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The global non-entropic arrow of time: from global geometrical asymmetry to local energy flow

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Abstract Since the nineteenth century, the problem of the arrow of time has been traditionally analyzed in terms of entropy by relating the direction past-to-future to the gradient of the entropy function of the universe. In this paper, we reject this traditional perspective and argue for a global and non-entropic approach to the problem, according to which the arrow of time can be defined in terms of the geometrical properties of spacetime. In particular, we show how the global non-entropic arrow can be transferred to the local level, where it takes the form of a non-spacelike local energy flow that provides the criterion for breaking the symmetry resulting from time-reversal invariant local laws.

Keywords Global arrow of time · Non-entropic arrow of time · Local energy flow · Time-symmetric twins

1 Introduction

Since the birth of thermodynamics in the nineteenth century, the problem of the arrow of time has been traditionally analyzed in terms of entropy. Even at present many authors still relate the direction past-to-future to the gradient of the entropy function of the universe. However, entropy is not a fundamental property but a phenomenological thermodynamic magnitude, whose value is compatible with many different configurations of a system. In this paper, we shall follow a different path by considering the arrow of time as a property of time and not as a derived feature reducible to

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more basic properties of physical systems. In particular, we shall adopt a global and non-entropic approach to the problem, according to which the arrow of time can be defined in terms of the geometrical properties of spacetime.

In a recent work (Castagnino et al. 2003a), the problem of the arrow of time has been addressed from a comprehensive viewpoint, on the basis of a global and non-entropic approach. In turn, in a second paper (Castagnino et al. 2003b) the first part of that work is discussed by analyzing under what conditions it is possible to define the arrow of time for the universe as a whole only on the basis of the geometrical properties of spacetime, independently of any entropic considerations. The aim of the present paper is to develop in depth the second part of that initial comprehensive work. In particular, we shall focus on a point not usually considered in the literature on the subject, where the problem of the arrow of time is treated either in local terms or in global terms, but the connections between both approaches are not analyzed. Here, it will be shown how and under what conditions the global arrow of time defined by the geometrical properties of spacetime can be transferred to the local level, where it takes the form of a non-spacelike local energy flow that provides the criterion for breaking the symmetry resulting from the time-reversal invariant laws of local physics. In order to develop this program, the paper is organized as follows. Since the expressions 'arrow of time' and 'irreversibility' are usually confused in traditional discussions, in Sect. 2, we carefully distinguish between both concepts, arguing that the problem of the arrow of time has conceptual priority over the problem of irreversibility. In Sect. 3 we briefly justify our adoption of a global and non-entropic approach to the problem of the arrow of time, and in Sect. 4 we consider the conditions necessary for defining a global and non-entropic arrow of time. These initial sections will allow us to introduce our main points. Section 5 is devoted to define a global time-orientation on the spacetime in physical terms, considering the properties of the energy-momentum tensor. In Sect. 6 we present the conditions under which the global arrow of time can be transferred to the local level as a local energy flow. Finally, in Sect. 7 it is shown how the local energy flow provides the criterion for distinguishing between the temporally symmetric structures resulting from local physical theories.

2 Disentangling problems: irreversibility versus arrow of time

The problems of irreversibility and of the arrow of time have been traditionally identified, as if irreversibility were the only clue to understanding the origin and the nature of the arrow of time. In this section, we shall argue for the distinction between both questions, and this will allow us to supply a clear and concise formulation of the problem of the arrow of time as we shall conceive it in the rest of our argumentation.

2.1 The problem of irreversibility

Some authors view the problem of irreversibility as the problem of explaining the temporal asymmetry of our experience of processes in time: we never see two gases 'unmixing' or a candle 'unburning'. Since those processes are predominantly entropy-increasing evolutions, the question is transferred to thermodynamics: the puzzle seems to be to explain the irreversible entropy-increasing processes of thermodynamics on the basis of the time-reversal invariant laws of mechanics.

Here, we shall conceive the problem of irreversibility not in terms of what we experience but in theoretical terms. Our considerations will remain within the limits of the philosophy of physics and, in this context, we shall elucidate the concepts of irreversibility and of time-reversal invariance usually invoked in the discussions about the problem. Although many different characterizations of these concepts have been given, the simplest mathematical definitions will allow us to develop our argument.

Definition 1 A dynamical equation (law) is *time-reversal invariant* if it is invariant under the application of the time reversal operator $\mathbf{T}$, which performs the transformation $t \rightarrow -t$ and reverses certain relevant magnitudes.[1] As a result, for each solution (evolution) $e(t)$, $\mathbf{T}e(t)$ is also a solution.[2]

Definition 2 A solution of a dynamical equation (an evolution) is *reversible* if it has no attractors in finite or infinite points of the phase space.

It is quite clear that both concepts are different to the extent that they apply to different mathematical entities: time-reversal invariance is a property of dynamical equations and, *a fortiori*, of the *set* of its solutions; reversibility is a property of a *single* solution of a dynamical equation. Furthermore, they are not even correlated: for instance, time-reversal invariant equations can have irreversible solutions.[3]

This is not the place for considering all the definitions of reversibility and time-reversal invariance proposed in the literature; however, it is not difficult to see that the difference with respect to the entities to which they apply holds in any case. When such a difference is kept in mind, the problem of irreversibility can be easily stated: how to explain irreversible evolutions in terms of time-reversal invariant laws. Let us note that this definition of the problem includes the usual characterization in terms of thermodynamic concepts, since entropy increase is a feature of thermodynamic irreversible evolutions that should be explained by means of the time-reversal invariant laws of mechanics. In turn, our definition is more general than the usual characterization since it also include other interesting cases of irreversibility, like those studied in irreversible quantum mechanics (*cf.* Bohm 1979; Bohm and Gadella 1989).

On the basis of the above definitions, it is quite clear that there is no conceptual puzzle in the problem of irreversibility: there is no obstacle to obtain irreversible

[1] For a detailed discussion of how the time-reversal operator must be defined in each case, *cf.* Earman (2002).

[2] When the dynamical equation can be expressed as $e(t) = \mathbf{U}_t\, e(0)$, where $\mathbf{U}_t$ is a unitary evolution operator such that $\mathbf{U}_{-t} = \mathbf{T}^{-1}\mathbf{U}_t\mathbf{T}$, the equation is time-reversal invariant when $\mathbf{T}e(0) = \mathbf{U}_t\mathbf{T}\mathbf{U}_t\, e(0)$. It can be proved that the time-reversal invariance condition amounts to the fact that the set of the operators $\mathbf{U}_t$ forms a group: $\mathbf{U}_t\mathbf{U}_{-t} = \mathbf{I}$. In fact, $\mathbf{I} = \mathbf{U}_t\mathbf{U}_{-t} \Rightarrow e(0) = \mathbf{T}^{-1}\mathbf{U}_t\mathbf{T}\mathbf{U}_t\, e(0) \Rightarrow \mathbf{T}e(0) = \mathbf{U}_t\mathbf{T}\mathbf{U}_t\, e(0)$.

[3] For instance, let us consider the following autonomous system: $dq/dt = F(q, p), dp/dt = G(q, p)$, such that both equations are time-reversal invariant ($F(q, p) = -F(q, -p)$, $G(q, p) = G(q, -p)$). This system defines an attractor when: (i) there is a fixed point in the semiplane $p > 0$, (ii) the Jacobian matrix of the system computed in the fixed point has positive determinant ($\partial F/\partial q - \partial G/\partial p - \partial F/\partial p \partial G/\partial q > 0$), and (iii) the trace of the Jacobian matrix of the system computed in the fixed point is negative ($\partial F/\partial q - \partial G/\partial p < 0$). In this case, the fixed point is an attractor. When the transformation $t \rightarrow -t$, $p \rightarrow -p$ is applied, the determinant of the Jacobian matrix does not change, but the trace changes its sign and the attractor becomes a repulser.

evolutions from time-reversal invariant laws since time-reversal invariant equations may have irreversible solutions. But although the conceptual answer to the problem is simple, a great deal of theoretical work is needed for effectively obtaining irreversible evolutions from time-reversal invariant dynamics: this was the problem faced by the founding fathers of statistical mechanics when they sought to describe the irreversible evolutions of thermodynamics by means of the time-reversal invariant laws of classical mechanics. Nevertheless, it is worth noting that the question about the arrow of time does not have to be invoked for addressing the problem of irreversibility. In fact, when we talk about entropy-increasing processes, we are presupposing an entropy increase *towards the future*; or when we consider a process going from non-equilibrium to equilibrium, we implicitly locate equilibrium *in the future*. In general, any evolution that tends to an attractor is conceived as approaching the attractor towards the future. This means that the distinction between past and future is usually assumed in the traditional treatments of the problem of irreversibility.[4] But this is not a shortcoming of those treatments, since their aim is to explain irreversibility and not to seek a physical distinction between the two directions of time. On the contrary, such a distinction becomes the central point when the account of the arrow of time is the question at issue; this is precisely the problem in which we shall be interested in the rest of this work.

