J Supercomput (2011) 58:84–95
DOI 10.1007/s11227-010-0533-9

3D face recognition based on sparse representation

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Published online: 22 December 2010

Abstract In this paper, we present a novel 3D face recognition algorithm based on the sparse representation. First, a 3D face normalization approach is proposed to deal with the raw faces. Then, three types of facial geometrical features are extracted to describe the 3D faces. Meanwhile, in order to guarantee the feasibility of the sparse representation framework and promote the recognition efficiency, a novel feature ranking scheme based on Fisher linear discriminant analysis (FLDA) is designed to arrange the facial descriptors. Finally, the sparse representation framework is used to collect all the face features, and it addresses the recognition task. The experiments tested on the BJUT-3D and FRGC v2.0 databases demonstrate the validity of the proposed 3D face recognition algorithm, and the necessity of the FLDA ranking scheme in the sparse representation framework.

Keywords 3D face recognition · Sparse representation · Facial geometrical features · FLDA ranking scheme

1 Introduction

Automatic face recognition has become one of the most challenging research topics in pattern recognition and computer vision. Much progress has been made on its theories and algorithms [1, 2], and most of them are based on 2D face images. But it has been demonstrated that the expression, pose and illumination variances of face images influence the recognition performance greatly. Compared with 2D face images, the 3D faces contain more spatial information, which is an inherent property of human faces and is robust to uncontrollable environment, where the 2D appearance

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can be affected largely. So recently, the 3D face recognition has attracted much more attention [3–10].

However, 3D faces have their own difficulties. First, it needs special acquisition devices to get the 3D faces. Second, the raw 3D faces come with a large amount of points, which costs much time to normalize them. Third, the absence of a consistent parameterization among face meshes or point clouds makes it impossible to directly obtain registered features with a uniform sampling pattern. Moreover, facial expressions can also cause facial geometry distortion, which results in degraded recognition performance or heavy computation cost [8].

Recently, sparse representation, also known as compressed sensing [11, 12], has been applied to image-based face recognition [13] and demonstrates encouraging results. Compared with conventional methods, the method based on the sparse representation has several unique advantages. First, the sparse representation can find the only one optimal matching by solving an $L0$-norm minimization problem, which is better suited for face recognition. Second, due to the inherent source-and-error separation property, the presence of irrelevant features will not degrade recognition performance, as long as the remaining features are sufficiently informative. But the calculational cost will increase with the more features presence. Third, in the sparse representation framework, increasing the number of individuals in the training set in fact improves the sparsity and thus will not likely degrade recognition performance, which is in contrast to conventional methods [14].

In this paper, we make more efforts on the sparse representation framework for 3D face recognition. First, a uniform sampling approach is proposed to normalize the 3D faces, and it gives all the faces uniform topology. Then, three types of face geometrical features are extracted to describe the 3D faces. Due to the large amount of extracted 3D features, which goes against the application of sparse representation, we develop a feature ranking scheme based on Fisher linear discriminant analysis (FLDA) to pick up the most useful face features for recognition. This scheme not only selects a few robust face features, which improves the recognition performance and makes low computation cost, but also it eliminates the necessity of constructing the extensive set for reference faces of each individual and guarantees the applicability of the sparse representation framework on 3D face recognition. Finally, the sparse representation framework is used to collect the face features, and addresses the recognition task. The experiments tested on the BJUT-3D [15] and FRGC v2.0 [16] databases achieve encouraging results, and demonstrate the efficacy of the sparse representation framework for 3D face recognition.

2 3D face normalization

The raw 3D faces employed in our experiments usually come with a large amount of data and noises, and cannot be used in experiments directly. So we need to normalize them (Fig. 1).

Because the acquired 3D faces are noisy and spinose, they need removal of these spikes, to fill the holes resulting from removing spikes by interpolation and to smooth the face surface. Then, the faces are trimmed from the whole scanned data by cutting

Fig. 1 3D face normalization process

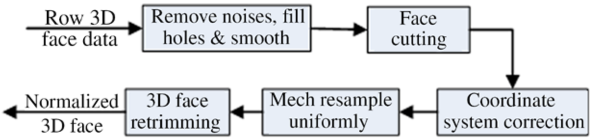
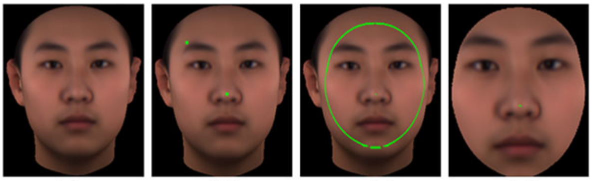


