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Nonflat surface level pyramid: a high connectivity multidimensional interconnection network

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Abstract Parallel machines are extensively used to increase computational speed in solving different scientific problems. Various topologies with different properties have been proposed so far and each one is suitable for specific applications. Pyramid interconnection networks have potentially powerful architecture for many applications such as image processing, visualization, and data mining. The major advantage of pyramids which is important for image processing systems is hierarchical abstracting and transferring the data toward the apex node, just like the human being vision system, which reach to an object from an image. There are rapidly growing applications in which the multidimensional datasets should be processed simultaneously. For such a system, we need a symmetric and expandable interconnection network to process data from different directions and forward them toward the apex. In this paper, a new type of pyramid interconnection network called Non-Flat Surface Level (NFSL) pyramid is proposed. NFSL pyramid interconnection networks constructed by L-level A-lateral-base pyramids that are named basic-pyramids. So, the apex node is surrounded by the level-one surfaces of NFSL that are the first nearest level of nodes to apex in the basic pyramids. Two topologies which are called NFSL-T and NFSL-Q originated from Trilateral-base and Quadrilateral-base basic-pyramids are studied to exemplify the proposed structure. To evaluate the proposed architecture, the most important properties of the networks are determined and compared with those of the standard pyramid networks and its variants.

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1 Introduction

Computation needs have been surprisingly grown with recent advancements in new technologies of defense, aerospace, automotive, etc., and their simplest problems need hours for TFLOPs computing power. Computer systems should now be designed with computational power in order of PFLOPs to solve such problems [1, 2]. Parallelism is the main approach for reaching the required processing power. A parallel machine is comprised of several processing units connected together through interconnection network. The processing nodes use messages to communicate with each other. So, the main task of the interconnection network will be transmission of the messages among nodes with the minimum possible delay [1, 3]. An interconnection network can be modeled using a graph, where a node refers to a processing unit and an edge refers to a physical channel between two corresponding processing units. Topology, node addressing, and routing algorithm affect the performance of the interconnection networks significantly [4–8].

Many topologies have been proposed for interconnection networks, each one support some specific types of applications, ranging from the simplest one like n-node ring, R_n, in which each node has two neighbors, to the fully-connected network where each node is connected to all other nodes [1]. Increase in the number of the nodes, originated from huge computation needs, results in needing the large scale structural networks. Thus, more complex topologies have been proposed, such as cubic networks (e.g., Mesh, Tori, Hypermesh, k-ary n-cube, Hypercube), hierarchical networks (e.g., Tree and Pyramid) and recursive networks (e.g., RTCC, OTIS, WK-recursive, and star graph) [9–12].

2D-mesh topology is one of the first interconnection networks, which was used in many scientific applications. Furthermore, many parallel computers use this simple and effective topology [13]. One extension on 2D-Mesh is adding an apex node and relating edges on top of the mesh, forms a simple pyramid. A more complete pyramid, which is usually called the standard pyramid, has some intermediate levels that are smaller 2D-meshes, between the base and the apex.

One improvement on the standard pyramids was replacement of the mesh with other topologies. In [14], a recursively connected cyclic networks called RTCC that was originally proposed in [15] is used as the base of the pyramid, results in RTCC-pyramid. In [16], torus is used as the base, to form EPM or enhanced-pyramid, and in [17], as a general form, different types of grids includes Torus, Hypermesh, and WK-recursive networks, used as the base to make the grid-pyramid. There are some other pyramid variants that try to change the intralayer connections of the standard pyramid such as the star-pyramid [18]. In some other topologies, the dimension of the mesh located in the base is changed, such as Tripy that employs triangular mesh as the base, introduced in [9]. In Tripy, each node may have more than one parent that can be utilized to handle more communication load in parallel applications. Meanwhile, the higher connectivity can contribute to the fault-tolerance properties.

Pyramid is a network which is capable of efficiently handling communication requirements of different problems such as image processing [19, 20], visualization [21], and data mining [22]. The main advantage of the pyramids for such applications is its inherent hierarchical structure that concentrates toward a unit node which is the apex. Node degree, connectivity, symmetry, and scalability are major properties to be considered because they affect the performance of the pyramids in various applications.

Recently, the growing set of data-intensive applications in collections and analysis of very large multidimensional datasets are widely appeared in industry and even public applications in the internet. Petroleum reservoir simulations [23], full-scale water contamination studies and surface/subsurface, satellite data processing [24], visualization of large-scale data [21], and data mining [22] are some of these applications. An example of multidimensional dataset is multiview image, which is a collection of images from the same scene, can be widely used for the purposes of free viewpoint TV (FTV) and 3DTV [25].

To serve a system for parallel processing the multidimensional datasets simultaneously, in this paper, we propose a symmetric and expandable interconnection network based on pyramid, to process data from different directions and forward it toward the apex. This new type of pyramid, which is called Non-Flat Surface Level (NFSL) pyramid, is constructed by L-level A-lateral-base pyramids that are named basic-pyramids. NFSL are expected to show scalability in making many views from different directions; all of them send data toward a unique central node. Also, according to the basic-pyramids, the nodes may be connected to more than one parent in NFSL.

