KEN GEMES

# VERISIMILITUDE AND CONTENT

ABSTRACT. Popper's original definition of verisimilitude in terms of comparisons of truth content and falsity content has known counter-examples. More complicated approaches have met with mixed success. This paper uses a new account of logical content to develop a definition of verisimilitude that is close to Popper's original account. It is claimed that Popper's mistake was to couch his account of truth and falsity content in terms of true and false consequences. Comparison to a similar approach by Schurz and Wiengartner show certain advantages of this new approach.

## 1. INTRODUCTION

In *Conjectures and Refutations* Popper identifies the content of a statement with the set of its (logical) consequences (Popper 1963, 233). That is to say, for Popper,

D1 $\alpha$ is part of the content of $\beta = \mathrm{df.}\, \beta \vdash \alpha$.[1]

This notion of content is common to philosophers as different as Popper and Carnap.[2] Gemes (1990), and (1993) argued against D1 on the grounds that (a) it has many unintuitive consequences and, more importantly, (b) it is ill suited to many of the technical needs of philosophers of science. Gemes (1994) and (1997) developed on alterative account of (logical) content. The purpose of the current note is to apply the new notion of content to Popper's definition of verisimilitude in order to remove some long known objections.

## 2. PROBLEMS FOR POPPER'S DEFINITION OF VERISIMILITUDE

In Popper (1963, 233) verisimilitude is defined using the notion of class inclusion as follows:

Assuming that truth-content and falsity-content of two theories $H1$ and $H2$ are comparable, we can say that $H2$ is more closely similar to the truth, or corresponds better to the facts, than $H1$, if and only if, either

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(a) the truth-content but not the falsity-content of $H2$ exceeds that of $H1$,

or

(b) the falsity-content of $H1$, but not its truth-content, exceeds that of $H2$.

In the same place, Popper identifies the truth-content of a statement $\alpha$ as the class of all true consequences of $\alpha$, and the falsity content of $\alpha$ as the class of all false consequences of $\alpha$.

In other words, given Popper's identification of content with consequence class, the truth content of a statement $\alpha$ is the class of all true content parts of $\alpha$ and the falsity content of $\alpha$ is the class of all false content parts of $\alpha$. So, where $\ulcorner X_T \urcorner$ names the set of all true content parts of $X$, and $\ulcorner X_F \urcorner$ names the set of all false content parts of $X$, the above definition is equivalent to the following;

V1 Assuming that truth-content and falsity-content of two theories $H1$ and $H2$ are comparable, $H2$ has more verisimilitude than $H1$ iff

(a) $H1_T \subseteq H2_T \;\&\; H2_F \subset H1_F$

or

(b) $H1_T \subset H2_T \;\&\; H2_F \subseteq H1_F$.

Tichy (1974), Harris (1974), and Miller (1974), have shown that V1 has the consequence for any two false theories $H1$ and $H2$, $H1$ does not have more verisimilitude than $H2$.

As if this result were not bad enough, we should also note that V1 has the horrendous consequence that for any (finitely axiomatizable) $H1$, true or false, if $H1$ does not entail every truth then $H1$ does not have more verisimilitude than its negation – which, for convenience, we symbolize as '$\sim H1$'.[3]

*Proof.* Let $H1$ be any arbitrary (finitely axiomatizable) theory such that there is some truth $t$ such that it is not the case that $H1 \vdash t$. Now consider $(t \vee \sim H1)$. Since $t$ is true $(t \vee \sim H1)$ is true. So clearly $(t \vee \sim H1) \in \sim H1_T$. Yet $(t \vee \sim H1) \notin H1_T$, otherwise $H1 \vdash (t \vee \sim H1)$, and hence $H1 \vdash t$, contra our choice of $t$. Therefore it is not the case that $\sim H1_T \subseteq H1_T$. Therefore $H1$ does not have more verisimilitude than $\sim H1$.

The Tichy–Harris–Miller result flies in the face of Popperian philosophy of science, which holds that while all our scientific theories are false, some are closer to the truth than others. The second result, that no theory short of the complete truth has more verisimilitude than its negation flies in the face of common sense. After all, where the atomic sentence ‘p’ is true it is presumably closer to the truth than its negation.[4]

Popper has offered other definitions of verisimilitude (Cf., for instance, Popper (1972, 334) and these have elicited criticisms from Tichy (1974) and others. Tichy (1978), and Oddie (1986), among others, have offered more complicated definitions of verisimilitude.[5] Rather than investigate these various definitions I want to suggest a simple remedy which will allow us to save Popper's original definition from the above noted infelicities.