2.2 The problem of the arrow of time

The problem of the arrow of time owes its origin to the intuitive asymmetry between past and future. We experience the time order of the world as ‘directed’: if two events are not simultaneous, one of them is earlier than the other one. Moreover, we view our access to past and to future quite differently: we remember the past and predict the future. The ultimate metaphysical nature of time has been one of the traditional interests of philosophy since its birth. There seems to be something essentially evasive in our experience of time and its ‘flow’ from past to future through the present. Some authors attempt to ground the intuitive asymmetry between past and future on the increase of entropy in isolated systems. An argument that has been presented in this context concerns to traces: there are many traces of the past but none of the future. The famous ‘footprint in the sand’, discussed by Grünbaum (1963) and Smart (1967), is a typical example (*cf*. the criticisms of Sklar 1993). In other cases, the experience of the ‘directedness’ of time is attempted to be explained in terms of biology: the functioning of the biological mechanisms which underlie our perception of time order and our sense of time depend on the entropic behavior of isolated systems. But there seems to be little or no evidence in support to this claim; Earman (1974) even adds that scientific research suggests that the perception of time order is more subtle and complex than what that position assumes.

Here, we shall not discuss questions related with the time-asymmetry of our experience or perception of time. As in the case of irreversibility, we shall address the

[4] When the distinction between past and future is not presupposed, time-reversal invariant dynamical laws lead to the ‘time-symmetric twins’ problem, which will be discussed in Sect. 7.

problem of the arrow of time within the limits of the philosophy of physics. Nevertheless, within these restricted limits a specific question can still be posed as an interesting matter of reflection. In this context, the problem of the arrow of time arises when we seek a *physical correlate* of the intuitive asymmetry between past and future: do physical theories pick out a preferred direction of time?

The main difficulty to be encountered in answering this question relies on our anthropocentric perspective: the difference between past and future is so deeply rooted in our language and our thoughts that it is very difficult to shake off these asymmetric assumptions. In fact, philosophical discussions around the question are usually subsumed under the label ‘the problem of the direction of time’, as if we could find an exclusively physical criterion for singling out *the* direction of time, identified with what we call ‘the future’. But there is nothing in physical theories that distinguishes, in a non-arbitrary way, between past and future as we conceive them in our ordinary language. It might be objected that physics implicitly assumes this distinction with the use of asymmetric temporal expressions, like ‘future light cone’, ‘initial conditions’, ‘increasing time’, and so on. However, this is not the case, and the reason relies on the difference introduced by the concepts of *conventional* and *substantial*.

Definition 3 Two entities are *formally identical* when there is a symmetry transformation between them that does not change the properties of the system to which they belong or in whose description they are involved.

In physics it is usual to work with formally identical entities: the two semicones of a light cone, the two spin senses, etc.

Definition 4 We establish a *conventional* difference between two formally identical entities when we assign different names to them.

This is the case when we assign different signs to the two spin senses or to the two poles of a magnet.

Definition 5 The difference between two entities is *substantial* when they are not formally identical.

In this case, we assign different names to them in virtue of such a substantial difference (*cf*. Penrose 1979; Sachs 1987). For instance, the difference between the two poles of an ideal magnet is conventional since both poles are formally identical, but the difference between the two poles of the Earth is substantial because in the North Pole there is an ocean and in the South Pole there is a continent, and the difference between ocean and continent remains substantial even if we conventionally change the names of the poles.

When this point is accepted, it becomes clear that physical theories use the labels ‘past’ and ‘future’ in a conventional way. Therefore, the issue cannot yet be posed in terms of singling out the future direction of time: the problem of the arrow of time must be conceived as the problem of finding a substantial asymmetry of time that allows us to distinguish between both temporal directions. But if this is our central problem, we cannot project our independent intuitions about past and future for solving it without begging the question. In other words, when we want to address the problem of the arrow of time from a perspective purged of our temporal intuitions, we must avoid the conclusions derived from subtly presupposing time-asymmetric notions. As Huw Price (1996) claims, it is necessary to 'stand' at a point outside of time, and thence to regard reality in atemporal terms.[5] This atemporal standpoint prevents us from using the asymmetric temporal expressions of our ordinary language in a non-conventional way. When we accept these constraints the problem of the arrow of time becomes the question about the possibility of establishing a *substantial* difference between the two directions of time on the basis of *exclusively physical* arguments, with no appeal to our temporal intuitions.

But, where shall we find such a substantial difference? Of course, if the two directions of time were related *solely* by a time-reversal symmetry transformation, then there would be no substantial difference but only a conventional difference between them. Then, what does 'the arrow of time' mean? It is quite clear that the traditional expression coined by Eddington has only a metaphorical sense: its meaning must be understood by analogy. We recognize the substantial difference between the head and the tail of an arrow on the basis of its geometrical properties; therefore, we can distinguish between both directions, head-to-tail and tail-to-head, independently of our particular perspective. Analogously, we shall conceive the problem of the arrow of time as the problem of establishing a substantial difference between the two directions of time grounded on *the geometrical properties of spacetime*. If such a difference exists, both temporal directions are not related solely by a time-reversal symmetry transformation: there is a substantial geometrical difference between them, which does not rely on our anthropocentric perspective but only on certain theoretical arguments provided by physics. But before considering how and under what conditions the geometrical arrow can be defined, we shall briefly justify our adoption of this global and non-entropic approach to the problem of the arrow of time.

3 A global and non-entropic approach to the problem of the arrow of time

3.1 Why global?

We shall call the attempt to solve the problem of the arrow of time by considering only the local laws of physics '*local approach to the problem of the arrow of time*'.[6] An example of this position is given by Geoffrey Matthews, who claims:

> Much of the recent literature on the problem of the direction of time assumes that if the problem is to be solved at all, it must be solved globally, for the entire universe at once, rather than locally, for some small part or parts of the universe. [...] I believe that this view is false (Matthews 1979, p. 82).

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[5] Our agreement with Price's general proposal of adopting an atemporal viewpoint does not amount to a complete agreement: as the present paper will show, we do not accept the conclusions drawn by Price in the cosmological context to the extent that they are based on an entropic approach to the problem of the arrow of time.

[6] As usual, we are using the expression 'local laws' to denote the laws that can be applied to physical systems confined to sufficiently small regions of the spacetime. By contrast, general relativity is a global theory since it applies to the universe as a whole.

According to the author, boundary conditions and the non-gravitational laws of physics are the basis for the direction of time; therefore, the global structure of the universe is of little or no importance to the problem of the arrow of time.

The traditional local approach owes its origin to the attempts to reduce thermodynamics to statistical mechanics: in this context, the usual answer to the problem of the arrow of time consists in defining the future as the direction of time in which entropy increases. However, already in 1912, Ehrenfest and Ehrenfest (1959) noted that, when entropy is defined in statistical terms on the underlying classical dynamics, if the entropy of a closed system increases towards the future, such increase is matched by a similar one in the past of the system. In other words, if we trace the dynamical evolution of a non-equilibrium system at the initial time back into the past, we shall obtain states that are closer to the equilibrium state. Gibbs' answer to this time-symmetry paradox was based on the assumption that "while probabilities of subsequent events may be often determined from probabilities of prior events, it is rarely the case that probabilities of prior events can be determined from those of subsequent events" (cited in Sklar 1993, p. 58). But this answer clearly violates the 'nowhen' standpoint since the formal theory of probability is blind to temporal direction: probability assignments based on previous events presuppose a sort of causality which, in turn, takes for granted the existence of asymmetric temporal relationships between events. Therefore, the appeal to the distinction between prior and subsequent events commits a *petitio principii* by presupposing the arrow of time from the very beginning.

The point can also be posed in different terms. Let us assume that we have solved the irreversibility problem; so we have the description of all the irreversible evolutions, say, decaying processes, of the universe. However, since we have not yet established a substantial difference between both directions of time, we have no way to decide towards which temporal direction each decay proceeds. Of course, we would obtain the arrow of time if we could coordinate the processes in such a way that all or the majority of them parallelly decay towards the same temporal direction. But this is precisely what local physics cannot offer: only by means of global considerations all the decaying processes can be coordinated. This means that the global arrow of time plays the role of the background scenario where we can meaningfully speak of the temporal direction of all or the majority of the irreversible processes of the universe, and this scenario cannot be built up by means of local theories that only describe phenomena confined in small regions of spacetime.

