Synthese (2014) 191:1375–1390
DOI 10.1007/s11229-013-0333-x

What logical pluralism cannot be

Rosanna Keefe

Received: 3 April 2013 / Accepted: 4 August 2013 / Published online: 6 September 2013

Abstract Logical Pluralists maintain that there is more than one genuine/true logical consequence relation. This paper seeks to understand what the position could amount to and some of the challenges faced by its formulation and defence. I consider in detail Beall and Restall's Logical Pluralism—which seeks to accommodate radically different logics by stressing the way that they each fit a general form, the Generalised Tarski Thesis (GTT)—arguing against the claim that different instances of GTT are admissible *precisifications* of logical consequence. I then consider what it is to *endorse* a logic within a pluralist framework and criticise the options Beall and Restall entertain. A case study involving many-valued logics is examined. I next turn to issues of the applications of different logics and questions of which logic a pluralist should use in particular contexts. A dilemma regarding the applicability of admissible logics is tackled and it is argued that application is a red herring in relation to both understanding and defending a plausible form of logical pluralism. In the final section, I consider other ways to be and not to be a logical pluralist by examining analogous positions in debates over religious pluralism: this, I maintain, illustrates further limitations and challenges for a very general logical pluralism. Certain less wide-ranging pluralist positions are more plausible in both cases, I suggest, but assessment of those positions needs to be undertaken on a case-by-case basis.

Keywords Logical pluralism · Logical consequence · Non-classical logics · Reasoning

Is there a unique correct logic—a single consequence relation that settles which conclusions are logical consequences of which premises? If you think not, you may seek

R. Keefe (✉)
Department of Philosophy, University of Sheffield, 45 Victoria Street, Sheffield S3 7QB, UK
e-mail: r.keefe@shef.ac.uk; R.Keefe@sheffield.ac.uk

to defend logical pluralism—the position, roughly stated, that there is more than one genuine/true logical consequence relation. Filling out the rough statement of pluralism is not straightforward, however, at least not if it is to be an interesting and controversial position about true consequence relations rather than, for example, simply the recognition of multiple relations of the right form that are candidates for the consequence relation. A characterisation of a pluralist position needs to explain what it is to endorse all of the various consequence relations the pluralist accepts and how they relate to an intuitive notion of logical consequence. This paper explores these and related questions, focusing in particular on the position Beall and Restall defend in several papers and in their influential monograph, *Logical Pluralism*.[1] I argue that Beall and Restall don't succeed in identifying a coherent and sustainable pluralist position. Some other alternative ways to understand logical pluralism are also shown to be problematic and their discussion helps us provide a clearer sense of constraints on and challenges for any position of logical pluralism.

1 Beall and Restall's logical pluralism

"Crudely put, a pluralist maintains that there is more than one relation of logical consequence," say Beall and Restall (p. 25). The less crude statements of their position involve a general principle, GTT (short for Generalised Tarski Thesis) which they advocate as providing the correct account of logical consequence.

> GTT: An argument is valid$_x$ if and only if, in every case$_x$ in which the premises are true, so is the conclusion.

Different logical consequence relations will result from employing different notions of a case or a different understanding of "every case"; and by spelling out the notion of a case, you produce an *instance* of GTT. Beall and Restall then state that "*Logical pluralism* is the claim that at least two different instances of GTT provide admissible precisifications of logical consequence" (p. 29). There are additional conditions on *admissible* precisifications corresponding to crucial features of the notion of logical consequence, namely the requirements that the relations display features of necessity, formality and normativity.[2] Beall and Restall argue that various different logical consequence relations each meet all the criteria and are all true logical consequence relations. In particular classical logic, intuitionistic logic and relevance logic are all defended as providing admissible instances of GTT, with cases captured by Tarskian models, stages of constructions and situations (including incomplete and complete ones), respectively.

The logical pluralist's position looks striking. The vigorous debates between advocates of different logics (say classical logic vs. intuitionistic logic or vs. relevant logics)

[1] E.g. Beall and Restall (2000, 2001) and Beall and Restall (2006); see also Restall (2002). Unless otherwise stated, all references are to Beall and Restall (2006).

[2] For concerns about Beall and Restall's employment of these constraints, see, for example, Bueno and Shalkowski (2009a; especially on the necessity constraint) and Paseau (2007) (especially on the formality constraint; he also suggests further possible constraints that they ignore).

could then be neutralised by accepting that both parties succeed in capturing a true consequence relation. And there wouldn't always then be a unique answer to whether a given conclusion follows from some premises, e.g. whether A follows from ¬¬A, or whether B follows from A&¬A. It may both follow and not follow from the premises, and drawing the conclusion from the premises may be both right and not right. Before exploring how this could be so, it is important to distinguish the target view from some other positions.

Independent of debates about which is the/a true consequence relation, we can accept many different logical systems for abstract study. The question what it takes for something to be a logic is not a trivial one (hence debates about whether second-order logic is really logic, for example), but you can reject pluralism while acknowledging that the conditions for being a logic are met by a wide range of cases. Some logical systems may fail to capture logical consequence and, even with the best possible rules of translation showing how we evaluate ordinary language arguments using the logical system, they don't always give true answers to what follows from what. Pluralism maintains that various alternative systems all *do* deliver genuine consequence relations.

