REINHARD KAHLE and PETER SCHROEDER-HEISTER

INTRODUCTION: PROOF-THEORETIC SEMANTICS

According to the model-theoretic view, which still prevails in logic, semantics is primarily denotational. Meanings are denotations of linguistic entities. The denotations of individual expressions are objects, those of predicate signs are sets, and those of sentences are truth values. The meaning of an atomic sentence is determined by the meanings of the individual and predicate expressions this sentence is composed of, and the meaning of a complex sentence is determined by the meanings of its constituents. A consequence is logically valid if it transmits truth from its premisses to its conclusion, with respect to all interpretations. Proof systems are shown to be correct by demonstrating that the consequences they generate are logically valid. This basic conception also underlies most alternative logics such as intensional or partial logics. In these logics, the notion of a model is more involved than in the classical case, but the view of proofs as entities which are semantically dependent on denotational meanings remains unchanged.

Proof-theoretic semantics proceeds the other way round, assigning proofs or deductions an autonomous semantic role from the very onset, rather than explaining this role in terms of truth transmission. In proof-theoretic semantics, proofs are not merely treated as syntactic objects as in Hilbert's formalist philosophy of mathematics, but as entities in terms of which meaning and logical consequence can be explained.

The programme of proof-theoretic semantics can be traced back to Gentzen (1934). Seminal papers by Tait, Martin-Löf, Girard and Prawitz were published in 1967 and 1971.[1] An explicit formulation of a semantic validity notion for generalized deductions with respect to arbitrary justifications was given by Prawitz (1973). Much of the philosophical groundwork for proof-theoretic semantics was laid by Dummett from the 1970s on, culminating in Dummett (1991). Martin-Löf's type theory, whose philosophical foundation is proof-theoretic semantics, became a full-fledged theory in the 1970s as well (see Martin-Löf 1975, 1982). The term "proof-theoretic semantics" was proposed by the second editor in a lecture in Stockholm in 1987.[2]

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Since proof-theoretic semantics has reached some status of maturity, we considered it appropriate to organize a conference with that title at the University of Tübingen in January 1999.[3] The papers presented at this conference were the following:

- Dag Prawitz: Meaning explained in terms of proofs: A comparison of some approaches
- Lars Hallnäs: Defining the semantics
- Patrizio Contu: The justification of the logical laws revisited
- Gabriele Usberti: Towards a semantics based on the notion of justification
- Michael Dummett: Reply to Warren Goldfarb
- Göran Sundholm: Inference *versus* consequence
- Roy Dyckhoff: Permutation-free sequent calculi
- Jörg Hudelmaier: A semantical sequent calculus for intuitionistic logic
- Robert Stärk: Proof-theoretic semantics of logic programs
- Grigori Mints: Partial proofs and cut introduction
- Per Martin-Löf: The distinction between sense and reference in constructive semantics
- Kosta Došen: Models of proofs
- Peter Schroeder-Heister: Frege's sequent calculus
- Reinhard Kahle: A proof-theoretic view of intensionality
- Michael Rathjen: The role of ordinals in proof theory
- William Tait: Beyond the axioms: The question of objectivity in mathematics

The present collection grew out of this conference but is not intended as a volume of proceedings. Our idea was, by means of various basic papers, to shed some light on central topics of proof-theoretic semantics to enable researchers from other branches of logic to gain some insight into a subject which we think has a bright future.

The first topic of these papers are approaches giving proofs a semantic value without reference to denotations: Prawitz philosophically elucidates his meaning theory based on proofs, and Schroeder-Heister, Contu and Hallnäs deal affirmatively and critically with validity notions developed in the tradition created by Prawitz. Tait, in a type-theoretic framework, shows that a non-denotational approach does not necessarily lead to non-classical (intuitionistic) logic. Then there are contributions which reflect on the framework in which proofs should be dealt with: Sundholm compares different forms of natural deduction from a

meaning-theoretic point of view, and Došen puts forward categorical logic as a framework particularly appropriate for proof-theoretic semantics. Two papers develop applications: Kahle uses proof-theoretic semantics in order to clarify the notion of necessity, while Usberti carries over proof-theoretic semantics to the justification of empirical sentences. Finally we have two contributions dealing with the background to proof-theoretic semantics: Mints presents some basic ideas of Russian constructivism, and Rathjen gives an overview of theories of ordinals which have dominated proof theory for quite some time.

Due to various circumstances, editing this collection stretched over a period of several years. We received the first manuscripts in 1999 and the last update of a paper in 2004. We apologize for this delay to those authors who submitted their contributions early.

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NOTES

[1] See Tait (1967), Girard (1971), Martin-Löf (1971), Prawitz (1971).

[2] First in press in Schroeder-Heister (1991). Whether this term had already occasionally been used in Stockholm at that time he cannot recall, although he does not want to rule this out. – As early as 1968 Kutschera used the term "Gentzen semantics" ["Gentzensemantik"] (see Kutschera 1968).

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Reinhard Kahle
Departamento de Matemática
Universidade de Coimbra
Apartado 3008
P-3001-454 Coimbra
Portugal
E-mail: kahle@mat.uc.pt

Peter Schroeder-Heister
Wilhelm-Schickard-Institut
Universität Tübingen
Sand 13
72076 Tübingen
Germany
E-mail: psh@informatik.uni-tuebingen.de