Once it is accepted that the problem of the arrow of time cannot be addressed from a local perspective, general relativity comes into play. As it is well known, in this theory the only basic properties are the geometrical properties of spacetime, embodied in the metric tensor, and the distribution of matter-energy throughout spacetime, embodied in the energy-momentum tensor, both related by the Einstein's field equations. Nevertheless, the question could be whether the arrow of time refers to a property of time itself or to a property of the arrangement of things in time. But in the context of general relativity both cases are closely related: the field equations express the lawlike connection between the geometrical properties of the spacetime and the distribution of matter-energy throughout that spacetime.

3.2 Why non-entropic?

We shall call the attempt to define the arrow of time in terms of entropy increase *'entropic approach to the arrow of time'*. When, in the late nineteenth century, Boltzmann developed the probabilistic version of his theory in response to the objections raised by Loschmidt and Zermelo (for historical details, *cf*. Brush 1976), he had to face a new challenge: how to explain the highly improbable current state of our world. In order to answer to this question, Boltzmann offered the first global entropic approach to the problem. Since that seminal work, many authors have adopted a global entropic approach to the problem of the arrow of time by defining the temporal direction past-to-future as the direction of the entropy function of the universe (*cf*., for instance, Feynman et al. 1964; Davies 1974, 1994; Layzer 1975). Perhaps the most philosophically influential work in the traditional entropic approach is Reichenbach's (1956) *The Direction of Time*, where the future direction of time is defined as the direction of the entropy increase of the majority of *branch systems*, that is, systems which become isolated from the main system during certain period.[7]

But, why is entropy supposed to be so relevant to the problem of the arrow of time? A given value of entropy is compatible with many configurations of a system: entropy is a phenomenological property. Thus, the question is whether there is a more fundamental property of the universe which allow both temporal directions to be distinguished. In other words, if the arrow of time reflects a substantial difference between both directions of time, it is reasonable to consider it as an intrinsic property of time, or better, of spacetime, and not as a secondary feature depending on a phenomenological property. For this reason we follow Earman's *"Time Direction Heresy"*, according to which the arrow of time, if it exists, is an intrinsic feature of spacetime "which does not need and cannot be reduced to non-temporal features" (Earman 1974, p. 20).

But there are further arguments for abandoning the traditional entropic view. The global entropic approach rests on two assumptions: that it is possible to define entropy for a complete instantaneous cross-section of the universe, and that there is an only time for the universe as a whole. However, both assumptions involve difficulties. In the first place, to confidently transfer the concept of entropy from the field of thermodynamics to cosmology is a controversial move. The definition of entropy in cosmology is still a very problematic issue, even more than in thermodynamics: there is not a consensus among physicists regarding how to define a global entropy for the universe. In fact, it is usual to work only with the entropy associated with matter and radiation because there is not yet a clear idea about how to define the entropy due to the gravitational field. But even leaving aside this problem, if the entropy of the universe is considered as an entropy out of equilibrium, there are different definitions for it (*cf*. Mackey 1989). In the second place, when general relativity comes into play, time becomes a dimension of a four-dimensional structure: it is not yet acceptable to conceive time as a background parameter which, as in pre-relativistic physics, is used to mark the evolution of the system. In fact, in order to recover the parametrical time of non-relativistic theories, on which a meaningful notion of entropy gradient for the

[7] On the basis of a detailed analysis of Reichenbach's argument for parallelism, Sklar (1993) concludes that it merely imposes parallelism on the systems without explaining it.

universe can be defined, the spacetime must satisfy certain *geometrical* properties (we shall return to this point in Subsect. 4.2). Therefore, the problem of the arrow of time cannot be posed, from the beginning, in terms of an entropy gradient computed on the linear and open time of the universe as a whole.

Summing up, the geometrical approach to the problem of the arrow of time has conceptual priority over the entropic approach, since the geometrical properties of the universe are more basic than its thermodynamic properties: the definition of entropy and the calculation of the entropy curve for the whole universe are possible only if the spacetime has certain definite geometrical features.

4 Conditions for a global and non-entropic arrow of time

4.1 Time-orientability

In a Minkowski spacetime, the set of all the light semicones can be split into two exhaustive and disjoint sets: the set of the past light semicones and the set of the future light semicones (where the labels 'future' and 'past' are conventional). In general relativity the metric can always be approximated, in small regions of spacetime, to the Minkowski form. However, on the large scale we do not expect the manifold to be flat because gravity can no longer be neglected. Many different topologies are consistent with Einstein's field equations; in particular, the possibility arises of spacetime being curved along the spatial dimension in such a way that the spacelike sections of the universe become the three-dimensional analogue of a Moebius band; in technical terms it is said that the spacetime is non-time-orientable. In general relativity, a *spacetime* is an ordered pair (M, g), where M is a pseudo-Riemannian manifold, and g is a Lorentzian metric for M such that $\nabla g = 0$.

Definition 6 A spacetime (M, g) is *time-orientable* if there exists a continuous non-vanishing vector field $\gamma^{\mu}(\mathrm{x})$ on M which is everywhere non-spacelike.

By means of this field, the set of all light semicones of the spacetime can be split into two equivalence classes, C_+ and C_-: the semicones of C_+ contain the vectors of the field and the semicones of C_- do not contain them.

Definition 6 implies that, in a non-time-orientable spacetime, it is possible to transform a future pointing timelike vector into a past pointing timelike vector by means of a continuous transport that always keeps timelike vectors timelike (e.g., going around a spacelike 'Moebius band'); therefore, the distinction between future and past light semicones is not univocally definable on a global level. This means that the time-orientability of spacetime is a precondition for defining a global arrow of time, since if spacetime is not time-orientable, it is not possible to distinguish between two temporal directions for the universe as a whole.

Earman (1974) was one of the first authors who emphasized the relevance of time-orientability to the problem of the arrow of time. However, his position was challenged by Matthews (1979), who argued that a spacetime may have a regional but not a global arrow of time if the arrow is defined by means of local considerations: in this sense, Matthews follows the traditional view of Boltzmann and Reichenbach, according to which only certain regions of the universe have temporal directions and these directions may be not the same in all the regions. Despite of its apparent plausibility, this claim can be objected from the viewpoint of contemporary cosmology (for details, *cf*. Castagnino et al. 2003b). Let us suppose that there exists a local non-time-reversal invariant law L, which defines regional arrows of time that disagree when compared by means of continuous timelike transport. The trajectory of the transport will pass through a frontier point between both regions: in a region around this point the arrow of time will be not defined in an univocal way, and this amounts to a breakdown of the validity of L in such a point.[8] But this fact contradicts the methodological principle of universality, unquestioningly accepted in contemporary cosmology, according to which the laws of physics are valid in all points of the spacetime.[9] This means that the possibility of arrows of time pointing to opposite directions in different regions of the spacetime not only demands a local definition of these arrows, but also requires to ignore every global consideration coming from cosmology.

Summing up, in a time-orientable spacetime we can speak of two temporal orientations of the manifold, since both orientations can be defined in a globally consistent way. However, in this case we are not still entitled to speak of two directions of *time*, as if there were a single time for the whole universe. As we shall see in the next sections, additional topological conditions must be added in order to coordinate all the clocks of the universe by a single time whose 'arrow' is to be defined.

The fact that time-orientability is a precondition for defining the arrow of time in a meaningful way supplies an argument against the local entropic approaches which try to define the arrow of time in terms of local entropy gradients. In fact, the above argument against Matthews directly applies to those approaches since, in a non-time-orientable spacetime, the non-time-reversal invariant law of entropy increase would be not valid in some point of the spacetime. When the possibility of non-orientable spacetimes is recognized, it is difficult to deny the conceptual priority of considerations about the geometrical structure of spacetime over entropic considerations in the context of the problem of the arrow of time.

4.2 Global time and cosmic time

As it is well known, general relativity replaces the older conception of space-through-time by the concept of spacetime, where time becomes a dimension of a four-dimensional manifold. However, when the time measured by a physical clock is considered, each particle of the universe has its own *proper time*, that is, the time registered by a clock carried by the particle. Since the curved spacetime of general relativity can

[8] A similar argument can be found in (Earman, 2002, p. 257).

[9] The strategy to escape this conclusion would consist in refusing to assign any meaning to the timelike continuous transport. This strategy would only be acceptable if the two regions with different arrows were physically isolated; this amounts to the disconnectedness of the spacetime. But this fact would contradict the methodological principle of uniqueness, according to which there is only one universe and completely disconnected spacetimes are not allowed. It is worth stressing that the adoption of the principles of uniqueness and universality does not imply conventionalism: they are not adopted by convention but as a consequence of certain very general principles of simplicity and uniformity of nature.

be considered locally flat, it is possible to synchronize the clocks fixed to particles whose parallel trajectories are confined in a small region of spacetime. But, in general, the synchronization of the clocks fixed to all the particles of the universe is not possible. Only in certain particular cases all the clocks can be coordinated by means of a *cosmic time*, which has the features necessary to play the role of the temporal parameter of the evolution of the universe.