## 3. TOWARDS A SOLUTION: THE NEED FOR A NEW NOTION OF CONTENT

Part of the reason why the above mentioned problems arise for V1 is that, given the D1 notion of content, it follows that for any theory $H1$ and for any arbitrary truth $t$, there will always be some content part of $H1$ which contains a non-vacuous occurrence of $t$. Given such a wide notion of content it is hardly surprising that for just about any arbitrary theories $H1$ and $H2$, provide $H2$ does not entail $H1$, we can always find some true content part of $H1$ that is not part of $H2$.

Let us consider a concrete case.

Suppose the atomic sentences ‘$p$’, ‘$q$’ and ‘$r$’ are all true. Now consider the true theory $\{p, q]$ and the false theory $\{\sim p, \sim q\}$. Then, given the D1 notion of content, $\{\sim p, \sim q\}$ has the true content part ‘$(\sim p \vee r)$’ not shared by $\{p, q)$.

Now, intuitively, ‘$(\sim p \vee r)$’ is not really part of the content of $\{\sim p, \sim q\}$, let alone part of its truth-content. If ‘$(\sim p \vee r)$’ counts as part of the content of $\{\sim p, \sim q\}$ then on the evidence of ‘$r$’ we would have to say that part of the content of $\{\sim p, \sim q\}$, has been conclusively confirmed. Indeed, if we are to follow D1 and let any arbitrary consequence of $\{\sim p, \sim q\}$, count as a content part of $\{\sim p, \sim q\}$, then on the evidence of ‘$q$’ we would have so say that part of the content of $\{\sim p, \sim q\}$, has been conclusively confirmed, since ‘$q$’ entails $\{\sim p, \sim q\}$'s consequence ‘$(\sim p \vee q)$’.

The lesson to be learned here is that, contra D1, we should not let such arbitrary disjunctions count as content parts.

While this is not the place to fully rehearse the notion of logical content developed in In Gemes (1994) and (1997) we can give a brief account. Presuming the notions of logical consequence and atomic well-formed formulae (wffs) are defined for the language in question, letting $\alpha$ be a variable for wffs and $\beta$ be a variable for wffs and sets of wffs of the language in question, we define content as follows,

D2 $\alpha$ is part of the content of $\beta$ iff $\alpha$ and $\beta$ are contingent, $\beta \vdash \alpha$, and every relevant model of $\alpha$ has an extension that is a relevant model of $\beta$.

A relevant model of arbitrary wff $\alpha$ is a model of $\alpha$ that assigns values to all and only those atomic wffs relevant to $\alpha$. An atomic wff $\alpha$ is relevant to wff $\beta$ iff there is some model $m$ of $\beta$ such that where $m'$ differs from m only in the value it assigns $\alpha$, $m'$ is not a model of $\beta$.[6] In the case of quantificational wffs the quantifiers are treated substitutionally in order to determine content parts. So the atomic wffs relevant to, for instance, '$(x)Fx$' are '$Fa$', '$Fb$', '$Fc$', etc. So, '$GbvFa$' is not part of the content of '$(x)Fx$' since that relevant model of '$GbvFa$' that assigns '$Fa$' the value $F$ and '$Gb$' the value $T$ cannot be extended to a model of '$(x)Fx$'. '$Fa$' is a content part of '$(x)Fx$', since the sole relevant model of '$Fa$', namely that which makes the single assignment of $T$ to '$Fa$', can clearly be extended to a relevant model of '$(x)Fx$', by adding the assignment of $T$ to '$Fb$', '$Fc$', '$Fd$', etc.

## 4. THE SOLUTION

Recall, for arbitrary theory $H1$, Popper identifies the truth-content of $H1$, that is, $H1_T$, with the set of all true consequences of $H1$, and the falsity-content of $H1$, that is, $H1_F$, with the set of all false consequences of $H1$. Now if we reject the D1 conception of content we need no longer identify the truth-content (falsity-content) of $H1$ with the set of all true (false) consequences of $H1$. In particular, if we accept the D2 notion of content we will not count every true (false) consequence of $T$ as part of the truth-content (falsity-content) of $T$ because we will not count every true (false) consequence of $T$ as part of the content of $T$.

