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Why does coherence appear truth-conducive?

Tomoji Shogenji

Abstract   This paper aims to reconcile (i) the intuitively plausible view that a higher degree of coherence among independent pieces of evidence makes the hypothesis they support more probable, and (ii) the negative results in Bayesian epistemology to the effect that there is no probabilistic measure of coherence such that a higher degree of coherence among independent pieces of evidence makes the hypothesis they support more probable. I consider a simple model in which the negative result appears in a stark form: the prior probability of the hypothesis and the individual vertical relations between each piece of evidence and the hypothesis completely determine the conditional probability of the hypothesis given the total evidence, leaving no room for the lateral relation (such as coherence) among the pieces of evidence to play any role. Despite this negative result, the model also reveals that a higher degree of coherence is indirectly associated with a higher conditional probability of the hypothesis because a higher degree of coherence indicates stronger individual supports. This analysis explains why coherence appears truth-conducive but in such a way that it defeats the idea of coherentism since the lateral relation (such as coherence) plays no independent role in the confirmation of the hypothesis.

Keywords   Coherence · Coherentism · Bayesian epistemology · Truth-conduciveness

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1 Introduction

When a claim is supported by multiple independent pieces of evidence, the degree of their coherence seems relevant to the probability of the claim's truth. Strong coherence among them boosts our confidence in the truth of the claim, while we start questioning the claim when the pieces of evidence clash with each other. The following case illustrates the point:

> Sarah is having persistent nausea and starts suspecting pregnancy. She goes over her life of the past several weeks and calls up memory of an activity that is conducive to pregnancy. Sarah visits a clinic and hears from the doctor that the preliminary test result indicates pregnancy.

Here Sarah has three coherent pieces of evidence—perceptual, memorial, and testimonial, respectively—in support of her pregnancy. Moreover, the pieces of evidence seem to be independent of each other in the sense that there is no *direct* link among them. For example, there is no particular reason to expect nausea from the pregnancy-conducive activity unless they are indirectly linked via pregnancy. There is indeed no particular reason to expect the three pieces of evidence together in the absence of pregnancy. So, Sarah is very well justified to believe in her pregnancy and an important part of justification seems to be the high degree of coherence among the independent pieces of evidence. If, for example, Sarah recalled no pregnancy-conducive activity or recalled only a very weakly pregnancy-conducive activity in the recent past, the pieces of evidence would be less coherent than in the original case, and Sarah would not be justified to believe in her pregnancy to the same degree as in the original case.

Bayesian coherentists have been trying to capture this role of coherence in formal terms of the probability calculus. Initially it looked as though the challenge was to find an appropriate probabilistic measure of coherence (a truth-conducive measure of coherence) such that other things being equal, the more coherent the independent pieces of evidence are, the more probable the supported claim is. However, negative results (the impossibility results) have emerged in the past few years to the effect that there cannot be a truth-conducive measure of coherence (Bovens & Hartmann, 2003, Ch. 1; Olsson, 2005, Ch. 7). This is puzzling. Is our common sense about coherence misguided, or is something amiss in the way the formal results are obtained? This paper attempts to reconcile the common sense and the formal results about the role of coherence in confirmation.[1]

2 The simple model

This section proposes a simple model (hereafter "the Simple Model") to analyze Sarah's pregnancy case. I will describe the Simple Model in comparison with models used by Bovens and Hartmann (hereafter "B & H") and by Olsson in their impossibility proofs. It is not my contention that the Simple Model is superior to those models. I adopt the Simple Model for the reason that it presents the conflict between the common sense and the formal result in a clear and stark form, and is helpful when we try to understand the reason for the impression that coherence is truth-conducive.

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[1] Throughout this paper the term "confirmation" means incremental confirmation. In other words, evidence E confirms hypothesis H if and only if E raises the probability of the hypothesis—i.e. if and only if $P(H|E) > P(H)$.

B & H's model draws a distinction between two layers of propositions about evidence. The two layers are (E) the layer of the *possessions* of the evidence, and (A) the layer of the direct *contents* of the evidence.[2] In Sarah's pregnancy case the propositions in layer (E) would be:

E_1: Sarah possesses the evidence in support of the proposition that she has persistent nausea.