The rest of paper is organized as follows. NFSL is introduced in Sect. 2 as well as reviewing previous variant of pyramid that may employ as basic-pyramids. In Sects. 3 and 4, NFSL-T and NFSL-Q originated from Trilateral-base and Quadrilateral-base basic-pyramids are studied to exemplify the proposed structure. In Sect. 5, to evaluate the NFSL, its properties are compared with the ones of standard pyramid interconnection networks and their variants. Section 6 concludes the paper.

2 Nonflat surface level pyramid topologies

Apex node in all pyramids is located at the top level. We call this first level, level-0, and other levels are located under it. The network connection between the nodes at each level may form conventional mesh, triangular mesh, RTCC, and so on [14–16, 23–26]. The nodes in all suggested pyramids are located on a flat page. In order to reach to a symmetrical and expandable structure, we merge P basic-pyramids so as their apexes are merged to form a central node. Each basic-pyramid has only one common face with its neighbor basic-pyramid. Hence, we are going to exploiting parallelism intrapyramid takes in hierarchical reductive property of the basic-pyramids, beside with another parallelism interpyramid for processing the multidimensional dataset trough the base of the basic-pyramids.

Unlike other pyramids in which the apex node is located at the top level, in the NFSL pyramid, it is located inside as a central node; we called it level-0. Other levels are formed around it. Each level is composed of a surface with some faces that are really the intermediate or main base of basic-pyramids. The structure of connections in each level and between levels is inherited from the basic-pyramids structure.

To describe the NFSL pyramid, we called the nodes according to their position, in each level. The nodes located at corners and edges are called corner nodes (CNs) and edge nodes (ENs), respectively, while other nodes are called interior nodes (INs). Moreover, child and parent nodes refer to the nodes with a connection to the corresponding node, located at next and previous levels of NFSL pyramid respectively; even they have been in different basic-pyramids. The neighbor node is referred to the node which is located in the same corresponding node level and connected to it; again even they have been in different basic-pyramids.

In the rest of this section, we formally define the basic-pyramids that may be exploited in constructing the NFSL pyramid, and try to discuss about their main topological properties.

Definition 1 $a \times b$ mesh, $M_{a,b}$, includes a set of nodes $V(M_{a,b}) = \{(x, y) \mid 1 \le x \le a,\ 1 \le y \le b\}$ in which any two nodes of (x_1, y_1) and (x_2, y_2) are connected together with a link iff $|x_1 - x_2| + |y_1 - y_2| = 1$, where x_i and y_i are coordinates of a given node in the network [26]

Definition 2 L-level mesh pyramid, PM_L, is a set of nodes $V(P_L) = \{(k, x, y) \mid 0 \le k \le L, 1 \le x, y \le 2^k\}$ that any node at level k is denoted by (k, x, y). L is the number of levels of the pyramid network. All nodes of level k form a $2^k \times 2^k$ 2-D mesh network. A total number of $N = \sum_{k=0}^{L} 4^k = (4^{L+1} - 1)/3$ nodes are expected to exist in a PM_L. In addition to the mesh connected between nodes at level k, any node (k, x, y) is connected to the nodes of $(k + 1, x - 2,\ 2y)$, $(k + 1,\ 2x,\ 2y - 1)$, $(k + 1,\ 2x - 1,\ 2y - 1)$, and $(k + 1,\ 2x, 2y)$ at level $k + 1$ as the child nodes and the node $(k - 1, [\frac{x}{2}], [\frac{y}{2}])$ at level $k - 1$ as the parent node [26].

Definition 3 A radix-n triangular mesh, T_n, is formed by a set of nodes $V(T_n) = \{(x, y) \mid 0 \le x + y \le n\}$. Any two nodes (x_2, y_2) and (x_1, y_1) are connected together through a link provided that $|x_1 - x_2| + |y_1 - y_2| < n - 1$. The number of nodes and links in a T_n is represented by $n(n + 1)/2$ and $3n(n - 1)/2$, respectively [9].

Definition 4 A triangular pyramid, Tripy, is made using both triangular mesh and mesh pyramid. L-level Tripy, TP_L, consists of a set of nodes $V(TP_L) = \{(k, (x, y)) \mid 1 \le k \le L, 0 \le x + y < k\}$ in which all nodes at level k are connected in T_n structure. Hence, there is a total number of $N = \sum_{k=0}^{L} k(k + 1)/2 = L(L + 1)(L + 2)/6$ nodes and $L(L + 1)(L - 1)$ links in a TP_L. Each node at level k, $(k, (x, y))$, is connected to the nodes of $(k + 1, (x, y))$, $(k + 1, (x + 1, y))$ and $(k + 1, (x, y + 1))$ at level $k + 1$ as the child nodes. It is also connected to the node of $(k - 1, (x - 1, y))$ provided that $x \ne 0$, and to the node of $(k - 1, (x, y - 1))$ if $y \ne 0$, and to the node of $(k - 1, (x, y))$ if $x, y < k - 1$ at level $k - 1$ as the parent nodes [9].

In the next two sections, we formally define two NFSL pyramids, in which their basic-pyramids are L-level trilateral and quadrilateral pyramids, and we call them NFSL-T_L and NFSL-Q_L respectively, and derive their topological properties.