The issue can also be posed in geometrical terms. A spacetime may be such that it is not possible to partition the set of all events into equivalence classes—disjoint and exhaustive—such that: (i) each one of the disjoint classes is a spacelike hypersurface, and (ii) the hypersurfaces can be ordered in time. There is a hierarchy of conditions which, applied to a time-orientable spacetime, avoid this kind of 'anomalous' features (*cf*. Hawking and Ellis 1973). Taking the spacetime (M, g) to be time-orientable, and using the words 'future' and 'past' in a conventional way, one can define:

- for $x \in M$, the *chronological future* (*past*) of x, $I^+(x)(I^-(x))$, is the set of all $y \in M$ such that y can be reached from x by a future directed (past directed) timelike curve;
- for $x \in M$, the *causal future* (*past*) of x, $J^+(x)(J^-(x))$, is the set of all $y \in M$ such that y can be reached from x by a future directed (past directed) non-spacelike curve (timelike or lightlike).

The first condition of the hierarchy is the *chronology condition*, which holds on (M, g) iff $\forall x \in M$, $I^+(x) \cap I^-(x) = \varnothing$; this first condition rules out closed timelike curves. The second condition, stronger than the previous one, is the *causality condition*, which holds on (M, g) iff $\forall x \in M$, $J^+(x) \cap J^-(x) = \varnothing$; this condition rules out closed non-spacelike curves. If we also want to exclude cases where there are non-spacelike curves which return arbitrarily close to their point of origin, we must strengthen the requirement imposed on the spacetime: the *strong causality condition* holds on (M, g) iff $\forall x \in M$, every neighborhood of x contains a neighborhood of x which no non-spacelike curve intersects more than once. We may also want to exclude situations with spacelike curves which pass arbitrarily close to another spacelike curve which passes arbitrarily close to the origin of the first curve. As Hawking and Ellis (1973) point out, there is an infinite hierarchy of higher degree causality conditions depending on the number and order of the limiting processes involved. Even if the strong causality condition rules out closed or almost closed non-spacelike curves, it does not exclude the possibility that the slightest variation of the metric of the spacetime leads again to such temporal 'pathologies'. In order to avoid this possibility, the spacetime must have some form of stability, defined as a property of 'nearby' spacetimes. The *stable causality condition* holds on (M, g) iff strong causality condition holds on (M, g), and for every metric h which is sufficiently close to g, (M, h) has not closed timelike curves. It can be proved that the stable causality condition holds on (M, g) iff (M, g) possesses a *global time function*:

Definition 7 A spacetime (M, g) possesses a *global time function* t if there exists a function $t : M \to \mathbf{R}$ whose gradient is everywhere timelike.

The value of the global time function increases along every future directed non-spacelike curve. The existence of such a function guarantees that the spacetime is globally splittable into hypersurfaces of simultaneity which define a foliation of the spacetime (*cf.* Schutz 1980).[10] This means that the existence of a global time function not only avoids temporal 'pathologies', but also excludes the possibility that the slightest variation of the metric leads again to them.

The weaker conditions of the hierarchy permit to define a globally consistent time order, that is, an order relationship between any two points of the manifold. However, such a time order is still not sufficient for meaningfully speaking of the two directions of *time*, as if a single time existed for the whole universe. Given the equivalence between the stable causality condition and the existence of a global time function, if the spacetime is not stably causal, then it is impossible to find a smooth function which attaches to each event a real number, the time of the event, such that the number assigned to e_1 is less than that assigned to e_2 whenever there is a causal signal propagable from e_1 to e_2. This means that only the existence of a global time function guarantees the existence of a single time for the universe as a whole, whose two directions may be conventionally of substantially different. In this sense, the existence of a global time is a precondition for meaningfully speaking of a global arrow of time.

Earman (1974) points out the relevance of the existence of a global time function to the problem of the arrow of time; however, he does not take into account that such a function does not yet permit to define a notion of simultaneity in an univocal manner and with physical meaning. In order to avoid ambiguities in the notion of simultaneity, we must choose a particular foliation of the spacetime. The foliation τ such that there exists a continuous set of worldline curves which are orthogonal to all the hypersurfaces $\tau = \text{const.}$ is the proper choice, because orthogonality recovers the notion of simultaneity of special relativity for small regions—tangent hyperplanes—of the hypersurfaces $\tau = \text{const.}$ (for the necessary formal conditions, *cf.* Misner et al. 1973). However, even if this condition selects a particular foliation, it permits the proper time interval between two hypersurfaces of simultaneity to be relative to the particular worldline considered for computing it. If we want to avoid this situation, we must impose an additional constraint: the proper time interval between two hypersurfaces $\tau = \tau_1$ and $\tau = \tau_2$ must be the same when measured on any orthogonal worldline curve of the continuous set (for details, *cf.* Castagnino et al. 2003b). In this case, the metric results:

$$ds^2 = dt^2 - h_{\mathrm{ij}} dx^{\mathrm{i}} dx^{\mathrm{j}} \tag{4.1}$$

Definition 8 When the metric of the spacetime can be expressed as $ds^2 = dt^2 - h_{\mathrm{ij}} dx^{\mathrm{i}} dx^{\mathrm{j}}$, t is the *cosmic time* and $h_{\mathrm{ij}} = h_{\mathrm{ij}}\left(t, x^1, x^2, x^3\right)$, with $i, j = 1, 2, 3$, is the three-dimensional metric of each hypersurface of simultaneity.

[10] By a *foliation* F of the Riemannian n-manifold M of codimension $q (1 \leq q < n)$ we mean a partition of M into submanifolds $(n - q)$-dimensional called '*leaves*'. A foliation of spacetime into hypersurfaces is therefore a foliation of codimension 1. There are other definitions of foliation which require the path-connectedness of the *leaves* (*cf.* Torretti 1983).

Although the global arrow of time can be defined on a spacetime having a global time, only the existence of a cosmic time guarantees that all the processes of the universe can be coordinated by a single time which can be considered as the parameter of the evolution of the universe as a whole. In other words, the cosmic time is the time in which the history of the universe can be conceived as the temporal sequence of its instantaneous states univocally defined.

These considerations supply a strong argument against the global entropic approaches to the problem of the arrow of time. By defining the time direction past-to-future in terms of the gradient of the entropy function of the universe (*cf.*, for instance, Price 1996), those approaches take for granted the possibility of defining such a function. But this amounts to the assumption that: (i) the spacetime can be partitioned in spacelike hypersurfaces on which the entropy of the universe can be defined, and (ii) the spacetime possesses a global time on which the entropy gradient can be computed. Once again, the conceptual priority of the geometrical properties of the spacetime over its entropic properties becomes clear in the light of the possibility of spacetimes with no global time.

5 Time-orientation

5.1 Time-asymmetry

As Grünbaum (1963) correctly points out, time-orientability is merely a necessary condition for defining an arrow of time since it assumes only the globally consistent mere oppositeness of the two directions of time. In fact, time-orientability does not provide a substantial distinction between both temporal directions. Such a distinction could be nomologically established by means of a fundamental non-time-reversal invariant law which, at a point x of the spacetime, introduced the substantial difference between the two light semicones $C_+(x)$ and $C_-(x)$ at that point; in turn, in a time-orientable spacetime, this difference could be transferred by continuous timelike transport to the whole manifold. As it is well known, the CPT theorem establishes that the only general symmetry of fundamental physics is the one resulting from combining charge-conjugation, space-reflection and time-reversal; then, there is no theoretical reason to expect that time-reversal invariance will hold universally. In particular, the laws of certain elementary processes involving the K-meson are non-time-reversal invariant. These theoretical facts would provide the desired substantial difference between the two directions of time. However, this argument can be questioned by arguing that the effects of the elementary processes described by non-time-reversal invariant laws are so extraordinarily weak that they cannot be responsible for the arrow of time as manifested at the macroscopic level; therefore, the problem of defining the arrow of time has not yet been solved.

This kind of discussions relies on the assumption that the arrow of time inescapably depends on the existence of non-time-reversal invariant laws; then, the fact that almost all the fundamental laws of physics are time-reversal invariant is considered as a theoretical obstacle to define such an arrow. Our aim in this section is to reject this traditional opinion by arguing that, even if all the laws of physics were time-reversal invariant, the global arrow of time could be defined on the basis of the time-asymmetry of spacetime.

Let us begin by elucidating the general concept of time-symmetry.

Definition 9 A solution $e(t)$ of a dynamical equation is *time-symmetric* if there is a time t_S such that $e(t + t_S) = e(t - t_S)$.