Second, Beall and Restall insist that their position is not Carnapian pluralism, or pluralism about the meanings of logical constants (p. 79 and p. 102; also Restall 2002). Carnap's pluralism allows for multiple different logical languages which dictate the rules of the connectives contained within the language and between which you can choose on pragmatic grounds. There is no answer to whether an argument such as "¬¬ A, therefore A" is valid when considered outside any particular language: there are different connectives in different languages which deliver different verdicts on the version of the argument within that language, mirroring the different meanings of "¬" in those languages. By contrast, in accepting both classical and intuitionistic logic, Beall and Restall are maintaining that those apparently conflicting logics both do govern the same connectives of negation, conjunction etc. It is the one same argument which is declared valid by one system and invalid by the other.[3]

Third, more generally, logical pluralism is not a form of relativism about logics (see e.g. p. 88). It does not claim that arguments are only valid *relative to* a logical system. Validity is not relative to the choice of logical constants nor relative to the particular system of rules for the constants. The different accepted logics all capture validity simpliciter. Nor is the claim that arguments are valid relative to contexts, so, for example, the idea is not that an argument form is valid in the context of mathematical reasoning but not for reasoning from observational statements. There is, Beall and Restall are claiming, more than one true logic governing *the same arguments*, involving the same subject-matter and taken in the same context.

Having seen here a range of positions that Beall and Restall want to distinguish from their logical pluralism, the challenge is to pin down their alternative. Section 2 explores their claim that their admissible instances are *precisifications* of logical consequence: they correspond to different ways of pinning down that vague notion and are all candidates for the *same* relation. We may hope that this allows us to preserve a univocal

[3] Field (2009a, pp. 346–347) argues that this idea of "inter-theoretic sameness of meaning of connectives" is not so straightforward, as he illustrates by considering the translations of negation between different competing logics.

notion of "follows from" (our ordinary imprecise notion) without this implying that there must be only one account. Perhaps, then, we can accommodate the multiplicity of relations that all still count as logical consequence relations by invoking vagueness and precisifications. I will argue, however, that this approach cannot work.

2 Logical consequence relations as precisifications

We will here put aside the possible concern that GTT does not succeed in capturing the notion of logical consequence, that for example, there may be candidates for consequence relations that are not instances of GTT.[4] Let us suppose that GTT does correctly pin down the ordinary concept of logical consequence (in so far as it makes sense to speak of such a thing) without settling the interpretation of "case". Are Beall and Restall right to take instances of GTT as *precisifications* of logical consequence? And does treating them as precisifications give the features of logical *pluralism* that one would expect and require for it to be appropriately so called?

What is a precisification, in general? It is vague or indeterminate notions that have precisifications, where each precisification settles imprecision in the notion in such a way as to agree with the vague/indeterminate notion on the aspects of it that are determined (in particular, the clear cases) and to settle those elements that are not determined. For example, a term such as "thin" is vague and although it determines that certain people are definitely thin and certain others are definitely not, it leaves borderline cases undetermined either way. A precisification of this notion draws a sharp boundary within the borderline cases and thereby settles those undetermined cases one way or the other while preserving the determined ones. Precisifications play a central role in the supervaluationist theory of vagueness, which maintains that a sentence is true if and only if it is true on all precisifications (see Fine 1975; Keefe 2000). As we will see, Beall and Restall are not quantifying over precisifications in this way, but, rather, they are treating the multiplicity of precisifications of logical consequence as establishing the multiplicity of logical consequence relations.

Note, first, that at least some of Beall and Restall's examples do not fulfil a basic constraint on being a precisification, namely that of agreeing with the settled cases (even recognising the uncertainty over which cases should be counted as settled). Among the examples of acceptable consequence relations that Beall and Restall cite is the relation yielded by the propositional calculus (e.g. p. 90), for which the relevant definition of cases is as valuations. But employing this account of cases treats arguments such as "John is tall so someone is tall" as not valid, since it is not valid in virtue of the propositional connectives. Whatever the indeterminacy in our notion of consequence, *this* argument is surely paradigmatically valid.[5]

[4] Beall and Restall (2006), p. 91 briefly addresses the possibility of non-transitive logics (e.g. Tennant's) which can't fit the form of GTT; see also Read (2006) for discussion of the limitation. Bueno and Shalkowski (2009a) advocate a form of pluralism employing a modalist view whereby the necessity involved in logical consequence isn't to be spelled out quantificationally, so GTT is no longer central.

[5] Similarly, of course, predicate calculus does not recognise the validity of, say, arguments of the form "necessarily p, therefore p" and this problem generalises for any logic that does not recognise all candidate logical connectives (e.g. temporal or epistemic operators). It may be easier to bite the bullet on such cases

Now, the usual approach is to refrain from treating an English-language argument as invalid simpliciter just because its propositional calculus formalisation is not valid according to that logic. Verdicts of invalidity can be overthrown by recognition of validity by some richer system. GTT might then naturally be seen as giving conditions for being valid *in some particular system*, so that all the propositional calculus instance of the principle yields is that "John is tall so someone is tall" is not valid in propositional calculus. But, as we have seen, Beall and Restall do not treat GTT this way, as this would be identified as a form of relativism. GTT is offered as an analysis of validity simpliciter, and it is a biconditional—an argument is valid simpliciter if *and only if* it meets the conditions.[6]

Beall and Restall could adapt their account to rule out as genuine logical consequence relations those logics with a level of complexity less than first-order logic (but allowing variation over other connectives). That may look rather ad hoc, but we can put that aside. The examples of alternative logics that they discuss most are ones that employ a different analysis of "cases" to yield a different set of valid arguments for the same set of logical constants, as classical and intuitionistic logics do. Could *these* be seen as precisifications of logical consequence?

As we have seen, Beall and Restall's official characterisation of their logical pluralism is as the claim that:

(A) At least two different instances of GTT provide admissible precisifications of logical consequence (p. 29).

