 891132 million 94619. But these are not all independent. From the answers to some the answers to others can be deduced. The total number of really *independent* and really difficult questions, requiring each some days of laborious consideration is only 1073 million 741794. So that if I should have the good fortune to live 82 or 83 million years longer I might hope to exhaust the subject. It is not to be expected that I should live long enough to answer them all with positive assurance.

I will say that I assume at the outset a certain definition of a sign. No assumption made at the outset of so broad an inquiry could have any scientific value; and I claim none for this definition. Yet I may say that although I say I assume it at the outset of my inquiry, this means at, perhaps the twenty-first revision of my inquiry. I had not truly this definition distinctly in mind when I first began. I will say that a sign is a something which is on the one hand caused or otherwise determined by something else which is not utterly and altogether unreal,—this something else being the object of the sign. When I say that the object is not altogether unreal, I mean this. If a thing has whatever characters it has utterly regardless of what any men existing either now or at any assignable future date may opine that its characters are, that thing is, by definition, perfectly real. But in so far as it is whatever the thinker may think it to be, it is unreal. Now I say that the object of a sign must resist in some measure any tendency it may have to be as the thinker thinks it. Now a thing of which any predicate whatever should have been from all eternity true in case anybody should take it into his head to say it had been true, would confirm accurately to the best logical definition of nothing, and could not be the direct object of any sign. From this a very important corollary might be deduced; but I pass it by. But a sign is not only on the one hand determined by a more or less real object but on the other hand it determines something,—which I call its *interpretant*,—to be through it determined as it is by the object of the sign. The interpretants of the signs with which logic chiefly has to do are themselves signs. For every cognition is a sign as Leibniz and other nominalists have sufficiently shown and all deliberate meditation is of the nature of a dialogue as Plato represented it to be. But it is important to recognize that there are signs whose interpretants are not *ipso facto* signs. Such is the command of a captain of infantry "Ground arms!" The interpretant is the fact that the arms are grounded, which may take place without any thought; and though the action may serve as a sign,—for example as a sign of the perfection

of the company drill,—it is not *ipso facto* a sign. It is not a sign merely in being the interpretation of the signal. From this, again, important corollaries can be drawn.

I trust that, although nice questions still remain that could be started, yet in a general way I have made it clear what I mean by the object and what by the interpretant of a sign. The object is the sign's *determinant*; the interpretant is the *determinand* of the sign.

[MS 490:] Recent Developments of Existential Graphs and their Consequences for Logic.[2] In working with Existential Graphs, we use, or at any rate imagine that we use, a sheet of paper of different tints on its two sides. Let us say that the side we call the *recto* is cream white while the *verso* is usually of somewhat bluish grey, but may be of yellow or of a rose tint, or green. The recto is appropriated to the representation of existential, or actual, facts, or what we choose to make believe are such. The verso is appropriated to the representation of possibilities of different kinds according to its tint, but usually to that of subjective possibilities, or subjectively possible truths. The special kind of possibility here called subjective is that which consists in ignorance. If we do not know that there are not inhabitants of Mars, it is subjectively possible that there are such beings.

The sheet we employ, called the sheet of assertion, in any one state of it, expresses a proposition which may be complexus we choose to make it; for a complex of assertions is an assertion. The transformations of the sheet represent a course of reasoning.

Any truth we may come to recognize, or what we choose to make believe is truth, is to be recorded by writing or drawing, which we shall call "scribing", on the appropriate side of the sheet. A proposition as expressed according to the conventions of the System of Existential Graphs is called a *Graph* which is an abbreviation for existential graph. What we thus place upon the sheet is necessarily a graph-instance and not a graph. For it would be as inaccurate and absurd to speak of a graph as being placed on a sheet of paper as it would be to speak of *word* being so placed. Every page of a book may have on it *instances* of the word 'the'; but 'the', however, is a single word, and if it lies on any page of the book, which is that page, rather than another? A word is not an image nor a figure. It is a general type, which *governs* instances but is not identical with any of them; and so it is with a *graph*. Being a type, it cannot be put in any place. Yet just as we speak of writing a word on a sheet of paper, so we shall give the word *scribe* such a signification that to place a graph-instance on either side of the sheet shall be correctly described as "scribing" the graph itself of which what is placed on the sheet is an instance on that side.

We begin with a blank sheet; and that blank sheet must have the force of a proposition according to whatever is taken for granted in every discussion and does not need to be explicitly set down. (3 min)

A heavy dot marked on the recto shall stand for an individual object existing in that universe which the utterer or graphist and the interpreter well understand one

[2] [On the notebook cover is written: "For the National Academy of Sciences 1906 April Meeting in Washington". Peirce prepared the following paper to be a 40 min presentation, as the timing that appears in the marginal, reproduced here, make it clear. The timing is quite accurate, assuming that the graphs were projected or drawn on the blackboard prior to the presentation. If Peirce also read MS 499 first, it would have added some 25 min to the total reading time, making the entire presentation certainly consistent with what conference report in the Nation described as the last and "a long paper by Mr. C. S. Peirce"].

another to be discoursing about.[3] Any object we please may be looked upon as logically indivisible or individual, and the *dot*, considered in itself, implies nothing but such individuality. But the act of scribing any sign upon the *recto* involves assuming the responsibility of *asserting* that the object denoted by that sign *is* in the universe of existents. As to the place on the sheet where the dot lies, that only refers quite vaguely to the occasion on which one is prompted to make the assertion. Two dots, therefore, denote each an individual object asserted to exist; but whether the two objects are distinct in their existence or identically the same is neither asserted nor denied:

• •

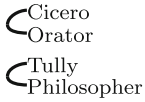
If it be desired to *describe* the individual denoted by a dot, we shall write the description or scribe it in any way we please, but so that the describing graph-instance shall be in contact with the dot. For we assume as a first principle of the system that one dot cannot denote two different individual objects.

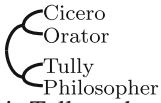
Another principle of this system is that whatever is asserted by being scribed on the recto is so asserted, regardless of anything else that may be scribed thereto.

It follows that if two descriptions are in juxtaposition with one dot as •-Cicero -orator there is an assertion that one individual exists of which both descriptions are true. (5 min)

From this it further follows that a limited and heavily drawn line, since such a line is nothing but a continuum of juxtaposed dots, can be nothing but the graph of identity of the objects denoted by its extremities. For juxtaposition as just said signifies identity. And further it follows that a point of branching of such a line is the graph of *teridentity*:

Y

Thus 

means Cicero is an orator and Tully is a philosopher. But 

means that there is something that is at once Cicero and an Orator and is Tully and a philosopher.

[3] [Alt. from MS S-36:] …be discoursing about. Any object we please may be looked upon as logically indivisible or individual; and the *dot* considered in itself has no other meaning than that its object is so apprehended. But to scribe it on the recto is so apprehended. But to scribe it on the recto is to assume a responsibility. It is to *assert* that this object exists in the universe. The place on the sheet refers to the individual occasion at which the assertion takes place. Two dots, then, denote each an individual object; and they present themselves to the asserting mind on different occasions. Whether or not they are different existents does not appear.

Now if we wish to describe that occasion, we shall naturally write the description along by it. Suppose for instance that this dot • denotes the present moment and this part of Washington. Then if we wished to make believe that it now and here rains we should scribe •it rains or •rains.

A dot on another part of the sheet will denote some other existential occasion, say Peking at this instant; and if we wished to assume that it is snowing there, we should write against that dot •snows.

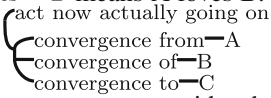
Now suppose we say that any such graph always asserts the same thing, no matter what may be scribed on distant parts of the sheet. If then both graphs are scribed both will be asserted.

We now come to consider the *verso*, which is usually appropriated to imparting information about *subjective possibilities* or what may be true for aught we know.

Now since the act of scribing a graph records an item of information, and since every item of information consists either in adding to the number of facts that we know to exist or else in subtracting from the subjective possibilities, it follows that a graph scribed on the recto must be affirmed while a graph scribed on the verso must be denied.

Of course  is the same as  or 

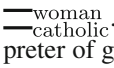
Another very important fact, that will especially please those logicians who are in love with my algebra of Dual Relatives, is that no other triad than that of teridentity seems to be needed.[4]

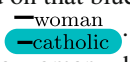
I call a ~~simple~~ non-relative term such as 'woman' a monad. —woman means that something is a woman. A dual relative is a dyad. It has two places on it called 'hooks' that are appropriated to the two members of the pair to which it refers. Thus A—loves—B means A loves B. A relative like A—gives—B / to—C is a triad. This can be written 

.

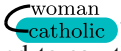
We now come to consider the *verso*, which is usually appropriated to imparting information about *subjective possibilities* or what may be true for aught we know. To scribe a graph is to impart an item of information; and this item of information does one of two things. It either adds to what we know to exist or it cuts off something from our list of subjective possibilities. Hence, it must be that a graph scribed on the *verso* is thereby denied.