Suppose then that we define the notions of truth and falsity-content using the D2 notion of content. In this case the Tichy–Harris–Miller result does not obtain because the proofs all involve constructing a putative member of theory $H1$'s truth content by disjoining some false consequence $F$ of $H1$ with some arbitrary truth that is not a consequence of $H1$. But any such disjunction will not be a content part of $H1$, according to the D2 notion of content. The proof, given above, that if finitely axiomatizable $H1$ does not entail every truth it does not have more verisimilitude than its negation, fails since $(t \vee \sim H1)$ is not a D2 content part of $\sim H1$.

## 5. BUILDING A BETTER SOLUTION

While Popper's V1 when combined with the new account of content D2 does not have all the shortcomings of V1 supplemented with D1 it does share some of its problems. For instance, both share the presumably unfortunate consequence that where $H2$ is false and $H1$ does not entail $H2$, $H2$ can not have more verisimilitude than $H1$. This follows since under either understanding of content, where $H2$ is false and $H1$ does not entail $H2$, $H2 \in H2_F$ and $H2 \notin H1_F$, and hence it is not the case that $H2_F \subseteq H1_F$. Let us consider a concrete case.

Suppose that the atomic sentence '$p$' is true and the atomic sentence '$q$' is false. Let $H2$ be the theory consisting of the two claims '$p$' and '$q$' and let $H1$ be the theory consisting of '$q$' alone. Now intuitively, I suppose, $H2$ has more verisimilitude than $H1$. The problem here is that while both $H1$ and $H2$ share the same basic falsity content, namely '$q$', we can manufacture extra falsity-content for $H2$ over $H1$ by simply conjoining any content part of $H2$, not shared by $H1$, onto '$q$'. In particular, $H2$ has the falsity-content '$(p \& q)$' not shared by $H1$.

Now perhaps if we could make good on the notion of basic falsity-content we could avoid this result. Note, where $H2$ contains only the true atomic statement '$p$' and the false atomic statement '$q$', while '$(p \& q)$' is part of the falsity-content of $H2$, it is only part of that falsity-content because its second conjunct is false. The first conjunct, being true, plays no part in getting '$(p \& q)$' into $H2$'s falsity-content. Informally, the second conjunct is doing all the work while the first is merely a free rider. Perhaps then we can get at the basic falsity-content of a theory by demanding that each member of that content contains no such free riders. More formally, a statement that is part of a theory's falsity-content has a free rider when it has a content part that is true. The basic falsity-content of a theory should consist of all those false content parts of the theory that contain no such free riders as content parts. Thus we have the following definition, where '$\alpha \in \beta_{\mathrm{BF}}$' abbreviates '$\alpha$ is part of the basic falsity-content of $\beta$',

D3 $\alpha \in \beta_{\mathrm{BF}} =_{df.} \alpha$ is part of the content of $\beta, \alpha$ is false and no content part of $\alpha$ is true.

On this analysis, where '$p$' is true, '$q$' false, and $H2$ contains the two claims '$p$' and '$q$' and $H1$ consists of '$q$' alone, $H2$ and $H1$ share the same basic falsity-content namely '$q$', and its logical equivalents.

Note, we cannot define a notion of basic truth-content by strict analogy with this notion of basic falsity-content. The disanalogy occurs because while a member of $\beta$'s falsity-content may itself have a true content part, a member of $\beta$'s truth-content cannot have a false content part. While false statements can contain true content parts, true statements cannot contain false content parts. Now, in fact, it is not clear that we need to recast the Popperian V1 in terms of basic truth-content as well as basic falsity-content. There is no problem of free riders in truth-contents analogous to the problem of free riders in falsity-contents.

Now we are in a position to offer an improved version of the Popperian V1,

V2 Assuming that truth-content and falsity-content of two theories $H1$ and $H2$ are comparable, $H2$ has more verisimilitude than $H1$ iff

(a) $H1_T \subset H2_T \;\&\; H2_{\mathrm{BF}} \subseteq H1_{\mathrm{BF}}$

or

(b) $H1_T \subseteq H2_T \;\&\; H2_{\mathrm{BF}} \subset H1_{\mathrm{BF}}$.