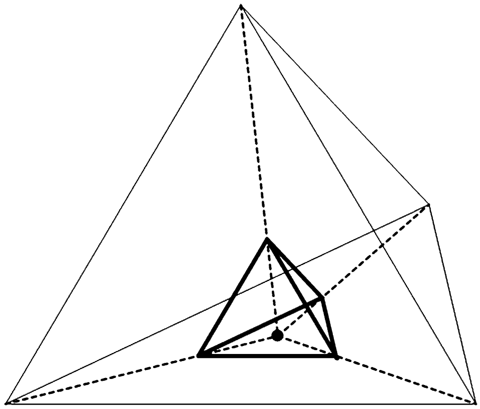


Fig. 1 Typical levels of NFSL-T pyramid

3 NFSL for trilateral-base pyramid

In this section, the NFSL-T pyramid has been described as a special case of NFSL pyramid. NFSL-T constructed from four trilateral pyramids, basic-pyramids, in which the nodes of each level includes the bases of the pyramids. The trilateral pyramids may be any of previous topologies for triangular pyramids include those defined in Sect. 2. In the rest of this section, we use T_n topology introduced in Definition 3, as the basic-pyramids in the first case study for NFSL.

Various levels of NFSL-T pyramid are illustrated in Fig. 1. The first level, level-0, (central node) is shown by a solid point whereas the second and the third levels are shown with thick and thin lines, respectively. Dashed lines represent the edges of the basic pyramids.

Definition 5 L-level NFSL-T pyramid structure, NFSL-T_L, is made by 4L-level trilateral basic-pyramids with a given number of nodes at level-n $(0 \leq n \leq L)$.

In the level-n of NFSL-T, which consists of four bases of basic pyramids, there are 4 CNs, $6(n-2)$ ENs and $4(n-2)(n-3)$ INs. Total node at level n of NFSL-T, $B(n)$, is equal to $n(n+1)/2$ nodes. Nodes in each level are connected together in a T_n structure and incorporate some child nodes at next levels, as well as, some parent nodes at previous levels.

Four distinct faces are distinguished in each pyramidal surface. Both the CNs and ENs bear 4 child nodes but the INs have 3 child nodes, while the parents of CNs, ENs and INs are 1, 2, and 3 respectively. Moreover, each CN, EN, and IN has 12, 8, 12 neighbors, respectively, in the internal levels.

The necessary information for calculating the total number of nodes and links in NFSL-T pyramid are presented in Table 1. If we consider the bases of four basic pyramids separately, we would have $4(\frac{n(n+1)}{2})$ nodes $(1 \leq n \leq L)$ as the number of nodes in level n of NFSL-T_L. In this case, the nodes located on the pyramids' edges considered twice except the vertexes which considered three times. Finally, for level-n $(1 \leq n \leq L)$, the number of nodes at level n, $S(n)$, and the total number of

Table 1 NFSL-T pyramid characteristics

Parameter	Quantity
Number of base sides (n)	3
Number of pyramid's surfaces except the base (s)	3
Number of pyramid's surfaces together with the base (f)	4 ($= n+1$)
Number of base's nodes at level $n(B(n))$	$\frac{n(n+1)}{2}$
Number of necessary pyramids in the NFSL (P)	4
Number of pyramid's edges (E)	6
Number of pyramid's vertexes (V)	4
Number of nodes at level n of the NFSL ($S(n)$)	$2n^2 - 4n + 4$
Number of links at level n of the NFSL ($C(n)$)	$6n^2 - 12n + 6$

nodes for NFSL-T$_{\mathrm{L}}$ pyramid, S_{Total}, can be obtained from Eqs. (1) and (2), respectively.

$$S(n) = \begin{cases} 1; & n = 0 \\ 4\frac{n(n+1)}{2} - 6n + 4; & 1 \le n \le L \end{cases} \tag{1}$$

$$S_{\mathrm{Total}} = \begin{cases} 1; & n = 0 \\ \sum_{n=1}^{L} S(n); & 1 \le n \le L \end{cases} \tag{2}$$

At any level, each node has 6 neighbors except the corner nodes which have 3 ones. The number of links at level-n ($1 \le n \le L$), $C(n)$, and the total number of links at level n in an NFSL-T$_{\mathrm{L}}$ pyramid, C_{Total}, can be obtained from Eqs. (3) and (4), respectively.

$$C(n) = \begin{cases} 0; & n = 0 \\ \frac{6S(n) - 3V}{2}; & 1 \le n \le L \end{cases} \tag{3}$$

$$C_{\mathrm{Total}} = \begin{cases} 0; & n = 0 \\ \sum_{n=1}^{L} C(n); & 1 \le n \le L \end{cases} \tag{4}$$

As an example of the NFSL, NFSL-T is illustrated in Fig. 2. In this figure, the nodes are represented by solid points with the links of second and third levels being specified by thick and thin lines, respectively. Dashed lines represent the links between different levels.

Some widely used metrics for comparing the topological properties of different networks are node degree (d), diameter (D), network cost ($d \times D$), and connectivity degree.