Now the denial of a subjective possibility usually, if not always, involves the assertion of a truth of existence; and consequently what is put upon the *verso* must usually have a definite connection with a place on the recto. The way this is managed in the System of Existential Graphs is as follows:

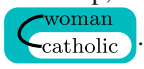
To express that there exists a woman and that a catholic likewise exists we scribe 

. But suppose it being well understood between the graphist and the interpreter of graphs that their discourse refers exclusively to the conditions prevailing in Hell, the graphist should desire to express that there exists a woman but that there should be a catholic is subjectively impossible. He will take a knife and proceed to make an oval or other self-returning cut through the sheet, and will then turn over the piece thus cut out, so as to bring the blueish side uppermost, and on that blue area of the verso he will scribe what he wishes to say is impossible thus 

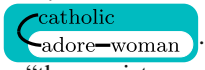
.

Suppose however he only wished to express that there exists a woman who is not a catholic. Then he will scribe 

.

Suppose, however, he wished to say that it was impossible there should exist a woman who was not a catholic. He would then cut out a piece, and then cut out a piece from the middle of that piece and turning the annular piece over, blueish side up, he would leave the interior piece with the creamy side up, and scribe thus: 

. (10 min)

Or if he wished to say "*Any* catholic there may be will adore some woman *or other*", this would be his graph 

.

Or if he wished to say "there exists a certain woman whom any possible catholic will adore" he will scribe 

.

[4] [Ernst Schröder (1841–1902). See Peirce's letter to William James, 25 Dec 1909, and MS 478, 1903, *Syllabus of Logic*, REMARK ON THE GAMMA RIMS].

You will observe that these interpretations follow apodictically. Those two graphs can mean nothing else. But I had better tell you that practically, I content myself with performing these cuts in my imagination, merely drawing a light line to represent the cut. The *blue* tint, however, of the area within the cut is a great aid to the understanding. How great I have only recently discovered. In my former exposition of Existential Graphs, I said that there must be a department of the System which I called the Gamma part into which I was as yet able to gain mere glimpses, sufficient only to show me its reality, and to rouse my intense curiosity, without giving me any real insight into it. The conception of the System which I have just set forth is a very recent discovery. I have not had time as yet to trace out all its consequences. But it is already plain that, in at least three places, it lifts the veil from the Gamma part of the system.

The new discovery, which sheds such a light is simply that, as the main part of the sheet represents existence or actuality, *so* the area within a cut, that is, the *verso* of the sheet, represents a kind of possibility.

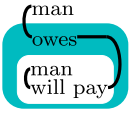
From thence I immediately ~~perceive~~ infer several things that I did not understand before, as follows:

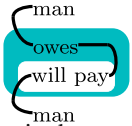
First, the cut may be imagined to extend down to one or another depth into the paper, so that the overturning of the piece cut out may expose one stratum or another, these being distinguished by their tints; the different tints representing different kinds of possibility.

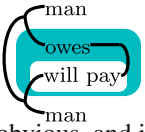
This improvement gives substantially, as far as I can see, nearly the whole of that Gamma part which I have been endeavoring to discern.

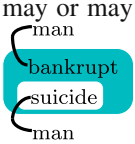
Secondly, in a certain partly printed but unpublished 'Syllabus of Logic', which contains the only formal or full description of Existential Graphs that I have ever undertaken to give, I laid it down, as a rule, that no graph could be partly in one area and partly in another; and this I said, simply because I could attach no interpretation to a graph which should cross a cut.[5] As soon, however, as I discovered that the verso of the sheet represents a universe of possibility, I saw clearly that such a graph was not only interpretable, but that it fills the great lacuna in all my previous developments of the logic of relatives. For although I have always recognized that a possibility may be *real*, that it is sheer insanity to deny the reality of the possibility of my raising my arm, even if, when the time comes, I do *not* raise it; and although, in all my attempts to classify relations, I have invariably recognized, as one great class of relations, the class of *references*, as I have called them, where one correlate is an existent, and another is a mere possibility; yet whenever I have undertaken to develop the logic of relations, I have always left these references out of account, notwithstanding their manifest importance, simply because the algebras or other forms of diagrammatization which I employed did not seem to afford me any means of representing them. (15 min) I need hardly say that the moment I discovered in the *verso* of the sheet of Existential Graphs a representation of a universe of possibility, I perceived that a *reference* would be represented by a graph which should cross a cut, thus subduing a vast field of thought to the governance and control of exact logic.

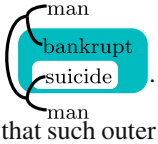
[5] [MS 478, 1903, the unpublished parts are to be published in Pietarinen, A.-V. (ed.), *Logic of the Future: Peirce's Writings on Existential Graphs*, to appear].

Thirdly, my previous account of Existential Graphs was marred by a certain rule which, from the point of view from which I thought the system ought to be regarded, seemed quite out of place and unacceptable, and yet which I found myself unable to dispute. I will just illustrate this matter by an example. Suppose we wish to assert that there is a man every dollar of whose indebtedness will be paid by some man or other, perhaps one dollar being paid by one man and another by another man, or perhaps all paid by the same man. We do not wish to say how that will be. Here will be our graph: 

. But if we wish to assert that one man will pay the whole, without saying in what relation the payer stands to the debtor, here will be our graph 

. Now suppose we wish to add that this man who will pay all those debts is the very same man who owes them. Then we insert two graphs of teridentity and a line of identity thus: 

. The difference between the graph with and without this added line is obvious, and is perfectly represented in all my systems. But here it will be observed that the graph "owes" and the graph "pays" are *not only* united on the *left* by a line *outside* the smallest area that contains them both, but likewise on the *right*, by a line *inside* that smallest common area. Now let us consider a case in which this *inner* connection is lacking. Let us assert that there is a man A and a man B, who may or may not be the same man, and if A becomes bankrupt then B will suicide: 

. Then, if we add that A and B *are* the same man, by drawing a line outside the smallest common area of the graphs joined, 

. which are here bankrupt and suicide, the strange rule to which I refer is that such outer line, because there is no connecting line within the smallest common area, is null and void, that is, it does not affect the interpretation in the least. It seems monstrous to say that these two come to the same thing, that, on the one hand, there is a man, B, who will commit suicide if a certain man, A, not necessarily a different man, becomes bankrupt, and, on the other hand, to say that there is a man who will commit suicide if *he* becomes bankrupt. But here is the reasoning. A conditional proposition is false only if the condition of it is satisfied, while the consequent is falsified. For the proposition asserts nothing at all in case the condition is *not* satisfied. So then it is only if the condition *is* satisfied, while the consequent is falsified, that the conditional proposition is false. But a proposition that is not false is true. So, then, this proposition about A and B will be false only in case, whatever *two* men or whatever one man, be chosen to be called A and B, A *will* go bankrupt while B *will not* suicide. That is, it will be false only in case *every man*

goes bankrupt, and *no man* suicides. By the same reasoning, the proposition that there is a man who if *he* goes bankrupt will commit suicide is false *only* in case, taking *any man you please*, he *will* go bankrupt, and *will not* suicide. That is, it is falsified only if every man goes bankrupt without suiciding. But this is the same as the state of things under which the other proposition is false; namely, that every man goes broke while no man suicides. This reasoning is irrefragable as long as a mere possibility is treated as an absolute nullity. Some years ago, however, when in consequence of an invitation to deliver a course of lectures in Harvard University upon Pragmatism, I was led to revise that doctrine, in which I had already found difficulties, I soon discovered, upon a critical analysis, that it was absolutely necessary to insist upon and bring to the front, the truth that a mere possibility may be quite real. That admitted, it can no longer be granted that every conditional proposition whose antecedent does not happen to be realized is true, and the whole reasoning just given breaks down.

I often think that we logicians are the most obtuse of men, and the most devoid of common sense. As soon as I saw that this strange rule, so foreign to the general idea of the System of Existential Graphs, could by no means be deduced from the other rules, nor from the general idea of the system, but has to be accepted, if at all, as an arbitrary first principle,—I ought to have poked myself, and should have asked myself if I had not been afflicted with the logician's *bêtise*, What compels the adoption of this rule? The answer to that must have been that the *interpretation* requires it; and the inference of common sense from that answer would have been that the interpretation was too narrow. Yet I did not think of that until my operose method like that of a hydrographic surveyor sounding out a harbour, suddenly brought me up to the important truth that the *verso* of the sheet of Existential Graphs represents a universe of possibilities. This, taken in connection with other premisses led me back to the same conclusion to which my studies of Pragmatism had already brought me, the reality of some possibilities. This is a striking proof of the superiority of the System of Existential Graphs to either of my algebras of logic. For in both of them the incongruity of this strange rule is completely hidden behind the superfluous machinery which is introduced in order to give an appearance of symmetry to logical law, and in order to facilitate the working of these algebras considered as reasoning machines. I cannot let this remark pass without protesting, however, that in the construction of no algebra was the idea of making a calculus which would turn out conclusions by a regular routine other than a very secondary purpose.

This rule being corrected, the permissible modes of transformation of existential graphs become more simple and unitary. When the revision is complete this feature, I am confident will be still more marked. Every elementary transformation is either an insertion or an exclusion, two operations that can hardly be conceived to be complex. The rules, in the form in which I am accustomed to employ them, are as follows. (25 min)

First: RULE OF INSERTION AND OMISSION. From any recto area any graph-instance, total or partial, that stands there, can be erased, this permission including, of course, that of severing of any line of identity at any recto part of it. In any verso area, on the contrary, any graph whatever can be inserted and given any connexions by lines of identity. Moreover, for the purposes of this rule every point of a cut must

be considered as lying on the area WITHIN that cut. But observe, that this does not give any right to insert a new verso area in a recto area.