Fig. 2 3D face preprocess

(a) Face cutting (b) Face correction

Fig. 3 The 3D face trimming process. (**a**) The aligned 3D face; (**b**) two key points; (**c**) the trimmed boundary; (**d**) the normalized face



(a) (b) (c) (d)

boundary and removing the 3D data lying on the hair and shoulder (Fig. 2(a)). For pose correction, the cutting faces are corrected to a uniform coordinate system, by fitting the discrete 3D face vertices to a cylinder (Fig. 2(b)). To facilitate the 3D faces on experiments and make them have the same topology, they are aligned with pixel-wise correspondence according to face features by the mesh resampling method [17]. After that, the resulting 3D faces keep alignment based on face features, and they have the same topology. That means the vertices and triangular patches of every 3D face are recorded by the same rule.

In face recognition, we pay more attention to the facial center area, such as the eyes, brow, nose, mouth, and so on, so the aligned 3D faces are trimmed as reference [18] (Fig. 3 shows the process). First, two key feature points (the nose tip and outboard vertex on the left brow) are located. Then a sphere with radius r and center at the nose tip is used to trim the 3D faces, where r is the Euclidean distance between the two key points. All the experimental faces are processed by this method, and the normalized 3D face can be obtained.

3 3D face feature extraction and ranking scheme

3.1 3D face feature extraction and representation

In order to describe the 3D facial essential attributes, we analyze the properties of facial curve surface, and extract three types of geometrical features to represent the 3D face, the area of every triangular patch, the normal of every triangular patch, and the geodesic distance between any two vertices on the 3D face. All the normalized 3D faces are arranged by discrete vertices and triangular patches, which provide us with the coordinates and the adjoining vertices information for every vertex. So we can get these three types of features easily.

Triangle area (TA) The face vertices and the according triangular patches form a face surface. After normalization, the face areas of different 3D samples vary greatly, which can characterize the unique individual. So the area of every triangular patch on the face surface is used to describe the 3D face, which can be calculated by Heron's formula [19] as

$$S_i = \sqrt{s(s-a)(s-b)(s-c)} \tag{1}$$

where $s = (a + b + c)/2$, $a = Dist(A, B)$, $b = Dist(A, C)$, $c = Dist(B, C)$. $Dist(M, N)$ means the Euclidean distance between the spatial nodes M and N. The spatial nodes $A(x_1^i, y_1^i, z_1^i)$, $B(x_2^i, y_2^i, z_2^i)$, and $C(x_3^i, y_3^i, z_3^i)$ are the three vertices of the ith triangular patch. S_i $(i = 1, \ldots, m)$ is the ith triangular patch area, and m denotes the number of triangular patches for the given 3D face. So the given 3D face F^k can be represented by all the triangular patch areas formed in a certain sequence as

$$F_S^k = \left[S_1^k, S_2^k, \ldots, S_m^k\right]^{\mathrm{T}} \tag{2}$$

Triangle normal (TN) For a face mesh, the normals of all the triangular patches not only describe its concavo-convex attribute, but also depict the curvature variety, which are inherent properties of the mesh, and contributive to mesh recognition. So the triangle normals of 3D faces are extracted. In the space rectangular coordinate system, for a plan α, if we can find two non-parallel vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ on it, the normal $\boldsymbol{n}$ of α can be solved as

$$\boldsymbol{n} \perp \alpha \Rightarrow \begin{cases} \boldsymbol{n} \bullet \boldsymbol{a} = 0 \\ \boldsymbol{n} \bullet \boldsymbol{b} = 0 \end{cases} \tag{3}$$

For a certain triangular patch i on the given face mesh, we can get its three vertices as $A(x_1^i, y_1^i, z_1^i)$, $B(x_2^i, y_2^i, z_2^i)$, and $C(x_3^i, y_3^i, z_3^i)$, and also the two intersectant vectors $\boldsymbol{AB}$ and $\boldsymbol{AC}$. So according to (3), we can get the normal N_i of the triangular patch i. So the given 3D face F^k can be represented by triangle normals as

$$F_N^k = \left[N_1^k, N_2^k, \ldots, N_m^k\right]^{\mathrm{T}} \tag{4}$$

Geodesic distance (GD) The geodesic distance, between any two points located on the surface of an occlusor, is robust to isometric deformation, and can describe the property of the occlusor accurately. The human face deformation (usually caused by facial expression) can also be regarded as an isometric deformation approximately, so the geodesic distance can be used to depict the facial uniqueness, which is suitable to the task of face recognition.