V2 combined with D2 gives the result that where '$p$' is true and '$q$' is false, and $H2$ contains both claims and $H1$ contains only the latter claim, $H2$ has more verisimilitude than $H1$.

One defect of V2 when combined with D2 is that where $H2$ is true and $H1$ is merely a watered-down, that is to say, weaker, version of $H2$, the combination of V2 and D2 does not always entail that $H2$ has more verisimilitude that $H1$. Let us consider a concrete example.

Suppose everything has the property $M$ and nothing has the property $T$. Let $H2$ consist of the true claim '$(x)(Mx)$' and $H1$ consist of the true claim '$(x)(Mx \vee Tx)$'. Then, I suppose, intuitively, $H2$ has more verisimilitude than $H1$. However V2 does not render this verdict. The problem here is that, while $(x)(Mx \vee Tx) \in H1_T$, $(x)(Mx \vee Tx) \notin H2_T$. '$(x)(Mx \vee Tx)$' is not a member of $H2_T$ because, under D2, '$(x)(Mx \vee Tx)$' is not a content part of $H2$. Though '$(x)(Mx \vee Tx)$' is a consequence of $H2$, it is not a content part of $H2$, because, for instance, none of those relevant models of '$(x)(Mx \vee Tx)$' which assigns the value $T$ to '$Ta$', '$Tb$', '$Tc$', etc., and $F$ to '$Ma$' can be extended to a relevant model of '$(x)Mx$'.

Now note, Popper's original V1, combined with the traditional notion of content D1, does have the consequence that in the above case $H2$ has more verisimilitude than $H1$. The difference here lies not in the difference between V1 and V2, but in the difference between D1 and D2. According to D1, $H1$'s true content part '$(x)(Mx \vee Tx)$' is part of the content of $H2$. According to D2 it is not. It now seems that D2, which was touted as saving us from the calamities noted in Section 2. above, brings on problems of its own. The very virtue of D2, that it does not allow any arbitrary disjunctive consequence of a theory to count as a content part of the theory, seems to bring trouble in its wake.

But wait, while under D2 not every true content part of $H1$ is part of the content of $H2$, every true content part of $H1$ is a consequence of some true content part of $H2$. Maybe, then, we can still use our new D2 notion of content in order to avoid the problems of Section 2, but recast V2 in terms of the members of $H1$'s truth-content being consequences of $H2$'s true content parts rather than being members of $H2$'s truth-content.

Now presumably, something like this problem for truth-contents also arises in the case of falsity-contents. Let us see.

Suppose a domain of discourse consisting of four objects, $a$, $b$, $c$, and $d$. Further suppose that $a$, $b$, and $c$ have the property $T$ but lack the property $B$, while $d$ lacks both $B$ and $T$. Then presumably where $H2$ contains the single claim '$(x)(Bx \vee Tx)$' and $H1$ contains the single claim '$(x)Bx$', $H2$ has more verisimilitude than $H1$. Yet, in fact, while '$(Bd \vee Td)$' is part of the basic falsity content of $H2$ it is not part of the basic falsity-content of $H1$, hence according to V2, $H2$ does not have more verisimilitude than $H1$. '$(Bd \vee Td)$' is not part of $H1$'s basic falsity-content because, under D2, it is not even part of $H1$'s content, since that relevant model of '$(Bd \vee Td)$'

which assigns '$Bd$' the value $F$ and '$Td$' the value $T$ cannot be extended to a relevant model of '$(x)Bx$'.

Now, this problem may be similarly avoided if instead of demanding that $H2$'s basic falsity-content be part of $H1$'s basic falsity-content we simply demand that every member of $H2$'s basic falsity content be a consequence of $H1$'s basic falsity content. Thus we have the following definition,

V3 Assuming that truth-content and falsity-content of two theories $H1$ and $H2$ are comparable; $H2$ has more verisimilitude than $H1$ iff

(a) $H2_T \vdash H1_T$ and, $H1_T \nvdash H2_T$, and $H1_{\mathrm{BF}} \vdash H2_{\mathrm{BF}}$

or

(b) $H2_T \vdash H1_T$, and $H1_{\mathrm{BF}} \vdash H2_{\mathrm{BF}}$ and $H2_{\mathrm{BF}} \nvdash H1_{\mathrm{BF}}$