Secondly: RULE OF ITERATION AND DEITERATION. Any graph that is scribed on any area whatever can be iterated upon that area or upon any area immediately or mediately contained within that area, the *iteration* consisting in the insertion of a copy of the original graph-instance, together with lines of identity connecting by the aid of points of teridentity all the different parts of the replica with the very identical lines and hooks with which the corresponding points of the original graph-instance were connected. And note that this gives the right to prolong a line of identity in any way within that same area provided no new connections are made and no cut be traversed by the extension of the line. Moreover, if any graph on any area should be already iterated on the same or on a contained area, then the replica may be erased from the latter area, provided no connection of the graph instance that remains is broken.

Moreover, for the purposes of this Rule a point on a cut must be considered as lying on the area WITHOUT that cut.

Thirdly, there is a rule for the CREATION AND DESTRUCTION OF AREAS. I may possibly find that the new generalized interpretation calls for some modification of this rule. But if so, it cannot affect the special purpose of this paper. And even if it should not require modifications I must say that the statement of it which I am about to make straddles the irreconcilable purposes, and only imperfectly satisfies either.[6]

One is to make the form of statement conform to the condition that postulates should be independent of purpose, which several logico-mathematicians have so much insisted upon of late, and the other is to put the rule into a shape not too inconvenient for direct practical application.

The rule as I shall state it has four clauses. Clause 1 is that a new cut may be made, provided there be made at the same time a cut in its area, and provided this inner cut contains no graph-replica that might not have been inserted by iteration from without the outer cut. Clause 2 is that if there be on any area a cut which encloses, together with no matter what, another cut which latter contains nothing but iterates from without the larger cut, then this larger cut with all that it encloses may be abolished. Clause 3 is that any graph-instance on any area may be encircled by a new cut, provided this cut be itself encircled by a second new cut which shall contain, beside the second cut and its contents, nothing but iterates of graphs scribed outside. The fourth Clause is that if the annular space between two cuts contain nothing but iterates of graphs scribed outside, then those two cuts may be obliterated leaving the scribed contents of the two undisturbed.

Finally, there are two minor rules to which I will briefly refer. One gives a list of special graphs which can be inserted in or omitted from any area. Such for example

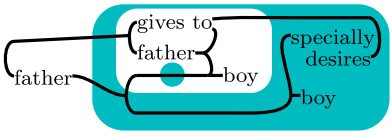
[6] **[Del.:]** Thirdly, any part of a ligature or line of identity, including any points of teridentity, but excluding any extremities may be erased provided that attached to all the new extremities so formed there be scribed one and the same singular monadic graph, which must not occur anywhere else; and the original line may be restored at any time. This device is a mere superfluity which occasionally helps in puzzling cases. It needs no special permission. It here serves as an introduction to the following.

Any annular area which [is] entirely blank unless for lines of identity each reading from the outer cut to the inner one can be anywhere suppressed by allowing the outer and inner areas to merge, the two cuts being closed up, and such an annular area may be created in any area. (30 min)

is a detached line of identity. The fifth rule is strictly speaking superfluous. The chief use of it is to enable the student to see how the other rules apply to lines of identity that run across cuts, especially to a line of identity which twice crosses the same cut or two different cuts. The rule is that any portion of a line of identity may be erased provided that in place of it each of the two loose ends so made be attached to a monad graph not used anywhere else and well understood to denote a single individual. And after any desired and permitted transformations the two graphs so introduced may be removed and the loose ends so left joined by a line of identity.

I am now coming to the point of this paper, which is that there are certain very interesting theorems of logic which I have discovered by the use of existential graphs. But before giving them I had better give one more illustration of the interpretation of graphs:

There is a father and if any boy of his specially desires anything the father will give that thing to some other boy of his.

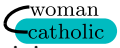


Every graph expresses some proposition. Almost every part of such a diagram as [above] is a graph. A blank place is a graph. What proposition does it express? It expresses "something exists" or "something coëxists with something", that is, "something is in existential relation to something". For woman – catholic expresses that "Some woman coexists with some catholic", for that is the same as saying some woman exists and some catholic exists. What does the entire recto express before anything is scribed upon it? It expresses all that is taken for granted at the outset. The sheet of the graphs in all its states collectively, together with the laws of its transformations, corresponds to and represents the Mind in its relation to its thoughts, considered as signs. That thoughts *are* signs has been more especially urged by nominalistic logicians; but the realists are, for the most part, content to let the proposition stand unchallenged, even when they have not decidedly affirmed its truth. The scribed graphs are *determinations* of the sheet, just as thoughts are *determinations* of the mind; and the mind itself is a comprehensive thought just as the sheet considered in all its actual transformation-states and transformations, taken collectively, is a graph-instance and taken in all its permissible transformations is a graph. Thus the system of existential graphs is a rough and generalized diagram of the Mind, and it gives a better idea of what the mind is, from the point of view of logic, than could be conveyed by any abstract account of it.

Now let us see how graphs are compounded.

sage – simpleton sage (simpleton sage (loves – simpleton

Sage and simpleton are in these three graphs combined in the same general way. That is, they are united in each case by being the relate and the correlate of a relation. The blank signifies merely *some existential relation*. The other relations, that is, *loving*, and *being identical with* are special determinations of the blank, just as every graph-instance is a special determination of the blank sheet, and every idea is a special determination

of the mind. In the following, 

woman and catholic are equally the relate and correlate of a relation; only it is not an existential relation, but is the relation of an existent to a possibility. For the proposition does not assert that a catholic exists, but merely that a woman exists who is not and could not be identical with any possible catholic. What are combined are —woman, i.e. "something is a woman" and —catholic, i.e. "something is other than any possible catholic". These two somethings determine each other, describe each other. The relation of *other than* or *not* is here a relation ~~between two possibilities or~~ between an existent and a possibility. It is signified by that part of the ligature that crosses the cut and is an example of a graph that crosses a cut.

It thus appears that the System of Existential Graphs recognizes but one mode of combination of ideas, that by which two indefinite propositions define, or rather partially define, each other on the *recto* and by which two general propositions mutually limit each other upon the *verso*; or in a unitary formula by which two indeterminate propositions mutually determine each other in a measure. I say in a measure, for it is impossible that any ~~knowledge~~ sign whether mental or external should be perfectly determinate. If it were possible such ~~knowledge~~ sign must remain absolutely unconnected with any other. It would quite obviously be such a sign of its entire universe, as Leibniz and others have described the omniscience of God to be, an intuitive representation amounting to an indecomposable feeling of the whole in all its details, from which those details would not be separable. For no reasoning, and consequently no abstraction, could connect itself with such a sign. This consideration, which is obviously correct, is a strong argument to show that what the system of existential graphs represents to be true of propositions and which must be true of *them*, since every proposition can be analytically expressed in existential graphs, equally holds good of concepts that are *not* propositional; and this argument is supported by the evident truth that no sign of a thing or kind of thing—the ideas of signs to which concepts belong,—can arise except in a proposition; and no logical operation upon a proposition can result in anything but a proposition; so that non-propositional signs can only exist as constituents of propositions. But it is not true, as ordinarily represented, that a proposition can be built up of non-propositional signs. The truth is that concepts are nothing but indefinite problematic judgments. The concept of *man* necessarily involves the thought of the possible being of a man; and thus it is precisely the judgment, "There may be a man". Since no perfectly determinate proposition is possible, there is one more reform that needs to be made in the system of existential graphs. Namely, the line of identity must be totally abolished, or *rather* must be understood quite differently. We must hereafter understand it to be *potentially* the graph of *teridentity* by which means there always will virtually be at least one loose end in every graph.

In fact, it will not be truly a graph of *teridentity* but a graph of indefinitely multiple identity.

Graph of indefinitely multiple identity.

We here reach a point at which novel considerations about the constitution of knowledge and therefore of the constitution of nature burst in upon the mind with cataclysmal multitude and resistlessness. It is that synthesis of tychism and of pragmatism for which I long ago proposed the name, Synechism, to which one thus returns; but this time with stronger reasons than ever before. But I cannot, consistently with my own convictions, ask the Academy to listen to a discourse upon Metaphysics. (40 min)

CHAPTER 2

Charles Peirce: Assurance Through Reasoning (MS 669 & MS 670, 1911)

The following text reproduces two manuscripts of the same title, MS 670 written in June 7–17, 1911, and an earlier draft MS 669 written from May 25 to June 2, 1911. These two manuscripts may have been drafts for the paper Peirce planned to deliver in the upcoming autumn meeting of the National Academy of Sciences at which he was invited to give two presentations. Manuscripts 669 and 670 are his last (the ninth and the tenth) attempts to complete the series of papers on the grounds and rationale of reasoning he had worked on since the summer of 1910, in view of publishing a collection of essays on reasoning (MSS 651–670, entitled "Assurance" since November 1910, MSS 661–670).

Perhaps only days before the November meeting, Peirce trod on papers lying on a waxed floor at his home, Arisbe, causing a serious injury which would have made him unable to attend the Academy session. It was not until much later in 1913, though now terminally ill from cancer, that he recovered some strength to continue working on these topics.[7] Consequently, the announced paper, "The Reasons of Reasoning, or Grounds of Inferring", together with his "A Method of Computation", were read by title on November 22, 1911.

