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MODELING CRACKS AND CRACKING MODELS: STRUCTURES, MECHANISMS, BOUNDARY CONDITIONS, CONSTRAINTS, INCONSISTENCIES AND THE PROPER DOMAINS OF NATURAL LAWS

ABSTRACT. The emphasis on models hasn't completely eliminated laws from scientific discourse and philosophical discussion. Instead, I want to argue that much of physics lies beyond the strict domain of laws. I shall argue that in important cases the physics, or physical understanding, does not lie either in laws or in their properties, such as universality, consistency and symmetry. I shall argue that the domain of application commonly attributed to laws is too narrow. That is, laws can still play an important, though peculiar, role outside their strict domain of validity. I shall argue also that, by way of a trade-off, while the actual domain of application of laws should be seen as much broader. At the same time, what I call 'anomic' representational elements reveal themselves as central to the descriptive and explanatory power of theories and models: boundary conditions, state descriptions, structures, constraints, limits and mechanisms. I conclude with a brief consideration of how my discussion has consequences for discussion of understanding, unification, approximation and dispositional properties. I focus on examples from physics, macroscopic and microscopic, phenomenological and fundametal: shock waves, propagation of cracks, symmetry breaking, and others. This law-eccentric kind of knowledge is central to both modeling the world and intervening in it.

1. INTRODUCTION

A law of nature is, in the broadest sense, a general relation that holds between physical quantities, or properties of physical objects or of systems of a certain kind. If one asks for examples in physics, common answers are Newton's laws of motion, the law of conservation of energy, Schrödinger's equation for quantum wave functions, Maxwell's laws of electromagnetism, Hooke's law of elasticity, Boyle's law of gases, Snell's law of optical refraction, and Stokes' equation of motion of fluids.

Philosophers of science have commonly attributed to natural laws a number of characteristic properties: one is their universal validity;

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another is their accuracy, as they consistently and accurately describe and predict the facts; another is their power to explain those facts, whether or not the laws describe causal relations; and another is the role of natural laws in our conceptual understanding of actual properties and dispositions of things. It has also prevailed in the work of philosophers such as Fred Suppe, Nancy Cartwright and Ron Giere, who argue that the application of scientific theories and their laws to the world is mediated by idealized representations called models. In this they partially conform to the semantic view of theories, which attaches laws to models and treats models as the primary form of theoretical knowledge.[1] Incidentally, some philosophers speak of models in relation to phenomena or real systems and events, and others in relation to data.[2] The distinction between phenomena and data will be of little relevance to my discussion.

The emphasis on models hasn't eliminated laws from scientific discourse and philosophical discussion. Even in Cartwright's and, especially, Giere's account, the role of law is constitutive; their account of models is law-centered. Instead, I want to argue that much of physics is not in the laws. Elsewhere I have argued that a lot of physical understanding comes, in an abstract form, in preconditions of mathematical formalisms, and, in a more concrete form, in the interpretation of formalisms over and above their application in specific theories/laws or to specific phenomena.[3] I have argued also that even in law-centered or nomic concrete physical modeling, inconsistencies of different kinds have important heuristic and cognitive consequences.[4] In this paper I shall continue this exploration further and focus on the role of laws in the characterization of models of real systems and phenomena in order to highlight the cognitive value of 'auxiliary' elements in mathematical representation. I shall argue that in important cases the physics, or physical understanding, does not lie either in laws or in their properties, such as universality, consistency and symmetry. I shall argue that the domain of application commonly attributed to laws is both too broad and too narrow. That is, laws can still play an important, though peculiar, role outside their strict domain of validity. I shall argue also that, by way of a trade-off, while the actual domain of application of laws should be seen as much broader, in the broader domain of application their validity, centrality and consistency may be seriously compromised.

At the same time, what I call 'anomic' representational elements reveal themselves as central to the descriptive and explanatory power of theories and models: boundary conditions, state descriptions, configuration of parts, constraints and mechanisms.[5] In my view models can be either mediators – between theory and phenomena – or the ultimate representation. I hesitate to refer to representation. I will not consider explicitly prima facie non-representational elements in theories or models. But, at the end of the day, I would like my account to accommodate modes of using physical formulations that might be non-representational. Insofar as physical formulations are about something, one may talk, more loosely, of *characterizations*, which might include elements of representation, elements of prediction, or elements serving more pragmatic aims.[6] With reference to the specific case of models, from the claims above I shall argue that laws cannot always specify completely and consistently what models should describe and, connectedly, that laws might not be true of the relevant kind of models either. I shall argue that an adequate account of successful science needs notions of models that can accommodate and take seriously the relevant characterization of phenomena or reality that anomic elements offer.[7] I will conclude with a consideration of how my discussions has consequences for discussions of unification, approximation and dispositional properties.

I will focus on examples from physics, macroscopic and microscopic, phenomenological and fundamental. All are the topic of the accounts of models and laws. Given the common philosophical bias towards placing stakes on the most fundamental level of physical theory, my examples will include fundamental theories. Examples from non-physical sciences also abound; this kind of scientific characterization is becoming even more and more central to the kind of understanding provided by such sciences, as they have motivated forms of explanation that replace accounts motivated by applications to physics. But such focus on those sciences erects a misleading barrier between them and physics. To examine the place of this 'law-eccentric' kind of knowledge in actual physics is, then, even more important now. This kind of knowledge or characterization is central to both understanding the world and intervening in it. I am suggesting something of a gestalt shift in our philosophical attention. Our understanding of science needs to change in order to accommodate many kinds of successful practices.

2. LAWS AND MODELS

Laws have been understood in a number of ways. But the most popular interpretation since Hume and well into the twentieth century – with Hempel – is the regularity view: laws describe empirical regularities in matters of fact. All these views share one assumption: natural laws are universally valid; that is, that scientific knowledge applies to the world in general and that laws apply to the behavior and properties of all entities of any given kind and in every situation, without exception.

Part and parcel of the attack against logical empiricism in the 1960s was precisely an attack against the alleged universality of scientific laws. If laws truly describe the facts, the regularities they describe cannot be universal. For philosophers such as Michael Scriven the point was uncontroversial. Thus he began the essay 'The Key Property of Physical Laws – Inaccuracy' with the remark that 'the most interesting fact about laws of nature is that they are virtually all known to be in error.'[8] Laws have exceptions. We accept scientific laws as true in the sense of the simplest useful approximation to the actual facts. Scriven's view is a defense of a cognitive and pragmatic balance between restricted generality and descriptive or predictive power. This raises the question, how can natural laws be broken? For the broadest possible domain we decide the range of goodness of fit or approximation that meets our demands and purposes. But this decision is not epistemically or theoretically arbitrary. The willingness to accept a degree of approximation has to match our willingness to explain the deviation from whatever might count as the strict truth. Therefore, mere approximation doesn't guarantee appropriateness. On similar epistemic and pragmatic grounds one may argue also that for theoretical approximations represented explicitly by a law, it is the case both that not all approximations constitute violations, and that not all violations can be reduced to approximations. Alternatively, a new notion of approximation must be introduced.

A similar view has been defended recently by Marc Lange in order to justify talk of laws, however restricted, in biology. For the argument to work lawfulness should not require universality.[9] So he defends that the laws of physics, less controversial, can be both restricted and lawful. His example is Hooke's law of elasticity. In the simplest and original formulation, Hooke's law asserts that upon a body attached to a spring a restoring force is exerted proportional to the displacement of the spring from its initial equilibrium length,

$$F = -kx,$$

where the constant of proportionality depends on the specific material and design of the spring. For Lange Hooke's law is a law because within its limits, for any given amount of empirical data, it can support counterfactuals, and it is the simplest, broadest and most reliable rule of inference about the elastic behavior of springs.

In general, however, the domain of strict truth or truth-preserving equations is too narrow: the higher-order form of the behavior of actual materials – in terms of x^2, x^3, and so on – varies too widely from one to another. The price of literal truth about springs, if available, is then too many laws with too narrow domains. As in Scriven's view, there is a balance between scope and accuracy. Each particular project has to set what degree of approximation or accuracy is tolerable or desirable in correspondence with the operative dimension or function of the characterization at hand. In any case, the laws can be more or less approximately true, but they are not universal.

Nancy Cartwright addressed the issue in her controversial book *How the Laws of Physics Lie.*[10] There she claimed that there is never a good fit between our mathematical theories and the complex messy real world. The phenomena we observe are at best covered by 'ceteris paribus' generalizations, that is, generalizations that hold only under special conditions. When these conditions are required to be absent, it is more accurate to speak of 'ceteris absentibus' generalizations. If such qualifications are introduced, laws can retain their lack of exceptions. But there is a cost. This reformulation restricts their domain of validity. Cartwright focuses on interfering factors. But the account can be trivially extended to include critical conditions. The relevant conditions are ideal conditions that are most often realized in highly controlled experimental settings. In such settings possible interfering factors do not usually occur. But outside such situations the conditions are not typically satisfied.[11]

Stuart Glennan has argued that non-fundamental laws and regularities in the behavior of systems can be explained in terms of underlying causal mechanisms below I will challenge this view both as law-centered and ultimately law-reductionistic.[12] An alternative would be to claim that laws describe possible worlds or even Platonic universals, although the latter view, especially, sheds little insight into the ins and outs of scientific practice and has very little currency among philosophers of science.[13]

Now, if our simple fundamental and empirical laws are at best approximately true, and if taken universally, literally false, what are they true of? As law statements they must be a function of some description.[14] Scriven, Cartwright and also Ron Giere agree that laws describe scientific models. Cartwright emphasizes the case of fundamental laws, although she would not claim that all successful models and explanations must satisfy available laws.[15] Scriven and Giere, and more strict proponents of the semantic view are more radical. For them both fundamental and more phenomenological laws, no matter how complicated, describe scientific models. Models are fictional descriptions of ideal cases. Scriven acknowledges that Boyle's law of gases is true only for ideal gases. And Giere argues that the law of the pendulum, or a spring, does not represent truly the behavior of actual systems.[16] But models also tie laws and theories to reality. When we, or scientists, say that theories or laws apply to reality or explain phenomena we mean that laws are literally true of models and then that models bear a certain degree of similarity in certain respects with the more complex real systems. To say that a law is approximately true of entities in a controlled experimental setting is to say that the experimental situation bears enough resemblance to the ideal case or model. Alternatively, Suppe argues that the link of experimental phenomena and the theory is a counterfactual interpretation of the ceteris paribus or idealization conditions.[17]

In all these views models are nomic or law-centered, and, more strongly still, laws are constitutive of models. This law-centered view, so pervasive in the literature, is what I want to challenge. Given the exhaustive character of the views I am criticizing, my examples will include phenomenological laws.