This is given as the working out of the looser, more natural characterisation, according to which a pluralist maintains that

(B) There is more than one relation of logical consequence. (p. 25)

But it is far from clear that a theory holding (A) also yields (B), and without (B), the view does not look like a form of logical pluralism.

To see the problem here, consider someone who maintains that there are several admissible precisifications of logical consequence–they are equally good candidates for the notion—but who denies that they are all logical consequence relations. He/she may say, for example, that our notion of logical consequence determines that there is only one logical consequence relation (perhaps for reasons to do with normativity: there cannot be many conflicting ways that we *should* reason: see Sect. 3 below on

Footnote 5 continued
than it is for the quantificational arguments discussed in the text: "necessarily p, therefore p" could be declared not definitely valid, so only valid in some senses and on some precisifications.

6 Tarski's definition of logical consequence takes GTT to involve models that differ in their assignments to non-logical constants. He expressed scepticism about the possibility of providing grounds on which to draw a sharp boundary between logical constants and other expressions, though he did assume that the universal quantifier was definitely among the constants (1936, p. 418). He acknowledges the possibility that "we shall be compelled to regard such concepts as 'logical consequence'… as relative concepts which must, on each occasion, be related to a definite … division of terms into logical and extra-logical", taking this to reflect the fluctuation in the common usage of the concept of consequence (p. 420). Although this allows for several acceptable logics, it is not pluralism as advocated by Beall and Restall, as it is relativist in ways they reject and does not allow divergence between logics with the same logical constants (e.g. classical and intuitionistic logic).

this issue). The availability of several *precisifications* is compatible with the denial of the claim (B). Indeed, this combination of views is in line with other debates in which precisifications have been employed. Consider Mount Everest. There is no sharp boundary at the base of the mountain that delineates the exact area of Mount Everest: different lines could be drawn and each would provide a precisification of "Mount Everest". But many theorists would deny that it follows that there are many mountains (or many Mount Everests) here, one for each precisification (see, e.g., Lewis 1993). There may be several equally good *candidates* for the referent of our name "Mount Everest", but it might be a settled matter that there is only one Mount Everest. This combination of views can be achieved either by accepting that something determines a uniquely correct candidate, even if we cannot tell which one it is—a form of epistemicism—or by employing a form of supervaluationism. According to the latter, "there is only one mountain there" (with the appropriate demonstration) is true since that is true on all precisifications, even though which precisification is the mountain is different according to the different precisifications. Similarly, though there are many precisifications of "bald", it does not look natural to conclude that there is more than one property of baldness: none of the precisifications are actually *the* (or *a*) property of baldness, even if they are candidates for being it.

Supervaluationism, then, clearly invalidates analogues of the move from (A) to (B). Consider a supervaluationist-type proposal to determine the truth-value of consequence claims by quantifying over the various precisifications, calling an argument valid iff it is valid according to all those admissible logics. This approach may be unappealing because the wide extent of variation among candidate logics would mean that a very small set of arguments would count as valid and there would be a large range of borderline valid ones (where different logics disagree) about which it is neither true nor false to say they are valid. More importantly, it would not be a version of pluralism, since there would always be a single answer to whether an argument is valid.[7]

You don't have to accept supervaluationism to be suspicious of the move from (A) to (B), however. We should distinguish between saying that it is *indeterminate* which notion of logical consequence is the right one (or is our notion) and saying that they are all correct notions. You may maintain that nothing pre-theoretic or intuitive settles the choice between some alternative systems: it is indeterminate which is right—which, *pace* logical pluralism, is not to say that they all are—but this indeterminacy is to be characterised in a non- supervaluationist way.

Even if the commitment to multiple precisifications of 'logical consequence' doesn't *guarantee* a commitment to multiple logical consequence relations, is it possible to maintain that in this case, there are nonetheless both many precisifications and many logical consequence relations? Beall and Restall draw on some analogies

[7] Beall and Restall (p. 92) consider and reject a closely related view, prompted by an envisaged objector proposing that validity requires truth-preservation in all cases of all kinds. (The view is not the same, however: arguments that are not valid according to some logics but valid according to others count as invalid on that view and borderline valid on the supervaluationist one.) Bueno and Shalkowski (2009a, p. 300) argue that Beall and Restall's necessity constraint drives them to this view, and that once we consider the full range of alternative logics, this would result in logical nihilism whereby nothing is valid.

in an attempt to illustrate the kind of claim involved in logical pluralism. The third of these is the vagueness case (p. 27), which as I have argued, works against them rather than in their favour. For different reasons, the other analogies don't help here either, I argue. The first is computability, where the Church-Turing thesis makes precise an informal notion of computability by equating it with recursiveness (pp. 26–27). Even if this helps illustrate how a formal condition can be used to make precise an informal notion (recursiveness for computability or instances of GTT for logical consequence), in the case of computability, there is a single precise notion and so the multiplicity of precisifications does not arise. The analogy cannot help with understanding the multiplicity.