E_2: Sarah possesses the evidence in support of the proposition that she recently had an activity conducive to pregnancy.

E_3: Sarah possesses the evidence in support of the proposition that her preliminary test result indicates pregnancy.

The propositions in layer (A) would be:

A_1: Sarah has persistent nausea.

A_2: Sarah recently had an activity conducive to pregnancy.

A_3: Sarah's preliminary test result indicates pregnancy.

B & H's approach is to measure the degree of coherence at the layer (A) of the content propositions, and then try to determine the effect of coherence on the hypothesis, which they take to be the conjunction of the content propositions.

I follow B & H's policy of measuring the degree of coherence at the layer (A) of the content propositions, but I think their choice of the hypothesis is not suitable in many ordinary cases of confirmation. In Sarah's pregnancy case, for example, B & H's hypothesis would be the conjunction: *Sarah has persistent nausea, and she recently had an activity conducive to pregnancy, and her preliminary test result indicates pregnancy*. But what Sarah wants to know is whether she is pregnant. She is not interested in the truth of the conjunction of the content propositions. So, I take the hypothesis in the case to be the following proposition H:

H: Sarah is pregnant.

More generally, I will assume no logical relation (such as conjunction) between the contents of the evidence and the hypothesis to confirm. Of course, in some cases the hypothesis of interest may happen to be the conjunction of the content propositions. That is fine. My suggestion is to avoid any logical restriction on the hypothesis that may preclude the proposition of our true interest from being the hypothesis to confirm.

Once we remove the logical restriction on the hypothesis to confirm, we have three layers of propositions—viz. the two layers (E) and (A) of evidence and then the layer of the hypothesis that is logically independent of (E) and (A). In order to reduce complexity, I am going to ignore the distinction between (E) and (A), assuming that we possess a piece of evidence if and only if its direct content is true. In other words, $P(E_i) = P(A_i)$. In Sarah's pregnancy case, this amounts to assuming the complete reliability of perception, memory, and testimony—i.e. given her perceptual, memorial and testimonial evidence, it is indeed true that Sarah has persistent nausea, had a pregnancy-conducive activity, and the preliminary test result indicates pregnancy.

This is a strong idealization but harmless in the present context of inquiry, which is to seek an explanation why coherence appears truth-conducive, for when there is no doubt about the reliability of the relevant perception, memory, and testimony, it

still seems to our common sense that coherence of evidence plays a positive role in the confirmation of the hypothesis. Under this idealization the pieces of evidence in Sarah's pregnancy case are still coherent and strongly support the hypothesis. If, on the other hand, Sarah recalled (with complete reliability) no pregnancy-conducive activity or recalled (with complete reliability) only a very weakly pregnancy-conducive activity in the recent past, the pieces of evidence would be less coherent than in the original case, and she should not believe in her pregnancy as strongly as in the original case. The idealization of complete reliability does not affect the impression that coherence is truth-conducive, which is the central focus of our discussion. Thus, I will assume that there is no epistemic gap between the layers (E) and (A), and let the term "pieces of evidence" refer to the content propositions in the layer (A).

As for the independence of the pieces of evidence $A_1, \ldots, A_N$ with regard to the hypothesis H, I follow the standard formulation of evidential independence by conditional probabilities, namely:

$A_1, \ldots, A_N$ are independent pieces of evidence for H if and only if $A_1, \ldots, A_N$ are probabilistically independent of each other both on condition of H and on condition of ∼H.

The idea is that once the truth or falsity of the hypothesis is given, the independent pieces of evidence are probabilistically independent of each other since there is no direct link between them. In the case of Sarah's pregnancy we can spell out the condition fully as follows:

$$P(A_1 \& A_2 | H) = P(A_1 | H) \times P(A_2 | H) \qquad P(A_1 \& A_2 | {\sim}H) = P(A_1 | {\sim}H) \times P(A_2 | {\sim}H)$$

$$P(A_2 \& A_3 | H) = P(A_2 | H) \times P(A_3 | H) \qquad P(A_2 \& A_3 | {\sim}H) = P(A_2 | {\sim}H) \times P(A_3 | {\sim}H)$$

$$P(A_3 \& A_1 | H) = P(A_3 | H) \times P(A_1 | H) \qquad P(A_3 \& A_1 | {\sim}H) = P(A_3 | {\sim}H) \times P(A_1 | {\sim}H)$$

$$P(A_1 \& A_2 \& A_3 | H) = P(A_1 | H) \times P(A_2 | H) \times P(A_3 | H)$$

$$P(A_1 \& A_2 \& A_3 | {\sim}H) = P(A_1 | {\sim}H) \times P(A_2 | {\sim}H) \times P(A_3 | {\sim}H)$$