3. DOMAINS AND INCONSISTENCIES

To make my claims more precise and clear I want to distinguish eight types of domains (scopes) of laws. Most generally once can speak of their *domain of validity* and their *domain of application*. The domain of application is the domain of systems or phenomena the law is said to characterize, or to be about and to which the law is applied by playing some role in the attainment of, for instance, theoretical, heuristic and cognitive outcomes. The domain of validity is the proper domain of laws in which they are said to be instantiated or satisfied.[18] Some types of domains can be either strict or approximate. And in turn, each type can be either theoretical or real.[19]

1. The *strict real* domain of Hooke's law, say, is the domain of real systems that not only don't yield or fracture, but obey the linear proportionality law perfectly. This is the domain that perfectly accurate laws would have. But Scriven, Lange and Giere agree that perhaps no physical law is accurate in that sense.
2. The *strict theoretical* domain of Hooke's law is the set of descriptions or fictions that satisfy and illustrate the law perfectly; by construction, as Giere says. This is what proponents of the semantic view claim is the domain of models or ideal cases. This is the strict domain of validity.
3. The *approximate real* domain is the collection of real materials that obey non-linear laws of elasticity between stress and strain with Hooke's law as a first-order approximation for very small strains. This is an example of what Scriven, Lange and others consider the real domain of most laws. This is the approximate domain of validity.
4. The *approximate theoretical* domain would be the collection of models satisfying the law within a certain degree of approximation and describing more accurately systems in the approximate real domain.

Both theoretical domains lie within of the *constitutive domain* or framework of conditions of their conceptual possibility. In the narrowest case, the constitutive framework is set by Hooke's law and the precondition of continuity. Here one can speak also of the role of law that Pap has called functional *a priori* and analytic, namely, as a definitional criterion, in this case of elasticity.[20]

Finally, one can speak of a *critical* or *boundary domain*. This is where validity and application conflict or, equivalently, their respective domains no longer intersect other than at the very boundary, however fuzzily construed, of the domain of validity.[21] Some of the examples I will mention fall within this domain.

To speak of lying outside a particular type of domain can be equivalent to speaking of inconsistency. One may distinguish five main types of inconsistency affecting a theory, three external and two internal:

The three external types of inconsistency are: (1) inconsistency between two competing theories, (2) inconsistency between a theory and other principles or theories at work in the same domain of application, and (3) inconsistency between a theory and the model of phenomena or the data it is meant to capture. Two types of internal inconsistency are: (4) inconsistency between two statements, or elements, of the theory and (5) inconsistency between a statement of the theory, such as a law, and a precondition of its validity or formulation.[22] In some cases I will present the evolution of the system under a law yields states of the system which introduce inconsistencies in the model. I refer to such types of cases as instances of dynamical inconsistency.

Inconsistency of types (3) and (4) involves falling outside, at least, the strict real domain of validity. Inconsistency of type (5) involves falling outside the strict theoretical domain and a constitutive domain. In as much as some of my examples involve inconsistencies here I draw conclusions that are not those typical of the literature on inconsistencies. The latter emphasizes the heuristic value of inconsistencies in terms of computational value or suggestion of a consistent successor, or the need for a new logic or the isolation of consistent subsets of a theory's statements.[23] Instead, I will point to a peculiar role of laws in models and the cognitive role of anomic elements of representation in description and explanation of phenomena.

4. INCONSISTENCIES BETWEEN LAWS AND BOUNDARY CONDITIONS

The first anomic elements of theory or mathematical representation I will consider are initial and boundary conditions. In the mathematical treatment of most phenomena – especially in idealized, modeled form – scientists give more epistemic privilege to equations representing laws than to the context-specific information in the form of initial and boundary conditions in the case at hand. By contrast, a majority of scientists and philosophers alike appear to neglect to give cognitive credit to the contribution to the applicability of such laws from the modeling of the specific circumstances chosen to represent the phenomena at hand.

Still, the most persuasive case for the active cognitive role of boundary conditions in the understanding of the operations of the physics for a given phenomenon is probably to point to situations in which laws and boundary conditions actually conflict. In particular, the laws in those situations happen to be expressed by differential equations. The revealing character of the conflict is an intuition held by mathematicians Courant and Hilbert as well as Hadamard and put to philosophical work especially by Mark Wilson.[24]

The conflict between equations, especially partial differential equations, and their proper boundary conditions stems from a conflict between rigor and applicability of mathematical formalism. In order to link the two, Courant and Hilbert set a standard for when a problem is *properly posed*: 'A mathematical problem which is to correspond to physical reality should satisfy the following basic requirements:

(1) The solution must exist.
(2) The solution should be determined uniquely.
(3) The solution should depend continuously on the data.[25]

Each requirement is in turn duly motivated. The first is the logical demand that the solution should present no inconsistencies in the form of mutually contradictory properties. The second demands that ambiguity be excluded unless it inheres in the physical situation (e.g., degenerate states).[26] The third, of stability, guarantees realisticness with regards observable phenomena. It is essential for the empirical purpose of approximation and effective measurement.

Courant and Hilbert admit, however, that the arsenal of mathematical tools includes cases such as Cauchy's initial value problem for the potential equation, which is not properly posed since it violates the first and third requirements.[27] This is a case famously discussed by Hadamard, who pointed to the possibility of mismatches between differential equations and their proper boundary conditions (the debates on problems raised by the use of partial differential equations date back to the 1840s in connection with potential theory).[28] Moreover, Courant and Hilbert admit that it is a mistake to hold 'properly posed problems are by far the only ones which appropriately reflect real phenomena'[29], pointing to the cases of non-linear phenomena, quantum theory and numerical methods of computation as appropriate for describing reality.

Wilson has also echoed Hadamard's notice and has analyzed the problem in the application of differential equations to a physical system occupying a certain region of space. According to Wilson, the problem arises from a conflict between internal and external requirements upon the boundary enclosing the region. The internal requirements are established by the law governing the behavior of the system in the region. If the system is an iron bar obeying laws of elasticity, its displacement or strain function must be twice differentiable along the boundary in order to satisfy laws involving second derivatives. Or in the case of a bar obeying Fourier's heat equation, the law might demand that the temperature at every point including at opposite ends be uniformly constant. The external requirements contain the prior local knowledge of the system in that region. But these conditions might clash with the uniformly constant solutions to these equations. For instance, the external boundary conditions that set fixed different temperatures of opposite ends of the bar.[30]

Other examples include jumps or discontinuities in external boundary conditions – even if they are internal, in a topological sense and not in the sense above: gaps in velocity, viscosity or pressure distributions violating hydrodynamic laws of flow of a compressible fluid. In many cases the active role of boundary conditions is revealed by the need of defining smooth boundary limits in order to remove the conflict. But the situation is often such that the limit is itself hopelessly riddled with singularities.[31]

The lesson I hope this discussion suggests is twofold. First, with Wilson I want to defend that boundary conditions have a place of their own in our mathematical representation of phenomena. More generally, I want to extend the claim to all sorts of anomic elements in mathematical models of physical phenomena. In the rest of the paper I will provide additional support for my extended claim. Second, the examples above also problematize the law-centered notion of modeling. Models do not always accommodate laws consistently. In the next two sections I will make the case that certain types of inconsistency reveal a peculiar role of laws in modeling phenomena.

5. SELF-DESTRUCTIVE EQUATIONS AND DYNAMICAL INCONSISTENCY: SINGULARITIES, SHOCKS AND CRACKS

In the cases mentioned above the inconsistency involves the boundary conditions falling outside the domain of validity of the law and a domain of constitutivity, namely, the domain in which the continuity requirement is satisfied. The same situation is manifested in several kinds of physical examples: shocks, cracks and cosmological singularities. Here I would like to add that they illustrate a type of inconsistency, namely dynamical inconsistency, which is not brought out merely by a clash with external boundary conditions. In such cases the inconsistency is the product of the action of the dynamical law itself; in Wilson's own term, the equations 'self-destruct'.[32] The formation of black holes in cosmological models of the theory of general relativity is an example from fundamental theory. Here I will focus on the cases of shocks and cracks.

Shock waves are discontinuity curves that appear in fluids described by quasi-linear or non-linear equations with conservation laws expressed by divergence equations.[33] Small solutions develop discontinuities after a finite amount of time. In the case of linear conservation laws, those solutions can be obtained as limits of smooth solutions. The discontinuity depends on the discontinuity of the initial state. In non-linear cases the discontinuity appears despite smooth initial conditions. In the case of a small wave separating regions with different entropy values the squared speed – of sound, say – increases with increasing density. As a result, waves in regions of high density travel faster than those in regions of low-density. The wave fronts of one type overtake those of the other and form, irreversibly, a singular front separating regions of different pressure, velocity, density and entropy. In non-linear cases the dependence of solutions from initial conditions is destroyed and the unique solution that describes the physics of the system needs to be picked up by supplementing the theory with the domain of a new framework, for example, by adding a thermodynamic entropy condition, or replacing the differential equations with integral equations as fundamental.