Second, Beall and Restall appeal to an analogy with different precisifications of "necessary" (pp. 26–27). Their treatment of necessity follows their treatment of logical consequence very closely, with GLT—*p* is necessarily true iff *p* is true in every case—playing the role of GTT and again allowing different interpretation of "case". Here, one so-called precisification has "case_x" range over all worlds (giving metaphysical necessity) and other so-called precisifications restrict the worlds, for example—for physical necessity—to just those satisfying our laws of nature. This promotes a view whereby there is an absolute sense of necessity and various weaker notions which may be legitimate senses (even precisifications) of "necessity". If this is a form of *pluralism*, it isn't an interesting one, just as allowing a legitimate use of "all" in a restricted quantification such as "all the beer is in the fridge" doesn't reveal a problem for the claim that there's a unique quantifier expressing unrestricted quantification. On the logical analogy, we get no further than the suggestion that in some contexts we might only demand truth-preservation across a restricted class of cases rather than across all cases.[8]

As another analogy, Beall and Restall claim that "the notion of *follows from*, as it applies to reasoning, may be made precise in at least two distinct ways" (p. 28), namely as deductive consequence or as inductive consequence. We may agree that there are here two ways of settling the "goodness" of an argument (though put in terms of the conclusion "following from" the premises, the inductive reading sounds wrong). Nonetheless, these do not correspond to two different ways of making the same notion precise. There are, rather, two different informal notions here, with different connections to other notions, even if we don't always distinguish between them and sometimes use the same term—"good argument"—for both. It is relatively easy to get people to recognise the distinction and to get them to clarify in what sense they mean that something was a good argument, for example. This looks more like ambiguity in the notion, then, and not pluralism about a univocal notion, so, again, it does not provide a good analogy for logical pluralism. Recognition of a range of different notions that can each play their different roles (as in the deductive/inductive reasoning case) as an

[8] This paragraph appears to ignore any types of necessity—perhaps conceptual necessity—that can't be effectively treated in this way as restricted necessity in this way. Any potential types of necessity that don't fit the form of (GLT) will be of no use to Beall and Restall here, though. Considering other possible types—perhaps a two-dimensional treatment of conceptual necessity—would take us well beyond the scope of this paper. The point at present is that, at the very least, Beall and Restall's treatment of necessity (which also ignores these kinds of case) does not provide an analogy that helps illuminate their logical pluralism.

acknowledgement of a kind of ambiguity does not give us a form of *pluralism* about one thing.[9]

Beall and Restall have not offered us an understanding of how the different formal consequence relations relate to the informal notion of logical consequence and the official statement of their pluralism, by (A), does not yield a form of pluralism about consequence. We need to return to the question of what exactly the pluralist is saying about their multiple logical relations. They are saying more than that they are logics, because they are *endorsing* them as true consequence relations. But what does endorsing a logic amount to? In the next section, I first criticise Beall and Restall's answer to this question. I then show that considering the applicability of logics (allowing different logics for different applications) might seem to help, but in fact not only offers no answer to our question, but also poses further problems for constructing a sustainable form of logical pluralism.

3 Endorsing and using logical consequence relations

A pluralist accepts several logics and denies that there is a need to select just one privileged one. But what is it for them to *accept* each logic?

Beall and Restall distinguish between strongly endorsing a logic and weakly endorsing it and say that to endorse several logics in either sense is to be a pluralist (p. 82–83). You merely weakly endorse a logic if you regard it as providing an admissible instance of GTT without satisfying the *actuality constraint*, where "an instance of GTT satisfies the actuality constraint if and only if the actual case is in the domain of its quantifier". To strongly endorse a logic is to take it additionally to satisfy the actuality constraint.[10] To take their example, a dialetheist—someone who believes that there actually are true contradictions—can only strongly endorse logics that recognise cases containing true contradictions (paraconsistent logics). Classical logic would not then satisfy the actuality constraint, since, according to that dialetheist, the actual case is inconsistent whereas no models of classical logic are. So a dialetheist can weakly endorse classical logic, but not strongly endorse it. By contrast, if the actual case is consistent after all and classical logic is to be strongly endorsed, the actual case will also be one of the cases quantified over within a paraconsistent logic, so someone who denies that there are any inconsistent situations can still strongly endorse the paraconsistent logician's consequence relation as well as the classical one.

As another example, which Beall and Restall don't discuss, consider the relation between classical logic and many-valued logics (where the latter can be shown to

[9] Beall and Restall sometimes write as if the pluralist's different consequence relations correspond to different senses of "consequence" and several commentators have taken their position as such (see, e.g., Beall and Restall 2001, p. 1, Field 2009a and Hjortland 2013). I cannot here explore different, more subtle types of ambiguity, but we can say, at the very least, that drawing an analogy with other standard cases of ambiguity does not help in understanding the logical pluralist position in question.

[10] Note that on pp. 82–83, the definition of "strongly endorsing" a consequence relation requires it to be "an instance" of GTT, whereas weak endorsement of a consequence relation requires it to be an "admissible instance" of GTT. I take it that the admissibility of the instance is also required for strong endorsement as the authors are clear that strong endorsement entails weak endorsement.

provide admissible instances of (GTT) within Beall and Restall's scheme). If you strongly endorse a many-valued logic because you claim that some sentences actually take non-classical values, then you'll say that the actual case isn't in the domain of the quantifier of classical logic so you cannot strongly endorse classical logic. Again, there is a striking and unappealing asymmetry: an opponent who denies that there are any non-classical truth-values can still count as strongly endorsing a many-valued logic, since the case they take to be the actual one (some two-valued valuation) is among the cases quantified over within a many-valued logic.

Weak endorsement cannot be the key notion for logical pluralism, I maintain. A valid argument guarantees that if the premises are true, then the conclusion is also true. But if the actual case is not in the domain of quantification then the fact that an argument is truth-preserving in all the cases quantified over does not even ensure that it is *actually* truth-preserving. A compelling normativity constraint should rule this out. Consider how Beall and Restall put it early on: "if an argument is valid then you somehow go *wrong* if you accept the premises but reject the conclusion" (p. 16). If the validity of an argument does not guarantee truth-preservation in the *actual* case then why think that you *do* go wrong in accepting the premises and denying the conclusion of a valid argument in that actual case? There's no guarantee that that isn't the way things are. Weakly endorsing a logic looks like merely recognising it as a logic. A logic that fails the actuality constraint surely fails to capture the true consequence relation even if it has enough formal features to count as a logic.[11]

Although strong endorsement looks more promising in this respect, as the asymmetry pointed out above illustrates, you count as strongly endorsing any logic contained within another logic that you endorse, for the weaker logic will automatically pass the actuality constraint if the former does. For example, you can count as strongly endorsing three-valued logic while denying that there are or could be sentences that have non-classical truth-values.[12] In such a case, it may be natural to say that you are not a pluralist because you see no *use* for a many-valued logic. (It requires truth-preservation over irrelevant cases and misclassifies an argument as invalid for failing to preserve truth in those cases.) Maybe a better way to restrict the accepted logics among those satisfying GTT is to ask which logic(s) we should *use*. That brings us on to the *applications* of various logics.