While the content of the latter presentation has not been preserved, it is likely that Peirce wanted with his former paper to communicate to the audience many of the ideas surviving in the following pages. These last two manuscripts concerning "Assurance through Reasoning" present what may be Peirce's most successful attempt to explain the logic of existential graphs, and the philosophy concerning the notation of diagrammatic syntax in particular. The notions of identity, teridentity, composition of graphs, plurality, conditional, scroll, and the derivation of the idea of negation as a consequence of the scroll, all get their fair shares of exposition. Peirce then makes the remarkable observation in MS 670 that in the diagrammatic syntax, logical constants ought to be such that can serve both the roles of (i) *collectional signs* capable of expressing their own scope, and (ii) the *truth function* that those constants have in the context of making assertions. He then notes that "there is no reason why a single sign", "as it is seen by the mind's eye", "should not perfectly fulfill both these purposes". Moreover, tinctures are reintroduced to enable logic to assert, among others, modalities such as necessities and metaphysical possibilities, that call for changes in the nature of the universes of discourse.

[7] See Peirce to F. A. Woods, October 14, 1913, where Peirce recounts his accident to have happened "23 months ago", that is, in November. In the August 28, 1913 letter to Paul Carus he dates the accident to 13 December 1911.

Two further noteworthy observations that Peirce draws in the final paragraph of MS 669 are: (a) that graph transformation rules, together with the double cut rule, are semantically complete: they "will suffice to enable any valid deduction to be performed"; and (b) that even in the presence of such a simple set of rules, any attempt at automated or mechanized theorem proving, unaided by a "living intelligence", would face enormous complexities.

In 10 August 1911 Victoria Welby writes to George F. Stout that Peirce is still planning to submit his paper, which he originally conceived to be "an abstract" of his "entire system of logic", to the *Essays on Significs* which Stout and John W. Slaughter were editing in Lady Welby's honour; a project that was abandoned following Welby's death in late March 1912. Peirce had just written to Welby that he now has to limit his chapter to "*Logical Critics*; that is, to the quality of grade of assurance that the three classes of reasoning afford". Peirce's accident in late 1911 would mean that the submission was put on hold indefinitely. As neither the Academy presentation, nor Peirce's writings on Logical Critics or indeed any of these planned books materialised, one is led to wonder what the subsequent course in logic and its philosophy might have been under only slightly less unfavourable circumstances.

MS 670. June 7–17, 1911. Houghton Library.

DEDUCTIVE REASONING. The word "Deduction" will here be used, in a generalized sense, to include any necessary, or mathematical, reasoning,—any reasoning of which the premisses,—or, as they will here often be called, the "Copulate Premiss",—having been asserted, the conclusion cannot consistently be denied by the same assertor. But though all deduction is thus necessary reasoning it will, nevertheless, be convenient to divide deduction into "Necessary Deduction" and "Probable Deduction", the latter expression denoting any deduction concluding that, under stated conditions, a given kind of "Event" would have a stated probability, whether the statement of it be numerically precise or be as vague as it may; while by a Necessary Deduction is to be understood a Deduction which simply reaches the conclusion that a certain state of things would necessarily result from the facts asserted in its copulate premiss without resorting to any calculation of probabilities. Thus, the kind of reasoning herein termed Necessary Deduction is no more necessary reasoning than is Probable Deduction, the reasoning of which, being mathematical, is of course necessary. The designation here applied to the former ought, strictly speaking to be, "deduction which concludes a necessary, or apodictic, conclusion, without the introduction of the concept of probability in the process of deriving that conclusion"; but it is presumed that the indulgent reader will be willing to put up with the name "Necessary Deduction", in view of its brevity and of the fact that nothing whatever would be gained in discussing this kind of deduction, by reminding the reader in almost every sentence that there is a complication of Deduction with which, for the time being, he has nothing whatever to do.

The reader's study of the reason of the validity of deduction will be greatly facilitated by practicing the expression of the copulate premiss and conclusion of ~~every~~ a variety of deductions ~~that comes under his consideration in such study~~, not in the ~~ordinary~~ syntax of any spoken tongue, but in a certain *diagrammatic syntax* which has been

specially devised for such purpose, and which shall forthwith be here described.[8] This system supposes that every single word ~~expresses~~ it uses is an assertion. Thus, if the word "man" is put upon a sheet appropriated to expressions in this syntax, it will be understood to mean "There is a man", i.e. "Something is a man". So if "loves" appears on the sheet, it will be interpreted as asserting that "Somebody at some time did love or will love somebody or something". Of course, one cannot *speak* in this syntax, since it is diagrammatic; for nobody can talk a diagram. It is essentially superficial: a surface ~~on which assertions are to be made~~ must be devoted to the purpose of receiving the diagrammatic expressions, and this surface is called the Phemic Sheet. Are those expressions written, drawn, or incised? All three of these methods are employed; though the last is only ~~used in pretence~~ a make-believe incision represented by a fine line. Because we generally both write, and draw, and else pretendedly incise, in almost every ~~expression~~ assertion made in this syntax, we cannot say we write it, or that we draw it, or that we incise it. We say that we *Scribe* it; and that assertoric form to which, when we say we "scribe" it, we mean that we "give it a local habitation and a name", that we *embody* it, i.e. give it existence in a single "instance", though in itself it is only a *kind*, and so a "May-be", not a definite individual person or thing. Just so, the word "the", though it be printed twenty times on an ordinary page, is only one single word, those twenty occurrences of it being so many *instances* of the single word. In this diagrammatic syntax in which every word is an assertion, the form which *would*, were it scribed be embodied in an instance, is called a "Graph". It is a graph whether it be composed of many graphs, or whether it ~~is an atomic~~ be a "Graph-atom". ~~The sheet upon which it is pretended the student of logic exercises himself by scribing the premisses that making believe to scribe he makes believe he asserts is called the "Phemic Sheet."~~

The fact that according to this syntax any single word is an assertion classes the syntax, but does not characterize it. With that understanding, "man" by itself would most naturally mean what this syntax makes it mean, and the same may be said of "loves", "gives", "sells", etc. Yet "man" might mean "Everything is a man"; and "love", "everything loves everything" etc. This latter was, indeed, the first proposal which led up to the syntax here to be described. That first proposed may be named "the Syntax of Entitative Graphs" and that which is here to be described is "the Syntax of Existential Graphs".

There are but three peculiar signs that the Syntax of Existential Graphs absolutely requires. ~~The first of these is a Sign which shall deny Graph or a scribed assertion. Such a sign is required, since "Not man" would only mean, "There is something besides some man", or else, "There is something that is not a man".~~ The first of these is a sign of identity. An ordinary word or abbreviation would not answer this purpose because it would not show what two objects were said to be identical, which is the sole end of

[8] I must acknowledge that my assertion that the reader's study of the ~~reason of~~ critic of necessary deduction will be greatly facilitated by the ~~study~~ practice of this diagrammatic syntax is merely an analogical inference from my own experience. I had studied and practiced the whole theory of reasoning for many years and perhaps was unconsciously puffed up with my acquisitions as ~~a true~~ the ideal student never will be. All I can say for certain is that after some years' acquaintance with this syntax I found it had taught me a good deal more than I had at all suspected that it could. Beyond that I cannot know how much a reader will gain from similar practice.

Fig. 1

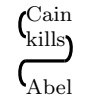


Fig. 2

Fig. 3

a sign of identity. To write one before and the other after the graph of identity could only be suggested by a person who had no experience of the cases in which such a sign is needed. How, for example, should we express "Somebody loves himself"? A practicable suggestion and one which may occasionally be used is to attach duplicate marks of reference, such as asterisks, obelisks, double daggers, sections, parallels, and paragraphs. But a much more diagrammatic method is commonly used. ~~Namely, when every graph except a "Medad", i.e. one expressing an entire assertion ought to carry a heavy dot, at a point of its periphery to be agreed upon to be settled by a convention to denote each of its "Subjects", a word used in this syntax to denote indifferently a Subject nominative or any variety of object. If a graph is a "Monad", that is, has but~~ Namely, every "Simple Graph", or "Spot", that is to say, every graph that is not composed of other graphs, unless, indeed, it be a "Medad" (i.e. unless it be, in itself, a complete assertion, such as "rains", meaning "it rains", and as such has no subject), ought to carry at a point of its periphery to be decided upon by convention, a heavy dot to denote quite indefinitely, each of its subjects. Thus, "•dies" will naturally mean "Something dies", this something being denoted by the dot; and •kills• will naturally mean "Something kills something", the left-hand dot denoting the killer, and the right-hand one the killed. So •$\overset{\bullet}{\text{gives}}$• may very well be used to mean "Somebody gives something to somebody", the left-hand dot denoting the giver, the right-hand dot the gift, and the dot below the recipient of the gift. For to give is not a simple physical act, like to illumine, to hear, to push, to pull. If it were it would be "dyadic", that is, the assertion of it would be sufficiently completed by two subjects. But it is a transfer of something created by the mind, to wit, a legal right; and such actions are mostly "triadic", at their simplest. These heavy dots attached to spots and indefinitely denoting their different subjects are called the "pegs" of the spots. They have their analogues in chemical graphs. Now we have only to stretch such a heavy dot into a heavy line, and it automatically becomes an assertion of the identity of the two graph-subjects denoted by its two extremities. Thus, Fig. 1 asserts "Mary is a woman". Fig. 2 asserts "Cain kills Abel". Fig. 3 asserts "Somebody kills himself".