The example of fracture of materials, in intuitive detail, will help readers with little technical inclination, or background. The first concept is the concept of stress. Imagine a block made out of rubber attached to the surface of a table. If we apply a force or a weight to any side, the stress it is subject to is the amount of weight, the load, per unit area, that is the ratio of load to surface area of the side,

$$s = \text{load}/\text{area} = P/A.$$

If the load acts in the direction of the block, by pushing, the stress is a pressure; if it is in the opposite direction, by pulling, it is a tension. Stress is then a measure of how hard or with how much force a surface element of matter anywhere in the system is being pushed or pulled in any direction.

The next concept is strain. When we push or pull the block of rubber it suffers a compression or an extension. Remember the particular case of the spring. Strain is the amount it can be extended per unit length. In other words, it is the ratio of the displacement in the spring or block under a given load to the original length in any direction,

$$e = \text{extension}/\text{original length} = l/L.$$

Now we can formulate the general expression of Hooke's law of elasticity. Hooke's law holds that changes in stress are directly proportional to changes in strain,

$$s = Ee,$$

where the constant of proportionality, E, is a property peculiar to each material. It is a measure of the elasticity, or conversely, of stiffness or hardness of the material. The strain is a continuous variable, so Hooke's law requires the system that obeys it to be a perfect continuum.

The hardness of different materials is represented by different stress–strain functions – and different lines in the stress–strain diagrams. But in the case of real materials the lines are not always straight as Hooke's law predicts. At certain levels of stress, the lines curve or bend. That means that the material displays behavior outside the domain of validity of Hooke's law. The violation or failure of the law can have two different physical meanings. If there is a region where the line curves, the material displays plastic yielding or deformation; when the force is discontinued, the material doesn't return to its original size. That's the mechanical property that characterizes the ductility of metals and makes your car relatively safe. If there is a point or stress level at which the line stops and the system looses continuity, the material breaks. This is precisely the mechanical property relevant to the characterization of strength.

Mechanical laws such as Hooke's law can act as constraints but cannot determine the concrete distribution of stresses in a material. Internal structural properties do a lot of work here. My example of a structure or geometrical boundary condition is a crack. A crack concentrates stress. Assume a certain amount of stress applied to a system in a direction perpendicular to the direction of a crack. The effect on the crack is to produce much higher levels of stress at the location of its tip or tips. The longer the crack and the sharper its tip the higher the concentration of stress at the tip. In 1913 the British engineer C.E. Inglis calculated the concentration factor at the tip of a crack. The calculation is based on modeling a crack as the limit of a very narrow elliptical hole in an elastic solid (see Figure 1 a–c).

The concentration factor depends – taking the relevant approximations – on the length of the crack and the shape = radius of curvature – of its tip, $C = 2\sqrt{(L/r)}$.[34] As a consequence, the longer the crack and the sharper its tip the higher the concentration of stress at the tip. In the case of a radius approaching zero the stress goes to infinity. To avoid this problem, the calculation of the stress is performed by integration along a convenient path. Alternatively, the radius is set above a critical minimum dependent on the critical value of stress that causes fracture of the material. Microscopic cracks can multiply applied stresses by a factor of a hundred. Fracture is simply

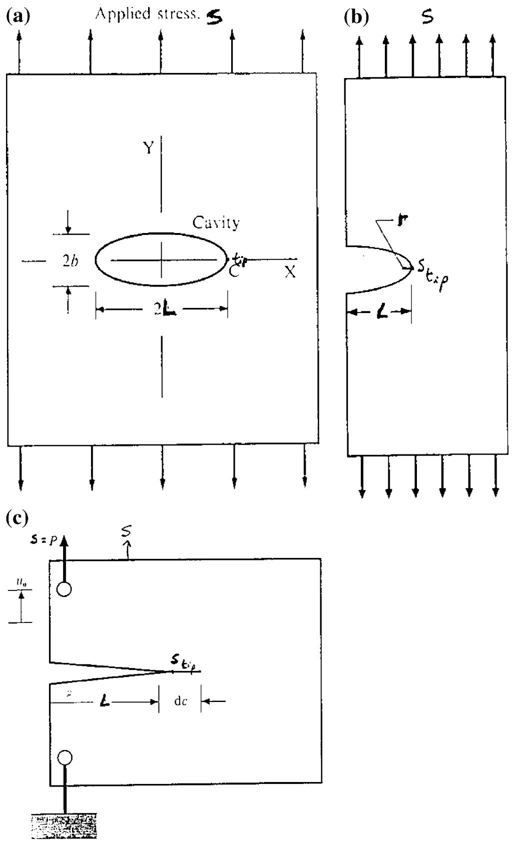

Figure 1.

the inseparable formation and propagation of crack-type structure. Thus, the application of Hooke's law in the presence of the local structural condition leads to the local rupture of the material throwing it outside the validity and constitutive domain requiring continuity.

Naturally the example of cracks further supports my point about the active role of boundary conditions. The geometry sets the descriptive and explanatory focus. It also illustrates my negative claim that a law can violate or be an inconsistent part of a model of the phenomenon. Remember that the model represents a system that cracks up and fractures, but this is a system which is not strictly continuous, as this law implies. And continuity is a constitutive – precondition-property of the law and its reasonable and useful approximations. Moreover, a system that cracks up and fractures is not manifesting, almost by construction, what for Scriven, Cartwright and Giere, is the ideal elastic behavior that the models of Hooke's law would describe. Yet the law is essential to the derivation of the critical properties of strength and fracture that the model generates.

It may be argued that the model can be divided into spatial or temporal parts, some of which obey the law and fall either in its strict or in its approximate real domain, whereas the others clearly do not. This would be an application of the divide-and-conquer strategy suggested by some authors in order to eliminate inconsistencies of theories by isolating contexts with consistent subsets of their claims.[35] But in what sense can some spatial and temporal parts obey Hooke's law? For the spatial parts to obey the law means that for any force they will undergo a linearly proportional displacement at any time; but sooner or later they might fail to do so. For the temporal parts, to say that they obey the law until they don't, is to hold either a vacuous or a peculiar notion about the validity of a law. Ultimately, it could be said that the process of fracture simply modifies the conditions of validity of Hooke's law understood as a ceteris paribus law. The crack brings the system to a level of stress that falls outside the condition of validity of the law.[36] But the distinction between parts that do and parts that don't obey the law simply avoids the issue at hand. From the point of view of explanation and prediction of empirical data, the elements of the model that in some sense obey the law cannot be dissociated from the elements that break it. Hence, the model as a description of the whole of the relevant phenomenon falls within none of the strict domains listed above. Likewise, it cannot be protested that the law describes the phenomenon of its violation by negation; that is, that it is the denial of the law, not-L, rather than L, that appears in the description of the model and explains the phenomenon of fracture. In the next section I will clarify this point further. I will show in my characterization of the dynamical models that Hooke's law is essential not only to the derivation of the critical conditions from which predictions are made but also to the energy-based considerations that explain the very process of fracture that breaks the law.

The role of laws in this domain demands more attention. I believe there exist other cases. For example, in quantum field theory of elementary particles, the theory makes inferences about energy balance in interaction processes involving elementary particles from the law of conservation of energy. Yet in the analyses of many such interactions with perturbation expansions and Feynman diagrams they are understood in terms of the description of so-called virtual particles, which in fact violate the law of conservation.[37] If I am right, the semantic view that considers theories as collections of models and laws as tools to construct models must be modified in order to accommodate my examples. As it stands, it cannot place all models in the strict or approximate domains of validity of laws.

We can say, at least, that the law is representing a phenomenon that extends right outside the bounds of its domain of validity; in this rather vague sense, we may speak of a boundary phenomenon. In addition, and far from raising a problem, in Wilson's terms, of a conflict of 'analysis *versus* geometry', or, more generally, of nomic *versus* anomic parts of the model, the example of cracks already reveals the sense, anticipated above, in which boundary conditions in the form of structures cover up the action of a mechanism. The new role of laws and the importance of mechanism in this context are the subject of the next section.

6. THE ROAD TO STRUCTURE AND MECHANISM (1): FROM INCONSISTENCY TO NEW FRAMEWORKS AND A NEW ROLE FOR LAWS AT THE EDGE OF OLD MODELS

The violation of Hooke's law, rather than its validity, is central to modeling fracture. This is not paradoxical. As a point of conceptual analysis, it is the cracking that is central to grasping the notion of the bonafide property of physical strength. Strength is a measure of the resistance to fracture. It is the stress needed to break an element of material. Strength then is different from hardness. Thus cookies are hard but weak, nylon is not flexible (low value of E) but it is strong, strawberry jelly is both flexible and weak, and steel is both hard and strong.[38] This difference is important to the choice of the kind of material we need for a given purpose. In this section I want to show how in a new domain of validity the violation of Hooke's law illustrates the critical role of a law in its boundary domain in a model of a physical phenomenon.

The central viewpoint of contemporary fracture dynamics is that "the crack is an entity which itself behaves according to a "law" of mechanics expressed in terms of a relationship between driving force and motion."[39] The critical stress criterion is empirically inadequate: the notion that a solid should break at a characteristic critical stress level is not based on a sound physical framework. In this sense the history of the physical understanding of the strength of solids is the history of models placing cracks at the cognitive center, and embedding it in constitutive theoretical frameworks of, for instance, dynamics – based on energy – and thermodynamics – based on energy and entropy – which identify what feature of cracks acts as their 'driving force'.