Note, moving from the claim that each member of $H1$'s truth content be a member of $H2$'s truth content to the claim that each member of $H1$'s truth content be a consequence of $H2$'s truth content does not represent a complete abandoning of Popper's original intuition that what is true in $H1$ occurs in $H2$. While it is true that from the fact that $\alpha$ is a true content part of $H1$ and $\alpha$ is a consequence of $H2$ it does not follow that $\alpha$ itself is a content part of $H2$, it does follow that some content part of $H2$ entails $\alpha$. Thus suppose $H2$ contains the true content part '$p$' and $H1$ contains the true content part '$(p \vee q)$'. Then while $H1$'s content part '$(p \vee q)$' is not a content part of $H2$, $H2$ contains a content part, namely '$p$' that entails that true content part of $H1$. The point here is that V3 demands that for each true content part of $H1$ there be a corresponding, possibly even stronger, true content part in $H2$. Similarly it demands that for each falsehood in $H2$ there be a corresponding, possibly even stronger, falsehood in $H1$. This seems pretty much in line with the intuitions behind Popper's original notion of verisimilitude.

## 6. SOME GENERAL PROPERTIES OF V3

Presumably any definition of greater verisimilitude should have the properties of being non-reflexive, non-symmetrical and transitive.

**Theorem 1.** For any $H1$, $H1$ does not have more verisimilitude than $H1$.

*Proof.* For any $H1$, if $H1$ has more verisimilitude than $H1$ by clause (a) of V3, then, per impossible, $H1_T \nvdash H1_T$. If $H1$ has more verisimilitude than $H1$ by clause (b) of V3, then, per impossible, $H1_{\mathrm{BF}} \nvdash H1_{\mathrm{BF}}$.

**Theorem 2.** For any $H2$ and $H1$, if $H2$ has more verisimilitude than $H1$ then $H1$ does not have more verisimilitude than $H2$.

*Proof.* Suppose $H2$ has more verisimilitude than $H1$. Then if this is so by clause (a) of V3 then $H2_T \vdash H1_T$ and, $H1_T \nvdash H2_T$ but then $H1$ does not have more verisimilitude than $H2$. If it is so by clause (b) of V3 then $H2_{\mathrm{BF}} \nvdash H1_{\mathrm{BF}}$, and so $H1$ does not have more verisimilitude than $H2$.

**Theorem 3.** For any *H3*, $H2$ and $H1$, if $H3$ has more verisimilitude than $H2$ and $H2$ has more verisimilitude than $H1$ then $H3$ has more verisimilitude than $H1$.

*Proof.* Suppose $H3$ has more verisimilitude than $H2$ and $H2$ has more verisimilitude than $H1$.

Then there are four cases to consider

*Case 1*:

$H3$ has more verisimilitude than $H2$ by clause (a) of V3 and $H2$ has more verisimilitude than $H1$ by clause (a) of V3. In this case $H3_T \vdash H2_T$ and $H2_T \vdash H1_T$ and $H1_T \nvdash H2_T$ and $H1_{\mathrm{BF}} \vdash H2_{\mathrm{BF}}$ and $H2_{\mathrm{BF}} \vdash H3_{\mathrm{BF}}$ So $H3_T \vdash H1_T$ and $H1_T \nvdash H3_T$ and $H1_{\mathrm{BF}} \vdash H3_{\mathrm{BF}}$ so by clause (a) of V3, $H3$ has more verisimilitude than $H1$.

*Case 2*:

$H3$ has more verisimilitude than $H2$ by clause (a) of V3 and $H2$ has more verisimilitude than $H1$ by clause (b) of V3. In this case $H3_T \vdash H2_T$ and $H2_T \vdash H1_T$ and $H2_T \nvdash H3_T$ and $H1_{\mathrm{BF}} \vdash H2_{\mathrm{BF}}$ and $H2_{\mathrm{BF}} \vdash H3_{\mathrm{BF}}$ So $H3_T \vdash H1_T$ and $H1_T \nvdash H3_T$ and $H1_{\mathrm{BF}} \vdash H3_{\mathrm{BF}}$ so by clause (a) of V3, $H3$ has more verisimilitude than $H1$.