To summarize, the Simple Model consists of the hypothesis H and the pieces of evidence $A_1, \ldots, A_N$ that are independent of each other with regard to H. In order to make the case meaningful, we assume that $0 < P(H) < 1, 0 < P(A_i) < 1$, and that each piece of evidence is consistent with the hypothesis—i.e. $0 < P(H \& A_i)$ and thus $0 < P(H | A_i)$.

The Simple Model is similar to Olsson's model that consists of the hypothesis H and the two pieces of evidence E_1 and E_2 that are independent of each other with regard to H.[3] The difference is that Olsson's two pieces of evidence, E_1 and E_2, belong to the layer (E) of the possessions of the evidence, while H is the direct content of both E_1 and E_2 and thus H belongs to the layer (A) of the contents of the evidence. This is because what Olsson intends to capture is a case where two independent pieces of evidence E_1 and E_2 have identical contents, e.g. two independent witnesses produce identical reports. As a result, coherence in Olsson's model is a relation between the content H of E_1 and the content H of E_2. In the Simple Model, on other hand, $A_1, \ldots, A_N$ are already the contents of the evidence, while H is not logically tied to the contents. So, coherence in the Simple Model is a relation among $A_1, \ldots, A_N$, and not a reflexive relation on H.

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3   Olsson focuses on the basic case where there are only two pieces of evidence.

Olsson's model is useful for its intended case where the pieces of evidence have identical contents, but it is not helpful when the pieces of evidence have different contents. In the case of Sarah's pregnancy, for example, coherence of evidence is, intuitively, a relation among the three propositions (A_1) Sarah has persistent nausea; (A_2) Sarah recently had an activity conducive to pregnancy; and (A_3) Sarah's preliminary test result indicates pregnancy. It is not a reflexive relation on the proposition (H) Sarah is pregnant. I grant that even in this case we *could* take H to be the common content of E_1, E_2, and E_3, setting aside their direct contents A_1, A_2, and A_3, if we wanted to. But such modeling fails to capture the intuitive sense of coherence that we are trying to understand. For example, if Sarah recalled no pregnancy-conducive activity or recalled only a very weakly pregnancy-conducive activity, the pieces of evidence would be less coherent, intuitively, than in the original case. This difference is lost if we take H to be the content of the evidence regardless of the *direct* contents of perception, memory, and testimony.

As mentioned earlier, it is not my contention that the Simple Model is superior to models used by B & H and by Olsson in their impossibility proofs. My point is that the Simple Model is one reasonable model for Sarah's pregnancy case, and it is helpful—as we will see below—when we try to understand the reason for the impression that coherence is truth-conducive.

3 Irrelevance of the lateral relation

This section uses the Simple Model to analyze the role of coherence among independent pieces of evidence in the confirmation of the hypothesis. The hope is to isolate three apparent factors of confirmation: (a) the prior probability of the hypothesis H (hereafter "the prior probability"), (b) the individual vertical relations between each piece of evidence A_i and the hypothesis H (hereafter "the individual vertical relations"), and (c) the lateral relation, such as coherence, among the pieces of evidence $A_1, \ldots, A_N$ (hereafter "the lateral relation"). We would then be able to examine how changes in (c) the lateral relation affect the conditional probability of the hypothesis given the total evidence $P(H|A_1 \& \cdots \& A_N)$ (hereafter "the conditional probability") while holding (a) the prior probability and (b) the individual vertical relations equal, to see the role of coherence *per se*.