This way of asserting identity, along with other important advantages, has that of automatically furnishing *the second general sign required by the syntax of existential graphs*. This is the "Spot of Teridentity". At first blush it is likely to seem to the reader

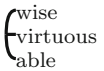


Fig. 4

Fig. 5

that the identity of three subjects is nothing but the simple identities of the first with the second, and of this with the third; whence follows the identity of the first and the third. But in the Critic of Deduction it is needful to think more accurately than that. It is true that if the three can be definitely and distinctly identified, their teridentity is given in two simple identities. But from the premisses "Some wise man is identical with some virtuous man" and "Some virtuous man is identical with some able man", it by no means follows that "Some wise man is identical with some able man", much less does it follow that because "Some wise is identical with some virtuous, some virtuous is identical with some able, and some able is identical with some wise", that some wise, some virtuous, and some able are coidentical; and consequently the existence of coidentity is far from consisting in the existence of two or in three simple identities.

It thus becomes evident that we must have a Spot of Teridentity, and that no Existential Graph of Teridentity can be built up of instances of the graph of simple identity. However, a simple point of branching on a line of identity automatically signifies teridentity, as in Fig. 4, and teridentity once ~~scribed~~ conquered we have the means, by it alone, of expressing any grade of multi-identity, as in Fig. 5.

We thus perceive that the essence of Plurality is not perfectly realized in the number two, while it [is] in three; which, by the way, indicates the truth of those numerous languages of every type that distinguish the grammatical dual number from plural. Moreover, it furnishes a hint toward explaining why perfectly comprehended and essential logical divisions are often trichotomies, but never result in a number of parts having any prime factor greater than three. There is more to be said about the Line of Identity, after the third ~~essential~~ indispensable peculiar sign of the Syntax of Existential Graphs has been considered.

This third is one that shall deny a Graph-instance, or scribed assertion. For without that we could not, in Existential Graphs, express any proposition [in] that form which is, *par excellence*, Critical; namely, the conditional form, "If Antecedent then Consequent". No more could we in Entitative Graphs (whose development starts from the conditional form) express a proposition of the particular form with which the development of Existential Graphs begins. But that Conditional Proposition just written may be expressed by the aid of a sign of Negation in the form, "That the Antecedent is true and the Consequent not true, is not true". This example shows us that along with the sign of negation we require one of those *Collectional Signs*,—"Klämmen",

Schröder calls them,—such as in algebra are the parentheses, brackets, braces, the vinculum, ~~and, in the principal effect~~ period. The functional signs when more than single letters are attached to them belong to this class of Collectional Signs, which class is the most important,—it would, indeed, be strictly true to say they are the only indispensable,—signs of algebra. But the whole of the strict truth is, in this case, not important. What *is* important is to understand that the essential power of algebra is due exclusively to collectional signs. It will not be surprising, then, to find that logical critic cannot be firm upon its legs and able to progress until it is provided with convenient collectional signs. However, they are only first needed in Existential Graphs after this syntax has been provided with a sign of negation; for whether one says "To assert that the assertion of A would be *true*, and that the assertion of B would be true, and that the assertion of C would be true is to assert nothing but the truth", or whether one says "To assert that the assertion of C would be true is to assert nothing but the truth", or whether one says "To assert that the assertion of A would be true and that the assertion of B and C together would be true", or whether one says, "To assert that the assertion of A and B together would be true and that the assertion of C would be true" is perfectly indifferent: they all come to the same thing.

On the other hand to say that "To assert that the assertion of A would be *false*, and that the assertion of B would be false, and that the assertion of C would be false" would be to assert nothing but the truth", would amount to much more than either to say "To assert that the assertion of A would be false, and that the assertion of B and C together would be false, would be to assert nothing but the truth" or to say "To assert that the assertion of A and B together would be false and that the assertion of C would be false would be to assert nothing but the truth" would be very different. But while the syntax of existential graphs thus needs both a sign of negation and an endless series of collectional signs, there is no reason why a single sign should not perfectly fulfill both these purposes. Here immediately follows an explanation of this sign as it is seen by the mind's eye; and this shall be supplemented by an account of how this ideal sign can be conveniently represented on paper.

Let the descriptions of things or of events of each of which one wishes to say that something of the sort exists or sometimes occurs be scribed in black upon the white surface of the phemic sheet. Then if of some description of object one wishes to deny that it ever exists, let a part of the sheet be blackened but with white spot left in it, so shaped as to compose in their entirety an instance of a description in question, and let the interpretation be that no such thing or event is ever actualized as that description in its entirety. Thus, Fig. 6 will affirm A and affirm B while denying C and denying D. But Fig. 7, while affirming A and B will only deny that *both* C and D are true; that is it will assert that no C ever occurs at the same time, or occasion, as D. On precisely the same principle Fig. 8 must be interpreted as asserting that if A is ever true either C or D will sometimes be true; for this is precisely the same as to assert that to say that A is sometimes true and that C and D are each sometimes false would be to assert something not altogether true.

It is needless to say that the process of making a black surface with a diagram in white upon it is insufferably inconvenient. For that reason in practice one substitutes Fig. 9 or Fig. 10 in place of Fig. 8. This sufficiently overcomes the inconvenience of making the diagrams; but it is accompanied by a serious danger. For though the

Fig. 6

Fig. 7

Fig. 8

Fig. 9

bounding ovals be drawn in as fine lines as they may,—and they must be so drawn,—one is but too apt to think of these ink-lines as signs, like the graphs, and to forget that they have no meaning at all, but are mere boundaries between black and white. They are called "cuts",—that is, girdling-edges; for the black areas usually carry white areas within them, and may be thought of as taking away the presence of that which the graphs they carry denote. It is only the color of the area itself which has the force of affirming, if it be white or *evenly-enclosed* (i.e. is enclosed by an even number of cuts, or by none) or of denying if it be shaded or *oddly-enclosed*. It is a help to shade the oddly-enclosed areas and omit the lines that represent the cuts, as in Fig. 11, which is equivalent to Fig. 10.

Though the cuts in themselves have no meaning whatever, but are, at most, mere punctuation-marks separating "particular" affirmations from general negations, yet an *enclosure*, that is, an area together with all that it carries, including whatever other enclosure or enclosures it may carry is a graph-instance. The area upon which a given area lies is called *the place* of the latter area, or *the place* of the cut which is its outer boundary; while an area within a cut is sometimes called *the area* of the cut. The "place" of any graph-instance is the outermost of the areas which any part of it enters. But an instance of a simple graph, i.e. one not having parts that are themselves graphs, must be wholly in one area: and this is not an unimportant fact.

Let us now exercise ourselves in interpreting a few graph-instances. Figure 12 reads "If it thunders it must have lightened"; for it denies an occurrence of thunder without

Fig. 10

Fig. 11

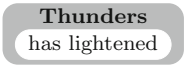


Fig. 12

Fig. 13

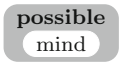


Fig. 14

Fig. 15

Fig. 16

its having lightened. Figure 13 reads "If there is anything there is a mind". Figure 14: "If anything is possible there must be a mind". Figure 15 is an absurdity; for it denies that which does not assert anything. Figure 16: "Either there is a mind or there is an absurdity".

This may be considered as asserting that the existence of a mind is an absolute necessity. It does not quite say this because it does not say there *would* be a mind under all circumstances. To express that explicitly, it would be necessary to indicate, upon the very outermost, or border, area, [that] the logical universe is that of metaphysical possibility, so that oddly-enclosed graphs express necessities. The nature of the universe or universes of discourse (for several may be referred to in a single assertion) in the rather unusual cases in which such precision is required, is denoted either by using modifications of the heraldic tinctures, marked in something like the usual manner in pale ink upon the surface, or by scribing the graphs in colored inks. In the former method it is usual to employ the different metals (or, argent, fur, and plomb) to ~~signify~~ mark the different kinds of existence or actuality, the different colours (azure, gules, vert, purpur) for the different kinds of possibility,—possibility consisting of ignorance, of variety, of power, of futurity; and the furs (sable, ermine, vair, potent), for the different kinds of intention.

But when what is scribed has not to go to press, nothing else is so simple as the use of colored inks, of which excellent mauve, crimson, scarlet, maroon or reddish brown, buff or yellowish brown (the so-called bismarck brown) olive or greenish brown (in *appearance*; for it is really nothing but darkened gamboge), blue, violet, and royal purple, are easy to be had. The pale colors may be used for possibility, the dark for actuality, and the high for intentions. To express the metaphysical necessity of mind, one may on a pale reddish ground make a deep red area bearing a white spot in its turn on which is written "mind". This will express, "It is metaphysically impossible that

there should not actually exist a mind". But it is to be remembered that an assertion,—although in itself, either a command, usually obeyed (like other ~~customary~~ commands that the interpreter is ~~habituated~~ accustomed to obey), by force of habit; or else, taken as the total~~ity~~ composed of the appearance of the assertor, and all the interpreter's experience of him, of that assertor's tone and manner, together with the proposition considered as asserted and by whom it is to be regarded as a retroductive argumentation tending to persuade its interpreter of the ~~breadth~~ reality of its substance,[9]—yet, when it is to be made the premiss of a yet undeveloped necessary deduction, it ought to be regarded merely in its syntactical *form*, carrying significant matter, it is true,—but that matter packed up as concisely as it may, and not to be undone and scrutinized until the deductive conclusion shall have been drawn, so that the energy of attention may not be wasted upon it.