As we have seen, time-reversal invariance is a property of dynamical equations (laws). On the contrary, time-symmetry is a property of a single solution of a dynamical equation (an evolution). When this point is understood, the apparent obstacle to the definition of the arrow of time begins to vanish, since the time-reversal invariance of an equation does not imply the time-symmetry of its solutions: a time-reversal invariant law may be such that all or most of the possible evolutions relative to it are individually time-asymmetric. Huw Price (1996) illustrates this point with the familiar analogy of a factory which produces equal numbers of left-handed and right-handed corkscrews: the production as a whole is completely unbiased, but each individual corkscrew is asymmetric.

It is quite clear that these considerations are not applicable to the field equations as originally stated. Nevertheless, in a spacetime with cosmic time, the issue can be expressed in familiar terms: under this condition, Einstein's field equations are time-reversal invariant in the sense that if $h_{ij} = h_{ij}(t, x^1, x^2, x^3)$ is a solution, $h_{ij} = h_{ij}(-t, x^1, x^2, x^3)$ is also a solution. But the time-reversal invariance of these equations does not prevent us from describing a time-asymmetric universe whose spacetime is asymmetric with respect to its geometrical properties along the cosmic time. This idea can also be formulated in terms of the concept of time-isotropy.

Definition 10 A time-orientable spacetime (M, g) is *time-isotropic* if there is a diffeomorphism d of M onto itself which reverses the temporal orientations but preserves the metric g.

However, when we want to express the time-symmetry of a spacetime having cosmic time, it is necessary to strengthen the definition.

Definition 11 A time-orientable spacetime (M, g) having a cosmic time t is *time-symmetric* with respect to some spacelike hypersurface $t = t_S$, where t_S is a constant, if it is time-isotropic and the diffeomorphism d leaves fixed the hypersurface $t = t_S$.

Intuitively this means that, from the hypersurface $t = t_S$, the spacetime looks the same in both temporal directions. Therefore, if a time-orientable spacetime having cosmic time is time-asymmetric, we shall not find a spacelike hypersurface $t = t_S$ which splits the spacetime in two 'halves', one the temporal mirror image of the other one with respect to their intrinsic geometrical properties.

But, how does this time-asymmetry allow us to choose a time-orientation of the spacetime? First, it is necessary to clarify what we shall understand by time-orientation.

Definition 12 If a spacetime (M, g) is time-orientable, a continuous non-vanishing non-spacelike vector field $\gamma^{\mu}(x)$ can be defined all over the manifold. The choice of one of the vector fields $\gamma^{\mu}(x)$ and $-\gamma^{\mu}(x)$ as containing the future pointing non-spacelike vectors is the choice of a *time-orientation* for the spacetime.

Of course, the choice of a time-orientation for a time-orientable spacetime does not solve the problem of the arrow of time. If the distinction between $\gamma^{\mu}(x)$ and $-\gamma^{\mu}(x)$ is just conventional, the mere choice does not express a substantial difference between the two directions of time. But when time-asymmetry comes into play, time-orientation can be established on a substantial basis. In fact, in a generic time-asymmetric spacetime, any time $t = t_A$ splits the manifold into two sections that are different to each other: the section $t > t_A$ is *substantially* different than the section $t < t_A$ with respect to their geometrical properties.[11] This means that the temporal direction $t < t_A$-to-$t > t_A$ is *substantially* different than the temporal direction $t > t_A$-to-$t < t_A$, analogously to the substantial, geometrically grounded difference between the direction head-to-tail and de direction tail-to-head in an arrow.

Once the substantial difference between the two temporal direction has been established, we can chose any point x_0 with $t = t_A$ and conventionally consider that $-\gamma^{\mu}(x_0)$ points towards $t < t_A$ and $\gamma^{\mu}(x_0)$ points towards $t > t_A$ or vice versa: in any case we have established a substantial difference between $-\gamma^{\mu}(x_0)$ and $\gamma^{\mu}(x_0)$ and, therefore, between the direction of $-\gamma^{\mu}(x_0)$ and the direction of $\gamma^{\mu}(x_0)$. We can conventionally call the direction of $\gamma^{\mu}(x_0)$ 'future' and the direction of $-\gamma^{\mu}(x_0)$ 'past' or vice versa, but in any case past is substantially different than future. Now we can extend this difference to the whole continuous fields $\gamma^{\mu}(x)$ and $-\gamma^{\mu}(x)$: in this way, the time-orientation of the spacetime has been established. Since the field $\gamma^{\mu}(x)$ is defined all over the manifold, it can be used locally at each point x to define the future and the past light semicones: for instance, if we have called the direction of $\gamma^{\mu}(x)$ 'future', the semicone $C_{+}(x)$ contains $\gamma^{\mu}(x)$ and the semicone $C_{-}(x)$ contains $-\gamma^{\mu}(x)$.

5.2 The arrow of time as energy flow

As we have seen, in a time-orientable spacetime having a cosmic time, time-asymmetry establishes a substantial difference between the two temporal directions represented by the continuous non-vanishing non-spacelike vector fields $\gamma^{\mu}(x)$ and $-\gamma^{\mu}(x)$. Then, we can define a substantially founded time-orientation on this spacetime by deciding that, for instance, $\gamma^{\mu}(x)$ points to the future: in this case, any vector of the vector field $\gamma^{\mu}(x)$ represents the arrow of time. However, up to this point $\gamma^{\mu}(x)$ was characterized exclusively in mathematical terms as the vector field that must exist for the time-orientability of spacetime. Now the question is whether the arrow of time can be defined in a physical way, that is, by means of some mathematical object that can also be interpreted in terms of the more 'familiar' magnitudes of physics.

As it is well known, the energy-momentum tensor $T_{\mu v}$ represents the density and the flow of energy and momentum at each point of the spacetime. Then, it would be desirable to define the vector field $\gamma^{\mu}(x)$ in terms of $T_{\mu v}$ in order to endow it with a physical meaning. Although this task cannot be accomplished in a completely

[11] A simple argument proves the vanishing probability of perfect time-symmetry in the case of FLRW (Friedmann-Lemâitre-Robertson-Walker) models, by demonstrating that time-symmetric solutions of the universe equations have measure zero in the corresponding phase space (*cf.* Castagnino et al. 2003a).

general case, it is possible to define the arrow of time in terms of $T_{\mu\nu}$ if two conditions are satisfied: (i) the density of matter-energy is non-zero everywhere, and (ii) the energy-momentum tensor satisfies the *dominant energy condition* everywhere. The first condition means that there is perfect vacuum in no region of the universe; this seems to be a physically realistic situation since in any region of the universe there exists, at least, blackbody cosmic radiation. The second condition is based on the following definition (*cf*. Hawking and Ellis 1973; Visser 1995).

Definition 13 The energy-momentum tensor satisfies the *dominant energy condition* if, in any orthonormal basis, the energy component dominates the other components of $T_{\alpha\beta}$:

$$T^{00} \geq \left|T^{\alpha\beta}\right| \quad \text{for each } \alpha, \beta$$

This means that to any observer the local matter-energy density appears non-negative and the energy flow is non-spacelike. The requirement that the energy-momentum tensor satisfies the *dominant energy condition* everywhere does not impose a very strong constraint; according to Hawking and Ellis (1973, p. 91), such a condition "holds for all known forms of matter and there is in fact good reason for believing that this should be the case in all situations." There are, of course, strange cosmological 'objects' whose existence would lead to universes where the dominant energy condition is not satisfied in certain regions of the spacetime. For instance, in wormhole spacetimes, the dominant energy condition is violated in the vicinity of the wormhole throat since the wormhole is threaded by negative 'exotic' matter (*cf*. Visser 1995). Nevertheless, it is plausible to suppose that universes containing such kind of objects will surely not satisfy the stronger conditions necessary for defining the arrow of time, that is, time-orientability and existence of cosmic time.