We start with the case of Sarah's pregnancy, where there are three independent pieces of evidence A_1, A_2, and A_3 for the hypothesis H. The analysis is straightforward. We use Bayes' Theorem to calculate the conditional probability $P(H|A_1 \& A_2 \& A_3)$, and then plug in the condition of evidential independence, as follows:

$$P(H|A_1 \& A_2 \& A_3)$$

$$= \frac{P(A_1 \& A_2 \& A_3|H) \times P(H)}{P(A_1 \& A_2 \& A_3|H) \times P(H) + P(A_1 \& A_2 \& A_3| \sim H) \times P(\sim H)}$$

$$= \frac{P(A_1|H) \times P(A_2|H) \times P(A_3|H) \times P(H)}{P(A_1|H) \times P(A_2|H) \times P(A_3|H) \times P(H) + P(A_1|\sim H) \times P(A_2|\sim H) \times P(A_3|\sim H) \times P(\sim H)}.$$

More generally, the following equation (1) holds under the condition of evidential independence:

$$P(H|A_1 \& \cdots \& A_N)$$
$$= \frac{P(A_1|H) \times \cdots \times P(A_N|H) \times P(H)}{P(A_1|H) \times \cdots \times P(A_N|H) \times P(H) + P(A_1|\sim H) \times \cdots \times P(A_N|\sim H) \times P(\sim H)}. \quad (1)$$

The equation reveals that the conditional probability of the hypothesis is completely determined by (a) the prior probability, and (b) the individual vertical relations. In other words, changes in (c) the lateral relation make no difference in the confirmation of the hypothesis.

This is the most negative result one can think of about the role of the lateral relation (such as coherence) among the pieces of evidence in confirmation—it plays no role at all. Bovens and Hartmann (2003, Ch. 1) and Olsson (2005, Ch. 7) have presented formal arguments to claim that there is no probabilistic measure of coherence that captures the apparent positive correlation between the degree of coherence and the conditional probability of the hypothesis, but according to the Simple Model the trouble is not just there is no truth-conducive measure of coherence, but that there is no room left for the lateral relation, such as coherence, to influence the conditional probability of the hypothesis.

I also want to show (for use in later discussion) that the conditional probability of the hypothesis, $P(H|A_1 \& \cdots \& A_N)$, is a strictly increasing function of $P(H|A_i)$ for each $i = 1,\ldots, N$. Note first that by Bayes' Theorem, for any $i = 1,\ldots, N$:

$$P(A_i|H) = \frac{P(H|A_i) \times P(A_i)}{P(H)}.$$
$$P(A_i|\sim H) = \frac{P(\sim H|A_i) \times P(A_i)}{P(\sim H)} = \frac{(1 - P(H|A_i)) \times P(A_i)}{1 - P(H)}.$$

Thus, we can rewrite (1) above as follows:

$$P(H|A_1 \& \cdots \& A_N)$$
$$= \frac{P(H|A_1) \times \cdots \times P(H|A_N)/[P(H)]^{N-1}}{P(H|A_1) \times \cdots \times P(H|A_N)/[P(H)]^{N-1} + (1 - P(H|A_1)) \times \cdots \times (1 - P(H|A_N))/[1 - P(H)]^{N-1}}. \quad (1^*)$$

We can see from (1*) that the conditional probability $P(H|A_1 \& \cdots \& A_N) = y$ is a strictly increasing function of $P(H|A_i) = x$ since:

$$\frac{dy}{dx} = \frac{AB}{((A - B)x + B)^2} > 0,$$

where:

$$A = P(H|A_1) \times \cdots \times P(H|A_{i-1}) \times P(H|A_{i+1}) \times \cdots \times P(H|A_N)/[P(H)]^{N-1}.$$
$$B = (1 - P(H|A_1)) \times \cdots \times (1 - P(H|A_{i-1})) \times (1 - P(H|A_{i+1}))$$
$$\times \cdots \times (1 - P(H|A_N))/[1 - P(H)]^{N-1}.$$

This means that, other things being equal, the more strongly one piece of evidence individually supports the hypothesis, the more strongly the total evidence supports the hypothesis.

4 Coherence and the individual strength of evidence

We have obtained a formal proof in the Simple Model that the lateral relation (such as coherence) among the pieces of evidence plays no role at all in the confirmation of the hypothesis, but it still appears, intuitively, that a higher degree of their coherence makes their support for the hypothesis stronger. Is our intuition misguided, or does the simple model miss something important?