The number of physical channels which connect a node to its neighborhood is called the node degree. Since the node degree is a measure for input and output complexity of the node, it is better to make node degree as small as possible. The network is regular when all nodes of a given network show the same node degree, otherwise the network is deemed irregular and average node degree should thus be used instead

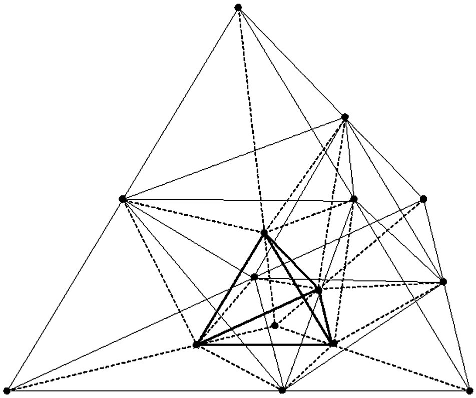


Fig. 2 Three levels of the NFSL-T with the links

Table 2 The average node degree of NFSL-T pyramid for levels 1 to L-1

	CNs	ENs	INs
With parents	1	2	3
With children	4	4	3
With neighbors	3	6	6
Total	8	12	12

[1, 9]. The average node degrees of CNs, ENs and INs of the NFSL-T pyramid and the total average node degree of NFSL-T pyramid for levels 1 to L-1 are presented in Table 2. The central node (Level-0) has 4 children and the nodes, which are located in the last level (Level-L) have no child node, so the node degree of each CNs, ENs, and INs in the last level are obtained as 4, 8, and 9, respectively. Since the average node degree is the twice of the total number of links divided by the total number of nodes, it can be derived from Eqs. (2) and (4) as follows:

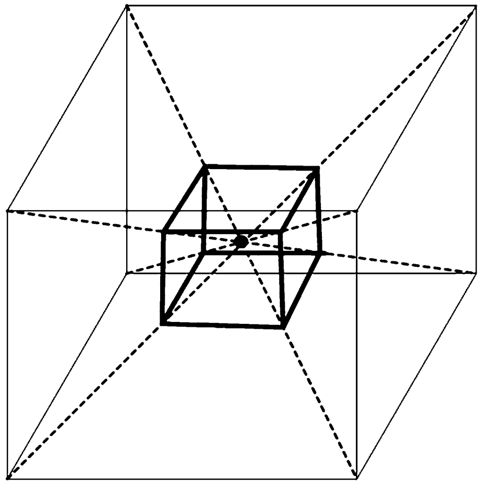
$$\mathit{Average\ node\ degree} = \frac{2(\sum_{n=1}^{L} 6n^2 - 12n + 6)}{\sum_{n=1}^{L} 2n^2 - 4n + 4} \tag{5}$$

The network diameter is used to evaluate performance of the static interconnection network. Diameter is defined as the longest path between the nodes of a network. In other words, diameter is a path with the shortest length and the maximum number of links in comparison with other shortest paths. The diameter of the NFSL-T_L pyramid is defined with Eq. (6). Having applied Eqs. (5) and (6), the network cost of the NFSL-T pyramid can be calculated as the product of multiplying the average node degree with the network diameter and it is derived from Eq. (7). It should be noted that the network cost is preferred to be as small as possible [9].

$$\mathit{Diameter} = 2L - 1 \tag{6}$$

$$\mathit{Network\ Cost} = (2L-1)\frac{2(\sum_{n=1}^{L} 6n^2 - 12n + 6)}{\sum_{n=1}^{L} 2n^2 - 4n + 4} \tag{7}$$

Fig. 3 Different levels of an NFSL-Q pyramid network



The connectivity degree of NFSL-T_L pyramid can be defined with Eq. (8).

$$\textit{Connectivity Degree} = \frac{2[\sum_{n=1}^{L} 6n^2 - 12n + 6]}{[\sum_{n=1}^{L} 2n^2 - 4n + 4][(\sum_{n=1}^{L} 2n^2 - 4n + 4) - 1]} \quad (8)$$

4 NFSL for quadrilateral-base pyramid

In this section, NFSL-Q pyramid has been described as a special case of NFSL pyramid. NFSL-Q constructed from six quadrilateral pyramids, basic-pyramids, in which the nodes of each level includes the bases of the pyramids. The structure of the basic-pyramids may be any of previously proposed quadrilateral ones, such as standard pyramid, RTCC-pyramid [14], EPM [16], or Grid-pyramid [17]. In the rest of this section, we use mesh topology introduced in Definition 1 as basic-pyramids for a NFSL-Q pyramid which is our second case study for NFSL pyramid structure.

Various levels of NFSL-Q pyramid are illustrated in Fig. 3. The first level, level-0, (central node) is shown by a solid point whereas the second and the third levels are shown with thick and thin lines, respectively. Dashed lines represent the edges of the basic pyramids.

Definition 6 L-level NFSL-Q pyramid structure, NFSL-Q_L, is made by 6 L-level quadrilateral basic-pyramid with a given number of nodes at level-n $(0 \le n \le L)$.

In level-n of NFSL-Q which consists of six bases of basic pyramids, there are 8 CNs, $12(n-2)$ ENs and $6(n-2)^2$ INs. Total nodes at level n of NFSL-Q, $B(n)$, is equal to n^2 nodes. Nodes in each level are connected together in a mesh structure and incorporate some child nodes at next levels, as well as some parent nodes at previous levels.

Six distinct faces are distinguished in each level of surface in NFSL-Q_L. CNs, ENs, and INs have 7, 6, and 4 child nodes, respectively. Moreover, CN, EN, and IN has 12, 11, and 12 neighbors, respectively, in the internal levels.