*Case 3*:

$H3$ has more verisimilitude than $H2$ by clause (b) of V3 and $H2$ has more verisimilitude than $H1$ by clause (a) of V3. In this case

$H3_T \vdash H2_T$ and $H2_T \vdash H1_T$ and $H1_T \nvdash H2_T$ and $H1_{\mathrm{BF}} \vdash H2_{\mathrm{BF}}$ and $H2_{\mathrm{BF}} \vdash H3_{\mathrm{BF}}$ So $H3_T \vdash H1_T$ and $H1_T \nvdash H3_T$ and $H1_{\mathrm{BF}} \vdash H3_{\mathrm{BF}}$ so by clause (a) of V3, $H3$ has more verisimilitude than $H1$.

*Case 4*:

$H3$ has more verisimilitude than $H2$ by clause (b) of V3 and $H2$ has more verisimilitude than $H1$ by clause (b) of V3. In this case $H3_T \vdash H2_T$ and $H2_T \vdash H1_T$ and $H1_{\mathrm{BF}} \vdash H2_{\mathrm{BF}}$ and $H2_{\mathrm{BF}} \vdash H3_{\mathrm{BF}}$ and $H3_{\mathrm{BF}} \nvdash H2_{\mathrm{BF}}$ So $H3_T \vdash H1_T$, $H1_{\mathrm{BF}} \vdash H3_{\mathrm{BF}}$ and $H3_{\mathrm{BF}} \nvdash H1_{\mathrm{BF}}$. So by clause (b) of V3, $H3$ has more verisimilitude than $H1$.

Furthermore it should be the case that for any true theories $H2$ and $H1$ if $H2$ is stronger than $H1$ then $H2$ has more verisimilitude that $H1$. The following proves a limited version of this result. In the proof we use $\ulcorner \wedge\alpha \urcorner$ to signify the conjunction of all the members of $\alpha$.

**Theorem 4.** For any $H2$ and $H1$, if $H2 \vdash H1$ and $H1 \nvdash H2$ and $H1$ and $H2$ are true and finitely axiomatizable then $H2$ has more verisimilitude than $H1$

*Proof.* Suppose $H2 \vdash H1$ and $H1 \nvdash H2$ and $H1$ and $H2$ are true and finitely axiomatizable. Since $H2 \vdash H1$, $H2 \vdash H1_T$. Now consider $\wedge H2$. Since $\wedge H2 \dashv\vdash H2$ and $H1 \nvdash H2$, $H1 \nvdash \wedge H2$. Gemes (1997) demonstrated that logical equivalents have the same content parts and every wff is a content part of itself, so $\wedge H2$ is a true content part of $H2$, so $\wedge H2 \in H2_T$. So $H1_T \nvdash H2_T$. And since $H2$ is true $H2_{\mathrm{BF}}$ is empty, so trivially $H1_{\mathrm{BF}} \vdash H2_{\mathrm{BF}}$. So $H2 \vdash H1_T$, $H1_T \nvdash H2_T$ and $H1_{\mathrm{BF}} \vdash H2_{\mathrm{BF}}$. So by clause (a) of V3, $H2$ has more verisimilitude than $H1$.

Provided the following plausible lemma is true

**Lemma 1.** Where true $T2$ is stronger than true $T1$, there is some true $\alpha$ such that $\alpha$ is a content part of $T2$ and $T1 \nvdash \alpha$,

the above proof can be generalized to include non-axiomatizable theories.

While V3, presumably, has its own drawbacks, it is, I believe, the best available definition of verisimilitude that keeps to the spirit of Popper's original definition.

## 7. A COMPARISON WITH THE SCHURZ-WEINGARTNER SOLUTION

Before concluding it is worth comparing the revision proposed above to Popper's original definition of verisimilitude to that proposed by Schurz and Weingartner.[7] Their approach, like that used above, basically rests on restricting the truth (falsity) content of a proposition to a subset of its true (false) consequences. To do this they first define a notion of relevant consequence, which like our notion of content is a partition on the consequence class relation. According to their definition,

> $\alpha$ is a relevant consequence of $\beta$ iff $\beta \vdash \alpha$ and there is no n-ary predicate in $\alpha$ replaceable in some of its occurrences in $\alpha$ by a predicate of the same arity (not occurring in $\beta$ or $\alpha$) saliva validitate of $\beta \vdash \alpha$.