We now return to Sarah's pregnancy case to take a closer look at how the impression arises that coherence is truth-conducive. In our story Sarah initially has only one piece of evidence for pregnancy (perception of persistent nausea), but soon the second piece of evidence emerges (memory of a pregnancy-conducive activity), which is coherent with the first piece. It seems obvious, intuitively, that their coherence is truth-conducive, especially when we think of an alternative scenario where Sarah had the same perceptual evidence (perception of persistent nausea) but could not recall any pregnancy-conducive activity in the recent past. The memorial evidence would then be incoherent with the perceptual evidence, and this would make Sarah dismiss the hypothesis of pregnancy. Whether the pieces of evidence are coherent or not does make a difference in the confirmation of the hypothesis. The *degree* of coherence is also relevant. If Sarah only recalled a very weakly pregnancy-conducive activity in the recent past, the memorial evidence would be less coherent with the perceptual evidence than in the original case. This would make her less confident about her pregnancy than in the original case. The same is true of the third piece of evidence from testimony. If the doctor's report were incoherent or less coherent with the perceptual and memorial pieces of evidence, e.g. if Sarah were told that the preliminary test result was negative, or told that it was only very weakly supportive of pregnancy, then Sarah should be less confident about her pregnancy than in the original case. These counterfactual considerations leave no doubt that the degree of coherence between the new and old pieces of evidence affects the confirmation of the hypothesis.

I have described above differences in the degrees of coherence in terms of new and old pieces of evidence, but the temporal order in the acquisition of evidence is not important. Suppose all three pieces of evidence are already in. One can still say in retrospect that if the perceptual evidence were incoherent or less coherent with the rest of the evidence, Sarah would be less confident about her pregnancy. Regardless of the temporal order of acquisition, if one piece of evidence were incoherent or less coherent with the rest of the evidence, it would have a negative effect on the conditional probability of the hypothesis. The degree of coherence between the *focal* piece (usually the new piece) of evidence and the rest of the evidence is a significant factor in the confirmation of the hypothesis.

We need to reconcile this observation with the formal result of Sect. 3 that the prior probability and the individual vertical relations completely determine the conditional probability of the hypothesis. The key to the reconciliation is that in all the alternative scenarios we just considered, where the focal piece of evidence is less coherent with the rest of the evidence, the focal piece of evidence supports the hypothesis less strongly than in the original case. For example, if Sarah had no memory of pregnancy-conducive activity, the memorial evidence (taken by itself in isolation from the rest of the evidence) would cast strong doubt on the hypothesis of pregnancy. It is no surprise then that the total evidence (the perceptual, memorial, and testimonial pieces of evidence taken together) would no longer support the hypothesis of pregnancy strongly. Similarly, if Sarah only recalled a very weakly

pregnancy-conducive activity, the memorial evidence (taken by itself in isolation from the rest of the evidence) would support the hypothesis of pregnancy less strongly than it does in the original case. It is again no surprise that the total evidence (the perceptual, memorial, and testimonial pieces of evidence taken together) would not support the hypothesis of pregnancy as strongly as it does in the original case. The same is true of the third piece of evidence from testimony. If Sarah were told either that the preliminary test result were negative or that they indicated pregnancy only very weakly, the testimonial evidence (taken by itself in isolation from the rest of the evidence) would either cast doubt on the pregnancy hypothesis or support it less strongly than in the original case. There is again no surprise that the total evidence (the perceptual, memorial, and testimonial pieces of evidence taken together) would not support the hypothesis of pregnancy as strongly as it does in the original case.

What the formal result of Sect. 3 shows is that these differences in the *individual strength* of the focal evidence fully account for the differences in the conditional probability of the hypothesis. A high degree of coherence among the independent pieces of evidence does not raise *further* the conditional probability of the hypothesis as determined by the prior probability and the individual vertical relations.

This, however, is not the whole story, for it is not coincidence that when the focal evidence is less (more) coherent with the rest of the evidence, the focal evidence individually supports the hypothesis less (more) strongly. As we now see formally, the degree of coherence is associated with the individual strengths of the pieces of evidence. Thus, the degree of coherence is relevant to the confirmation of the hypothesis, after all, via its association with the individual strengths of the pieces of evidence. I am going to show the association in the case of two pieces of evidence, first, and then extend it to the case of N pieces of evidence.

