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On three arguments against categorical structuralism

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Abstract Some mathematicians and philosophers contend that set theory plays a foundational role in mathematics. However, the development of category theory during the second half of the twentieth century has encouraged the view that this theory can provide a structuralist alternative to set-theoretical foundations. Against this tendency, criticisms have been made that category theory depends on set-theoretical notions and, because of this, category theory fails to show that set-theoretical foundations are dispensable. The goal of this paper is to show that these criticisms are misguided by arguing that category theory is entirely autonomous from set theory.

Keywords Set theory · Category theory · Foundation of mathematics · Categorical structuralism

1 Introduction

Employing only one non-logical symbol for the membership relation, the Zermelo–Fraenkel axioms for set theory (henceforth, ZF) not only allow us to express mathematical theorems from different branches—such as analysis or topology—but they also provide a unified and rigorous notion of proof for mathematics.[1] Because of this, some mathematicians and philosophers contend that set theory plays a foundational role in mathematics.[2] In the preface of *Notes on set theory*, Moschovakis gives a nice description of this view:

[1] See Levy (1979) about how theorems from different branches in mathematics can be formulated in the language of ZF.
[2] See, e.g., Maddy (1997, pp. 22–35).

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> At the same time, *axiomatic set theory* is often viewed as a *foundation of mathematics*: it is alleged that all mathematical objects are sets, and their properties can be derived from the relatively few and elegant axioms about sets. Nothing so simple-minded can be quite true, but there is little doubt that in standard, current mathematical practice, "making a notion precise" is essentially synonymous with "defining it in set theory". Set theory is the official language of mathematics, just as mathematics is the official language of science. (Moschovakis 2005, p. vii)

Using techniques from a different axiomatic theory, those of category theory (henceforth, CT), Mac Lane (1986) makes room for the development of a *structuralist* view about mathematics as an alternative to the set-theoretical foundations. Mac Lane (1996) compares these two views as follows:

> All mathematics can indeed be built up within set theory, but the description of many mathematical objects as structures is much more illuminating than some explicit set-theoretic description. (ibid., p. 182).

An important motivation for adopting mathematical structuralism is provided by Benacerraf's argument that numbers are not identical to objects; more specifically, they are not *particular* sets. Benacerraf's conclusion is based on two observations. Firstly, there are different ways of identifying numbers with sets. For instance, we can identify the number 2 with the set $\{\emptyset, \{\emptyset\}\}$ or with the set $\{\{\emptyset\}\}$. Furthermore, there is no principled basis that decides which set is the correct one. The sets $\{\emptyset, \{\emptyset\}\}$ and $\{\{\emptyset\}\}$ are equally good candidates to be taken as referents of the number 2. However, Benacerraf argues, this consequence is absurd. Consequently, numbers cannot be sets. As an alternative, Benacerraf motivates a structuralist view of mathematics by defending that 'in giving the properties (that is, necessary and sufficient) of numbers you merely characterize an *abstract structure*—and the distinction lies in the fact that the "elements" of the structure have no properties other than those relating them to other "elements" of the same structure' (Benacerraf 1965, p. 70).

Although different versions of structuralism agree with Benacerraf's *motto* that number properties characterize an 'abstract structure', they disagree just about what the correct conception of mathematical structure is.[3] Following in Mac Lane's steps, some structuralists maintain that CT provides the necessary tools to formulate a proper account of mathematical structure without having to appeal to set theory. McLarty (1993) for instance employs CT to define what numbers are:

> No model of the Peano axioms (or of any axioms) in ZF has only the properties that all have in common. That is Benacerraf's point. But the point fails for categorical set theory. Sets there, like Benacerraf's numbers, have only structural relations. (McLarty 1993, p. 495)

And Awodey (1996), after defining the notion of *product* in a category, remarks that:

3 See Reck and Price (2000) about different conceptions of mathematical structure in the literature.

> The definition [of product] above provides a uniform, structural characterization of a product of two objects in terms of their relations to other objects and morphisms [arrows] in a category, in contrast to 'material' set-theoretic definitions which depend on specific and often irrelevant features of the objects involved, introducing unwanted additional structure. Indeed it is just this material aspect of conventional set theory that gives rise to such pseudo-problems as whether the number 1 is 'really' the set {Ø}, or whether the real numbers are 'really' cuts in the rationals. (Awodey 1996, p. 220)

Categorical structuralists articulate the relation between mathematical structure and CT in different ways. For instance, Bell (1986) employs CT to argue against the view that mathematical concepts have, as their referents, a fixed universe of sets. In particular, he develops the topos theoretic view that the meaning of mathematical concepts are relative to *local* frameworks, and that these local frameworks can change. Awodey (2004) emphasizes how the categorical perspective of mathematics differs from the 'foundationalist' one. For rather than looking for the single foundation for mathematics, Awodey's categorical structuralism is concerned with investigating structures as they occur in different branches of mathematics independent from their particular set-theoretic interpretations. As Awodey phrases it, categorical structuralism has more to do with 'bridge-building than foundation-building'.[4]

Nevertheless, criticisms have been made that no particular version of categorical structuralism fares better than the set-theoretic foundations (Feferman 1977; Hellman 2003; Shapiro 2005). Hellman (2003) argues that CT depends on the set-theoretic concept of function (Sect. 2). Feferman (1977) argues that CT cannot provide an account of two indispensable notions to formulate a structuralist view of mathematics: the notions of operation and collection (Sect. 3). Shapiro (2005) argues that CT cannot provide an account of mathematical existence without relying on set theory (Sect. 4). The goal of this paper is to argue that these three different criticisms fail to show that CT depends on set theory. Accordingly, if CT provides a unified notion of structure for mathematics, it stands without any commitment to set theory.

2 CT axioms require the conception of function

2.1 Hellman's claims

A *category* consists of the following data: a class of *arrows*; a class of *objects*; and two mappings from the class of arrows to the class of objects called *domain* and *co-domain*. These data satisfy the following axioms:

(A) To each object A there is associated an arrow $1_A : A \to A$, the *identity arrow*.
(B) To each pair of arrows $f : A \to B$ and $g : B \to C$ there is an arrow $gf : A \to C$, the *composition* of f and g.
(C) The following equations hold, for all $f : A \to B$, $g : B \to C$, and $h : C \to D$:

[4] Cf. also McLarty (2004, 2005). See Landry and Marquis (2005) for a survey of different versions of categorical structuralism available in the literature.

$$f 1_A = 1_A f \tag{1}$$

$$(hg) f = h(gf)^5 \tag{2}$$

If the above definition of category provides a structuralist alternative to the set-theoretical foundations, then the CT axioms are not dependent on set-theoretic concepts. Nevertheless, Hellman (2003) argues that the notion of a category employs the notion of function, and so CT cannot provide an independent account for this notion. Because of this Hellman concludes that CT axioms are not autonomous in relation to set theory.

More precisely, Hellman's charge against categorical structuralism is based on the following argument:

(H1) '[T]he notion of *function* is presupposed, at least informally, in formulating category theory' (Hellman 2003, p. 133);
(H2) CT cannot provide an account of function without relying on set-theoretic concepts.[6]
Therefore,
(H) CT depends on set theory.

In the next section I will argue that (H1) is false and, thereby, argue that (H) lacks justification.

2.2 Arrows versus functions

Suppose we take out the expression 'at least informally' from (H1). If so, the modified version of (H1) would simply state that CT axioms presuppose the notion of function. So, the tendency here is to say that the notions of arrows and functors (i.e., arrows between categories) studied by CT are just a roundabout way of talking about functions.[7] Call this modified version of (H1) *mod*-(H1). I will argue that *mod*-(H1) is false.