Table 3 NFSL-Q pyramid characteristics

Parameter	Quantity
Number of base sides (n)	4
Number of pyramid's surfaces except the base (s)	4
Number of pyramid's surfaces together with the base (f)	5 $(= n + 1)$
Number of base's nodes at level $n(B(n))$	n^2
Number of necessary pyramids in the NFSL (P)	6
Number of pyramid's edges (E)	12
Number of pyramid's vertexes (V)	8
Number of nodes at level n of the NFSL ($S(n)$)	$6n^2 - 12n + 8$
Number of links at level n of the NFSL ($C(n)$)	$12n^2 - 24n + 4$

The necessary information for calculating the total number of nodes and links in NFSL-Q pyramid are presented in Table 3. If we consider the bases of six basic-pyramids separately, we would have $6n^2$ nodes $(1 \leq n \leq L)$, as the number of nodes in level n of NFSL-T_L, while the nodes located on the pyramids' edges considered twice except the vertexes which considered three times. Finally, for level-n $(1 \leq n \leq L)$, the number of nodes at level n, $S(n)$, and the total number of nodes for NFSL-Q_L pyramid, S_{Total}, can be obtained from Eqs. (9) and (10), respectively.

$$S(n) = \begin{cases} 0; & n = 0 \\ 6n^2 - 12n + 8; & 1 \leq n \leq L \end{cases} \tag{9}$$

$$S_{Total} = \begin{cases} 0; & n = 0 \\ \sum_{n=1}^{L} S(n); & 1 \leq n \leq L \end{cases} \tag{10}$$

At any level, each node has 4 neighbors except the corner nodes which have 3 ones. The number of links at level-n $(1 \leq n \leq L)$, $C(n)$, and the total number of links in an NFSL-Q_L pyramid, C_{Total}, can be obtained from Eqs. (11) and (12), respectively.

$$C(n) = \begin{cases} 0; & n = 0 \\ \frac{4S(n) - 3V}{2}; & 1 \leq n \leq L \end{cases} \tag{11}$$

$$C_{Total} = \begin{cases} 0; & n = 0 \\ \sum_{n=1}^{L} C(n); & 1 \leq n \leq L \end{cases} \tag{12}$$

As an example of the NFSL, NFSL-Q is illustrated in Fig. 4. Only the links in right side of the network have been shown for simplicity. In this figure, the nodes are represented by solid points with the links of second and third levels being specified by thick and thin lines, respectively. Dashed lines represent the links between different levels.

The average node degrees of CNs, ENs, and INs of the NFSL-Q pyramid and the total average node degree of NFSL-Q pyramid for levels 1 to L-1 are presented in Table 4. The apex node (Level-0) has 8 children and the nodes which are located in

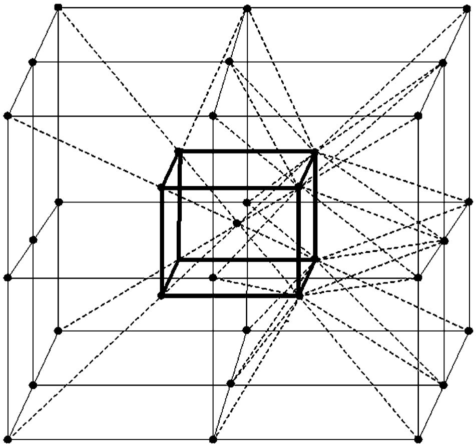


Fig. 4 Three levels of NFSL-Q pyramid

Table 4 The average node degree of NFSL-T pyramid

	CNs	ENs	INs
With parents	1	2	4
With children	7	6	4
With neighbors	3	4	4
Total	11	12	12

the last level (Level-L) have no child node, so the node degree of each CNs, ENs, and INs in the last level are obtained as 4, 6, and 8, respectively. From Eqs. (10) and (12), the average node degree can be calculated by the following equation:

$$\mathit{Average\ node\ degree} = \frac{2(\sum_{n=1}^{L} 12n^2 - 24n + 4)}{\sum_{n=1}^{L} 6n^2 - 12n + 8} \tag{13}$$

The diameter of $\mathrm{NFSL_L}$-Q pyramid can be obtained according to Eq. (14).

$$\mathit{Diameter} = (2L - 1) \tag{14}$$

The network cost of $\mathrm{NFSL_L}$-Q pyramid is calculated as below using Eqs. (13) and (14).

$$\mathit{Network\ cost} = (2L - 1)\frac{2(\sum_{n=1}^{L} 12n^2 - 24n + 4)}{\sum_{n=1}^{L} 6n^2 - 12n + 8} \tag{15}$$

The connectivity degree of NFSL-$\mathrm{Q_L}$ pyramid can be defined with Eq. (16).

$$\mathit{Connectivity\ degree} = \frac{2(\sum_{n+1}^{L} 12n^2 - 24n + 4)}{[\sum_{n=1}^{L} 6n^2 - 12n + 8][(\sum_{n=1}^{L} 6n^2 - 12n + 8) - 1]} \tag{16}$$

Table 5 Number of children and parents of ENs, CNs, and INs in NFSL-T pyramid

Node type	Num. of parents	Num. of children
Edge Nodes	2	2
Corner Nodes	1	3
Interior Nodes	n	n