Schurz-Weingarnter definition then introduces the notion of relevant consequence elements as follows:

> A is a relevant consequent element of T iff A is a relevant consequence of T and there exists no set of wffs $\alpha$ such that A is logically equivalent to $\alpha$ and the conjunction of all the members of $\alpha$ is a relevant consequence of T and each member of $\alpha$ shorter than A.

These definitions allow for the definition of what is essentially the basic truth and falsity content of a theory. Thus the basic falsity content of a theory $T$, $(T)_{rf}$ is the set of all relevant consequent elements of $T$ which are false, and the basic truth content of $T$, $(T)_{rt}$, is the set of all relevant consequent elements of $T$ that are true.

For reasons similar to those noted in Section 5 above the Schurz-Weingarnter definition does not proceed in terms of set inclusion but proceeds by use of the notion of logical entailment as follows,

V4 Assuming that truth-content and falsity-content of two theories $H1$ and $H2$ are comparable, $H2$ has more verisimilitude than $H1$ iff

(a) $(H2)_{rt} \vdash (H1)_{rt}$ and $(H1)_{rt} \nvdash (H2)_{rt}$ and $(H1)_{rf} \vdash (H2)_{rf}$

or

(b) $(H2)_{rt} \vdash (H1)_{rt}$ and $(H1)_{rf} \vdash (H2)_{rf}$ and $(H2)_{rf} \nvdash (H1)_{rf}$

V4 is effective in eliminating many of the counter-examples eliminated by V3. However, the Schurz/Weingartner V4 produces the following questionable results:

(1) where '$c$' refers to William Jefferson Clinton and '$P$' stands for 'was at sometime President of the U.S.A.', '$Pc$' does not have more verisimilitude than '$\sim Pc$'. This follows because '$(\exists x)\sim Px$' is a true relevant consequence of '$\sim Pc$' but not of '$Pc$'. More generally, they produce the result that for any atomic wff and its negation, neither has any more verisimilitude than the other if there are two objects in the domain of discourse such that one satisfies the predicate in the atomic wff and one does not.
(2) where wff $\alpha$ is true and wff $\beta$ is false, $\ulcorner\alpha\equiv\sim\beta\urcorner$ has no more verisimilitude than does $\ulcorner\alpha\equiv\beta\urcorner$. This follows because where wff $\alpha$ is true and wff $\beta$ is false, $\ulcorner\alpha\text{v}\sim\beta\urcorner$ is a true relevant consequence of $\ulcorner\alpha\equiv\beta\urcorner$, but not of $\ulcorner\alpha\equiv\sim\beta\urcorner$.
(3) where every object in the domain of discourse has property M and the domain of discourse contains at least two objects, the wff '$(x)Mx$' does not have any more verisimilitude than does the wff '$(\exists x)\sim Mx$', since '$Ma\supset(\exists x)(x\neq a)$' is a true relevant consequence of the later but not the former.

None of these results obtain where truth and falsity content are defined using the D2 notion of content. Result (1) does not hold because '$(\exists x)\sim Px$' is not a content part '$\sim Pc$; those relevant model of '$(\exists x)\sim Px$' which assign '$Pc$' the value true cannot be extended to relevant models of $\sim Pc$'. Result (2) does not hold because $\ulcorner\alpha\text{v}\sim\beta\urcorner$ is not a content part of $\ulcorner\alpha\equiv\beta\urcorner$; those relevant models of $\ulcorner\alpha\text{v}\sim\beta\urcorner$ that assign $\alpha$ the truth value $T$ and $\beta$ the truth value $F$ cannot be extended to a relevant model of $\ulcorner\alpha\equiv\beta\urcorner$. Result (3) does not hold because '$Ma\supset(\exists x)(x\neq a)$' is not a content part of '$(\exists x)\sim Mx$'; any relevant model of '$Ma\supset(\exists x)(x\neq a)$' which assigns '$a=b$' the value $F$ cannot be extended to a relevant model of '$(\exists x)\sim Mx$' because no relevant model of '$(\exists x)\sim Mx$' makes assignments to '$a=b$' since '$a=b$' is not relevant to '$(\exists x)\sim Mx$'.

