Set-Valued Var. Anal (2018) 26:1–4
https://doi.org/10.1007/s11228-018-0473-8

Preface

Heinz H. Bauschke[1] · Boris S. Mordukhovich[2,3] · Claudia S. Sagastizábal[4] · Michel A. Théra[5,6]

Received: 11 February 2018 / Accepted: 11 February 2018 / Published online: 16 February 2018

This two-volume issue of *Set-Valued and Variational Analysis* is dedicated to the life and work of Jonathan Michael Borwein who passed away suddenly in August 2016. Jon's contributions to numerous fields of mathematics and its applications are difficult to overstate. He was an absolutely outstanding, brilliant mathematician whose mathematical talent and abilities surpassed by far those of ordinary mathematicians.

This issue is devoted to several topics of set-valued and variational analysis that were among Jon's primary interests. But first we briefly recall some facts of Jon's biography and academic career.

Jonathan Borwein was born in 1951 in St. Andrews, Scotland into an academic family. His parents, Bessie and David, have been prominent scientists in anatomy and mathematics, respectively. David was Head of Pure Mathematics at the University of Western Ontario from 1967 to 1989 and President of the Canadian Mathematical Society (1984–1986). He and Jon co-authored many papers together. Jon's brother Peter is an outstanding mathematician in number theory who also wrote numerous papers and books with Jon.

Jon Borwein received his B.A. (Honours Math) from the University of Western Ontario in 1971 and his PhD from Oxford University in 1974 at the very prestigious position of Rhodes Scholar. At Oxford, he originally studied pure functional analysis but quickly moved

✉ Boris S. Mordukhovich
boris@math.wayne.edu

1 Mathematics, Irving K. Barber School, University of British Columbia Okanagan, Kelowna, British Columbia V1V 1V7, Canada

2 Department of Mathematics, Wayne State University, Detroit, MI 48202, USA

3 RUDN University, Moscow 117198, Russia

4 Instituto Nacional de Matemática Pura e Aplicada - IMPA, Rio de Janeiro, 22460-320, Brazil

5 Institut de Recherche XLIM, UMR CNRS 7252, Université de Limoges, Limoges, France

6 Federation University Australia, Ballarat, Australia

to more applied optimization topics and received his doctoral degree under the supervision of Michael Dempster. It is worth mentioning that in his Ph.D. thesis Jon formulated some questions to which he had returned with his students and collaborators over the years. In particular, with Xianfu Wang he answered in the late nineties a question he had first posed in the seventies: most nonexpansive functions are so badly behaved that they oscillate as much as they can at every point.

From Oxford Jon went to Dalhousie University in 1974 and came under the wing of Michael Edelstein, "an intense Polish-Israeli functional analyst who by example and engagement converted a generation of young analysts into full blooded independent researchers". In the early eighties, Jon moved to Carnegie-Mellon, where he had the opportunity to work with many exciting operations researchers and mathematical economists including Dick Duffin, a distinguished mathematician and engineer. From that period Jon said that "the problems of teaching large classes of management science and business students (at Carnegie-Mellon and after returning to Dalhousie in 1982) provided the impetus for my first involvement with computer assisted mathematics". With his collaborators, he started to write very simple software which could assist students in solving linear programs by the simplex method, then and now, one of the work-horses of the algorithmic world; and which are horrifyingly unpleasant to execute at the blackboard. Their "idiot-pivoter" was a huge success!

In 1982, Jon and his brother Peter decided to study the number π together. They thought it would be nice to write a little paper together on π, the area of a unit circle and how to compute it other elementary constants quickly. Over the years, that little paper has generated many joint publications, several shared prizes and four joint books.

Ten years later, in 1992, Jon and Peter co-founded the Centre for Experimental and Constructive Mathematics (CECM) at Simon Fraser University. The CECM was founded on the premise that we are in the midst of a massive paradigm in mathematics. In the *Notices of the American Mathematical Society* Jon wrote: "new subjects such as computational geometry, fractal geometry, turbulence, and chaotic dynamical systems have sprung up. Indeed, many second-order phenomena only become apparent after considerable computational experimentation. Classical subjects like number theory, group theory, and logic have received new infusions. The boundaries between mathematical physics, knot theory, topology, and other pure mathematical disciplines are more blurred than in many generations. Computer assisted proofs of "big" theorems are more and more common: witness the 1976 proof of the Four Color theorem and the more recent 1989 proof of the non-existence of a projective plane of order ten". Since that time Jon became a major figure in the field of *experimental mathematics.* He also had the vision and determination to build other research centres, including the University of Newcastle's centre for Computer Assisted Research Mathematics and its Applications (CARMA) where he served as founding director and held a very prestigious Laureate Professorship.