For the sake of simplicity, let us assume that a category is anything that satisfies the CT axioms whereas a set is anything existing in a universe that satisfies the ZF axioms. It is in this particular sense that we may say that CT axioms (or ZF axioms) *define* what a category (or a set) is. In this vein, *mod*-(H1) can be understood as the conjunction of two statements: (a) in order to define what a category is, CT has to make use of the notion of function; and (b) category theory cannot define what a function is without the aid of ZF. However, I would like to argue, (b) is false and, accordingly, *mod*-(H1) as well. The statement (b) is false because we can understand the notion of function presumed by a category as an arrow in some other category. For instance, take the category **Set** that has as objects all the small sets, and as arrows

5 Cf. Awodey (2006, p. 5) or Mac Lane (1998, p. 7).

6 Hellman argues that CT cannot provide an account of function because CT '*lacks substantive axioms for mathematical existence*' (ibid., p. 138). I will postpone the discussion about this argument to the Sect. 4. In this section, I will focus only on (H1).

7 Hellman apparently defends this modified version of (H1) when he affirms that CT investigates 'the behavior of families of functions under the operation of composition' (Hellman 2003, p. 134).

all the functions between these sets.[8] Now we can define a internal category $\mathscr{C}$ inside **Set**. By this means, the notion of function employed in the internal category $\mathscr{C}$ can be seen as an arrow in the ambient category **Set**.[9] Because of this, it follows that CT can provide an account of function autonomously, without having to be backed up by ZF.[10] Thus *mod*-(H1) is false.

So, if (H1) is to be claimed as true, the word 'informally' must be used to show why the above argument against *mod*-(H1) cannot be extended to (H1) as well. Hellman is not clear about what he means by 'informally'. A possible reading is to understand (H1) as saying that the metalanguage used to formulate CT employs the notion of function. But this reading makes (H1) susceptible to the same criticism as *mod*-(H1): the notion of function employed in the metalanguage can be understood in categorical terms as formulated in the previous paragraph.

Another reading for (H1) is to interpret it as stating that the notion of function "motivates" the categorical concept of arrow. But, if so, it paves the way to a point already mentioned by McLarty (2004): 'A very general notion of function, older than set theory, certainly does motivate category theory. But motivation is not presupposition' (p. 50). For instance, the development of the syntax for the first-order logic was motivated by the desired semantics. However, just from this historic fact we cannot infer that the syntax of the first-order logic presupposes semantic concepts.

To sum up, the use of the expression 'at least informally' makes (H1) an ambiguous thesis. If we take out this expression, (H1) turns out to be false. In addition, Hellman does not provide an interpretation of this expression that makes (H1) true. So, either (H1) overlooks the fact that functions can be defined in CT, or it confuses the distinction between the motivation of the theory and what the theory presupposes.

3 The priority of the notions of collection and operation

Feferman (1977) argues that a structuralist foundation for mathematics must present an account of the notions of operation and collection, and that CT fails to fulfill this requirement. Feferman's argument is then different from saying that CT must provide an account of the notions of set and function. In Feferman's view, NBG, ZF and his non-extensional type-free theory are different accounts of the notion of collection.[11] For instance, unlike ZF, in NBG we can talk about proper classes. Furthermore, because the notion of collection is more general than the notion of sets, unlike Hellman, Feferman is '*not* arguing for accepting current set-theoretical foundations of mathematics' (Feferman 1977, p. 154).

As was discussed in the previous section, to suppose that CT axioms presupposes the set-theoretic notion of function is problematic because it neglects the fact that CT

[8] Since **Set** contains all sets and all functions between these sets, **Set** is too 'large' to be considered as a set. A way of dealing with this difficulty is to make use of the distinction between classes and sets as formulated in NBG (Mac Lane 1998, pp. 21–24). Another strategy is to try to formulate, following Lawvere (1965), the category of categories. About these two strategies, see (McLarty 1992, pp. 107–111).

[9] See Mac Lane (1998, p. 267).

[10] I am grateful to an anonymous reviewer for suggesting this argument.

[11] See Feferman (1977, p. 154) about his own non-extensional theory on operations and collections.

can define functions without relying on ZF axioms. But, as the notions of collection and set are not necessarily the same, Feferman's position is not susceptible to the same kind of criticisms. His point is based on the priority of the notions of collection and operation, not the presumption of the set-theoretic notions of set and function.