Every part of the ~~composite~~ copulate premiss that does not need to be dissected for the purpose of drawing the conclusion is best represented by a single letter with the requisite pegs about its periphery; while at the nodal point of the argumentation no subtilty can be useful in drawing any part of the conclusion ought to be spared. But before we consider deductions, we need to exercise ourselves in correctly so scribing graphs that involve both enclosures and lines of identity as to express various familiar forms of thought.

In the first place, if the reader can put up with another technical term, it must be remarked that the line of identity, that heavy kind of line that when evenly enclosed affirms that that individual object that is denoted by one of its extremities *is identically the same as* the individual denoted by its other extremity. Since no concept stands less in need of analysis in the process of deduction than does that of identity, it ought to be regarded as a *simple graph* expressing that "Something (denoted by one of its extremities) *is* the *same individually as*" (that which its other extremity denotes); and consequently it must be scribed, if at all, upon some single area within the border of the phemic sheet. Any instance of it is, however, scribed as a continuous line, which is divisible into and consists of as many parts as one may like. Now it is not the question how it may be with a real line; for let us assume that our conception of a line is correct, or, at any rate, is all we are talking about. Then there can be no question that there is room upon it for a multitude of points at least as great as the multitude of all numbers, rational or irrational, between any two values,—say between 2 and 4.

The mathematicians assume that any simple line *is composed* of just so many points, and no more. The present writer ventures to disagree with them;—and let it be well understood that, as just said, it is a disagreement in regard to our notion of a line. His reason is that if we cut a line in two thus making two new ends, our notion is that there will be a point at each of those ends while if we cut the series of all real numbers in two so as to leave a number on one side next to the cut, then on the other side however close to the cut any number may be there will be an *endless* series of numbers yet nearer. For example if we cut the series of numbers so that 3 shall be the highest of all the numbers below the cut, then there will be above it 3.5 3.05 3.005 3.0005 and so on *endlessly*. For 3.00000 *et cetera ad infinitum* 1, is the same as 3, or, if it be

[9] By the "Substance" of an assertion is to be understood that state of things or event whose Reality it asserts.

not, then the number expressed by substituting 0001 for the last 1 is higher that 3 and lower than 3.00000 *et cetera ad inf.* 1. No competent mathematician will hesitate to endorse the statement that if the series of real numbers be cut so that there shall be a number nearest to the cut on one side of it, then the series as one approaches the cut on the other side is *endless*, whether the series embraces all real numbers or only all real rational numbers. It is one of the recognized truths of arithmetic, demonstrated by Euclid.

The cause of the misrepresentation by mathematicians of the concept of a line is ~~simply~~ that their representation is faithful to every measure of line; or at least is so nearly as faithful to the concept of a measured line as to result in no measurable error, and they would not correct their concept without going into logic deeper than it is pertinent to measurement to go. But Kant sometimes defines the continuity of time in a manner which solves the difficulty and is perfectly applicable to a line, although Kant himself fails distinctly to appreciate the merits of his definition. He says that the continuity of time consists in the fact that every part of a time is a time; that is time is not wholly nor partially composed of instants.[10] The continuous parts of a line, i.e. where there is neither a node, or point of branching, nor an extremity, nor any marked point, contains no point. But as soon as a point is marked upon a line the continuity is broken at that point and nowhere else; and a point on a line occupies no part of the line. To be sure, if you choose to *call* a point a part of a line, by an arbitrary misuse of the word "part", then you must define the continuity of a line differently. You may say that every *constituent part* of a continuous line is a line. Or you may say that every part of the *room* on a line affords *room* for a line. This is merely a variation of nomenclature. The multitude of rational numbers is demonstrably no greater than the multitude of whole numbers; but the multitude of real irrational numbers is greater.

There are still greater and greater multitudes endlessly. The reason the mathematician can conceive of no more points on a line is not at all that there is no room for more on the line; for no multitude whatever of points can fill any linear space; but the reason is that he has exhausted his vocabulary for distinguishing the units of a multitude. He can only approximate to distinguishing the irrational numbers, and for distinguishing the different individuals in higher multitudes than that of the irrational reals, the human mind is not capable even of approximating to doing so.

The line of identity does no more than assert the individual identity of the objects denoted by its extremities. Identity is a dyadic relation; and among dyadic relations it belongs to the type to which belongs the relative term "is loved by whatever loves". For to say that "some woman is identical with something that is beautiful" is precisely the same as to say that "some woman is identical with whatever there may be that a certain something that is beautiful is identical with".

The term "line of identity" is taken in such a sense as to limit it to *graphs*. Now a graph-instance must be scribed in some one area and be limited to that. But a line of identity may be continuous with another line of identity in another area, as in Fig. 17; and such a series of lines of identity thus continuous each with the next is called a *ligature*.

[10] C.d.r.V. 1st Aufl. S. 169ff.

Fig. 17

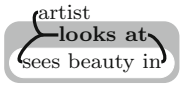


Fig. 18

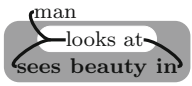


Fig. 19

Figure 17 may be read, "Some woman is identical with something which is not identical with anything unless it be with something that is other than everything except what is other than anything beautiful", which comes to saying, "Some woman is other than everything beautiful", or "Some woman is not beautiful". Thus a ligature which has its outer end evenly enclosed and its inner end oddly enclosed has no other effect on the whole than asserting the identity of something denoted by a graph in the outer area with some thing that is *not* of the description signified by a graph in the oddly enclosed area. Figure 18: "Some artist sees beauty in whatever he looks at". Figure 19: "Some man sees beauty only in things he looks at".

All the strictly synthetical elementary signs essential to this syntax have now been explained. But the writer has been led by fourteen or fifteen years experience of it to rate its utility so very high that he will here insert examples of all the propositional forms up to a certain limit of complexity, so as to familiarize the reader with the use of it, and following these will make the briefest possible restatement of the definitions of the signs.

It will be understood that the system is not a complete syntax, but is only a *syntax of assertions*. It is a question whether an assertion should be regarded as a command to believe, that is, as an action on the part of its Utterer,—a word which will throughout this writing be applied as much to one who puts forth a sign by writing or by any other kind of action as to one who does so by means of his voice,—as an action, then, on the part of its Utterer with the deliberate purpose of causing belief in the mind of its Interpreter, through a habit in that mind, much in the same way as an officer's command causes a soldier to obey with very little reflexion, if any at all; or whether, on the other hand, the assertion ought not rather to be regarded as the voluntary manifestation of the utterer's belief usually, but not necessarily, with more or less hope that the fact of belief so manifested will appeal to the interpreter's reason as a sufficient premiss to make him conclude that it is true; or whether finally it should be considered, neither as a command nor the manifestation of a premiss of reasoning, but rather as a third ~~genus~~ species of sign more or less resembling each of those others.

In order to solve this far from easy problem, let us turn our attention back to the essential nature of a *Sign*, in general, and consider whether that does not point out to us the nature of that fundamental division of signs which will furnish us with the concept

of the Assertion as one of its members. A Sign, then, is anything whatsoever,—whether an Actual or a May-be or a Would-be,—which affects a mind, its Interpreter,—and draws that interpreter's attention to some Object (whether Actual, May-be, or Would-be) which *has already* come within the sphere of his experience, and beside this purely selective action of a sign, it has a power of exciting the mind (whether directly, by the image or the sound or indirectly) to some kind of feeling, or to effort of some kind or to thought; and so far as it has any such effect *quâ* sign—for besides being a sign, it may also be a music,—but so far as it excites feeling, will, or thought *as a* sign, it connects the feeling, will, or thought in the mind of the Interpreter with its Object as due to it, as the interpretation of it.

The writer is not altogether satisfied with this attempt to analyze the nature of a sign; but he believes that the sign calls up its Object or Objects, for there may be several, and besides that excites the mind as if it were the Object that had this effect. If a person reads an item of news in a newspaper, its first effect on his mind will probably ~~depend on his habits; upon someone—upon anyone, perhaps, of little experiences—its first effect will~~ be to cause ~~him to believe the truth of what is asserted, that is, to believe the reality of the substance of it; upon another the first effect~~ something that may conveniently be called an "image" of the event, without any judgment as to its reality.

Appendix: Assurance Through Reasoning (MS 669)

MS 669. 25 May to 2 June 1911. Houghton Library.

DEDUCTIVE REASONING. The word Deduction will here be used in a generalized sense to include all necessary, or mathematical Reasoning,—every reasoning of which the premisses,—or, as they will here be termed, the *Copulate Premiss*, having been asserted, the Conclusion cannot consistently be denied ~~without self-contradiction~~; so that all that the latter asserts has really been already asserted in the former.[11] Although all Deduction~~, as I use the word,~~ is thus Necessary Reasoning, it will, nevertheless, be convenient to speak of "Probable Deduction" as distinct from "Necessary Deduction"; the term "Probable Deduction" being used to denote any Necessary Reasoning that concludes that under stated conditions a given kind of event would have a stated Probability. *Example*: That any homogeneous cubical die will, at any given throw, turn up six is ~~a chance against which the odds are 5–1.~~ decidedly improbable. Therefore, it is very improbable that a pair of quite disconnected good dice should turn up ~~sixes at any given throw~~ without referring to any calculation of Probabilities.