Beall and Restall say that "rivalry among logics ... enters at the level of *application*" (p. 44) and for each of the logics they endorse, they suggest some applications, including analyses of maths and semantics for classical logic (pp. 44–47), fictional discourse and truthmaking for relevance logic (pp. 56–58), and mathematical reason-

[11] What if, for example, you are not sure whether or not there are non-classical values or whether the classical models exhaust the possibilities? You might naturally be described as an agnostic not a pluralist (though for Beall and Restall, you count as weakly endorsing all the options and strongly endorsing none of them). By characterising their pluralism in terms of the logics that the advocate *endorses*, they leave no room for the debate between someone—surely a monist—who maintains there is a single correct logic, though they're not sure which and a pluralist who thinks there are several correct logics, but is uncertain about which pass the relevant tests.

[12] Beall and Restall's constraints of formality, normativity and necessity don't help here either: again, they are too easy to pass, as would be shown by replicating the arguments offered to show that their chosen logics satisfy them (e.g. p. 55, p. 69).

ing and constructive warrant for intuitionistic logic (pp. 71–74). The suitability and/or necessity of the various logics for these applications are controversial and to defend or undermine any of these applications in detail would go far beyond the scope of their book or this paper. (This weakens the argument for pluralism, since a monist committed to classical logic may well use as a central part of their defence the claim that they can accommodate all applications classically, but I put this aside). We need to understand, however, how the issue of the applications of a particular logic bears on the question whether it is among the logics you endorse as part of your pluralism. Are logics only *admissible* when they are applicable? For example, could an admissible logical consequence relation have *no* useful applications? If there are no applications, can the normative constraint be met? There is a way you go wrong with an invalid argument according to the logic in question, but if that logic has no applications, it is never the case that you should avoid going wrong in that way.

Are there wrong and right ways to reason (or use a consequence relation) in different applications (i.e. different contexts)? A dilemma threatens here. On the first horn, we maintain that, in different types of application, different logics give the right answers about consequences. But this amounts to a relativist position, where the right logical consequence relation varies with (and is thus relative to) different contexts. As explained above, Beall and Restall deny that their pluralism is a form of relativism; see e.g. p. 88, "we do not take logical consequence to be relative to languages, communities of inquiry, *contexts*, or anything else" (my italics). How is this quotation compatible with the observation that how we should reason (which logic we should use) depends on the context?

This prompts the second horn of the dilemma whereby any true consequence relation delivers a correct verdict in any context, but sometimes some choices are pragmatically more effective than others. For example, sometimes the simplicity of the propositional calculus helps in applications, and treating cases as possible worlds can often be less useful than other accounts of cases whose formal tests give us easier access to whether an argument is valid. The worry with this horn is that a candidate consequence relation can have certain sorts of applications without being a true consequence relation. A monist can benefit from the simplicity of some system that does not capture true consequence and, for example, trust its judgements of validity but not all its judgements of invalidity. And a relation that has been offered as a consequence relation can be co-opted for an account of a relation that requires more than consequence.

For example, one application of relevant logic that Beall and Restall cite is in the explanation of truthmaking (pp. 57–58). The standard account of what it takes for a to be a truthmaker for P is that the existence of a entails P, but, with a classical notion of entailment, anything would thereby count as a truthmaker for A if A is a theorem of classical logic. With a notion of relevant entailment, not everything entails a necessary truth, so it opens up the possibility of an illuminating story about the truthmakers of necessary truths. Even if this story works, however, it shouldn't (or at least needn't) be treated as involving the/a true consequence relation: perhaps the moral is that truthmaking cannot be understood in terms of entailment but involves a stronger relation (the "standard account of truthmaking" referred to above is far from uncontroversial). Similarly, they seek to use relevant logic to model inconsistent but non-trivial theories (pp. 56–57), and again we can see the relevantist's "consequence" relation as a useful

way of making sense of such a theory as extending beyond the axioms to just those consequences that satisfy certain further conditions.[13] So, the utility of a so-called consequence relation is not sufficient to show it is a true consequence relation.

Consider, next, a relatively ordinary context of reasoning, and suppose our subject, S, endorses Beall and Restall's pluralism. If S accepts premises Γ and is considering conclusion C, what logic should she call on to decide whether or not to accept C? I argue that she ought to endorse the argument and accept its conclusion if it is valid according to *any* of the acceptable relations in the plurality. Suppose "Γ therefore C" is valid on some acceptable logics and not on others: does the truth of Γ guarantee the truth of C? Yes, and no, because it depends what you mean by "guarantee", and there is no unique sense to the claim that conclusion is *guaranteed* to be true. But, *no* sense of "guarantee" is compatible with the premises being actually true and the conclusion actually false, even if there is variation over the putative cases the truth-transmission must travel across. So, if the argument is valid in any sense, that is enough to show that the conclusion is actually true, assuming the premises are.