In the next subsection I present Feferman's reasons for rejecting categorical structuralism. In the second subsection I offer an argument against his view.

3.1 Feferman's argument

According to Feferman (1977), a definition of what a structure is requires an account of the notions of operation and collection:

> The point is simply that *when explaining* the general notion of structure and of particular kinds of structures such as groups, rings, categories, etc. we implicitly *presume as understood* the ideas of *operation* and *collection* (Feferman 1977, p. 150)

> Thus at each step we must make use of the unstructured notions of operation and collection to explain the structural notions to be studied. The *logical* and *psychological priority* if not primacy of the notions of operation and collection is thus evident (ibid., p. 150).[12]

So, Feferman's argument against categorical structuralism can be stated as follows:

(F1) A structuralist view of mathematics relies on the notions of operation and collection.
(F2) ZF provides an account of the notions of operation and collection (see Feferman 1977, p. 151).
(F3) The notions of operation and collection are logically prior to the notions employed by CT.
Therefore,
(F) CT cannot provide a structuralist view of mathematics without relying on some theory of collections/operations like ZF.

In the next subsection, I will argue that if (F3) is true, then (F2) is false. So, Feferman's conclusion cannot follow from (F1)–(F3).

3.2 What is necessary for logical priority?

McLarty (2005) offers the following argument against Feferman:

> Obviously I agree with Feferman that foundations of mathematics should lie in a general theory of operations and collections, only I say the currently best general

[12] By 'logical priority' Feferman means the order of definition of concepts: 'For example, the concept of vector space is logically prior to that of linear transformation; closer to home, the (or rather some) notions of set and function are logically prior to the concept of cardinal equivalence.' (Feferman 1977, p. 153)

theory of those calls them *arrows* and *objects*. It is category theory (McLarty 2005, p. 49).

Therefore, McLarty contends that although (F1) is true, (F3) is false since category theory includes an account of the notions of operation and collection. Accordingly, it follows that CT can provide a structuralist view of mathematics. Unlike McLarty, my argument does not conclude that either (F1) is true or that (F3) is false. The argument that will be presented here leaves open the truth values of Feferman's premises. My point is just that (F2) and (F3) are mutually exclusive.

Osius (1974) proves that a special kind of category, the elementary topos of ZF sets (henceforth, ETS(ZF)), is logically equivalent to ZF.[13] Based on this result, a possible argument against Feferman is to insist that, if (F2) is true, the category defined by ETS(ZF) axioms can also provide an account of the notions of operation and collection. In this case, (F2) is true only if (F3) is false.

Feferman claims that this argument is not sufficient to show that ZF has no priority over ETS(ZF). He insists that his 'use of 'logical priority' refers not to relative strength of formal theories but to order of definition of concepts, in the cases where certain of these *must* be defined before others' (Feferman 1977, p. 152). So, in Feferman's view, logical equivalence has nothing to do with the 'order of definition'. However, this assumption seems false.

Suppose that ETS(ZF) and ZF are formulated by the languages $\mathscr{L}_C$ and $\mathscr{L}_\in$ respectively. As these two theories are logically equivalent, there is a translation mapping $\mathscr{I}\colon \mathscr{L}_C \to \mathscr{L}_\in$. If (F3) is true, then the following statement is also true:

(*) The predicates '... is a collection' or '... is an operation' cannot be defined by some formula φ such that $\varphi \in \mathscr{L}_C$.

And, as there is a translation between $\mathscr{L}_C$ and $\mathscr{L}_\in$ we could infer from (*) that the notions of collection and operation cannot be defined in terms of some formula $\mathscr{I}(\varphi)$ such that $\mathscr{I}(\varphi) \in \mathscr{L}_\in$. Therefore, if (F3) is true, then ZF cannot also define the notions of operation and collection.

Nevertheless one may reply: the above argument does not meet Feferman's criticism because it already employs set-theoretical notions such as the membership relation in (*). So, according to this reply, my conclusion contradicts my commitment to some theory of collections like ZF. However, in my view, the use of the notion of membership is not sufficient to show that (*) rests on ZF.