In this paper, the geodesic distances of any two vertices on the face mesh are calculated by the Dijkstra's algorithm with the prior knowledge of the facial mesh topology. And the given face sample F^k can be represented by the geodesic distance as

$$F_G^k = \left[G_1^k, G_2^k, \ldots, G_{C_n^2}^k\right]^{\mathrm{T}} \tag{5}$$

where n is the number of vertices for the given 3D face mesh, and C_n^2 denotes the number of any pair of vertices on the mesh.

3.2 Ranking scheme

Because the 3D faces are with large data volume, the feature dimension is so high that the computation cost is heavy. Under the sparse representation framework, adopting all the features for recognition is unrealistic. So we need to decrease the feature dimension, and remain the more discriminative information for recognition. But it is difficult to determine which features are more informative for face recognition. More importantly, the majority of these features can be distorted to a certain extent by facial expressions, and these distortions usually spread over all the features. It has been observed that such distortions can damage the sparsity of the representations and degrade recognition performance. So how to determine which features are more useful for face recognition is a crucial problem.

In this paper, we present a novel feature ranking scheme based on Fisher linear discriminant analysis (FLDA) to determine which features are more useful for face recognition. According to the FLDA criterion, the similarity between different samples from the same individual (or the same class) is superior to the similarity between different samples from different individuals (or different classes). Under FLDA theory, the better discriminative feature should have better discriminative capability and so should have a large between-class similarity mean value, a little within-class similarity mean value, and little within-class and between-class covariance mean values. So we measure the discriminative capability of face features by the ratio between the difference of within-class and between-class similarity mean values and the sum of within-class and between-class similarity covariances.

For a face feature, the similarity measure Ω_{ij} of two faces is defined as

$$\Omega_{ij} = \bigcap (F_i, F_j) \quad (i \neq j) \tag{6}$$

where $\bigcap (F_i, F_j)$ denotes the difference between the two faces F_i and F_j.

Face recognition is a multi-class classification problem. According to the FLDA, the within-class similarity measure is calculated on the different samples from the same individual and the between-class similarity measure is calculated on the different samples from different individuals for each feature. The mean value and covariance of within-class and between-class similarity measures are formalized as

$$m_w^{\Gamma} = \frac{1}{N_w^{\Gamma}} \sum \Omega_{ij} \quad (i, j \in \Gamma) \tag{7}$$

$$S_w^{\Gamma} = \frac{1}{N_w^{\Gamma}} \sum \left(\Omega_{ij} - m_w^{\Gamma}\right)^2 \quad (i, j \in \Gamma) \tag{8}$$

where m_w^{Γ} and S_w^{Γ} are the within-class similarity measure mean value and covariance of the class Γ, respectively. Samples i and j are all from class Γ, and N_w^{Γ} is the number of different samples from class Γ.

The mean value and covariance of between-class similarity measure are defined, respectively, as

$$m_b^{\Gamma} = \frac{1}{N_b^{\Gamma}} \sum \Omega_{ij} \quad (i \in \Gamma, j \notin \Gamma) \tag{9}$$

$$S_b^{\Gamma} = \frac{1}{N_b^{\Gamma}} \sum \left(\Omega_{ij} - m_b^{\Gamma}\right)^2 \quad (i \in \Gamma, j \notin \Gamma) \tag{10}$$

where m_b^{Γ} and S_b^{Γ} are between-class similarity measure mean value and covariance between class Γ and any other class. Samples i and j are from different classes, but i is from class Γ. N_b^{Γ} is the number of different sample pairs from different classes, but one is from class Γ.

Thus, the discriminant level ω can be defined as

$$\omega = \frac{(m_w - m_b)^2}{S_w + S_b} \tag{11}$$

This definition means that the larger is the difference of within-class and between-class similarity measure mean values, the larger is ω. And the better is the discriminative capability, but the smaller is the sum of within-class and between-class similarity measure covariances, the larger is ω. So ω reflects the discriminative capability of the face feature, and the feature with larger ω value can play a greater role in face recognition. Thus, the descriptors of every representation, F_S^k, F_N^k and F_G^k (referring to (2), (4), (5)), can be ordered by the proposed ranking scheme, and we can pick up the first length-l descriptors for each face feature, F_S^k, F_N^k and F_G^k, to describe the 3D face.