This implies, for example, that when the consequence relation of one logic is contained within that of another, we should work with the latter in preference to the former if we want to know whether our conclusion is true. For example, it is enough that A follows classically from not-not-A to warrant reasoning that way, despite the fact that it does not follow by intuitionistic logic. Even if there's no strongest relation among those relations that a pluralist admits, there will still always be a right answer to whether we should draw some conclusion from some premises, namely that we should if it follows according to any of the logics they accept.

This thus yields a privileged consequence relation. The central role of logic is to use it to infer truths and our privileged consequence relation fills that role in such a way that it is plausible to say that a conclusion found to follow in this sense from some premises but not "follow" in another, simply fails to satisfy some stronger condition (just as we might say that the denial of a law of nature is *possible*, though it does not count as possible in the stronger sense of physically possible.)[14]

[13] Bueno and Shalkowski (2009a) also focus on uses of logic of this kind, where, again, what we should say is that stronger logics are useful for capturing something other than a consequence relation. The modalist element of their theory—that necessity is taken as primitive and not to be understood in terms of quantification over cases—does not justify their claim that in different domains, different judgements of necessity are warranted. A database may represent a contradiction (a student both has and has not returned some library books) and it can be useful to have a system whereby we can discriminate among the things that we can take this to commit us to (perhaps that they owe a £3 fine, but not that they owe a £3000 fine, as would also be true if we took any arbitrary conclusion to follow from the contradiction). But that's not to say that there's a sense of *possibility* in which it is possible that the student both has and hasn't returned the same books. It's possible for both "he has returned the books" and "he has not returned the books" to be *recorded*, but there's no contradiction there, and it isn't a domain in which he has both returned and not returned the books. Similarly, the situation in which Bueno and Shalkowski judge a constructivist view to be warranted is in "intensional contexts": it allows us to model "I am not certain that the Yankees will not win the World Series" as not-not-p, where the inference to p (the Yankees will win) does not go through. But, again, this doesn't give a sense in which p fails to *follow* from not-not-p, just a situation where a different, but related inference doesn't hold. Modelling the above claim as not-not-p does not make it a genuine instance of that form.

[14] On pp. 101–102, Beall and Restall consider the interpretation of logical consequence whereby an argument is logically valid iff it is logically valid according to one of the accepted logics (i.e. according to one

Beall and Restall may reply that I am begging the question against pluralism and assuming a unique answer to what follows from what or how we should reason. Compare a moral pluralism that acknowledges several incommensurable values: we would beg the question against that position if we assumed a unique answer to how we should act. If one value dictates one thing and another value dictates another, then it is true that you should do the first and that you should do the second, even though they are in conflict. But the case with logical consequence is different because we focus on the central role of inferring *truths* common to all consequence relations (where a parallel of a pluralism about truth has been rejected). The privileged relation (as described above) delivers only truths from truths and the grounds for resisting a conclusion which does not follow according to some other relation are not that it does not follow that the conclusion is true. So the situation is not like the question what we should do in the moral case, where the requirements can only be formulated by reference to different values.

But does this line of thought ignore the fact that we should and do require more of our arguments than mere truth-preservation—typically *necessary* truth-preservation? Perhaps when a conclusion follows by one of our logics and not by another, we can't ignore the narrower relation, because it also appeals to a sense of necessary truth-preservation that it is legitimate to require. That, however, raises the question why we should insist on more than truth-preservation, when what we are interested in is the truth of our conclusions. Historically, there have been conceptions of consequence which demanded mere truth-preservation (e.g. Philo's conception; see Smiley 1998). As Smiley asks, why have a requirement of necessity regarding truth-preservation but not regarding the truth of the premises? Now, there are epistemic reasons not to focus on mere truth-preservation, for we cannot typically know that an argument accidentally preserves truth without already knowing the truth-value of the conclusion, whereas we *can* know that an argument is necessarily truth-preserving (by knowing its form) and know that the premises are true, and thereby come to know the truth-value of the conclusion. But, if those are the grounds for needing more than mere truth-preservation, then we can settle for the strongest consequence relation, which will give us truth-preservation if not as much more (in terms of necessity) as other alternatives guarantee.

Footnote 14 continued
admissible interpretation of "case"). They call this "a perfectly singular sense of 'logically valid'"—though it isn't clear what this means, given that each sense is surely singular—but deny that this supports monism if relations also count as logical consequence relations. I am arguing that they shouldn't so count given the availability of this "univocal" sense. Elsewhere, Beall and Restall address a related argument from Graham Priest which considers some inference that follows according to one logic but not according to another and asks whether we are entitled to accept the conclusion (p. 93). But they just take up a part of Priest's argument that pushes it further than necessary, namely to the end-point that "we should ... apply the notion of validity appropriate to the smallest class that s is in [i.e. {s}]". They respond that "all that Priest's argument shows is that the more we know about s, the more we know about what is true in s" (p. 93). Whereas they may be right that we will only be able to use Priest's limit case of quantification over the actual situation alone if we know enough about what's actually true, this doesn't address the point as explained above, according to which we should use the strongest consequence relation of those endorsed—where we *can* take ourselves to know the truth of the premises and know the validity of the argument independently of knowing the truth of the conclusion. See also Read (2006) on Priest's argument.