Clearly, the predicate 'is defined' does not occur in $\mathscr{L}_\in$. As this predicate is present in (*), this sentence cannot be a formula in ZF. Therefore, the notion of membership in (*) is not the same as the notion of membership that occurs in formulas of ZF. Then one cannot be committed to the notion of membership as defined in ZF without being committed to the rest of ZF theory. Metaphorically speaking, the ZF's notion of membership (as well as other notions in ZF) only has a life within ZF.

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13 A topos is a special kind of category in the sense that it satisfies some additional axioms. See Johnstone (1977) or McLarty (1992) on topos theory. About the specific axioms of ETS(ZF), see Lawvere (1964). As ETS(ZF) and ZF are equivalent, ZFC is also equivalent to an appropriate extension of ETS(ZF). This extension is presented in McLarty (2004).

If I am right, it is not any prior grasp of the notion of membership that makes us committed to the binary predicate for membership as characterized by ZF. Similarly, one may understand the notion of implication as employed in some sentence without being committed to the notion of material implication as defined in classical logic or to the notion of implication in relevance logic. The notions of implication in these logics have very specialized uses which are delimited by the axioms of the logic in which they are formulated.

So I do not exclude the fact that (F1) might be true. Surely, to comprehend the CT axioms it is necessary to master some concepts and possibly the notions of collection and operation are among these concepts. However, as I argued above, the equivalence between ETS(ZF) and ZF is sufficient to demonstrate that there is no 'logical priority' of one theory over the other.

4 Mathematical existence

4.1 Structuralism and meta-mathematics

As Shapiro (2005) observes, Frege and Hilbert held contrasting views about mathematical theories. He points out, 'Frege insisted that arithmetic and geometry each have a *specific* subject matter, space in the one case and the realm of natural numbers in the other' (ibid., p. 67). In contrast, Hilbert thought mathematics was about any system that satisfies a specific list of axioms such as the Peano axioms or the ring axioms.[14] Consequently, in Hilbert's view, mathematical theories are not about some domain fixed in advance as Frege had thought.[15] As Shapiro formulates this difference, Frege viewed mathematical theories as *assertory* and Hilbert as *algebraic*.

Based on the distinction between the assertory and algebraic conception of mathematics, Shapiro offers the following argument against categorical structuralism:

(S1) '[T]he meta-theory, whatever it is, must itself be assertory' (Shapiro 2005, p. 73).[16]
(S2) CT is algebraic.[17]
(S3) ZF is assertory.[18]
Therefore,
(S) CT depends on ZF.

Thus, with this argument above, Shapiro aims to show that mathematics cannot be expressed purely structurally (i.e., purely algebraically). More precisely, he thinks that

[14] Hilbert's position is subtler than that. In Hilbert's view, the existence of a model is not the only constraint that a list of mathematical axioms is supposed to fulfill. The choice of mathematical axioms is also constrained by the present practice of mathematics and by considerations of simplicity and productivity of the chosen axioms. See Zach (2007, pp. 428–431). I am grateful to an anonymous reviewer for pointing that out.

[15] See Heijenoort (1967).

[16] By 'meta-theory' Shapiro understands the theory which formulates meta-theorems like consistency, completeness, and so on.

[17] Cf. ibid., p. 71.

[18] Cf. ibid., p. 74.

the meta-theorems in mathematics cannot be understood structurally. In other words, they have a specific subject matter. Accordingly, since CT is a theory about structure, it cannot serve as a meta-theory for mathematics.

In the following section, I will argue that if (S3) is true, then (S2) is false. Therefore, in my view, Shapiro's above parallel is misguided.

4.2 A problem with Shapiro's parallel

As Shapiro defines it, a theory is assertory if it has a fixed domain. If set theory is assertory (i.e., if (S3) is true), then there is a domain that set theory is about. Because of Gödel's second incompleteness theorem we cannot prove the existence of a model for the set theory, like the cumulative hierarchy, only from our set-theoretic axioms. Otherwise, we could prove that ZF is consistent using only ZF axioms.[19] Therefore, it follows from Gödel's theorem that set theory by itself cannot determine what counts as a model for its axioms.