Now suppose our domain of enquiry contains at least three objects and every object in the domain that has property $R$ has property $B$, and there are some objects that have property $R$ and some that lack property $B$. Then, both our V3 and the Schurz/Weingartner V4 produce the result that the true statement '$(x)(Rx\supset Bx)$' does not have more verisimilitude than it's negation. This follow since both '$(\exists x)Rx$' and '$(\exists x)\sim Bx$' are true content parts and relevant consequences of '$(\exists x)(Rx \,\&\, {\sim}Bx)$', but not of '$(x)(Rx \supset Bx)$'. This is an arguably sound result.[8] However the Schurz/Weingartner solution also has the unsound result that, relative to the domain described above, '$(\exists x)(Rx) \,\&\, (\exists x) \sim Bx \,\&\, (x)(Rx \supset Bx)$' does not have more verisimilitude than does '$(\exists x)(Rx \,\&\, {\sim}Bx)$'. This follows because '$(Ra \,\&\, Ba \,\&\, {\sim}Rb \,\&\, {\sim}Bb) \supset (\exists x)(x \neq a \,\&\, x \neq b)$' is a true relevant consequence of '$(\exists x)(Rx \,\&\, {\sim}Bx)$', but not of '$(\exists x)Rx \,\&\, (\exists x) \sim Bx \,\&\, (x)(Rx \supset Bx)$'. V3 does not produce this questionable result since '$(Ra \,\&\, Ba \,\&\, {\sim}Rb \,\&\, {\sim}Bb) \supset (\exists x)(x \neq a \,\&\, x \neq b)$' is not a content part of '$(\exists x)(Rx \,\&\, {\sim}Bx)$'. It may be alright to say that, typically, true law-like universal statements do not have any more verisimilitude then their negations, on the grounds that, typically, their negations entail true existential claims not entailed by the law-like statements. However it seems unacceptable to claim that the conjunction of a true law-like universal statements and the true existential content parts of its negation does not have greater verisimilitude than the negation of the true law-like statement.[9]

## NOTES

[1] Presumably Popper also takes this to be an adequate account of the content of theories as well as statements. Note, for our purposes it makes no difference whether a theory is taken to be (i) any set of statements, (ii) any finite set of statements, or (iii) any set of statements closed under the relation of logical consequence.

[2] Actually Carnap typically identifies the content of a statement $\alpha$ with the class of $\alpha$'s non-tautologous consequences, Cf. Carnap (1935, 56). Sometimes Popper also excludes tautologous consequences, for example, Cf. Popper (1959, 120).

[3] Where $H1$ is a finitely axiomatizable theory we can understand '${\sim}H1$' as the negation of the conjunction of all the axioms of $H1$' where $H1$' is a finite axiomatization of $H1$.

[4] In fact, I think that in some cases it is arguable that a true claim does not have more verisimilitude than its negation. Thus suppose that the atomic sentences '$p$', '$q$', '$r$', and '$s$' are true and '$t$' is false. Then, arguably, the true disjunction '$({\sim}p \vee {\sim}q \vee {\sim}r \vee {\sim}s \vee {\sim}t)$', does not have more verisimilitude than its negation.

[5] Niiniluoto (1998) provides a good summary of some of the recent attempts at finding a viable definition of verisimilitude.

[6] In the case of languages with an identity operator a slightly more complicated definition of relevance is needed: An atomic wff $\beta$ is relevant to wff $\alpha$ iff for some partial interpretation $P$, every full interpretation $P'$ that is an extension of $P$ is a model of $\alpha$ and each such extension of $P$ assigns the same value to $\beta$, and there is no proper sub-interpretation $P''$ of $P$, such that for every full interpretation of $P'''$ that is an extension of $P''$, $P'''$ is a model of $\alpha$. Under this definition $p$ is relevant to $p \vee q$. To see this let $P$ be the interpretation that assigns $p$ the value $T$ and makes no other assignments. Clearly every full interpretation which is an extension of $P$ is a model of $p \vee q$. Now there is only one proper subinterpretation of $P$, namely the

null interpretation which makes no assignments whatever. But clearly not every full interpretation of the null assignment is a model of $p \vee q$, since any full interpretation that assigns both $P$ and $Q$ the value $F$ is an extension of the null assignment but is not a model of $p \vee q$. For a more extensive examination of the notions of relevance and content for languages containing an identity operator see Gemes (1997).

[7] See Schurz (1991) and Weingartner and Schurz (1987).

[8] But then perhaps we should reconsider our objection (1) above.

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School of Philosophy
Birkbeck College
University of London
London, WCIE 7HX
UK
E-mail: k.gemes@bbk.ac.uk