4 Other ways to be and not to be a pluralist

The discussion of the previous section suggested that the normative feature of logical consequence limits the range of consequence relations that can be simultaneously endorsed. It does not follow, however, that normative considerations would always get us to the sole correct consequence relation. If we were to reject bivalence and genuinely take seriously certain non-classical logics, it is less clear how we ought to spell out the normative role of validity.[15] Consider many-valued logics again. Even given the same choice of set of truth-values, and the same choice of the definitions of the connectives, there are different definitions of validity that can reasonably be adopted as alternatives to the instance of GTT requiring preservation of complete truth. For example, validity is sometimes equated with preservation of certain other selected "designated values" (e.g. truth plus the intermediate value in a three-valued scheme, or all values above 0.5 in an infinite-valued logic). Or, if the truth-values are seen as degrees of truth, we may be concerned to preserve degree of truth, where that is, for example, a matter of the value of the conclusion not dropping below the value of the lowest premise. These different definitions of validity may each correspond to ways of regimenting the normative requirements of a valid argument once we've moved away from the bivalent scheme. For the basic thought that we *should* believe the conclusion of a valid argument if we believe the premises may sometimes be best spelled out in terms of preservation of complete truth and other times in terms of preservation of degree of truth in a more general way. In my 2000, I propose that, "if we were to adopt a degree-theoretic approach to vagueness then we should take a *pluralist* attitude to the different candidate notions of validity, allowing several of them to give equally legitimate senses of 'valid"'. Similarly, in my 2001, I advocated a pluralism regarding several different accounts of validity within a supervaluationist framework.

These positions are forms of what Hjortland (2013) calls "intra-theoretic pluralism" —accommodating different candidate consequence relations within the same theory— which he illustrates with a choice of different designated values within a three-valued logic. These cases don't face the kind of objection outlined in the previous section. Just because one definition of validity tells us that if the premises are wholly true, then the conclusion is too, that doesn't settle whether we should infer that our conclusion is nearly true given that our premises are, or that it is as true as the premises are: appeal to other definitions of validity will be needed to answer those questions.

This is just a brief sketch of a route to a less general and less permissive form of pluralism than Beall and Restall's.[16] The viability and plausibility of some such more modest pluralism will need to be examined on a case-by-case basis in the context

[15] See e.g. Field (2009b) and Milne (2009) for some discussion of degree-theoretic treatments of the normative requirements of a valid argument, where a valid argument dictates, not just that you should (fully) believe the conclusion if you fully believe the premises, but also the degree to which you should believe the conclusion when you merely partially believe the premises.

[16] Could we accommodate a pluralism that allows both classical logic and intuitionistic logic in a similar way? Hjortland doesn't want to rule out a semantics that would accommodate both of those logics. Maybe this would help establish that pluralist claim and *maybe* some other story could be told that allows both to count as true consequence relations, but the devil would be in the detail.

of exploring each of the alternatives.[17] But it is a more promising approach than a defence of logical pluralism that attempts to vindicate all of the diverse range of logics that Beall and Restall consider.

These kinds of modest pluralisms have parallels in debates about pluralism in other areas, such as Religious Pluralism. I finish this paper by exploring more general comparisons of positions in the debate over Logical Pluralism with positions that might be taken regarding Religious Pluralism. Although, first, there are clearly disanalogies between the debates and, second, my treatment of religions is rather rough-and-ready, the comparison illustrates my case against the project of defending logical pluralism at the level of generality that Beall and Restall aim for, and it also helps bring out some options relating to pluralist positions that have not been raised in the discussion above.

Talk of "Religious Pluralism" can just mean (a) that there are many religions within a society. Often when it is advocated, though, it also involves (b) the commitment to treat all religions with similar respect. This could be based on respect for individuals and their right to believe what they want; or it (c) could be a kind of modesty (acknowledgement that although I have my own my religion, others in a comparable epistemic position have other religions), or perhaps (d) a scepticism (we can't know what the "right" religion is), or (e) an agnosticism (which could, but needn't be, prompted by scepticism). These latter views could (f) be combined with a kind of error theory across the board: no religion is true in all the claims it makes, but many are legitimate in other ways (perhaps in delivering instruction on how to live), where to get a pluralist element, it must be that several different options are equally legitimate. Relatedly, a theorist might advocate (g) some kind of anti-realism or non-cognitivism that declares that the claims of each religion are not eligible for truth or falsity (e.g. because they are unfalsifiable), where, again, this will be combined with some way of adjudicating between religions other than on grounds of truth and with the thesis that several options are equally successful.

The most radical position would be to maintain that h) all the religions in the relevant class are true together, despite their conflicting claims. This is very hard to make sense of, given the wildly opposing views associated with the different religions in question. If one religion maintains that there is a single deity and another denies it, for example, then how can they both be right? If the set of endorsed religions is small enough, the "pluralism" can be just i) a claim about a particular small sub-set of religions. Some two options *may* be compatible, but that's just a quirk of those two and not a general religious pluralism or a way of reconciling *conflicting* religions.

Let us turn, then, to the analogies to this range of positions in the consideration of Logical Pluralism. The analogue of (a) says there are many logics. This is the first position we put aside in Sect. 1 and is surely uncontroversial and not an interesting form of logical pluralism. Adding (b)—treating many alternatives with respect—may suggest that it is good to study multiple logics. This adds little of interest in relation to our topic. Options (c)–(e) fall a long way short of pluralism too. (c) advocates

[17] E.g. Varzi 2007 rejects my 2001 pluralist thesis by defending a particular definition of "validity" within the framework in question.

open-mindedness—let's look at the theories in full specificity before we try to judge—surely reasonable, but not yet pluralism. Nor is it pluralist, (d), to express scepticism about the possibility of finding the true logic; and (e)—the desire to keep various options in play because you're not sure which is right—is no form of logical pluralism either.