Let us consider a continuous orthonormal basis field $\{\boldsymbol{V}_{(\alpha)}(x)\}$ such that, at each point of the manifold, $\boldsymbol{V}_{(\alpha)}$ is a tetrad (or '*vierbein*' in Einstein's terminology) consisting of four unitary vectors $\boldsymbol{V}_{(\alpha)} = (V^0_{(\alpha)}, V^1_{(\alpha)}, V^2_{(\alpha)}, V^3_{(\alpha)})$ (with $\alpha = 0, 1, 2, 3$). In the basis $\{\boldsymbol{V}_{(\alpha)}\}$, $g_{\mu\nu} V^{\mu}_{(\alpha)} V^{\nu}_{(\beta)} = \eta_{\alpha\beta}$ are the coordinates of the local Minkowski metric tensor and $T_{\mu\nu} V^{\mu}_{(\alpha)} V^{\nu}_{(\beta)} = T_{\alpha\beta}$ are the coordinates of the energy-momentum tensor. Then, $T^{0\alpha} V_{(\alpha)}$ could be conceived as a vector representing the energy flow, whose coordinates in that basis are the $T^{0\alpha}$. Now, if the dominant energy condition holds ($T^{00} \geq |T^{\alpha\beta}|$), then $T^{00} \geq |T^{0\alpha}|$. In turn, $T^{00} \geq |T^{0\alpha}|$ implies that $T^{0\alpha} \boldsymbol{V}_{(\alpha)}$ is non-spacelike. On the other hand, if the matter-energy density is non-zero, then $T^{00} \neq 0$ and, as a consequence, $T^{0\alpha} \neq 0$. Moreover, if the manifold is continuous, $g_{\mu\nu}$ is continuously defined over it and, provided that the derivatives of $g_{\mu\nu}$ are also continuous, $T^{\mu\nu}(T^{\alpha\beta})$ and, then, $T^{0\mu}\left(T^{0\alpha}\right)$ are also continuously defined all over the manifold. Therefore, it seems that, under conditions (i) and (ii), we have found a physical correlate of the continuous non-vanishing non-spacelike vector field $\gamma^{\mu}(x)$ since the coordinates of $T^{0\alpha}(x) \boldsymbol{V}_{(\alpha)}(x)$ in the basis μ, ν can be considered as the components of $\gamma^{\mu}(x) : T^{0\alpha}(x) V^{\mu}_{(\alpha)}(x) = \gamma^{\mu}(x)$. The trouble with this conclusion is that $T^{0\alpha} \boldsymbol{V}(\alpha)$ is not strictly a vector. In fact, even though $T^{0\alpha} \boldsymbol{V}(\alpha)$ is usually referred to as the "energy flow vector" (e.g., Hawking and Ellis 1973), it is not transformed as a vector by the Lorentz transformations. Strictly speaking, at each point x of the spacetime, $T^{0\alpha}(x)\,\boldsymbol{V}_{(\alpha)}(x)$ is a tetra-magnitude which represents the energy flow at *x only in the basis* $\left\{\boldsymbol{V}_{(\alpha)}(x)\right\}$; thus, it cannot directly play the role of $\gamma^{\mu}(x)$ as it was initially desired.

Nevertheless, the fact that the energy flow cannot be represented by a vector is not an obstacle to define the arrow of time in terms of such a flow. The dominant energy condition poses a *covariant condition*: if the energy flow is non-spacelike in a reference frame, it is non-spacelike in all reference frames. This means that, no matter which orthonormal basis $\{\boldsymbol{V}_{(\alpha)}\}$ is chosen, the energy flow in that basis, represented by $T^{0\alpha}\boldsymbol{V}_{(\alpha)}$, can be used to define the arrow of time. In fact, we can conventionally decide that, at each point of the spacetime, the future light semicone $C_{+}(x)$ is the one which contains the energy flow or to which the energy flow belongs whatever basis was chosen: in this way, the time-orientation is defined for the whole spacetime.

Of course, the fact that the time-orientation of a spacetime can be defined by means of the energy flow does not imply that time-asymmetry becomes superfluous. Given a spacetime with its own geometry, something analogous to the case of the pair $\gamma^{\mu}(x)$ and $-\gamma^{\mu}(x)$ happens with the energy flow: in any orthonormal basis $\{\boldsymbol{V}_{(\alpha)}\}$, for each positive $T^{0\alpha}\boldsymbol{V}_{(\alpha)}$ there is a negative $-T^{0\alpha}\boldsymbol{V}_{(\alpha)}$ that can also adequately represent the energy flow, since the difference between them is, a priori, just conventional. It is the time-asymmetry of the spacetime what establishes the substantial difference between both pseudo-vectors. Once again, we can conventionally call the temporal direction of $T^{0\alpha}\boldsymbol{V}_{(\alpha)}$ 'future' and the temporal direction of $-T^{0\alpha}\boldsymbol{V}_{(\alpha)}$ 'past' or vice versa, but in any case past is substantially different than future. The usual convention in physics consists in calling the temporal direction of the positive energy flow 'future'. This means that, no matter which orthonormal basis we use, at any point x of the spacetime $T^{0\alpha}(x)\boldsymbol{V}_{(\alpha)}(x)$ is contained in or belongs to the future light semicone $C_{+}(x)$: energy temporally flows towards the future for any observer.[12] But the relevant point consist in noting that this sentence only acquires a non-conventional meaning when the substantial difference between past and future has been previously established.

At this point it is worth stressing again the difference between conventional and substantial. The particular names assigned to two entities are conventional, but the fact that we use different names is substantially grounded when the difference between the two entities is substantial. In our case, the use of the words 'past' and 'future' to label the two directions of time is conventional: we follow the usual convention in physics, which consists in calling the temporal direction of the positive energy flow 'future'. But the fact that we assign different names to those directions is not conventional but the result of the substantial difference between the two temporal directions in a time-asymmetric spacetime.

[12] In fact, $T^{0\alpha}\boldsymbol{V}_{(\alpha)}$ can be considered as representing the energy flow originated at the Big Bang in Big Bang FLRW cosmologies.

5.3 Arrow of time and symmetry transformations

In the previous subsection it has been shown that, under the dominant energy condition, the energy flow is always non-spacelike and, therefore, its direction can represent the arrow of time when the spacetime is time-asymmetric. However, we know that, under the conditions of time-orientability and existence of cosmic time, given the time-reversal invariance of Einstein's field equations, if $h_{\rm ij} = h_{\rm ij}(t, x^1, x^2, x^3)$ is a solution, $h_{\rm ij} = h_{\rm ij}(-t, x^1, x^2, x^3)$ is also a solution: the first case corresponds to $T^{0\alpha}$ and the second case corresponds to $-T^{0\alpha}$. In other words, we obtain two solutions, each one of them is the temporal mirror image of the other and which are both possible with respect to the laws of general relativity. At this point, the ghost of symmetry threatens again: it seems that we are committed to supplying a non-conventional criterion for picking out one of both nomologically admissible solutions. However, as we shall see, the threat is not as serious as it seems.

When we accept the need of choosing one between both solutions, we are assuming that each one describes a different possible universe: there are two possible objects, and we must decide which one of them corresponds to our actual universe. The question is: why the two possible universes are different? The obvious answer is: 'they are different because they are arranged in time in opposed directions'. This statement presupposes that there is a single time, common to both possible universes, with respect to which we can meaningfully say that one is temporally opposed to the other one. But this assumption is completely contrary to the standard interpretation of general relativity, according to which time—or better, spacetime—coexists with the universe: each universe has its own spacetime, and there is no *temporal* viewpoint, external to both possible universes, from where they can be compared. Therefore, the two solutions $h_{\rm ij} = h_{\rm ij}(t, x^1, x^2, x^3)$ and $h_{\rm ij}(-t, x^1, x^2, x^3)$ obtained by means of the field equations are different but *equivalent descriptions of one and the same possible universe*.[13]

Notwithstanding its plausibility, this equivalence thesis has been challenged by Earman (1974) on the basis of the difference between continuous and discrete symmetry transformations: spatial translation, spatial rotation, and temporal translation are continuous transformations; on the contrary, spatial reflection and temporal reversal are discrete transformations. Under the active interpretation, a transformation corresponds to a change from one system to another; under the passive interpretation, a transformation consists in a change of the point of view from which the system is described. According to Earman, the defence of the equivalence thesis should be based on the passive interpretation of time-reversal: the two solutions of the field equations would describe the same universe from two different points of view. But the usual position about symmetries assumes that, in the case of discrete transformations, only the active interpretation makes sense: "The passive interpretation of continuous symmetries, like spatial rotation, is meaningful since one can suppose at least in principle that an idealized observer can rotate himself in space in correspondence with the given spatial rotation [. . .] But how is an observer, even an idealized one, supposed

[13] This position has been adopted by Reichenbach (1956), Gold (1966), and Price (1996) among others, but from an entropic perspective.

to ‘rotate himself in time’?” (Earman 1974, pp. 26–27); the same position is adopted by Sklar (1974). Therefore, according to these authors, the assumption that the two symmetrical solutions of the field equations are equivalent descriptions of the same universe is misguided, to the extent that it relies on the passive interpretation of a discrete symmetry transformation like time-reversal.

Of course, the usual position about symmetries is adequate when the idealized observer is immersed in the same spacetime as the observed system: the observer can rotate himself in space but cannot rotate himself in time. But, when the system is the universe as a whole, how is the observer supposed to rotate himself in space in order to describe the system from a different spatial perspective? Since there is not space outside the spacetime, we cannot change our spatial position with respect to the universe: it is as impossible to rotate in space as to rotate in time. But this does not imply that the active interpretation is the correct one: what is the conceptual meaning of the idea of two identical universes, one translated in space or in time with respect to the other one? These observations point to the fact that both interpretations of symmetry transformations, when applied to the universe as a whole, collapse into conceptual nonsense. In general relativity, symmetry transformations are neither interpreted in terms of a change from one system to another one, nor in terms of a change of the observer’s point of view.