4 Experiments and discussions

4.1 Recognition

In this paper, the sparse representation framework is used to collect the face features, and it addresses the recognition task. Under this framework, each face is represented by a set of face descriptors, which sufficiently characterize the individual. With the prior knowledge that faces of the same individual are similar to each other, a probe face can be considered as being well approximated by linearly combining the k reference faces of the same individual in the training set. The process can be formulated as follows.

For each face feature, taking the triangle area for example, we select the first l descriptors with more discriminative capability to represent the 3D face k as

$$F_S^k = \left[S_1^k, S_2^k, \ldots, S_l^k\right]^{\mathrm{T}} \tag{12}$$

All the 3D faces in the training set can be organized into a training matrix $A \in R^{l \times m}$, under the representation of triangle area, denoted by

$$\mathrm{A} = \left[F_S^1, F_S^2, \ldots, F_S^m\right] \tag{13}$$

m is the number of face samples in the training set. For a given probe 3D face p, it can be represented as F_S^p under the representation of triangle area. The task of recognizing F_S^p under the sparse representation framework is to solve the following L0-norm minimization problem:

$$\widehat{x} = \arg\min \|x\|_0, \quad \text{s.t.} \quad Ax = F_S^p \tag{14}$$

where x is a length-m coefficient vector, with ideally at most k nonzero entries. That means F_S^p is k-sparse over A. However, (14) is well known to be an NP-hard problem. According to [13], if the solution to (14) is sufficiently sparse, (14) can be approximatively equal to an L1-norm minimization as

$$\widehat{x} = \arg\min \|x\|_1, \quad \text{s.t.} \quad Ax = F_S^p \tag{15}$$

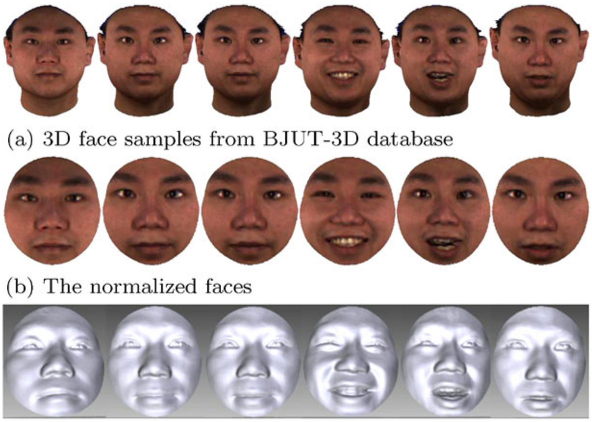
Equation (15) can be solved easily. In recognition, the Euclidean distances is used to find the nearest neighbors. We model the matching process as a minimization problem, and set the objective to be minimized as the L2-norm. With the solution $\widehat{x}$ to (15), we can compute the residual between F_S^p and each individual in the training set as

$$r_i = \left\| F_S^p - \sum \widehat{x}_i F_S^i \right\|_2 \tag{16}$$

and the training face with the minimal residual can be regarded as the optimal matching to the given probe 3D face.

4.2 Experiments

The proposed algorithm is evaluated on BJUT-3D face database, and 150 individuals are used in our experiments. Each has six samples, three neutral faces and the other three with expressions. All the samples are normalized by the method mentioned in Sect. 2. Figure 4 shows the faces of an individual from BJUT-3D face database and the normalized results.



(a) 3D face samples from BJUT-3D database

(b) The normalized faces

(c) The normalized face meshes

Fig. 4 3D faces from BJUT-3D database and the normalization results. The first three are neutral faces, while the other three carry happy, surprise and angry expressions

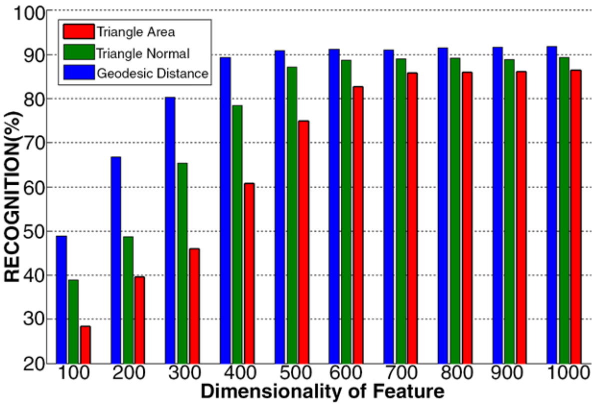


Fig. 5 Recognition performance of the three types of features for different dimensionality