If so, by 'ZF' in (S3) Shapiro does not only refer to the list of well-formed formulas that constitute this theory. Otherwise, we could not fix a domain in order to guarantee that set theory is assertory. Actually, Shapiro means something broader:

> One is to argue that for set theory to play the foundational role, it is *not* to be understood algebraically. On this view, set theory has a subject matter, the iterative hierarchy V. It is an *assertory* theory about how various structures relate to, and interact with, each other. (Shapiro 2005, p. 72)

Therefore, if (S3) is true, by 'ZF' Shapiro refers to the list of formulas that compose ZF *plus* an intended model—in this particular case, the iterative hierarchy V. So, the assumption that the iterative hierarchy V exists has to be a commitment that we add to our set theory.

But, like any other axiomatic theory, we can also talk about the CT axioms and their different interpretations.[20] If so, supposing that by 'CT' we mean the CT axioms plus a specific interpretation (like the category of groups), then the same argument that Shapiro used to warrant (S3) is applicable *mutatis mutandis* to show that (S2) is false. Therefore, (S2) and (S3) are mutually incompatible.

However, one might reply, ZF is assertory in the sense that this theory has an *intended domain*. Accordingly, as CT axioms do not have an intended interpretation, (S2) is true. Therefore, according to this reply, ZF is assertory not simply because we can fix a domain, but because ZF *intends* to model a specific domain, in particular, the iterative hierarchy V.

The above reply has some force if we consider the CT axioms. But, if we take into account a more particular category, the ETS(ZF) axioms for example, the iterative hierarchy can also be taken as an intended model for this theory. Both theories can thus be taken as intending to describe the same domain. Accordingly, once again, ETS(ZF) and ZF are in the same boat. Thus, Shapiro's distinction between assertory

[19] See Enderton (1977, p. 250).

[20] See McLarty (2004, p. 42).

and algebraic theories fails to show that CT cannot stand without a set-theoretical support.

5 Conclusions

As I mentioned earlier, in Osius (1974) there is a proof of the logical equivalence between ETS(ZF) and ZF. The three arguments against categorical structuralism presented here hold that CT has a specific sort of dependence on ZF. So, if these arguments are sound, they show that the dependence between CT and ZF is not threatened by Osius' result. To put it in Feferman's terms, the notions of 'logical equivalence' and 'logical priority' must be independent in order to warrant these criticisms. My goal here was to show that these argumentative strategies against categorical structuralism are flawed.

In this paper, I presented three ways of formulating arguments for the priority of ZF over CT. I began in the first section with Hellman's claim that there is some sort of dependence between the notion of arrow (in CT) and function (in ZF). On the one hand, I argued that functions can be defined in CT without the aid of ZF. On the other hand, if the notion of function has an important role to motivate the formulation of CT axioms, this is not sufficient to show the relation of dependence between CT axioms and ZF axioms. In short, as McLarty notes, motivation does not entail presumption.

One can also argue, like Feferman (1977), that (1) there are notions that are logically prior to the categorical notions; and (2) unlike CT, ZF provides an account of these notions. I claimed that, given the equivalence between ETS(ZF) and ZF, these two theories have to give an account of the same notions. So, although (1) might be true, (2) must be false.

In the previous section I argued that I do not see any reason to suppose that there is a difference between CT axioms and ZF axioms that support Shapiro's statement that CT is algebraic and ZF is assertory. The same reasons that Shapiro employs for arguing that ZF is assertory can be adopted to show that ETS(ZF) is assertory as well. Thus Shapiro's historical parallel is unwarranted.

Certainly, the arguments presented above do not suffice to show that CT offers us the necessary tools to formulate a structuralist view of mathematics. For instance, one might think that there are more in mathematics than structures. However, I hope that I have shown that if categorical structuralism is not an alternative to set-theoretical foundations, it is not because ZF has some kind of priority over CT.

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