5 Comparison between different pyramid networks

Some parameters of the NFSL are independent of the type and number of basic pyramids. In other words, the properties of all potential topologies of this structure, follows a systematic procedure. For example, the total number of nodes in a given NFSL-x structure always follow Eq. (17).

$$Total\ Num.\ of\ Nodes = \sum_{n=1}^{L} PB(n) - En + V \tag{17}$$

where P, E, and V are the number of necessary basic-pyramids in the NFSL-x, number of basic-pyramid's edges, and the number of basic-pyramid's vertexes, respectively. $B(n)$ denotes the number of base's nodes at level n $(1 \leq n \leq L)$. Another general property of the proposed structure is that the number of children and parents of ENs and CNs in an NFSL pyramid with an arbitrary base are fixed, and for INs, they are equal to the base's sides of the pyramid. This information is presented in Table 5. It should be noted that the nodes of level-0 do not have any parent and the nodes of level-L do not have any child.

In the rest of the section, a comparison between the proposed structure and some other well-known pyramids topologies, in terms of the properties and metrics determined in the previous sections is presented.

Various properties of mesh-pyramid, Tripy, P-RTCC, WK-pyramid, Star-pyramid and Hypermesh-pyramid and NFSL pyramid networks are summarized in Tables 6 and 7 [9, 11, 12, 17].

It has been tried here to compare topological properties of the proposed pyramid networks and other conventional pyramids, i.e., mesh-pyramid, Hypermesh-pyramid (HM-pyramid), triangular pyramid (Tripy), WK-Pyramid, Star-pyramid and RTCC-pyramid (P-RTCC). This comparison has been conducted in terms of network size (total number of nodes within the network).

The node degree of the pyramids is studied in two different views, interlayers and intralayer. In order to calculate the node degree of interlayer's nodes, we must consider the neighbors and the parents of those nodes. Regarding this description, the node degree of CNs, ENs, and INs at middle layers of the NFSL-T pyramid will be 3, 12, and 18, respectively. From the intralayer point of view, the node degree is calculated in a similar way with a difference, which is observing the children of the next level instead of parents of the previous layer. The node degree of intra-layer nodes in the NFSL-Q pyramid is also computed and the results are 4, 8, and 9 for mid-layer CNs, ENs, and Ins, respectively.

Table 6 Total nodes, total links, average node degree, and diameter of various pyramid networks

	Total nodes (N)	Total links (E)	Average node degree (d)	Diameter (D)
Mesh-pyramid (PM_L)	$(4^{L+1}-1)/3$	$\frac{(2^{2L+3}+4^{L+1})}{3}-2^{L+2}$	$\frac{2\times\mathrm{E}}{N}$	$2L-1$
WK-Pyramid ($\mathrm{P}_{WK,L}$)	$(4^{L+1}-1)/3$	$4^{L+1}-2^L-4$	$\frac{2\times\mathrm{E}}{N}$	$2L-1$
HM-pyramid ($\mathrm{P}_{HM,L}$)	$(4^{L+1}-1)/3$	$\frac{(8^{L+1}-1)}{7}-1$	$\frac{2\times\mathrm{E}}{N}$	$L-1$
Star-pyramid (SP_L)	$1+\sum_{k=2}^{L}k!$	$\sum_{k=2}^{L}\frac{(k+1)!}{2}$	$\frac{2\times\mathrm{E}}{N}$	$3(L-1)/2$
Tripy (TP_L)	$L(L+1)(L+2)/6$	$L(L+1)(L-1)$	$\frac{2\times\mathrm{E}}{N}$	L
P-RTCC(C, L)	$\frac{(C^{L+1}-1)}{C-1}$	$\frac{2.5(C^{L+1}-1)}{C-1}-\frac{CL}{2}$	$\frac{2\times\mathrm{E}}{N}$	$2L-1$
NFSL-T	$\sum_{n=1}^{L}2n^2-4n+4$	$\sum_{n=1}^{L}6n^2-12n+6$	$\frac{2(\sum_{n=1}^{L}6n^2-12n+6)}{\sum_{n=1}^{L}2n^2-4n+4}$	$2L-1$
NFSL-Q	$\sum_{n=1}^{L}6n^2-12n+8$	$\sum_{n=1}^{L}12n^2-24n+4$	$\frac{2(\sum_{n=1}^{L}12n^2-24n+4)}{\sum_{n=1}^{L}6n^2-12n+8}$	$2L-1$

Table 7 Network Cost and Connectivity degree of various pyramid networks

	Network cost	Connectivity degree
Mesh-pyramid (PM_L)	$D\times d$	$\frac{2\times\mathrm{E}}{N}$
WK-Pyramid ($\mathrm{P}_{WK,L}$)	$D\times d$	$\frac{2\times\mathrm{E}}{N}$
HM-pyramid ($\mathrm{P}_{HM,L}$)	$D\times d$	$\frac{2\times\mathrm{E}}{N}$
Star-pyramid(SP_L)	$D\times d$	$\frac{2\times\mathrm{E}}{N}$
Tripy (TP_L)	$D\times d$	$\frac{2\times\mathrm{E}}{N}$
P-RTCC(C, L)	$D\times d$	$\frac{2\times\mathrm{E}}{N}$
NFSL-T	$(2L-1)\frac{2(\sum_{n=1}^{L}6n^2-12n+6)}{\sum_{n=1}^{L}2n^2-4n+4}$	$\frac{2[\sum_{n=1}^{L}6n^2-12n+6]}{[\sum_{n=1}^{L}2n^2-4n+4][(\sum_{n=1}^{L}2n^2-4n+4)-1]}$
NFSL-Q	$(2L-1)\frac{2(\sum_{n=1}^{L}12n^2-24n+4)}{\sum_{n=1}^{L}6n^2-12n+8}$	$\frac{2(\sum_{n+1}^{L}12n^2-24n+4)}{[\sum_{n=1}^{L}6n^2-12n+8][(\sum_{n=1}^{L}6n^2-12n+8)-1]}$