First I want to introduce the concept of strain energy. Energy is the capacity to do work. Work is done against a force over a distance and is measured by the product of force and distance. Energy can take the form of kinetic energy or energy of motion when we do work, or it can be stored in a system as potential energy. The total amount of energy of an isolated system is always conserved,

$$\text{Energy} = K(\text{Kinetic}) + V(\text{potential}) = \text{constant}.$$

In other words, it cannot be increased or destroyed, only transformed. The work done against the elastic resistance of a spring, when it is strained, is stored up in the spring. This is the strain energy. Strain energy can also be stored as surface energy when work is done against the surface tension that tends to keep the surface of liquids to a minimum, as when we inflate soap bubbles or when drops of water tend to adopt spherical shapes and fuse together.

The measure of strain energy is the area under the stress–strain line,

$$U_{\mathrm{E}} = (1/2)se.$$

For an elastic system obeying Hooke's law, the strain energy is a quadratic function of the stress or the strain,

$$U_{\mathrm{E}} = (1/2E)s^2e = (E/2)e^2.$$

The dynamical models of fracture conceptualize cracks as mechanisms transforming elastic strain energy into energy surface of crack formation.

In 1920 another British engineer, A.A. Griffith, introduced the paradigmatic and first general dynamical framework for models of fracture.[40] Its main notion is energy equilibrium. A crack is a reversible thermodynamic system under the action of conservative forces (derived from a scalar potential function). The new framework remodels the phenomenon of fracture not under Hooke's law, which is violated, but under a more general dynamical principle of equilibrium or energy balance (first law of thermodynamics): In the general case, equilibrium is the configuration in which the total potential energy of the system is stationary, $\mathrm{d}U/\mathrm{d}l = 0$. In a stronger model, the system is said to seek a state of stable equilibrium if it tends toward the configuration that minimizes the total energy of work function of the system at the verge of extension, i.e., in virtual extension, $\mathrm{d}^2U/\mathrm{d}l^2 < 0$. For cases of unstable equilibrium, the same expression is >0.

The crack is the conceptual site of application of the general framework to the phenomenon of fracture and the lynchpin of its dynamical explanation. A crack is not merely a boundary condition but the structural site in which the macroscopic mechanical potential energy, U_{M}, is transformed into crack surface energy, U_{S}, for the work of surface separation – dynamical inconsistency in violation of Hooke's law – in a state of equilibrium. The total macroscopic energy, U, is the sum of external applied load or stress, U_{A}, and the stored elastic strain, U_{E}. So the critical condition that defines crack formation is

$$\mathrm{d}(U_{\mathrm{M}} + U_{\mathrm{S}})/\mathrm{d}l = 0.$$

Then the application of the conditions of equilibrium and of either stability or instability, will provide the critical length of a crack, so that the dynamical framework provides a picture of when, although not how, the crack initiates fracture. The event of shattering is the catastrophic propagation of the crack in unstable equilibrium.

In order to understand the crack as a site of physical activity, this expression can be decomposed in terms of $G = -\mathrm{d}U_{\mathrm{M}}/\mathrm{d}l$ and $R = \mathrm{d}U_{\mathrm{S}}/\mathrm{d}l$. G is the release rate of mechanical energy, and it plays the role of the crack's 'driving force'. R is the intrinsic work. U_{M} is the mechanical energy available, which favors crack extension, so $\mathrm{d}U_{\mathrm{M}}/\mathrm{d}l < 0$; that is, U_{M} decreases with crack extension. By contrast, U_{S} opposes it; it represents a measure of the dynamical cost of fracture. So, $\mathrm{d}U_{\mathrm{S}}/\mathrm{d}l > 0$, that is, U_{S} decreases with crack extension. If the external load is applied in conditions of fixed displacement at the aperture of the crack, U_{E} decreases and the release of elastic strain energy drives the crack. If the applied load is fixed but the displacement is variable, the potential energy decreases and the increase in strain elastic energy drives the crack. The increase or release of elastic strained energy is both based on the assumption that the system obeys a law of elasticity such as Hooke's law, as shown above, or a non-linear approximation, and the assumption that the change in stored energy at the tip of the crack requires fracture, hence violation of the elasticity law (linear or not). This is the critical role that a (Hooke's) law plays in the model. It's a role in its boundary domain, crossing the boundary of its domain of validity and constitutivity – continuum – into its critical domain of application. Interestingly enough, the mechanism of conversion of one kind of work into the other requires a singularity with infinite stress.[41] The singularity is precisely the representation of the physical interaction between the outer linear, say, continuum zone and the inner non-continuum crack tip.

But this *critical* role of laws in a model relies of course on the new framework that generalizes the model under a more abstract principle to, in this case, a dynamical model. The history of fracture theory is the history of the generalization and enrichment of such frameworks. For instance, in 1948, Mott introduced a kinetic energy term that captured the inertial resistance of crack sides at high speeds of propagation. Since Griffith's model the limit velocity of propagation has been associated with the speed of different kinds of elastic waves, from longitudinal elastic waves to Rayleigh surface waves.[42] The nature of this velocity has led to including the possibility of formation of shock waves, which, as I have discussed above, complicates matters even more. In 1948 Irwin introduced a term for energy dissipation extended from surface energy to include plastic deformation. In 1959 Barenblatt replaced the singularities in Griffith's model with the notion of finite cohesive forces in a discrete atomic arrangement at the crack's tip. In 1967 Rice introduced a non-linear integral generalization of the energy-release rate term G, for any linear or non-linear reversible deformation, which allowed for path independence and an integral contour however remote from the tip. A subsequent generalization by Rice brought the model under the second law of thermodynamics and included irreversible deformations. Others have explored more abstract formal mathematical generalizations of variational problems.[43] The alternative strategy has emphasized concrete or enrichment strategies: some models have added bridge principles to capture new phenomena within the same thermodynamical framework; other models have included all sorts of structural situations: dislocations, chemistry of surfaces, microcrack clouds, etc.[44] The role of structures is not exclusively macroscopic – even if unobservable: recent atomic models of fracture and ductility of materials base their predictions on the topological features of the arrangement of atoms in molecules of the material.[45]

To such extended models a new law or principle, or mathematical formalism is central. The main point of this section is that, contrary to what the accounts of models and their satisfaction of laws I discuss at the beginning of the paper, not all laws are central, nor do they play the same role. Models of different types of phenomena, such as fracture, are based on the critical violation of a law.[46] The ones that play this critical role are those that, crucially, often act as concrete empirical laws and bridge principles connecting abstract and concrete descriptions. More strongly, in the case of catastrophic fracture, one can still say that even the dynamical principle that explains the unstable development of fracture is nevertheless explanatory to us. The explanatory role of the more general law extends to its critical domain.

7. THE ROAD TO STRUCTURE AND MECHANISM (2): FROM FRACTURE OF SOLIDS TO SYMMETRY BREAKING; WHAT LAWS CANNOT DO

In this section I want to discuss another critical role of laws. A model may lie within the domain of validity of a law involving a certain variable and yet the cognitive demands of understanding – explanation and adequate description – rest on facts involving the elimination of a property of such law and variable. The property I will focus on is symmetry. I will examine the possibility that important cases, both phenomenological and fundamental, are best understood in terms of structural or state descriptions of broken symmetry and explanatory mechanisms of symmetry breaking.

Early in the 20th century, Pierre Curie put forward the notion that in physical phenomena the symmetry elements of a cause must be found in their effects, but not the converse. More interestingly for my purpose here, he also believed that asymmetry is what creates a phenomenon.[47]

Symmetry breaking can be of at least two kinds, explicit and spontaneous.[48] Assume systems whose dynamics and evolution is expressed by equations of motion involving an energy function such as a Hamiltonian (kinetic + potential energy) or a Lagrangian (kinetic–potential energy). Explicit symmetry breaking may involve an asymmetric term representing the asymmetric contribution of an external factor in interaction with the system. It may also involve the description of interactions that manifest empirical asymmetries (e.g., parity violation). By contrast, spontaneous symmetry breaking involves asymmetries in the states of systems – described by solutions of the dynamical equations or laws – that are not present in the Hamiltonian or Lagrangian functions, or in the equations of motion.

A great number of physical cases of spontaneous breaking of symmetries, classical and quantum, involve critical phenomena. The best known examples are the formation of turbulence in liquids, the transition between liquid and solid crystals, superconductivity, and the spontaneous magnetization of a ferromagnet. These phenomena involve the emergence of new properties by virtue of a change at some level of orderliness in the system. This state of order can be described as a structural property, namely, a special relationship between the components of the system in hand. The disorder–order transition is described by the non-zero value of the so-called order parameter. At the critical value of the order parameter the equilibrium (lowest energy) configuration, or ground state, of a system becomes unstable under small perturbations and a degenerate state, a set of stable but asymmetric solutions all of lowest energy.[49] In finite and infinite systems the degenerate states may be related by discrete and continuous symmetries (or symmetry transformations or mappings).