The most fitting analogue of (f)—the error theory position—is a nihilist position that declares that no arguments are genuinely valid, coupled—to provide endorsement of multiple logics—with a line on how a logic can nonetheless be useful. But, as regards the key question of the "true consequence relation" it then looks as if we have here a *monist* nihilist position, albeit combined with flexibility about other uses of logic. (Similarly, for the religious case, it looks like atheism has won out, where, though not a religion itself, this is naturally to be considered alongside the competing options). I suggest that the best ways to develop something close to this option—probably in the religious case as well as the logical one—will turn into a version of the next, anti-realist option, (g).

The anti-realist position might be worked out in different ways, but will not amount to the kind of view we have been considering or that advocated by Beall and Restall, who maintain that each of their favoured logics is a *true* logical consequence relation. One version of the anti-realist position is suggested by Hartry Field, who calls himself a "relativist expressivist" (2009a). Field rejects talk of the "true logic", instead focusing on the logic that individuals "express" in virtue of the way they reason, and allowing for different logics to be reasonably endorsed by different people or in different contexts.

Option (h) does correspond to the kind of pluralism we have been focusing on in this paper and that Beall and Restall advocate. I suggested that the initial response to the Religious Pluralist version of this option is, at best, puzzlement: how can sets of incompatible claims all be true? In the light of Beall and Restall's detailed defence of their position, it would, of course, be inappropriate to dismiss logical pluralism this quickly. But, my objections in the previous sections could be seen as filling out reasons for this natural presumption against such a pluralism and reasons to think they have not succeeded in showing how there can be multiple radically different consequence relations.

As we have seen, one interpretation of logical pluralism—an *ambiguity* interpretation—associates different logics with different senses of "valid". In the religious case, there is no single term in which we can locate the ambiguity, but we might see different religions as having different subject-matters. An unpromising way to make sense of this puzzling position might be to start from the thought that a religion tells you (something about) what you should believe and how you should behave. This suggests a mirroring of Beall and Restall's position on logics, in which we note that there are different senses of "should" and so different answers to what you should believe and how you should behave according to different religions, where each is right in its own way. But this clearly won't do: even if the idea of "what you should believe" isn't precise or clear-cut, there aren't different disambiguations/interpretations of the idea corresponding to each religion. What you should believe according to Christianity is incompatible with *any* sense in which you "should" adopt Islamic beliefs. This thought corresponds to my objections above to the ambiguity interpretation of logical pluralism.

Looking at particular religions individually may yield a very restricted "pluralist" claim reconciling several religions on grounds of a specific ambiguity or on grounds of compatibility, which brings us to option i) and the restricted forms of "pluralism". This position corresponds to the kind of limited pluralism discussed at the beginning of this section (including cases Hjortland labels "intra-theoretic" pluralism). As suggested above, there may be plausible forms of this kind of pluralism, but they need to be examined on a case-by-case basis. Such restricted pluralist positions are likely to be of interest in relation to the specific religions involved or the logical theory in which the different definitions of validity are accommodated, and detailed exploration of the positions may well help illuminate more general issues about, say, the scope of religion or about validity and its normative role. They will not, however, show how several radically conflicting theories can all be true, but I suggest that any attempt to fulfil this more general aim will be flawed.

Acknowledgments Many thanks to two referees for very helpful comments.

References

Beall, J. C., & Restall, G. (2000). Logical pluralism. *Australasian Journal of Philosophy*, *78*, 475–93.

Beall, J. C., & Restall, G. (2001). Defending logical pluralism. In Logical Consequence: Rival Approaches. In J. Woods, & B. Brown (Eds.), *Proceedings of the 1999 conference of the society of exact philosophy* (pp. 1–22). Stanmore: Hermes.

Beall, J. C., & Restall, G. (2006). *Logical pluralism*. Oxford: Clarendon Press.

Bueno, O., & Shalkowski, S. A. (2009a). Modalism and logical pluralism. *Mind*, *118*, 295–321.

Field, H. (2009a). Pluralism in logic. *Review of Symbolic Logic*, *2*, 342–359.

Field, H. (2009b). What is the normative role of logic? *Proceedings of the Aristotelian Society, Supplementary Volume*, *83*, 251–268.

Fine, K. (1975). Vagueness, truth and logic. *Synthese*, *30*, 265–300.

Hjortland, O.T. (2013). Logical pluralism, meaning-variance and verbal disputes. *Australasian Journal of Philosophy, 91*, 355–373.

Keefe, R. (2000). *Theories of vagueness*. Cambridge: Cambridge University Press.

Keefe, R. (2001). Supervaluationism and validity. *Philosophical Topics, 28*, 93–105.

Lewis, D. (1993). Many, but almost one. In J. Bacon, K. Campbell, & L. Reinhardt (Eds.), *Ontology, causality and mind: Essays in honour of D.M. Armstrong*. Cambridge: Cambridge University Press.

Milne, P. (2009). What is the normative role of logic? *Proceedings of the Aristotelian Society, Supplementary Volume*, *83*, 269–98.

Paseau, A. (2007). Review of logical pluralism. *Mind*, *116*, 391–396.

Read, S. (2006). Monism: The one true logic. In D. DeVidi, & T. Kenyon (Eds.), *The logical approach to philosophy*. Berlin: Springer.

Restall, G. (2002). Carnap's tolerance, meaning and logical pluralism. *Journal of Philosophy*, *99*, 426–43.

Smiley, T. (1998). Conceptions of consequence. In E. Craig (Ed.), *Routledge encyclopedia of philosophy*. London: Routledge.

Tarski, A. (1936). On the concept of logical consequence. In A. Tarski (Ed.), *Logic, semantics, metamathematics* (2nd ed.). Indianapolis: Hackett Publishing.

Varzi, A. (2007). Supervaluationism and its logics. *Mind, 116*, 633–75.