As mentioned before, using HM-pyramid network is not affordable due to its rather high costs. However, since Tripy is the only network, which has an acceptably high reliability after HM-pyramid network, it is usually applied to compare the suggested networks.

Figure 5 demonstrates average node degrees for different pyramids. It can be seen that the node degree of NFSL-Q and P-RTCC are smaller in all network sizes and they approach to 3 and 4, respectively. On the other hand, in the HM-pyramid, each node in its layer is in connection with other nodes of that row and column. As a result, node degree intensively grows by increasing the size of the network.

NFSL-T, WK-Pyramid, and Star-Pyramid nearly show the same node degree and take the next best places. For the network sizes over 100, NFSL-T and WK-Pyramid networks have a node degree of 5 and the Star-Pyramid network tends to a node degree of 6. So, our structure in its two cases beside WK-Pyramid and Star-Pyramid show an expectable node degree in any large networks, and this is a very important advantage. Also, the node degree of Tripy network tends to be 8 for large size net-

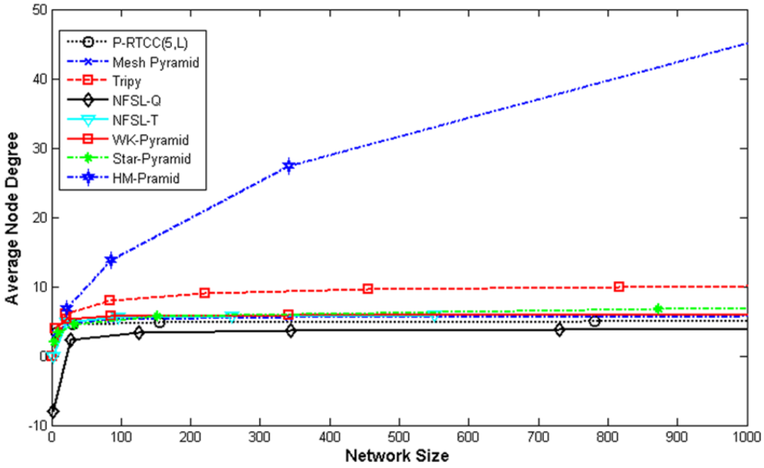


Fig. 5 Average node degrees for different pyramids

works. It can be observed that node degrees of the NFSL-T and NFSL-Q pyramids have been decreased at least 160 % and 260 % in comparison with Tripy, respectively. By comparing the average node degrees of different pyramids up to the size of 3000, it can be noticed that the node degree of NFSL-T pyramid has shown an average decrease of 25 % over that of Star-Pyramid. Also, meanwhile, the average node degree of NFSL-Q pyramid has been decreased about 200 % as comparison to that of NFSL-T pyramid.

As discussed before, in HM-pyramid, each node in its layer is connected with other nodes of that row and column. Thus, diameter of HM-pyramid is smaller than other pyramid networks. It can be also noted that diameter of the proposed networks is lower than that of Tripy network.

Diameter is the longest path which data traverse is a route in a network. In the NFSL pyramids, the diameter is equal to 2L-1, but it should be noted that data is traverse from the main bases of the basic pyramids to the central node; so the effective diameter for NFSL pyramids would be equal to the half of those determined in the Eqs. (6) and (14). Comparing the diameters obtained from different networks up to size of 3000 reveals that the average diameters of NFSL-Q and NFSL-T pyramids have been decreased 32 % in comparison with Tripy.

Figure 6 depicts network cost of various pyramids. As was expected before, increasing the size of the network in HM-pyramid will intensify the node degree of it. Therefore, cost of this network is more than others in spite of its diameter being minimum among other networks. Meanwhile, the cost of Tripy network is rather high because its node degree and diameter are rather high. In contrast, costs of the proposed networks are lower than that of Tripy, since diameters of other networks are smaller with node degree in these two networks being almost the same.

Comparing the network cost for pyramids up to size of 3000 uncovers that the average network cost of NFSL-T pyramid has been decreased 17 % over that of Tripy. Furthermore, it shows a 45 % decrease in network cost of NFSL-Q in comparison with the value for Tripy. It should be noted that the average network cost of NFSL-Q