[11] Any dispute on this point must, I think, be a dispute about words. For all I mean is that if a diagram, or model, or true representation of any kind that any being could make, should represent the Copulate Premiss to be true, it would *ipso facto* represent every Deductive Conclusion to be true, although, these being endlessly manifold, it could not *expressly* represent this of every such conclusion. That is it could not concentrate attention on them all; although it would *mean* all that consistency would make it mean; and being an assertion, it would *assert* all that.

It is not *words* that are "asserted", but *facts*: any dictionary ought to say that. Consequently, "Cain killed Abel", "Abel was killed by Cain", and "The death of Abel was directly due to the intentional agency of Cain" are not three assertions, but only three forms in which of one and the same assertion may be clothed.

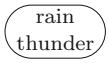


Fig. 1

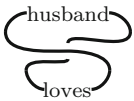


Fig. 2

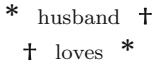


Fig. 3

The study of Necessary Deduction is much facilitated by expressing the Copulate Premiss, not in ordinary grammatical form, but in a sort of "diagrammatic syntax" of which an explanation shall forthwith be described, along with the terminology required in this description. A piece of paper having been taken for this use, and noun or verb written upon it is to be understood as *asserting* that the object, action, or state signified by what is written is actualized somewhere at some time, past, present or future. If two such words are written, both assertions are made. But if one or more of the words is surrounded by a fine line, that which would have been asserted if it had not been so enclosed is to be understood as being thereby, precisely and *as a whole*, denied. For example, Fig. 1 would assert that it does not both rain and thunder. Strictly it ought to mean that it either never rains or never thunders, but unless the reasoning turns on such an interpretation, it will not be worth while to be particular on such points, and when nothing to the contrary is asserted it is to be understood that all that is on any one of the sheet refers to some one place and some one time. Common sense on the part of the interpreter is supposed in this special respect, although in others a free rein must not be accorded to that useful servant to him who holds it with a firm hand.

A heavy dot before a verb will denote the perfectly indefinite subject of the verb. A similar dot after the verb will denote with the same indefiniteness its direct object; and such a dot close under the verb will usually denote the dative indirect object. But such conventions must yield to convenience. Similarly, places about other words may be appropriated, each to denote "something" in a particular relation to that which the word signifies. Any such heavy dot may be prolonged into a heavy line; and when this is done the whole line continues to denote the same identical individual. If such line joins another, the junction asserts the identity of two "somethings". But if it be desired to draw one such "line of identity" across another without joining them, there are two ways of doing so. The first is to make a little bridge, as in Fig. 2 which may be read "Somebody is husband of somebody that loves him". The other way is to do away with the line and use one of the usual marks of reference, such as, *, †, ‡, ‖, §, ¶, as in Fig. 3.

In either case, it is to be carefully observed that this syntax is *endoporeutic*. This means that a "line of identity" is to be understood as lying in the outermost of the

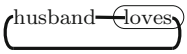


Fig. 4

Fig. 5

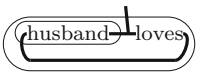


Fig. 6

"areas" within which any part of it lies, meaning by an "area" (here and everywhere), all of the surface that lies inside and outside of precisely the same "cuts", or fine oval lines.

Thus, Fig. 4 means "Somebody does not love some husband of hers"; Fig. 5, "Somebody is husband of whoever loves him" = "Somebody is loved, if at all, only by those (or some of them), of whom he is husband"; Fig. 6, "Somebody does not love anything that is not husband to her" = "Somebody has for her husband whatever there may be that she loves" = "Somebody if she love anybody, it is some husband of hers that she loves".

These two features, the finely drawn oval *cut* that denies as a whole, whatever it includes and the heavily drawn *ligature* that expresses the identity of whatever it abuts upon or is continuous with make the sum total of the essence of this Syntax. There is, therefore, no other difficulty in using it except that of knowing precisely what it is that one desires to express, without which one cannot think or do anything at all, to good purpose. There are, to be sure, a few signs besides those two, that are occasionally convenient; but they are never indispensable, and are not often wanted. The merit of this syntax is that when, by means of it, one has expressed one's premisses, with sufficient distinctness (i.e. analytically enough), it only remains to make, according to three general permissions, suitable insertions, followed by suitable deletions, the effect of such insertions and deletions amounting only to the omission of a part of what has been asserted, and one will be able to read in what will then be on the sheet, whatever sound deductive conclusion from those premisses that one may have aimed at in the insertions and deletions. A false logic has caused the habit of speaking of *the* conclusion from given premisses, as if there were but one. The truth is that the number of conclusions deducible from any proposition is strictly infinite.[12] It is, therefore, necessary to determine what sort of conclusion one desires to draw or by what sort of operation one proposes to proceed before one can deduce definite conclusion.

[12] The number of *interesting* propositions deducible from the definition of ~~such a single proposition as the def~~ series of positive integers is very great, while that deducible from the definition of projective space is enormous.

Before going further it will be well to define a few technical expressions that have been found almost indispensable in describing the properties of this Syntax. A full glossary shall be appended to this chapter.[13]

The *Phemic Sheet*, or the *Sheet* simply, is the surface on which the premisses are to be expressed, and from which, after insertions and deletions have been made, the deductive conclusion can be read off.

To *scribe* is to embody an *infima species* of *pheme*, or assertional sign, by writing or drawing.

A *Cut* is a fine oval line. It is called a Cut because, being the only kind of sign used in this system of syntax that does not, of itself express an assertion, and is in other ways *sui generis*, it is not convenient to speak of it as "scribed"; and besides imagining it to be cut through the sheet, we further imagine and speak of the part of the sheet within the "Cut" as if it were turned over so that what were exposed to view were the *Verso* side, unless the Cut in question be itself enclosed within another, or within any other odd number of others, in which case, of course, the even number of reversals will be imagined and spoken of as exposing the *Recto* again within it.

By an *Area* is meant so much of the exposed surface as lies wholly within and without the same identical Cuts. The Area within a Cut is called *the Area of* that Cut; while the Area in which the Cut has been made is called *the Place of* that Cut.

An Area that is enclosed in an even number of Cuts, or in none at all, is said to be "evenly enclosed". Any other is "oddly enclosed", since no Cut is allowed to intersect another.

The word "Graph" was introduced by the still lamented William K. Clifford to mean a diagram expressive of relations by means of lines abutting upon spots, after the fashion of those employed in organic chemistry. The syntax I am describing employs Graphs modified by Cuts. They are called *Existential* Graphs to distinguish them from another system of logical graphs[14] called Entitative Graphs. But ordinarily it will be convenient in the present essay, for the sake of abbreviating the long name Existential Graphs, the adjective is dropped. A *Graph*, then, as the word is used ~~by the present~~ when it is plain that an *Existential* Graph is meant, is *not* a sign or mark or any other existent or actual individual, but is a *kind* of sign ~~not *any instance* of~~ which if scribed on the Phemic Sheet (i.e. if an *Instance* of it stood on the Sheet) would make an assertion. The individual sign that results from the scribing of a Graph has been called an "*Instance*" of the Graph. This word "Instance" might conveniently be introduced into ordinary parlance. For example, only two words in our language are called articles; but one of these, the definite article, *the*, will commonly occur, on an average page of novel or essay, over twenty times. They are reckoned by the editor who asks for an article of so many thousand "words" as distinct words; but in fact they are only twenty or more *instances* of the same word; and if the editor takes any pleasure in speaking

[13] [Peirce either did not write this glossary or it has been lost as far as his present planned book on reasoning is concerned. See MS S-26 (1904), to appear in Pietarinen, A.-V. (ed.), *Logic of the Future: Peirce's Writings on Existential Graphs*, for an extensive glossary nearing 200 terms in total on the terminology he employed for the theory and method of existential graphs].

[14] See *Monist*, Vol. VII, pp. 168 *et seq*.

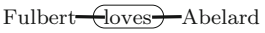


Fig. 7

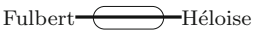


Fig. 8

Fig. 9

Fig. 10

accurately he should call for an article of so many thousand "word-instances". At any rate, it would be highly inconvenient to call "Graph-instances" Graphs.

The "line of identity" is a graph. For just as Fig. 7 asserts that "Fulbert does not love Abelard", so Fig. 8 asserts that "Fulbert is not identical with Héloise". But each Instance of a Graph must be either affirmative or negative, and consequently must lie wholly in one area. For that reason we must call the "*ligature*" of Fig. 8 not a Graph-instance, and consequently, not an Instance of the "Line of Identity", but as a composite of three graph-instances. This gives rise to a subtile and difficult doctrine about ligatures, with which common-sense finds it hard to have patience, because in its eyes a ligature is the simplest thing in the world. Namely, only the outermost, or least enclosed, part of it signifies anything, and the rest only serves to point out the two ~~individuals~~ objects, each of which the identity it signifies is affirmed or denied, according as that outermost part is *evenly* or oddly enclosed (i.e. enclosed in an even or an odd number of cuts, zero being, of course, divisible by two and so even). We may pat common sense on the back for this facile method of interpretation; yet there is no difficulty at all in the more formally logical view that a ligature consists of as many separate graph-instances as are the areas into which parts of it enter, these different graph-instances being connected by the *dot* or *dots* where they cross a cut or cuts. At such a dot—theoretically, a mathematical point—there can be no predication, since there can be neither affirmation nor denial of that which is neither within nor without the cut. It must be borne in mind that Figs. 9, 10 are both absurd. For Fig. 9 asserts that something is not identical with anything at all—not even with itself.[15] Now Fig. 11 denies, not A, B, *and* C, but *either* A or B or C. For it only denies the truth of A, B and C as one copulate. Figure 7 may therefore be read, "Fulbert is something that is something, that is something etc. that is either nothing (not even when it is), unless it be nothing except something that is nothing etc. but something that does not love anything that is something that is Abelard"; which comes to this that "Fulbert does not love Abelard". The number of times the ligature crosses the cut is immaterial, since

[15] [The graph in Fig. 10, for which Peirce did not provide a reading, asserts that it is not the case that something exists that is identical to itself, in other words, it expresses a denial of well-definedness: that the universe of discourse is empty].