In the case of a ferromagnet, according to Heisenberg's theory the transition consists in changing from a state in which the orientation of the spins of the electrons is random and shows no privileged direction – and alignment can be induced only by an external magnetic field – , to ferromagnetism, in which all spins are locked into alignment in one direction spontaneously. The transition is described by the values of mean magnetic moment of the atoms in the system. What is important to note in this example is that the law that characterizes the system is in both states rotationally invariant or symmetric, that is, the Hamiltonian, or energy function, $H = -\mathrm{M}\Sigma si.sj$, which describes the interaction between the constituent atomic spins, *si*, in the system is invariant under rotations. Yet, the order parameter is introduced by the fixed orientation of the ordered ferromagnetic state. The magnetized state is not symmetric. The rotational symmetry is said to be broken – law-centered accounts prefer to call it hidden. As Coleman has put it, a man living inside the ferromagnet would fail to detect the rotational invariance of the law.[50] Hence, neither the transition nor this crucial asymmetric property that characterizes ferromagnetism are captured by the law, but rather by a structural description of the state of the system. More generally, according to Anderson and Stein, three kinds of emergent properties can be linked to symmetry breaking: generalized rigidity, new dynamics and order parameter singularities.[51]

The significance of symmetry breaking applies also to systems that are considered fundamental, such as quantum fields.[52] In particular, in the Weinberg–Salam–Glashow model of unified electroweak forces, symmetry breaking explains how the unification both unifies and explains the distinctness of each interaction.[53] Despite the symmetry of the unifying Lagrangian function on which the theory is based – and the corresponding extremum principle from which the dynamical equations of motion derive – it is the asymmetrical states that are crucial to the mechanism of symmetry breaking known as the Higgs mechanism. The Higgs model postulates the action of a Higgs particle/field – Higgs boson – which acts on the ground state of the interaction and explains the differentiation among the fundamental forces of electromagnetic and weak interactions. Specifically, its presence explains the massiveness of some of the so-called gauge vectors – namely, Goldstone bosons – corresponding to the weak interaction (just like photons correspond to electromagnetic interactions) and its short range.[54] But the vacuum (ground state) state is no longer symmetric.[55] In the next section I discuss in what sense the Higgs mechanism might be thought of as a mechanism. Here the negative point regarding the role of a fundamental law and its properties stands: the unifying character of the symmetric law and original state do not bear the explanatory role. The Higgs symmetry-breaking mechanism and the asymmetric state do. The trumpeted unifying and explanatory values of such fundamental laws stand at odds with each other.[56]

8. STRUCTURES AND MECHANISMS

In the preceding sections I have hinted at the role of structures and mechanisms in yielding physical understanding at the expense of the traditional role attributed to laws. In this section I take stock of the previous conclusions and bring them together under a more abstract and general description. To that effect, first I want to establish an operative use of the terms structure and mechanism. As a result, the conclusion will be reasonably more precise and coherent and the examples above will help qualify the validity of some accounts of such notions. I will consider three notions: mechanical, causal and general. *Mechanical:* A narrow literal notion of mechanism appeared in a number of 19th century engineering and mechanics texts in an attempt to ground engineering on solid scientific foundation and, in turn, to ground the general theory of mechanics and of other kinds of

phenomena on concrete mechanistic conceptions. Thus, William Whewell asserts that *machines* are 'combinations of parts, when put together in order to move and produce certain notions, and thus to do work.'[57] Others such as Goodeve and Maxwell follow Whewell, Willis and Ampère in considering any elementary assembly establishing a *connection* among parts for the purpose of transmitting motion, force or energy, more generally, a mechanism.[58] In such a conception of a mechanism only mechanical properties and laws enter its characterization. If, by contrast, the assembly involves no motion and is meant to resist forces, it is a *structure*.[59]
Causal: Glennan has introduced a notion of a mechanical model in terms of a more general mechanism as causal interactions among parts: a mechanism is the explanation of a regular behavior or phenomenon in terms of the interaction of a number of parts according to direct causal laws.[60] All causal laws are explained in terms of underlying mechanisms except for fundamental laws. Cartwright has introduced a more reductive causal notion that is meant to explicate the nomological character of laws that explains regularities. Laws are descriptions of nomological machines, namely, properly shielded arrangements of entities with specific causal capacities.[61] Both views are law-oriented insofar as they explicate laws and, both explain laws and, in Glennan's case, he view reduces in turn reduce mechanisms to them. But they are not lawlike insofar as the mechanisms posited are not automatically replaceable – in their job – by a law.[62]
General dynamical: Machamer, Darden and Craver have introduced a general notion that preserves intuitions about the most basic paradigm cases and captures the use of the term 'mechanism' in many areas of science, even, unlike the previous type, without including an explicit role of laws. While it is quite general, it doesn't capture more pervasive and looser metaphorical generalizations that take up weaker meanings in different contexts. On their view mechanisms are entities and activities organized such that they are productive of regular changes from start or set-up to finish or terminal conditions.[63] Their notion is based on structural properties, namely geometrical properties relating properties, or entities – or their parts.[64]

Mechanisms, thus, are not in some cases totally separable from laws, and for the most part inseparable from structures. So, what are structures? In the typical sense adopted in, say, physical, chemical, biological descriptions, structures are geometrical or topological configurations of a collection of properties, or of entities or their parts. In some cases the elements involved are said to be organized, connected or in interaction: the molecular structure of DNA as modeled in biological descriptions and explanations, and, generally, the structure of molecules featured in chemical descriptions and explanations. In a weaker yet still spatial sense, a field description – a continuous distribution of properties over space – could be thought of as a structure. In the case of a single entity in which the description doesn't involve spatial relations, I prefer to talk, more generally, of a state description. Are there non-spatial structures? Only in the more general sense of entities being organized in space, metaphorically understood. An overly general formal notion that involved simply any mathematical 'structures' would be too weak to do the job in physical descriptions that separates it from that of laws.

It is now clearer in what sense a boundary condition can be a structure in a geometrical sense of a distribution of values in space. So it is clear that a crack is both an internal boundary condition and a structural property of a material or physical system. But a crack is also the site of a mechanism. The geometrical tip of the crack's structure acts as the lynchpin of a process of transformation and transmission of mechanical energies: elastic strain energy stored in the elastic matter around the crack is transformed into surface energy of the crack extension. This case illustrates Lanczos' insight that 'a boundary condition is always a simplified description of an unknown mechanism which acts upon our system from the outside', since 'a completely isolated system would not be subjected to any boundary conditions.'[65] Wilson has used this claim to elevate the physical status of boundary conditions in cases of clashes with differential equations.[66] However, as a matter of principle and practice, I am not prepared to endorse Lanczos' quantifier 'always.'

Both the mechanical and the general notion of mechanisms apply. Of the causal ones, only Cartwright's notion might apply insofar as that process would exercise a causal capacity if the material structure and yield a behavior that we associate with thermodynamical principles. The capacity could also be attributed to the material in a reversed sense, namely, that strength is a capacity or power but, unlike most accounts, not one of action, but rather one of resistance. Glennan's notion does not strictly apply because the connection between the parts is structural and not understood in terms of underlying known causal laws.

In the example of symmetry breaking I think I have established the negative point that it is not the law alone or any of its distinctive properties, such as symmetry, that bears the explanatory role. The importance of the asymmetry points to the importance of the state description as the only bearer of the asymmetry. In the non-fundamental cases, that the state description – e.g., the distribution of oriented spins in a ferromagnet – is a structure seems uncontroversial.

Liu has argued, that in general, at least in the classical case, the state of the system is only apparently asymmetric and that it is in fact one of many ground states associated with a degenerate solution to the equations, and that, taken together, they still manifest the original symmetry.[67] But this is a mere formal trick. I simply don't think that the sum of degenerate states, a formal property, has any relevant physical role in the phenomenon at hand. In the quantum case, the question is whether the unique linear superposition of degenerate asymmetric states (1) is the relevant physical description and (2) can be interpreted differently under different interpretations of quantum mechanics.[68]

The last question I want to address in this section is the sense in which the so-called mechanism of symmetry breaking is indeed a mechanism. Here it is worth mentioning that the phenomenon is also referred to as spontaneous symmetry breaking. In this regard, Liu has argued that there is no causal, or dynamical process in the theory that captures the phenomenon. This is similar to the premise of the original formulation of the measurement problem in quantum mechanics; it led to the introduction of the idea of a spontaneous collapse of the wave function. Liu, who doesn't make the analogy, claims that symmetry breaking is either a mathematical fact or an actual physical event related to the conjunction of the instability of the state of symmetric system and external boundary random fluctuations, with no causal value.[69] In other words, no real mechanism is at work, and the standard term 'mechanism of symmetry breaking' would be a misnomer just as, according to Liu, Coleman and others think that 'broken symmetry' is a misnomer and it'd better be replaced by 'hidden symmetry'.

Liu cannot deny that the focus on broken symmetry is a powerful conceptualization of a number of different phenomena. Thus my negative claim about the role of laws still stands, and so does the positive claim about the cognitive value of the asymmetric state, even as a structural description. It is true that the question of mechanism is far from straightforward. Liu points out that many models of spontaneous symmetry breaking do not involve asymmetric initial or boundary conditions. Just as in the case of a system reaching critical temperature, in the case of a bead sitting at the lowest point on a rotating circular wire, when the wire acquires critical angular velocity, the bead starts ascending and moves from the equilibrium ground state of zero angle or height to a different one that breaks the reflectional symmetry of the original arrangement while preserving the symmetry of the equation of motion. Liu asks whether in a cause or causal explanation can account for this type of process. He distinguishes the case without boundary conditions from the case with them. It is only the former, but not the latter, he adds, that scientists such as Coleman are entitled to consider a model of hidden symmetry. The problem is that Liu's demand is for something in the physical theory that would count as a cause and give the term 'breaking' causal explanatory content. But his causal criterion too narrowly requires a law-like property'.

The model with no boundary conditions or environmental causes would render the critical value of the property a cause without a separate law-like dynamical content within the theory. As in the case of quantum mechanics and the problem of measurement, meeting Liu's request might need to await a more specific causal interpretation of the theory. But the absence of such models doesn't exclude the possibility that, if the 'breaking' metaphor has cognitive value, the critical property must be understood as having a relevant causal role in the process by which the system has the variable change values.[70]

But even without a law for the specific case, the role of external factors cannot be dismissed as unexplanatory. It is in that sense that the role if causal must be understood as providing an explanatory mechanism. A weaker and more formal notion of explanation would of course be in line with my claim about the role of laws.