Definition 14 Two mathematical models for the universe, defined by (M, g) and (M', g'), are taken to be equivalent if they are *isometric*, that is, if there is a diffeomorphism $\theta : M \to M'$ which carries the metric g into the metric g' (Hawking and Ellis 1973).

In particular, symmetry transformations are *isometries*: models related by a symmetry transformation are isometric. Therefore, two models U and $\mathbf{S}(U)$, where $\mathbf{S}$ is any symmetry transformation, are considered as equivalent descriptions of one and the same universe.

Summing up, when we obtain two solutions of the field equations, one the temporal mirror image of the other, we are not committed to choose one of them to the extent that both describe a single possible universe. Therefore, the time-reversal invariance of Einstein’s equations is not an obstacle to consider the energy flow as representing the arrow of time.

6 From the global arrow to the local arrow

As we have seen, once it was decided that the vectors of the vector field $\gamma^{\mu}(x)$ point to the future, this field can be used locally to define the future and the past light semicones at each point of the spacetime. This argument can also be applied to the case of the energy flow. Let us suppose that, as usual in local physics, we call the temporal direction of the positive energy flow ‘future’. Since, in any orthonormal basis field $\{\boldsymbol{V}_{(\alpha)}(x)\}$, $T^{0\alpha}(x)$ is defined all over the spacetime, it transfers the global arrow into each local context: at any point x of the spacetime, the future light semicone $C_{+}(x)$ will be the semicone which contains $T^{0\alpha}(x)\boldsymbol{V}_{(\alpha)}(x)$ or to which $T^{0\alpha}(x)\boldsymbol{V}_{(\alpha)}(x)$ belongs.

In other words, the future light semicone at each point is defined by the positive energy flow at this point.

But, does $T^{0\alpha}$ really represent the energy flow as conceived by *local* physics? Let us remember that $T_{\mu\nu}$ satisfies the 'conservation' equation:

$$\nabla_\mu T_{\mu\nu} = 0 \tag{6.1}$$

However, this is not a true conservation equation since ∇_μ is a covariant derivative. The usual conservation equation with ordinary derivatives reads:

$$\partial_\mu \tau_{\mu\nu} = 0 \tag{6.2}$$

where $\tau_{\mu\nu}$, which is not a tensor, is defined as (*cf.* Misner et al. 1973):

$$\tau_{\mu\nu} = \sqrt{-g}\left(T_{\mu\nu} + t_{\mu\nu}\right) \tag{6.3}$$

The correction factor $t_{\mu\nu}$ is an homogeneous and quadratic function of the connection $\Gamma^\lambda_{\mu\nu}$:

$$\begin{aligned} t^{\mu\nu} = \mathrm{c}^4/(16\pi k) \Big\{ & \left(2\,\Gamma^\chi_{\alpha\beta}\Gamma^\delta_{\chi\delta} - \Gamma^\chi_{\alpha\delta}\Gamma^\delta_{\beta\chi} - \Gamma^\chi_{\alpha\chi}\Gamma^\delta_{\beta\delta}\right)\left(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\nu}\,g^{\alpha\beta}\right) \\ & + g^{\mu\alpha}g^{\beta\chi}\left(\Gamma^\nu_{\alpha\delta}\Gamma^\delta_{\beta\chi} + \Gamma^\nu_{\beta\chi}\Gamma^\delta_{\alpha\delta} - \Gamma^\nu_{\chi\delta}\Gamma^\delta_{\alpha\beta} - \Gamma^\nu_{\alpha\beta}\Gamma^\delta_{\chi\delta}\right) \\ & + g^{\nu\alpha}g^{\beta\chi}\left(\Gamma^\mu_{\alpha\delta}\Gamma^\delta_{\beta\chi} + \Gamma^\mu_{\beta\chi}\Gamma^\delta_{\alpha\delta} - \Gamma^\mu_{\chi\delta}\Gamma^\delta_{\alpha\beta} - \Gamma^\mu_{\alpha\beta}\Gamma^\delta_{\chi\delta}\right) \\ & + g^{\alpha\beta}g^{\chi\delta}\left(\Gamma^\mu_{\alpha\chi}\Gamma^\nu_{\beta\delta} - \Gamma^\mu_{\alpha\beta}\Gamma^\nu_{\chi\delta}\right)\Big\} \end{aligned} \tag{6.4}$$

Now we can consider the coordinates $\tau^{0\mu}$, which satisfy the usual conservation equation:

$$\partial_\mu \tau^{0\mu} = \partial_0 \tau^{00} + \partial_{\mathrm{i}} \tau^{0\mathrm{i}} = 0 \tag{6.5}$$

where τ^{00} represents the energy density and the $\tau^{0\mathrm{i}}$ are the three components of the spatial energy flow, that is, the Poynting vector.

Analogously to the case of the energy-momentum tensor, in any orthonormal basis $\{\boldsymbol{V}_{(\alpha)}\}$, $\tau^{0\alpha}\boldsymbol{V}_{(\alpha)}$ is not a vector since it is not transformed as a vector by the Lorentz transformations. But in contrast to that case, the dominant energy condition cannot be expressed in terms of $\tau^{\mu\nu}$; this means that we cannot guarantee that $\tau^{0\alpha}\boldsymbol{V}_{(\alpha)}$ is non-spacelike in any orthonormal basis. However, in the particular case of *local inertial frames* $\tau^{0\alpha}$ acquires a relevant feature. In fact, in a local inertial frame $\Gamma^\lambda_{\mu\nu} = 0$. Thus, in this case $t_{\mu\nu} = 0$ (Eq. 6.4) and $\tau_{\mu\nu} = \sqrt{-g}T_{\mu\nu}$ (Eq. 6.5); therefore, the dominant energy condition implies that:

$$\tau^{00} \geq \left|\tau^{\alpha\beta}\right| \quad \text{for each } \alpha, \beta$$

As a consequence, if $\{\boldsymbol{W}_{(\alpha)}\}$ is the basis of the local inertial frame, where $\boldsymbol{W}_{(\alpha)} = (\boldsymbol{W}^0_{(\alpha)}, \boldsymbol{W}^1_{(\alpha)}, \boldsymbol{W}^2_{(\alpha)}, \boldsymbol{W}^3_{(\alpha)})$, then the energy flow represented by $\tau^{0\alpha}\boldsymbol{W}_{(\alpha)}$ and satisfying the usual conservation equation is non-spacelike.

Although this result cannot be generalized to all the reference frames of the whole spacetime, it is relevant in local contexts since, in small regions of the spacetime, the metric tends to the Minkowski form. In fact, any local region of the spacetime can be approximated by the tangent Minkowski space at any point of that region. If $\{\boldsymbol{W}_{(\alpha)}\}$ is the basis of an inertial frame on this flat tangent space, $\tau^{0\alpha}\boldsymbol{W}_{(\alpha)}$ represents the non-spacelike *local energy flow* satisfying the usual conservation equation. Moreover, at each point x of the local region, the local energy flow denoted by $\tau^{0\alpha}(x)\,\boldsymbol{W}_{(\alpha)}(x)$ is contained in or belongs to the same light semicone as the one which contains the global energy flow $T^{0\alpha}(x)\,\boldsymbol{V}_{(\alpha)}(x)$ or to which it belongs. Therefore, if we have adopted the usual physical convention in the global level, we can also meaningfully say that future is the temporal direction of the positive *local* energy flow: the local flow of energy emitted at x points to the future, that is, it is contained in or belongs to the light semicone $C_+(x)$.

This result is particularly relevant because local inertial frames are the reference frames in which the non-relativistic local theories of physics, like non-relativistic quantum mechanics, are valid. In turn, quantum field theory, although relativistic, is also formulated in a local inertial frame. This means that the local energy flow directed towards the future is the flow of energy as conceived in this kind of theories, where energy satisfies the usual conservation law expressed by means of ordinary derivatives. Summing up, $\tau^{0\alpha}$ inherits the time-orientation defined at the global level and, to the extent that it has a local physical meaning, it not only transfers the global arrow of time to local contexts, but also translates the global arrow into a usual magnitude of local physical theories.