We adopt the following measure C_S of coherence (Shogenji, 1999):

$$C_S(A_1, \ldots, A_N) = \frac{P(A_1 \& \cdots \& A_N)}{P(A_1) \times \cdots \times P(A_N)}.$$

The following equation (2) then holds under the condition of evidential independence:[4]

$$C_S(A_1, A_2) - 1 = \frac{(P(H|A_1) - P(H))(P(H|A_2) - P(H))}{P(H)(1 - P(H))}. \tag{2}$$

This means that when the two pieces of evidence are independent with regard to the hypothesis, their degree of coherence is completely determined by the prior probability of the hypothesis and the individual vertical relations between each piece of evidence and the hypothesis. Further, it follows immediately from (2) that:

$$P(H|A_2) - P(H) = \frac{(C_S(A_1, A_2) - 1)P(H)(1 - P(H))}{P(H|A_1) - P(H)}. \tag{3}$$

If we regard A_2 as our focal piece of evidence, equation (3) means that so long as the other piece of evidence A_1 positively supports the hypothesis[5]—i.e. $P(H|A_1) - P(H) > 0$—the more coherent the two pieces of evidence are, the more strongly the focal

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[4] See the Appendix for a proof. See also Shogenji (2003) for a closely related result on a condition for transitivity in probabilistic support.

[5] If the other piece of evidence *disconfirms* the hypothesis—i.e. $P(H|A_1) - P(H) < 0$—then we can regard the negation of the original hypothesis as the hypothesis to confirm. If the other piece of

evidence A_2 supports the hypothesis, ceteris paribus—i.e. provided that the prior probability $P(H)$ and the individual strength $P(H|A_1)$ of the other evidence are held equal.[6] The following diagram depicts this relation:

H

Fixed
(Positive)

Stronger
Support

A_1

More
Coherent

A_2
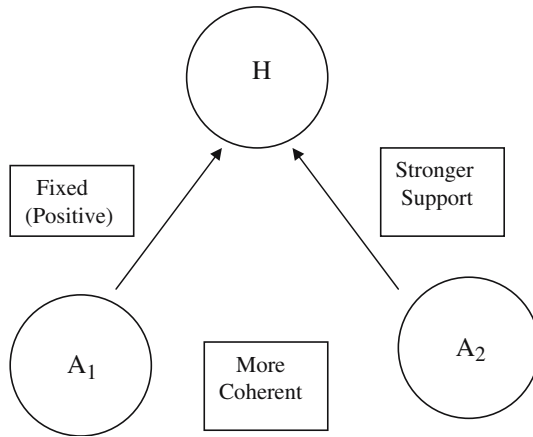
We have just observed the formal relation between the degree of coherence between the two pieces of evidence and the individual strength of the focal piece of evidence. What is its implication on the conditional probability of the hypothesis (given the total evidence)? Recall the observation in Sect. 3 that the conditional probability $P(H|A_1 \& \cdots \& A_N)$ is a strictly increasing function of $P(H|A_i)$. It follows from this observation that, other things being equal, the more strongly the focal evidence A_2 supports H, the more strongly the total evidence $A_1 \& A_2$ supports the hypothesis. As a result, the degree of coherence between the two pieces of evidence and the conditional probability of the hypothesis given the total evidence is linked indirectly through the individual strength of the foal piece of evidence. To be more precise, coherence between the two pieces of evidence is truth-conducive in the following sense:

> When one of the two independent pieces of evidence supports the hypothesis, the more coherent the two pieces of evidence are, the more probable the hypothesis is, ceteris paribus (provided the prior probability and the individual strength of the first piece of evidence are held equal).

To see what this means in concrete terms, regard the perceptual and memorial pieces of evidence in Sarah's pregnancy case to be the first and second (focal) pieces of evidence, respectively. Assume as before that the two pieces of evidence are independent with regard to the hypothesis of pregnancy, and the (non-focal) perceptual evidence supports the hypothesis. The thesis above implies that other things being equal (given the same prior probability and the same individual strength of the perceptual evidence),

Footnote 5 continued

evidence neither supports nor disconfirms the hypothesis—i.e. $P(H|A_1) - P(H) = 0$—then the two pieces of evidence are probabilistically independent regardless of the individual strength of the focal evidence.