Jon's steady interests in major parts of variational analysis and its applications can be perceived all along his academic career. Besides the fundamental Borwein-Preiss variational principle (1987), Jon and his collaborators strongly contributed in many areas of variational theory and generalized differentiation, particularly in their infinite-dimensional aspects. Among them we mention the introduction and study of compactly epi-Lipschitzian sets, pioneering results on metric regularity (this very name was coined by Jon), deep developments on metric projections and proximal analysis, duality and second-order generalized differentiation, viscosity subdifferentials and subdifferential calculus, nonconvex separation, etc. In 1988, Jon also co-invented the famous Barzilai-Borwein gradient method, one of the most utilized algorithms in optimization. He also pioneered the study of projection and Douglas-Rachford methods in both convex and nonconvex settings.

The contents of the first issue that Set-Valued and Variational Analysis publishes as a tribute to Jonathan Borwein is described below.

In “Quasidense monotone multifunctions” by Stephen Simons studies a new notion of quasidense monotone set-valued mappings/multifunctions acting from a Banach space into its topological dual. The author obtains various characterizations of such multifunctions and their maximal monotonicity properties.

“Contractive mappings on unbounded sets” by Simeon Reich and Alexander Zaslavski, is devoted to contractive properties of nonexpansive operators defined on unbounded sets of complete metric spaces. Using the Baire category approach, the authors study the asymptotic generic behavior of such operators. Their results are specified to the case of Banach spaces and applied to deriving some fixed-point theoretical and related consequences.

In “Uniform continuity and a new bornology for a metric space”,Gerald Beer, Isabel Garrido and Ana Merono introduce a new bornology on the space of mappings between arbitrary metric spaces. Among some nice properties the authors show for this bornology, is the fact that uniform convergence preserves uniform continuity of mappings.

“Nearly Chebyshev sets are almost convex” by Warren Moors concerns some long-standing problems on Chebyshev sets and proximality, which had been of Jon’s strong interest for many years. The author develops some new quantitative perspectives in the study of these and related geometric issues by considering certain “nearly Chebyshev” and “nearly uniquely remoted” sets with establishing their almost convexity and almost singleton properties, respectively.

“Maps with Radon-Nikodym property” by Luis Garcia-Lirola and Matias Raja revolves around the classical Radon-Nikodym property (RNP), which plays a crucial role in the geometry of Banach spaces, particularly in infinite-dimensional variational analysis. The focus is on deriving a nonlinear version of dentability and appropriately extending the RNP to a “less linear” framework. They show that many interesting results and applications can be kept under such an extension while some other important achievements (in particular, the fundamental Stegall’s variational principle) are lost in this general geometric setting.

In “Upper semismooth functions and the subdifferential determination property”, Marc Lassonde studies relationships between subdifferentials and subderivatives (generalized directional derivatives) of lower semicontinuous functions. The considered class, of upper semismooth functions, has subderivatives that can be recovered from subdifferentials. This class includes many previously known subclasses of functions that exhibit such a phenomenon. Furthermore, the author shows that upper semismooth functions are subdifferentially determined, i.e., two functions from this class are different by an additive constant if they have the same subdifferential.

“About intrinsic transversality of pairs of sets”, by Alexander Kruger, studies some qualification/transversality conditions ensuring a certain regular behavior of set pairs at intersection points. The main motivation comes from applications to the convergence analysis of the alternative projection algorithm for solving nonconvex feasibility problems. The author derives various verifiable characterizations of transversality-type conditions in Asplund spaces by using appropriate tools of generalized differentiation.

The main focus of “A partial answer to the Demyanov-Ryabova conjecture” by Aris Daniilidis and Colin Petitjean concerns the cycling in a finite family of polytopes, which is important for the exhauster technique in nonsmooth optimization. The authors formulate and affirmatively settle a stronger version of this conjecture under a certain well-posedness assumption on extreme points of polytopes.

"Equilibrium problems: Existence results and applications" by John Cotrina and Yboon Garcia is devoted to various equilibrium problems studied via a bifunction approach. In this way the authors establish novel existence theorems for equilibria and also propose and investigate five notions of regularization. The obtained results shed new light on monotonicity and continuity properties of bifunctions and equilibrium problems. Finally, applications of general results are developed and specified to particular equilibrium settings, including the Nash equilibrium problem.

The last paper presented in this volume is "Necessary optimality conditions for implicit control systems with applications to control of differential algebraic equations" by An Li and Jane Ye. The authors consider a general class of control systems, called of implicit type. This class includes, in particular, control problems for differential algebraic equations (DAEs), which is important for engineering applications. The main thrust of the paper is deriving necessary optimality conditions for optimal control of nonsmooth implicit systems and the so-called semi-explicit DAEs in the form of the Pontryagin Maximum Principle. The results are obtained under mild qualification conditions and improve those previously known in the literature.

We, the four Co-Editors of this SI, feel privileged for having shared part of Jon's tremendous appetite for life and maths. With this tribute we barely scratch the surface of the significance of his work.

To conclude this preface, we recall a few of Jon Borwein's many facets: he was a family man, plenty of love and affection for his wife Judith, their children and grandchildren; a wonderful friend; a prodigious mathematician; a mentor to many colleagues; a blogger; a historian; the quintessential *Homo Habilis Mathematicus*.

His legacy is enormous indeed.

Heinz Bauschke
Boris Mordukhovich
Claudia Sagastizábal
Michel Théra