7 Time-symmetric twins: breaking the symmetry

As we have seen in Subsect. 3.1, the Ehrenfests objected Gibbs' approach by pointing out that, if entropy is defined in statistical terms on the basis of the underlying time-reversal invariant classical mechanics, the increase of entropy towards the future is always matched by a similar increase in the past of the system. When considered in perspective, this old discussion turns out to be a particular case of a general problem arising from any local time-reversal invariant law. In fact, if $e(t)$ is a solution of a time-reversal invariant equation, $\mathbf{T}e(t)$ is also a solution. In general, any time-reversal invariant law gives rise to what we shall call '*time-symmetric twins*', that is, two mathematical structures symmetrically related by a time-reversal transformation: each twin is the temporal mirror image of the other. The traditional example of time-symmetric twins is given by electromagnetism, where dynamical equations always have advanced and retarded solutions, respectively, related with incoming and outgoing states in scattering situations as described by Lax–Phillips scattering theory (Lax and Phillips 1979). Another example is the mathematical structure resulting from irreversible quantum mechanics, a field largely studied in the last decades (*cf.*, for instance, Bohm 1979; Bohm and Gadella 1989): in this case, the analytical extension of the energy spectrum of the quantum system's Hamiltonian into the complex plane leads to poles in the lower half plane—usually related with decaying unstable states—and symmetric poles in the upper half plane—usually related with growing unstable states—(*cf*. Castagnino and Laura 1997).[14]

From the viewpoint of the local theory considered in each particular case, the difference between the time-symmetric twins is only conventional: both twins are nomologically possible with respect to the theory. The traditional arguments for discarding one of the twins and retaining the other usually invoke time-asymmetric notions that are not justified in the context of the local theory. In the field of electromagnetism, many discussions focus on the reasons for discarding advanced solutions, but usually retarded and advanced solutions are interpreted as waves propagating in the future direction; the difference between them is that whereas retarded waves spread outwards from a center, advanced waves evolve inwards toward a center. For instance, in a famous letter to *Nature*, Popper (1956) criticizes the commonly accepted view according to which thermodynamics provides the only significant time-asymmetry in physics: the arrow of radiation should be considered as fundamental as the entropic arrow. But when the problem is to explain the retarded nature of radiation, Popper appeals to *de facto* arguments that presuppose the application of initial conditions in the past: advanced solutions of wave equations correspond to converging waves that require a 'miraculous' cooperative emitting behavior of distant regions of space at the temporal origin of the process.

In the field of irreversible quantum mechanics, Arno Bohm analyzes scattering experiments on the basis of the *preparation-registration* arrow of time, expressed by the slogan 'no registration before preparation'. The key intuition behind this idea is that an observable property of a state cannot be measured until the state acting as the bearer of properties has been prepared (Bohm et al. 1994). As a consequence, some mathematical operations definable in Hilbert spaces became nonsensical; for instance, the expectation value of an observable before the start of detection has no physical meaning. This strategy allows Bohm to define two disjoint spaces: the space of states Φ_+ and the space of observables Φ_-. When the unitary quantum evolution operator $\mathbf{U}_t$, defined on the complete Hilbert space of the system, is restricted to Φ_+ or Φ_-, it becomes a semigroup that leads to non-time-reversal invariant evolution equations.

It seems quite clear that these arguments, even if admissible in the discussions about irreversibility, are not legitimate in the context of the problem of the arrow of time to the extent that they put the arrow 'by hand' by presupposing the difference between the two directions of time from the beginning. In other words, they violate the 'nowhen' requirement of adopting an atemporal perspective purged of temporal intuitions like, for instance, those related with the asymmetry between past and future or between preparation and measurement. Therefore, from an atemporal standpoint,

[14] In order to transform the poles in Gamov vectors it is necessary to introduce two subspaces Φ_+ and Φ_- of the Hilbert space. The states $|\varphi\rangle$ of $\Phi_+(\Phi_-)$ are characterized by the fact that their projections $\langle\omega|\varphi\rangle$ on the energy eigenstates $|\omega\rangle$ are functions of the Hardy class from above (below). It can be proven that incoming quantum states belong to the Hardy class from above and outgoing quantum states belong to the Hardy class from below (Castagnino et al. 2002). Subspaces $\Phi+$ and Φ_- are another example of time-symmetric twins since $K\Phi_{+(-)} = \Phi_{-(+)}$, where K is the Wigner antilinear time-inversion operator: in the context of the theory, the difference between Φ_+ and Φ_- is just conventional.

the challenge consists in supplying a non-conventional criterion, based on theoretical arguments, for choosing one of the time-symmetric twins as the physically meaningful one: such a criterion will establish a substantial difference between the two members of the pair.

The desired criterion, that can be legitimately supplied neither by the local theory nor by our pretheoretical intuitions, is given by the local energy flow expressed in terms of $\tau^{0\alpha}$. In fact, once we have conventionally decided to call the direction of the positive energy flow 'future', at each spatio-temporal point x of a small region of the spacetime the local energy flow defines the future light semicone: the energy emitted at x must be contained in or must belong to $C_+(x)$. Therefore, in any case the twin corresponding to this kind of energy flow is the member of the pair that must be retained as physically meaningful. For instance, in electromagnetism only retarded solutions fulfill this condition since they describe waves which cannot propagate outside of the future light semicone. In irreversible quantum mechanics, only poles in the lower half-plane are retained to the extent that they represent decaying unstable states providing a flow of energy contained in the future light semicone.

It is interesting to note that the need of breaking the symmetry between time-symmetric twins also arises in quantum field theory. In fact, axiomatic quantum field theory includes the Postulate A.3 (*cf.* Haag 1996), according to which the spectrum of the energy momentum operator P^μ is confined to a future light semicone, that is, its eigenvalues p^μ satisfy $p^2 \geq 0$ and $p^0 \geq 0$. This means that, when we measure the observable P^μ, we obtain a non-spacelike classical four-momentum p^μ contained in or belonging to a future light semicone. It is quite clear that condition $p^0 \geq 0$ selects one of the time-symmetric twins, $p^0 \geq 0$ and $p^0 \leq 0$, that would arise from the theory if Postulate A.3 were not included. But this choice is introduced from the very beginning, as a postulate of the theory;[15] then it only establishes a conventional difference between both twins. Here again, the substantial difference is established by the globally founded arrow of time. As it is well known, since the energy-momentum tensor is a symmetric tensor, $T^{0i} = T^{i0}$, where T^{0i} denotes the matter-energy flow and T^{i0} denotes the linear momentum density. As we have seen, $\tau^{\mu v}$ can be conceived as the local conservative version of $T^{\mu v}$; then, $\tau^{0\alpha} = \tau^{\alpha 0}$. This means that, if $\tau^{0\alpha}$ represents the local energy flow pointing to the future, the local linear momentum density represented by $\tau^{\alpha 0}$ also points to the future. But the local linear momentum density is precisely the relativistic magnitude corresponding to the classical four-momentum p^μ in quantum field theory. Summing up, the requirement that four-momenta point to the future can be justified by means of theoretical arguments instead of being imposed as a departing point of quantum field theory.

It is worth stressing again that we are completely free of using the labels 'past' and 'future' as we want: names are conventional. The substantial difference between the two directions of time at the local level is established by the local energy flow

[15] In ordinary (non-axiomatic) quantum field theory, the choice is introduced in the classification of one-particle states according to their transformations under the Lorentz group. Such a classification leads to six classes of four-momenta, but it is considered that only three of them have physical meaning (*cf.* Weinberg 1995). These are precisely the cases which agree with the Postulate A.3 of the axiomatic version of the theory.

represented by $\tau^{0\alpha}$. In turn, the local energy flow is what provides the substantial criterion for choosing one of the time-symmetric twins. In consequence, by endowing the arrow of time with a local physical meaning, the local energy flow can be used for breaking the symmetry of the set of solutions resulting from the time-reversal invariant laws of local theories.

8 Conclusions

The problem of the arrow of time is still at present one of the most controversial questions in the philosophy of physics: many attempts to solve it have been made in the past with scarce success. Perhaps these poor results are the consequence of conceptual confusions that contaminate many interesting debates. For this reason, we have elucidated the central concepts involved in the discussion from the very beginning, and this task has allowed us to supply a clear and concise formulation of the problem. On the other hand, the emphasis on the difference between conventional and substantial distinctions in physics has prevented us from confusing the conventional use of temporally asymmetric labels like 'past' and 'future' with the substantial, theoretically grounded difference between the two directions of time.

In this paper, we have argued for a global and non-entropic approach for addressing the problem of the arrow of time, according to which the arrow of time is a geometrical property of spacetime. Although this approach has been adopted in previous works, we have tried to clearly state the conditions that a spacetime must satisfy for having a global and non-entropic arrow. But our main aim has been to show how a substantial time-orientation results from the geometrical time-asymmetry of spacetime, and how this global time-orientation is transferred to local contexts where the local theories of physics are valid. Finally, we have argued that the globally founded local arrow may serve for con-conventionally and non-pretheoretically deciding which one of the two time-symmetric twins resulting from local time-reversal invariant theories has physical meaning. This might help to remove many puzzles and circularities encountered in the traditional attempts to solve the problem of the arrow of time in terms of exclusively local arguments.

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