[6]   Further, since the neutral point (the point of probabilistic independence) in Shogenji's measure of coherence is 1, the focal evidence A_2 positively supports the hypothesis if and only if A_1 and A_2 are coherent—i.e. $P(H|A_2) > P(H)$ if and only if $C_S(A_1, A_2) > 1$.

the more coherent the perceptual and memorial pieces of evidence are with each other, the more probable it is that Sarah is pregnant.

We can extend this result to cases involving three or more pieces of evidence. Let $A_1, \ldots, A_N$ be independent pieces of evidence for H. It follows from their independence that $A_1 \& \cdots \& A_{N-1}$ and A_N are pair-wise independent, and thus by replacing A_1 and A_2 of (3) with $A_1 \& \cdots \& A_{N-1}$ and A_N, respectively, we obtain the following result:

$$P(H|A_N) - P(H) = \frac{(C_S(A_1 \& \cdots \& A_N) - 1)P(H)(1 - P(H))}{P(H|A_1 \& \cdots \& A_{N-1}) - P(H)}. \qquad (3^*)$$

This means that when the rest of the evidence supports the hypothesis—i.e. $P(H|A_1 \& \cdots \& A_{N-1}) - P(H) > 0$—the more pair-wise coherent the focal evidence A_N is with the rest of the evidence, the more strongly the focal evidence individually supports the hypothesis, ceteris paribus. The ceteris paribus qualification here requires that the prior probability and the strength of the rest of the evidence $P(H|A_1 \& \cdots \& A_{N-1})$ be held equal. But we already know from the equation (1) of Sect. 3 that $P(H|A_1 \& \cdots \& A_{N-1})$ is completely determined by the prior probability $P(H)$ and the individual strengths $P(H|A_1), \ldots, P(H|A_{N-1})$ of the pieces of evidence. So, the ceteris paribus qualification only needs to hold equal the prior probability and the individual strengths of the non-focal pieces of evidence. Note further that:

$$\begin{aligned} C_S(A_1 \& \cdots \& A_N) &= \frac{P(A_1 \& \cdots \& A_N)}{P(A_1 \& \cdots \& A_{N-1}) \times P(A_N)} \\ &= \frac{P(A_1 \& \cdots \& A_N) \times P(A_1) \times \cdots \times P(A_{N-1})}{P(A_1 \& \cdots \& A_{N-1}) \times P(A_N) \times P(A_1) \times \cdots \times P(A_{N-1})} \\ &= \frac{C_S(A_1, \ldots, A_N)}{C_S(A_1, \ldots, A_{N-1})}. \end{aligned}$$

This means that if we also hold the degree of coherence among the non-focal pieces of evidence $C_S(A_1, \ldots, A_{N-1})$ equal by the ceteris paribus qualification, then the more coherent all the pieces of evidence $A_1, \ldots, A_N$ are, the more strongly the focal evidence A_N individually supports the hypothesis, ceteris paribus.[7]

As we observed in the case of two pieces of evidence, this relation between the degree of coherence among the pieces of evidence and the individual strength of the focal evidence, reveals the indirect relation between the degree of coherence among the pieces of evidence and the conditional probability of the hypothesis given the total evidence. Recall from Sect. 3 that the conditional probability of the hypothesis given the total evidence is a strictly increasing function of the individual strength of any piece of evidence. Applying this point to the focal piece of evidence A_N, we can conclude that coherence among all the pieces of evidence is truth-conducive in the following sense:

> When the non-focal pieces of evidence support the hypothesis, the more coherent all the pieces of evidence are, the more probable the hypothesis is, ceteris

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[7] As mentioned in footnote 5 above, in the case with only two pieces of evidence A_1 and A_2, the focal evidence A_2 positively supports the hypothesis H if and only if A_1 and A_2 are coherent—i.e. $P(H|A_2) > P(H)$ if and only if $C_S(A_1, A_2) > 1$—but in the case involving three or more pieces of evidence, the focal evidence A_N positively supports the hypothesis if and only if A_N makes the pieces of evidence *more* coherent—i.e. $P(H|A_N) > P(H)$ if and only if $C_S(A_1, \ldots, A_N) > C_S(A_1, \ldots, A_{N-1})$. The case with two pieces of evidence is a special case of this more general principle since $C_S(A_1, \ldots, A_{N-1})$ is trivially 1 (neutral) when $N = 2$.

paribus (provided the prior probability, the individual strengths of the non-focal pieces evidence, and the degree of coherence among the non-focal pieces of evidence are held equal).