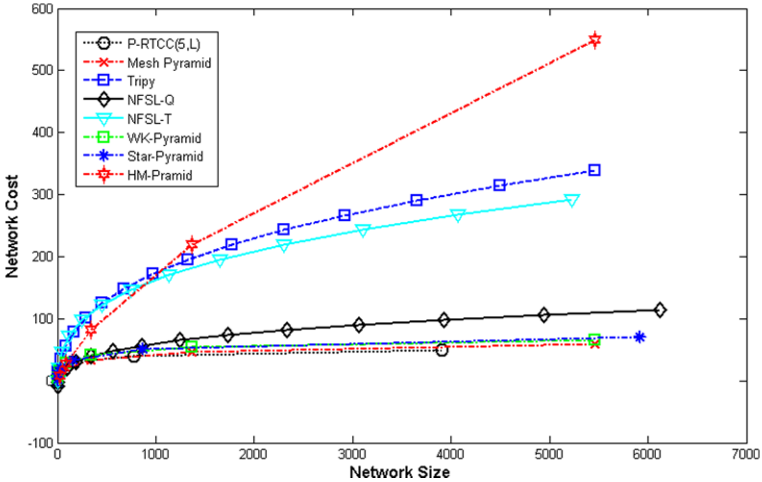


Fig. 6 Network cost for various pyramids

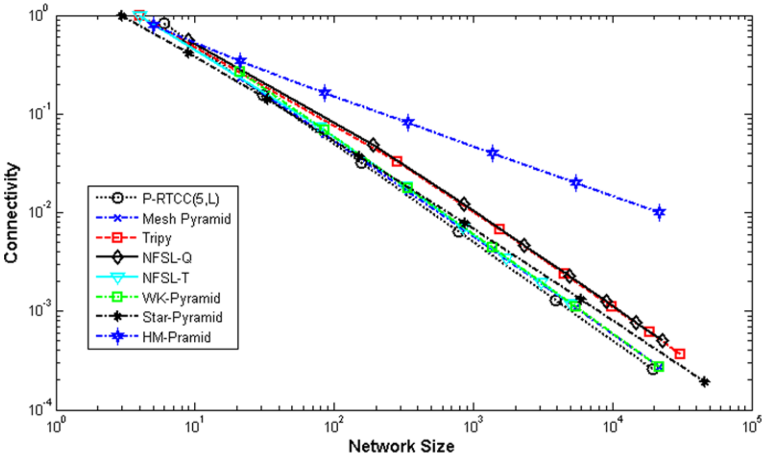


Fig. 7 Connectivity degree of different pyramids

pyramid seems more economic than NFSL-T pyramid as it has been reduced for about 29 %.

Figure 7 represents the connectivity degree for different networks. As expressed previously, connectivity degree is a reliability measure for networks. Taking into account that nodes of each layer are connected with other nodes in the same row and column of that layer in HM-pyramid network, connectivity degree of this network is thus greater than other networks. However, as was mentioned before, increasing size of the network will significantly rise node degree in this network. Thus, network cost is rather high in HM-pyramid and its application is uneconomic. On the other hand, in both cases of the proposed network, each node can bear more than just one parent, which will increase the number of the connections between different layers and will rise connectivity degree of the network as well.

As shown in this figure, after HM-pyramid, the NFSL-Q beside the Tripy network, in all network sizes yield the maximum connectivity degree in comparison with other pyramids. The obtained results show that the connectivity of NFSL-T pyramid is not as good as the NFSL-Q one and after the Star-Pyramid network it takes the third rank. Also, the results confirm that the connectivity of NFSL-Q pyramid is the best being 11 % greater than that of Tripy. After NFSL-Q pyramid, the connectivity degree of Tripy network has the next rank.

Increased connectivity degree of the network entails an increased number of links in this network. This can lead to a greater node degree and significantly raise costs of the network. Comparison of Figs. 6 and 7 reveals that although the connectivity degree of HM-pyramid is higher than other networks, cost of this network is also higher than other networks. It can also be seen that in our proposed network, NFSL-Q, has the highest connectivity degree after HM-pyramid. On the other hand, cost of this network is lower than those of Tripy and HM-pyramid networks. Thus, a significant reliability can be achieved by NFSL network, both NFSL-T and NFSL-Q, with lower cost than other pyramid networks.

6 Conclusion

In recent years, data-intensive applications, such as image processing, visualization, and data mining have been widely used in industry and even public applications in internet. Many of them require to collect and analysis very large multidimensional datasets. Pyramid topology which exploits its inherent property of reductive hierarchical toward an apex node, suitable structure for making parallelism to overcome the volume of the computations for one-dimensional datasets.

To process the multidimensional datasets in parallel manner, in this paper, we propose a new structure, NFSL pyramid formed by merging P L-level A-lateral-base pyramid, each one called basic-pyramid. The common apex node of basic-pyramids located as central node of NFSL and other levels being made around it.

Two topologies which are called NFSL-T and NFSL-Q originated from Trilateral-base and Quadrilateral-base basic-pyramids studied to exemplify the proposed structure. To evaluate the proposed architecture, different properties of NFSL pyramids such as connectivity, node degree, network costs are derived and discussed comparing previously propose variants of pyramid.

It is shown that the connectivity for NFSL is considerably better than other improved pyramids and is almost equal to Tripy network, while it was observed that the cost of NFSL networks is lower than Tripy. The derived equations show connectivity degree of NFSL-T and NFSL-Q pyramids 3 and 11 % better than that of Tripy, respectively, and at the same time, their network cost has been decreased 17 and 45 %, respectively in comparison with Tripy.

Future work can be concentrated on developing the routing algorithm for NFSL pyramid. Other area for future work is generalization and formulation of features and properties inherited from basic-pyramid to NSFL pyramid.

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