From a discussion by Balashov (see n. 68) we see that at the fundamental level it is the interaction of the different entities with their respective causal powers that renders the concept of symmetry breaking explanatorily powerful. In this sense, it could be considered a mechanism in the Cartwright sense, except that the law the mechanism here yields is not the symmetric law that unifies the different fundamental interactions, but the nomological explanatory value of the broken symmetry that Balashov explores and which explains at the cosmic level why the phenomena it explains can be postulated everywhere and at all times since the first breaking event. It is a mechanism also in the general dynamical sense. In addition, the field equation for the Higgs bosons in coupling with the other fields is employed as an interaction that allows the explanation of the massiveness and distinctiveness of quanta of weak interaction to go forth. However, it is also a counterexample to Glennan's notion, since the explanatory value of the mechanism does not lie in the underlying causal laws, even fundamental ones, and surely not in a way that preserves the phenomenon to be explained. All the same, I am not prepared to seek the action of mechanisms everywhere. My main point remains the contrast with the alleged emphasis on the action of laws alone as bearers of physical understanding.

Structures and mechanisms, in general, and not the irreducible rule of law alone, whenever present, are genuine and salient bearers of physical understanding in phenomenological and fundamental theory.

9. CONSTRAINTS BEYOND BOUNDARY CONDITIONS: FROM CONNECTED SYSTEMS TO QUANTUM GRAVITY

Many anomic elements in models or theoretic representations may be called conditions but not boundary conditions in the sense discussed above. Constraints are such cases. In a technical sense they appeared with the dynamical formalism of the least action principle. In the modeling of a macroscopic mechanical system, constraints are the connections or fixed relations between elements of a system or their properties at different places or times that enable one to reduce the number of degrees of freedom that describe it.

Thus if connected elements X and Y of a system under study are at a fixed distance, d, from each other, the position of only one, say, x is sufficient, since $y = x + d$. If the system is represented by a set of n configuration variables $\{xi\}$ $(1 \ldots n)$, and one can introduce m constraints $\{cj\}$ $(1 \ldots m)$, the set of generalized variables with be a set $\{qk\}$ with $k = 1 \ldots n{-}m$ variables. The $\{qk\}$, known as generalized variables, allow the system under that description to be subsumed under the Lagrangian or Hamiltonian form of the principle of least action. In a general form the principle, as a variational principle of stable or unstable equilibrium, yields the dynamical law the system's evolution obeys in terms of such controllable variables. The reduced number of variables forms the set known as the generalized variables of the system.

To the generalized variables correspond conjugate generalized momenta if each conjugate pair satisfies Hamilton's equations $\mathrm{d}q/\mathrm{d}t = \{p, H\}$ and $\mathrm{d}p/\mathrm{d}t = \{q, H\}$, in terms of the so-called Poisson bracket $\{A, B\} = (\mathrm{d}A/\mathrm{d}q)(\mathrm{d}B/\mathrm{d}p) - (\mathrm{d}A/\mathrm{d}p)(\mathrm{d}B/\mathrm{d}q)$, so that they determine the set of pairs of canonical variables ($q(t)$, $p(t)$) that characterize uniquely the dynamical trajectory or evolution of the system. The constants of motion satisfy the equation $\{C, H\} = 0$, since $\mathrm{d}C/\mathrm{d}t = 0$. If constraints in the form of relations between q's and p's, $f(q,p) = 0$, exist, they constrain the Hamiltonian and define a new $H' = H + cf(q.p)$. Such constraints, primary constraints, are preserved in time, $\mathrm{d}f/\mathrm{d}t = 0$. They also annihilate the Poisson bracket with H – as well as other functions. But this preservation condition might introduce new constraints independent of the primary ones – if so, they are labelled 'secondary constraints'. In quantum mechanics one must replace the Poisson bracket with the commutation relation, so that for corresponding operators P and Q, $[Q, P] = ih$, that is, they do not commute, except with themselves, $[Q, Q] = 0$ and $[P, P] = 0$. Operators representing constants of motion and constraints commute with the Hamiltonian, $[C, H] = 0$, and obey the equation for the wave function of the quantum system, W, $CW = 0$.[71]

What is interesting about Hamiltonian constraints is that at the cosmological fundamental level the distinction between time-dependent dynamical law and constraint vanishes. The quantized version of the General Theory of Relativity – via quantization of the metric field – is one of the research programs that aim at representing quantum gravity – canonical quantum gravity, the only fundamental force still not unified with the others. But it yields the dynamical equation, known as Wheeler–DeWitt equation, that turns out to be a Hamiltonian constraint equation for the wave function of the universe, $HW = 0$, with no place for time.[72] A distinction between a central law and an auxiliary constraint no longer holds. The constraint has become central.

10. PHYSICS IN THE LIMIT

The last type I want to introduce of anomic elements in theories or models of phenomena is limits. In many cases, taking the limit is a formal step in the mathematical apparatus that makes the theory or a description tractable, for instance, the central limit theorem and the law of large numbers in statistics, or else introduces a technical concept, for instance, that of a tangent. In other, important cases it is not only the law but also the limit that adds the relevant physics that connects with phenomena. Physical limits come in two types: an idealization that simplifies and characterizes the model, e.g., the quantum potential goes to infinity, and one that helps the model capture the description of a certain phenomenon. The latter case can be divided into two subtypes: the statement of a limit that has unproblematic physical meaning, e.g., the limit of velocity going to zero, and the limit that does not, e.g., the infinite-size limits: limit of volume or the number of particles going to infinity. The latter case often turns out to be singular, namely, the limiting value of the corresponding relevant parameter diverges. Each case of physical limit raises its own conceptual considerations.[73]

Note that limits are not to be identified with so-called bridge principles or correspondence rules. The latter connect abstract theoretical terms to more concrete terms describing the phenomena to which the abstract theory is to be applied. They might connect terms from different theories. They form an important class of anomic information. In many cases of reduction and explanation they contain physical information that's missing from they connected theories.[74] Physical limits form a more general class. They also might connect terms from different theories. But also, in the singular cases, they do not connect them at all. Hence, as Batterman insightfully argues, they fail to satisfy the criterion of certain models of reduction and explanation that require strict connection and derivation.[75] I will revisit this issue in the conclusion.

Krieger, Liu and, especially Batterman have focussed on problematic limits and defended their physical significance. My discussion will be cursory. Krieger takes limits seriously because he considers them an illustration of and evidence for his general claim that 'the technical details in the mathematical apparatus, and the mathematical moves employed in doing theoretical physics, are in fact physically meaningful, and not merely devices employed to do some auxiliary work.'[76] Moreover, according to him, such a consideration reveals the more general philosophical dimension of mathematical physics insofar as it points to the essential properties of our descriptions of physical phenomena.[77]

Another example concerns the puzzling infinite-size limit in statistical mechanics in order to recover the description of phenomena delivered by thermodynamics. This is discussed by Krieger, Liu and Batterman (see Appendix). For Batterman, more generally, emergent phenomena can be charcaterized and explained in terms of limits, namely, singular limits. The phenomena explained in such a way are "grounded" in the more fundamental theory but only in the sense that acknowledges the fact that the theory breaks down at the limit.

He then speaks of the explained emergent phenomena as being in the asymptotic domain. Here the limit has the critical role I have indicated in Section 3 above of applying the law of a theory outside its domain of validity. The asymptotic domain is, thus, a subset of the critical or boundary domain of law. Batterman here points to this asymptotic regime as an approximative scheme. But surely this is not the notion of approximation within a framework domain or the approximate strict domain of validity – as is the case for non-linear approximations to Hooke's law. This is a subtype of the type of approximation that involves criticality, namely, crossing the framework's boundary. It is worth noting the distinction between these two kinds. In the second case, only in the asymptotic regime the law is not just broken.

An interesting part of Batterman's account is that the asymptotic domain often falls within the domain of validity of another theory. He has in mind semiclassical theories such as some theories of quantum chaos, partaking of both classical and quantum mechanics, and other hybrid theories such as catastrophe optics, sitting between wave and ray optics.[78]

11. CONCLUSION: PHYSICS BEYOND LAWS

In this final section I will take stock of the main points and suggest some implications that I cannot discuss in detail. Models can be said to mediate the application of abstract theory to phenomena or data, or, alternatively simply to provide their understanding by way of representation or explanation. Some models of data may be said to possess only predictive power. Insofar as each model neglects aspects of what is modeled, it involves an element of idealization. In the last two decades, a number of accounts of models have located laws at their constitutive center, with only the relation between the model and what it meant to represent, explain or predict left as the only problem. I have argued otherwise. The relation between laws and models they help define is itself often problematic. In significant cases there exists a role for laws in those models furnishing understanding of data or phenomena that extends its application beyond the laws' strict and approximate domains of validity.

In such cases neither strict derivation nor subsumption, the once trumpeted law-centered pallbearers of explanation and unification, could be the notions that can conceptualize adequately the critical role of laws. The latter involves their critical or boundary domain. As a result, such notions cannot be said to conceptualize adequately the relation between laws and the modes of understanding, modeling, explanation and unification in which they participate. If the partial role of law in understanding is to be retained, I have suggested that, beyond masking quantitative notions of approximation, alternative notions be considered. I have mentioned more standard notions associated with theoretical and real domains, where the relation of approximation is the one relating the exact linear law, say Hooke's law, and a so-called non-linear approximation. For critical roles of laws, two additional types of approximations can be mentioned. In one type of case, the boundary domain will involve phenomena o models falling outside the strict domains – real and theoretical – of validity. In another type, the model and the phenomena will fall outside the more inclusive constitutive domain as well, for instance, the domain of continuity. Asymptotic explanations that Batterman discusses are of this type.