This principle explains why coherence appears truth-conducive.

5 Conclusion

We have uncovered that when the pieces of evidence are independent with regard to the hypothesis and the rest of the evidence supports the hypothesis, the more coherent the focal piece of evidence is with the rest of the evidence, the more strongly the focal evidence supports the hypothesis. This leaves us with the impression that coherence is truth conducive. When we focus on one piece of evidence and observe that its coherence with the rest of the evidence affects the probability of the hypothesis, it is natural to suppose that the lateral relation among the pieces of evidence is a third factor (other than the prior probability and the individual vertical relations) in the confirmation of the hypothesis. We may be tempted further by the idea of coherentism in epistemology that coherence is a *source* of epistemic justification. However, a careful analysis reveals that coherence among the pieces of evidence is related to the conditional probability of the hypothesis only indirectly through its relation to the individual strengths of the pieces of evidence.

It is worth noting here that a higher degree of coherence does not *strengthen* individual pieces of evidence. Rather, under the condition of evidential independence, the degree of coherence is simply a function of the individual strengths of the pieces of evidence. Thus, although is a sense in which coherence is truth-conducive, that is entirely consistent with the foundationalist view of epistemic justification since the lateral relation, such as coherence, has no independent role to play in the confirmation of the hypothesis.

Appendix: Proof of (2)

$$
\begin{aligned}
C_S(A_1, A_2) - 1 &= \frac{P(A_1 \,\&\, A_2)}{P(A_1)P(A_2)} - 1 \\
&= \frac{P(A_2|A_1) - P(A_2)}{P(A_2)}.
\end{aligned}
$$

However,

$$
\begin{aligned}
&P(A_2|A_1) - P(A_2) \\
&\quad = [P(A_2 \,\&\, H|A_1) + P(A_2 \,\&\, {\sim}H|A_1)] - [P(A_2 \,\&\, H) + P(A_2 \,\&\, {\sim}H)] \\
&\quad = [P(A_2|H \,\&\, A_1)P(H|A_1) + P(A_2|{\sim}H \,\&\, A_1)P({\sim}H|A_1)] - [P(A_2|H)P(H) \\
&\qquad + P(A_2|{\sim}H)P({\sim}H)] \\
&\quad = [P(A_2|H)P(H|A_1) + P(A_2|{\sim}H)P({\sim}H|A_1)] - [P(A_2|H)P(H) \\
&\qquad + P(A_2|{\sim}H)P({\sim}H)] \\
&\quad = P(A_2|H)[P(H|A_1) - P(H)] + P(A_2|{\sim}H)[P({\sim}H|A_1) - P({\sim}H)]
\end{aligned}
$$

$$= P(A_2|H)[P(H|A_1) - P(H)] + P(A_2|\sim H)[(1 - P(H|A_1) - (1 - P(H)))$$
$$= P(A_2|H)(P(H|A_1) - P(H)) - P(A_2|\sim H)[P(H|A_1) - P(H)]$$
$$= [P(H|A_1) - P(H)][P(A_2|H) - P(A_2|\sim H)].$$

And further,

$$P(A_2|H) - P(A_2|\sim H) = \frac{P(H|A_2)P(A_2)}{P(H)} - \frac{P(\sim H|A_2)P(A_2)}{P(\sim H)}.$$

So,

$$C_S(A_1, A_2) - 1$$
$$= [P(H|A_1) - P(H)] \left[\frac{P(H|A_2)}{P(H)} - \frac{P(\sim H|A_2)}{P(\sim H)} \right]$$
$$= \frac{[P(H|A_1) - P(H)][P(H|A_2)(1 - P(H)) - (1 - P(H|A_2))P(H)]}{P(H)(1 - P(H))}$$
$$= \frac{[P(H|A_1) - P(H)][P(H|A_2) - P(H)]}{P(H)(1 - P(H))}.$$

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