Fig. 11

Fig. 12

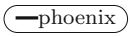


Fig. 13

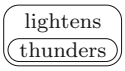


Fig. 14

at a crossing it merely transmits the identity from an outer line of identity to an inner one.

It will be well to give examples of the most frequently occurring forms; and the reader is counselled to reason out for himself the interpretation of each. Figure 12 reads "something is a dog"; Fig. 13: "Nothing is a phoenix".

Figure 14: "If it lightens, it thunders". Strictly this should mean "If it ever lightens, it sometime thunders". The truth is that it is very seldom requisite, in the critic of Deductive reasoning to observe distinctions of time. In order to do so, however, it would only be requisite to agree that a point on the upper part of the periphery of any graph of action or other change should denote an instant during such action or other change. Then Fig. 15 will read "It sometimes lightens and sometimes does not".

Figure 16: "If it ever lightens it will shortly after thunder". For were the outer cut of the figure not there, the interpretation would be "It sometime lightens without thundering at any shortly subsequent time". Figure 17: "Every multicellular animal dies". Figure 18: "There is a man who loves nothing but women". Figure 19: "There is a man who loves every woman". Figure 20: "Nothing but a man loves any woman". Figure 21: "The only positive integer that is not higher than One is One itself". Figure 22: "Philip is identical with whatever is Philip"; i.e. the name Philip denotes a definite

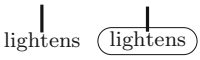


Fig. 15

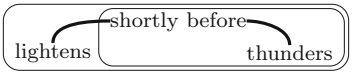


Fig. 16

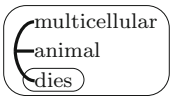


Fig. 17

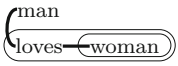


Fig. 18

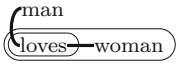


Fig. 19

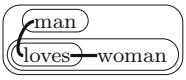


Fig. 20

Individual"[16] ("Some man" is an indefinite individual. "Any man there may be" is an indeterminate individual according to the terminology herein adopted.)[17]

The Syntax of Existential Graphs has thus been described. The elementary signs this syntax may require are, in the first place, such as, thunders, ∙man, ∙lover∙, ∙giver∙, ∙seller∙, ∙beauty, ∙water, ∙possessor∙ (as of a character or quality), ∙stands∙ (in the dyadic relation), ∙stands∙ (in triadic relation), ∙a collection that includes and excludes ∙is a collection, ∙is something that includes∙, ∙is something that excludes∙, ∙its members are in one to one correspondence to∙.

In addition to signs of this sort, which may be multiplied indefinitely and can cannot[18] be considered as constituent parts of the syntax, the only signs it so constantly requires that they may be individually regarded as almost inseparable parts of the syntax, are the following: First, the *cut*, or perhaps more properly, the "*Scroll*", which is a pair of cuts, the one enclosing the other, so that a scroll has two areas, its "*outer close*" and its "*inner close*". In the order of the actual mental evolution of the syntax of existential graphs, the Scroll was first adopted as a sign required before all others

[16] [Figures 21, 22, which Peirce did not draw in the main sequence of the manuscript, are found among the loose pages (p. 15) and have been added here and renumbered].

[17] **[Alt.:]** In order still further to make the reader acquainted with the *Syntax of Existential Graphs*, a series of problems shall at once be proposed, each requiring something to be expressed in this Syntax, with their solutions, as follows:
1. Required to express all those independent properties of Positive Integers, which are independent of any application of them. *Solution.* Let—w mean something is a positive integer; and let x–l–y mean x is lower than y. Let x–t–y mean x is true of y. [end of Alt.]

[18] [Peirce wrote both "can" and "cannot" here without, I presume, noticing to cross out "cannot"].

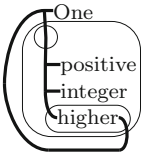


Fig. 21

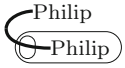


Fig. 22

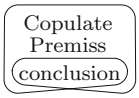


Fig. 23

Fig. 24

because it represented a necessary Reasoning, as in Fig. 23,[19] which reads: "If the Copulate Premiss is true, the Conclusion is true".

The Cut came to be thought of because of the immense frequency of occasions on which it was necessary to express the assertion "If *X* be true, then every assertion is true". It was forced upon the logician's attention that a certain development of reasoning was possible before, or as if before, the concept of *falsity* had ever been framed, or any recognition of such a thing as a false assertion had ever taken place. Probably every human being passes through such a grade of intellectual life, which may be called the state of paradisiacal logic, when reasoning takes place but when the idea of falsity, whether in assertion or in inference, has never been recognized. But it will soon be recognized that not every assertion is true; and that once recognized, as soon as one notices that if a certain thing were true, every assertion would be true, one at once rejects the antecedent that lead to that absurd consequence. Now that conditional proposition "If A is true, every proposition is true", is represented, in the model of Fig. 24, "If A is true, C is true" by blackening the entire inner close, as if there were no room, in reason, for any additional consequence. This gives Fig. 25: "If A be true whatever can be asserted is true", which is as much as to say that "A is not true and the inner close being cut very small", we get, first Fig. 26 and finally Fig. 27, in which the idea of flat falsity is first matured.

Beside the Syntax of Existential Graphs involves no other sign as essential to it except the Line of Identity, and the signs that grow out of that, such as the *Point of*

[19] [Figure numbering has been corrected from Fig. 22 to Fig. 23].

Fig. 25

Fig. 26

Fig. 27

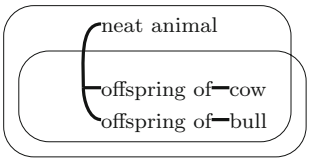


Fig. 28

Teridentity, where, as in Fig. 28,[20] a line of identity furcates, the *Peg*, or heavy dot which indicates that place on the periphery of a graph that denotes an individual kind of *Subject* of that graph (since, in this syntax, we have to recognize various other kinds of "Subjects" than the "subject nominative", as for example, the Object Accusative, which this Syntax shows to be as much a Subject as the other, if "subject" is to have any real meaning; the Object Dative; the Instrumental Object; the Locative; and others usually in the Aryan languages expressed by adverbial phrases). There is also the place on the cut where a Ligature crosses it.

All these are so many Graphs, whose general significations are forced upon them by that of the line of identity. The only other is the bridge, which is required simply to save the trouble of pasting the two ends of a paper ribbon on the sheet to make a real bridge.

It will be acknowledged that a simpler Syntax, capable of expressing any proposition, however intricate, would be difficult to imagine. ~~A proposition too intricate for any living human mind to grasp could be set down without ambiguity, by means of this Syntax with a very moderate amount of trouble.~~ A proposition too intricate to be clearly expressed in a single sentence in any living tongue can be expressed without ambiguity in this syntax as soon as it is distinctly apprehended.

It now only remains to ~~state~~ formulate those general permissions to modify what has already been scribed which express the ~~elementary~~ logicality of those several forms of elementary deductive inference, out of which all other deductions can be built up. There are but two of these general illative permissions; but before stating them there is one other thing that has to be said. Namely, it is to be imagined that every graph-instance anywhere on the sheet can be freely moved about upon the sheet; and since

[20] [The graph in Fig. 28 may be taken to read, "If there is a neat animal then it is offspring of a cow and offspring of a bull"].

a scroll both of whose closes are empty asserts nothing, it is to be imagined that there is an abundant store of empty scrolls on a part of the sheet that is out of sight, whence one of them can be brought into view whenever desired.[21] What is here said ought to be reckoned as a permission, but it is not an illative permission, i.e. a permission authorizing a species of inference.

First Illative Permission: the Rule of Deletion and Insertion. From any evenly enclosed area any enclosure or other graph-instance may be deleted, even if it involves the rupture of lines of identity on that area; and upon any oddly-enclosed area any enclosure or other graph~~-instance may~~ may be scribed and any connections made by lines of identity.

Second Illative Permission: the Rule of Iteration and Deiteration. Any graph may be iterated on the same area or within any number of cuts in that area, the graph-instance so inserted having identically the same connections as that which it imitates; and if a Graph occurs twice in the same area or in two areas one within a series of cuts within the area where the other occurs, and two having identically the same connections, then a deiteration may be executed by deleting the innermost of the two or if they are on the same area by deleting either of the two.

These two permissions will suffice to enable any valid deduction to be performed. The few examples that shall forthwith be given might tempt a lively mind to exclaim: Why, this syntax draws conclusions of itself, automatically. This would be extravagant; but one may say that the Syntax together with the application of the two illative permissions does so, provided it be ~~borne in mind~~ not overlooked that such application can only be made by a living intelligence.

[21] [This corresponds to the double-cut rule].