I have argued not only that laws help model phenomena beyond their standard domains, but also that they fail to bear the full burden of providing understanding of phenomena they help model. I have emphasized the active role of *anomic* elements of models, such as boundary conditions, constraints, structures and some types of mechanisms. In some of the examples above I have shown that these elements are often inter-related. The focus on one rather than the other depends on the relevant level of description and the information available.

Anomic elements challenge the view that applied mathematics is epistemically successful only insofar as it formulates laws of nature. With regards to laws, anomic elements also have in common the extent to which they replace laws and their properties as sources of understanding or other epistemic and pragmatic deliverances. In such cases, however, the benefits are not linked to the property of generality, the formal unifying and explanatory property that is minimally expected from laws. I say 'minimally' because it's the property common to a number of interpretations of laws, including, almost by definition, the most empiricist or Humean.

Anomic elements of theories or models can hardly be distinguished by their general value. As a consequence not all attitudes toward physical formalism will be compatible with their alleged role. Either fictional or more substantial, realist and causal, notions might be preferred. In general, though, any formal or instrumentalist notion of understanding involving derivation alone is relevant. The relevant derivation, however, will in many cases such as Batterman's examples of singular limits not be strict. He speaks of asymptotic explanations. For any given kind of preferred mode of understanding, the judgements on the relevant interpretation of the hypotheses at work should be settled or at least discussed on a case by case basis. With regards to the breaking of the law, I suggest it must be a matter of the data or phenomena falling outside any of the domains mentioned above, along with epistemic and pragmatic considerations about established goals and alternatives.

Anomic elements may contribute to generalized notions such as causality. Law-related notions of causality tend to emphasize power or capacities for action. By contrast, structure-based models of fracture suggest that we model the property of strength in terms of a capacity of resistance to fracture. The same can be said about dispositions, especially when the role of structures is ultimately reduced to that of laws.

The active role of anomic elements and the critical role of laws lead also to the problem of unity. The more local character of the application and usefulness of anomic elements complicates the relation between theories or parts of a theory that is often adopted as a preferred model of unification. This local character certainly complicates matters for global models of reduction based on linking generalities, typically in the form of laws, at different domains or levels. Similarly, one can speak of the explanatory unification they might be taken to provide as also a critical one. Batterman suggests focusing, instead, on the alternative notion of unity in terms of universality classes of critical phenomena.[79] From a global point of view, anomic elements of models will be seen as sources of complication. But in specific contexts, it is laws that should be seen, at most, as critical, in the sense above, mere frameworks or constraints; beyond them, it is the anomic elements of models that are the salient source of understanding. This should be the basis for a more specific positive suggestion about the form and function of models and approximations.

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APPENDIX

Krieger echoes Kramers' insight that thermodynamics is a limit science.[80] He relates the thermodynamical limit to the possibility of the formalization of a number intensive and extensive properties.[81] He also points to the ways in which the limit is linked to the dynamical independence of chunks of matter that make up the macroscopic system so that the energy of such chunks doesn't increase through composition.[82] But the weightiest consideration shared also by Liu and Batterman is another: not only the infinite size limit has an intrinsic physical meaning, namely, the macroscopic system has an infinite number of particles, but only in that limit phase transitions appear. That means that in its absence, statistical mechanics cannot generate the mathematical discontinuities, singularities or divergences in the values of critical parameters that characterize the thermodynamic idea of a phase transition, most importantly the correlation length among molecules. This apparent indispensability makes the physical meaning of the limit compelling but even more puzzling. Now, Krieger does not note that the puzzle results only from the conjunction of two operative assumptions: (1) the intrinsic meaning – infinite size – mentioned above and (2) the belief that the indispensability in explanatory success suggests something like correspondence to facts for all indispensable assumptions (a move analogous to the realist inference to the best explanation). The second assumption might be found questionable, but this move has difficulties: either a criterion is put forth to constrain the attribution of 'truth' or else indiscriminate instrumentalism or fictionalism about the explanatory theory follows.

Liu deals with the problem of the puzzling physical significance of the infinite-size limit in two connected ways. He thinks, in a way also suggested by Krieger, that the limit is a compensatory assumption for the formulation of assumptions such as stability – that only a finite number of parts can be crammed in a finite volume – and strong temperedness – that a system of large number of molecules won't explode. He then suggests, by way of an implicit replacement for assumption (2) above, a kind of positivist partial interpretation of the puzzling limit, namely, the kind of interpretation a hypothesis gets by inclusion in the conjunction of hypotheses that only together yield the phenomenological results.[83]

Batterman places the infinite-size limit in the context of the important class of singular limits and their explanatory value for emergent properties.[84] To this purpose he introduces the notion of asymptotic explanation. The models of explanation he rejects include causal-mechanical explanation and explanation through derivation from an underlying theory via non-singular limit – also similar to Nagel's well-known model of reduction. I cannot do justice here to the richness of this formal model. I can only mention a distinctive aspect. The model is based on the possibility of capturing stable, universal properties of a broad class of different types of emergent phenomena (hence the importance of explanations involving the application of the renormalization-group techniques, which eliminate uncontrollable details of descriptions at different levels of composition of microsystems).

An interesting case of an apparently benign type of limit has aroused philosophical controversy: it is the one involved in the notion of instantaneous velocity in the application of the differential calculus. The magnitude is defined by means of a limit of a temporal interval reducing to a set containing only the instant at which the velocity is determined. But what kind a property it is and whether it is a real physical property is the subject of recent debates.[85] Most physicists' intuitions concede that the property is physically real. A formally similar case is that of potential functions and forces or velocities defined at a point in terms of the value of a potential function in the neighborhood of that point.[86] In discussions of this case, however, it is the local function that is found to be physically significant whereas the potential function is taken to be either only of computational value or physically significant in a global or integral – not local and differential – form. An example in point is the magnetic vector potential in descriptions of the Aharanov–Bohm effect.[87]

NOTES

[1] From a set-theoretic point of view, the axioms of a theory satisfy the set of entities that count as its models.

[2] Cartwright (1983), Giere (1988, 1999), Suppe (1989), Bogen and Woodward (1983).

[3] Cat (2002), see my forthcoming book, *Physics Beyond Laws and Theories: the Limits of Unity, Universality and Precision.*

[4] Cat (2001).

[5] Mitchell (2000). My distinction between nomic and anomic elements complements the account defended by Sandra Mitchell of a multi-dimensional conceptual space for scientific laws. She speaks of different degrees of contingency, truth and universality, of abstraction, strength and stability.

[6] As I began writing parts of this essay in 1998 new notions of what models can do and how they can be constructed, especially in the social sciences, were emerging. See for instance, Morgan and Morrison (1999).

[7] Clark Glymour cautioned me against falling into a merely inconsistent use of the term 'model'.

[8] Scriven (1961, 91).

[9] Lange (1995).

[10] Cartwright (1983).

[11] In most cases we do not know all the relevant conditions that might interfere with the occurrence of the lawful regularity; and the true law cannot be precisely formulated. Cartwright then explains the regularity captured especially by phenomenological laws in terms of models of arrangements of systems and their causal capacities. She suggests with John Stuart Mill that outside the restricted domain of regularity in the real world laws must describe at best causal tendencies or capacities. See Cartwright (1983, 1989, 1997).

[12] Glennan (1995, 2000).

[13] Cartwright (1983, 1989).

[14] In Joseph (1980), from similar considerations Geoffrey Joseph concludes that the hypothetico-deductive method needs to be modified in order to accommodate the element of isolation and idealization necessaries to treat natural phenomena, and that different theories covering different aspects of real complex phenomena can only inconsistently be joined in a single unified theory – as their adequacy conditions are mutually incompatible – and suggests that extensionalist analyses of scientific language be abandoned.

[15] For Cartwright the model that makes the theory true relates to a more realistic description of phenomena through approximation and pragmatic considerations - and not deduction or formal principles.

[16] Instead, Giere claims, 'the equations are used to construct an abstract mechanical system…. By stipulation, the equations of motion describe the behavior of the model with perfect accuracy. We can say that the equations are satisfied or exemplified by the model or, if we wish, that the equations are true, even necessarily true, for the model. For models, truth, even necessity, comes cheap.' Giere (1999, 92).

[17] Suppe (1989).

[18] Some philosophers and scientists would speak here of truth, correspondence or conformation.

[19] I apologize to Robert Skipper for not giving a more satisfying and strict characterization of these kinds of domains beyond the requirement of my argument.

[20] Pap (1946, 30). I discuss the introduction of precise and fuzzy descriptive categories through the choice of mathematical and physical formalism in my forthcoming book, see n. 3.

[21] This characterization is a placeholder for the use of fuzzy-set membership and other notions of approximation.

[22] In Cat (2001) I point to several of the inconsistencies mentioned here in Maxwell's models and include an extra one involving different illustrative interpretations of the same theoretical quantity, e.g., the potential function.

[23] Meheus (2003).

[24] Courant and Hilbert (1937/1989, vol. 2), Hadamard (1932/1952) and Wilson (1990).

[25] Courant and Hilbert (ibid., p. 227).

[26] Anderson (1991), Cat (1995, 2001) and Howard (1996) for a discussion of this requirement.

[27] Courant and Hilbert (1989, 228). On this point, Michael Stoeltzner has pointed out to me that it is the reason for Hilbert's preference for integral equations.

[28] Hadamard (1952).

[29] Courant and Hilbert (1989, 230).

[30] Wilson, (p. 567). Under steady state conditions, the heat equation degenerates into Laplace's equation, whose only bounded conditions are uniformly constant.

[31] For examples and a general discussion of problematic limits see Batterman (2002).

[32] Wilson (1991, 205, f. 14).

[33] Lax (1957), Courant and Hilbert (1989, pp. 488–490) and for specific physical applications, see Fetter and Walecka (1980), and Smoller (1983).

[34] Inglis (1913).

[35] B.Brown (1992) and Meheus (2003).

[36] See n. 46, below.

[37] In terms of Feynman diagrams employed for the calculation of the vacuum to vacuum transition amplitude of a scattering process, external lines are associated with particles 'on mass shell', namely, that satisfy the constraint $p2 + m2 = 0$, where p represents the relativistic four-momentum and m, the mass of the particle. The internal lines, by contrast, represent the virtual particles off mass shell. How problematic this case is depends on whether one believes that virtual particles are physically real and carry explanatory value. See, for instance, Weingard 1988. This type of inconsistency is different from the purely calculational one in which Schroedinger derived his wave equation for quantum systems in which the energy was changed in interaction. Schroedinger proceeded from the equation for the case of constant energy by removing the constant energy term by assuming it constant and deriving with respect to time.

[38] Gordon (1976, 1978).

[39] Freund (1990, 2).

[40] Griffith (1920), Freund (1990) and Lawn (1995).

[41] Goodier (1968).

[42] Broberg (1960).

[43] Ball and Mizel (1985) and Ball (1995).

[44] See Lawn (1995) for a survey.

[45] See Eberhart and Giamei (1998).

[46] An alternative approach, based on the definitional role of laws, would conclude that the law hasn't been violated but the material has simply ceased to be elastic in the strict or approximate sense defined by Hooke's law. See n. 15, above.

[47] It is interesting that is a way Curie and Weyl believed in the constitutive a priori role of asymmetry and symmetry in physics, respectively.

[48] There exist a number of classifications of symmetries. I borrow this particular distinction from Castellani (2003).

[49] Another qualification typically made is that spontaneous symmetry breaking occurs in infinite many-body systems. In finite quantum systems the ground state becomes a unique superposition of the asymmetric degenerate states; but of course the physical role of the asymmetric state depends on the interpretation of quantum mechanics – a nd the corresponding interpretation of superpositions- one adopts.

[50] Coleman (1985) and Ryder (1985). The same law-centeredness occurs in philosophical discussion; a recent instance is Castellani 2003.

[51] Anderson and Stein (1987). Generalized rigidity can be associated with forces on the value of an order parameter at a point acting on the value at a distant point. Examples are both mechanical rigidity and phase correlations in superconductivity. An example of new dynamics is the Benard instability in the flow of a fluid: a layer of fluid between two horizontal rectangular plates is heated from below and when the heating rate – or thermal gradient – exceeds a critical value, it exhibits regular convective motion in the form of closed rolls with fixed size between the plates. An analogous phenomenon is the Couette flow, namely, the roll of vortical flow of a fluid that appears between two rotating cylinders with different rotational velocities around their axes when the velocity gradient exceeds a critical value. Finally, order parameter singularities play a critical role in dissipative processes, including stable dissipative structures, such as autocatalytic chemical reactions, which increase the rate of entropy production of their environment.

[52] It is typically assumed in physics texts that the dynamical symmetry breaking in unified electroweak theory, associated with the so-called Higgs mechanism, is an instance of spontaneous symmetry breaking in the sense presented above.

[53] For a discussion of unification and the different roles of symmetry breaking see Cat (1998).

[54] Begin with the Goldstone theorem: given a field theory obeying the usual conditions (Lorentz invariance, locality, a Hilbert space endowed with a positive-defined inner product, etc.), there is a locally conserved current (axiomatic version of the statement that the Lagrangian is invariant under some local continuous gauge transformation) such that the space integral of its time component does not annihilate the vacuum state, then the theory contains a massless spinless meson, with the same internal symmetry and parity properties as the time component of the current. The Goldstone bosons haven't been detected. Yet, in the Higgs mechanism they are part of the mechanism that explains the occurrence of detectable massive gauge particles. A gauge-symmetric vacuum state of a theory contains four massless gauge fields (through a minimal coupling motivated by the requirement of local gauge invariance). In a vacuum (ground state) with Goldstone/Higgs massless fields/ particles and increasing energy, then two (degrees of freedom) of the gauge fields combine with the one (degree of freedom of a) Golstone/Higgs scalar boson to make an (three degrees of freedom, the longitudinal polarization, of the) observable massive gauge fields, which in turn explains the shortness of range of the weak interaction.

[55] For details, see, for instance, Coleman (1985), O'Raifeartaigh (1986) and Ryder (1985).

[56] Cat (1998).

[57] Whewell (1841, 1).

[58] Goodeve (1860,1883, 1-2) and Maxwell (1873, vol. 2, art. 831), Willis (1841) and Ampere (1835).
[59] Whewell ibid.
[60] Glennan (1996). more recently Glennan has adopted Woodward's invariant-regularity-centered notion of mechanism, which for cases beyond physics, eschews the problem of laws-talk. According to Woodward, a necessary condition for a causal mechanism is a description (1) organized or structured set of parts or components; (2) the behavior of each is described by a generalization that is invariant under interventions; (3) the generalizations 'governing' each component are independently changeable (modularity); and (4) the representation allows us to see how by (1), (2) and (3) the output of the mechanism will vary under manipulation of the input to each component and changes in the components themselves. See Woodward (2002) and Glennan (2002).
[61] Cartwright (1997, 1999).
[62] In this category I might add that Salmon offered a mechanical model of physical causal explanation in terms of causal processes. The model of process was originally characterized by Reichenbach's mark method and subsequently by the interaction with other processes with exchange of conserved quantities. See Salmon (1998).
[63] Machamer, Darden and Craver (2000, 3). This notion can accommodate both Railton's type of law-based mechanical explanations and the notion of causal processes of the type at work in Salmon's model of causal explanation, capable of carrying marks induced by interaction with other processes and conserving quantities. See ibid., and Salmon 1998.
[64] Recently Tabery has proposed a model combining Glennan's interactionism (property-monism) with Machamer–Darden–Craven dynamical or productiver aspect (Entity-activity dualism): a mechanism is an occasion on which a change in a property of one part dynamically produces a change in a property of another part. The notion of productive or dynamical changes that yield regularities captures the actions mentioned in actual scientific models: synthesis, transmission, separation, etc. See Tabery (2004).
[65] Lanczos (1960, 504).
[66] Wilson (1990, 571–73).
[67] Liu (2003b). This is the current standard view.
[68] In a discussion of the nomological dimension of symmetry breaking, Balashov has argued that the nomological dimension of the phenomenon is not consistent with a universals-based notion of law and, instead, is to be located in the causal powers of the particles involved. Moreover since such fundamental particles have no elements, and thereby, no arrangements, those powers, unlike other dispositions, cannot be ascribed to any structure. See Balashov (1997). Balsahov is mistaken in reducing the phenomenon of symmetry breaking to a question of law and its nomological aspect. That a law is involved is clear but that the mechanism of symmetry breaking is about symmetry breaking and not the symmetry of the law, except in the critical role of law mentioned above, is also clear. The other issue is the state description in terms of structureless particles. Balashov here forgets that in the context of the theory in which symmetry breaking is described, we are talking about fields, and fields, I think, as spatial arrangements of field-properties are structures.This claim should be qualified in the light of discussions by Teller and Wayne of the distinction between field operators and field states in qft. See Teller (1995) and Kuhlman, Lyre and Wayne (2002).

[69] Liu, *ibid.*, and (2003a).

[70] The question of external perturbations through boundary conditions cannot be dismissed as unexplanatory. Anderson and Stein point out that it is through the mechanism of fixing boundary conditions with endplates or mirrors that enable a laser to oscillate in a single mode. See (Anderson and Stein, *ibid.*, p. 453). The question of random fluctuations is a vexing one indeed. Chance has little explanatory value to many, and even less causal substance. However, in phenomena such as the Lamb shift in the spectrum of energy levels of a hydrogen atom, it is the coupling of a quantum system with the fluctuations in an external electromagnetic field that adds an energy term of the Hamiltonian, or energy function of the system, and it is taken to physically explain the spectral phenomena. On the other hand, chance might not have statistical explanatory relevance on populations or types of events, but it can be accorded explanatory causal power in singular instances, namely, when the relevant fluctuation occurs.

[71] For details see Henneaux and Teitelboim (1992).

[72] Belot and Earman (2001) and Butterfield and Isham (2001), in Callender and Huggett 2001.

[73] See, for instance, Krieger (1996) and Batterman (2002).

[74] Cat (1998).

[75] Batterman, (*ibid*).

[76] Krieger (1996, 9). In Cat (2002) I develop a similar theme albeit from a different perspective

[77] Krieger (1996, *ibid.*) One of his arguments for the physical meaning of limits is based on the idea that the order in which limits are taken is physically meaningful. He gives the example of spontaneous magnetization of a crystal. The calculation of the capacity to be a permanent magnet involves taking two limits, one to infinite number of particles and the other to the zero value of the externally applied magnetic field. The zero-field limit describes precisely what to be a permanent magnet is: to maintain internal magnetization without an external field. But if the zero-field limit is taken first, the spontaneous magnetization is identically zero. The infinite-size limit guarantees the description of the necessary condensed or bulk matter that bears the internal magnetic capacity. See (ibid., 11).

[78] See also Cat (1998).

[79] See Batterman (2002) and also Cat (1998)

[80] *ibid.*, 101

[81] *Ibid.*, Chaps. 1 and 2

[82] *Ibid.*, 40

[83] Liu (2000).

[84] Batterman (2002).

[85] Arntzenius (2000) and Smith (2003).

[86] Anderson (1991), Cat (1995, 1998, 2001).

[87] Anderson (1991) and Cat (1995).

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