To demonstrate the efficacy of our ranking scheme, we first need to determine the value of l in (12). Three samples are randomly selected from each individual to form the training set, and the left for test. We set $l = 100, 200, 300, \ldots, 1000$, and recognize faces using every feature. Considering the time and calculation cost of a sparse representation, we only set l from 100 to 1000. Figure 5 shows the recognition performance under different l. It appears that the recognition performances of the three types of features all reach a better level when $l = 700$, and when $l > 700$, the recognition performances of them increase little. So in the following experiments, we pick up 700 descriptors for each type of face feature, F_S^k, F_N^k and F_G^k, to test our algorithm.

For each type of feature, we compare the recognition performances under two schemes. The first scheme uses a set of randomly selected (RS) 700 descriptors to represent the face. The second one uses the proposed FLDA ranking scheme to pick up the first 700 discriminative descriptors. Meanwhile, we also collect all the three types of features to form a combined feature pool, and all the descriptors in the pool are ranked together. Also 700 descriptors are picked up under the RS scheme and the FLDA ranking scheme to test the proposed algorithm.

Table 1 The recognition results for different feature types under the two schemes tested on BJUT-3D database

Method	Feature	Scheme	Neutral	Expression	Overall
PCA	All	–	68.4%	57.3%	61.7%
LDA	All	–	78.2%	63.6%	71.9%
Ours	TA	RS	86.7%	78.3%	81.2%
		FLDA	88.2%	85.4%	87.3%
Ours	TN	RS	89.3%	80.1%	84.8%
		FLDA	90.4%	88.7%	89.2%
Ours	GD	RS	91.8%	86.7%	89.0%
		FLDA	93.5%	91.1%	92.3%
Ours	All	RS	93.6%	87.9%	90.4%
		FLDA	96.2%	93.7%	95.3%

The recognition rates using different types of face features under the two schemes are presented in Table 1 for neutral faces, expression faces and all faces, respectively. We also test the PCA and LDA algorithms for comparison.

4.3 Discussions

Based on the results shown in Table 1, we have several important observations. A first point worth mentioning is that the proposed FLDA ranking scheme in fact facilitates the recognition. We conducted similar experiments using the FLDA ranking scheme and the randomly selected (RS) scheme, under the sparse representation framework. The recognition rates under the RS scheme are all lower than those under the FLDA ranking scheme using the same face features. For the feature type of triangle area, the recognition rates, using FLDA ranking scheme, achieves 88.2%, 85.4% and 87.3% for neutral, expression and overall faces, while 86.7%, 78.3% and 81.2% using the RS scheme. This is because the proposed FLDA ranking scheme sufficiently takes the within-class and between-class similarities for every descriptor into consideration, and successfully arranges the face descriptors according to their discriminative capability, which is more propitious to classification. Moreover, from Table 1, it is appears that the performances for neutral faces are nearly the same under the two scheme, while they are very different for expressions with the gap of about 7% between the two scheme. Our method with FLDA ranking scheme obviously outstands that with RS ranking scheme, which means the proposed FLDA ranking scheme is suitable to 3D face expressions. Furthermore, the 3D face recognition methods are usually subject to high dimensional face features, which makes them time-consuming, and sometimes the methods cannot solve the matching problem. However, our ranking scheme effectually deals with the puzzle. All the analyses above demonstrate that the FLDA ranking scheme plays a key role in our algorithm, and it is feasible to apply the sparse representation framework on 3D face recognition with our ranking scheme.

Secondly, three types of face features, triangle area, triangle normal and geodesic distance, are extracted to describe the 3D faces in this paper. Compared the recognition performances of them, the feature of geodesic distance achieves the best results, 93.5%, 91.1% and 92.3% for neutral, expression and overall faces under FLDA ranking scheme, which indicates that the feature of geodesic distance can depict the face properties accurately. That is because the face deformation caused by expressions can be regarded as an isometric deformation approximatively, and the geodesic distance is more robust to facial expression. So the feature of geodesic distance can preserve more inherent properties of faces, and it achieves better performance in face recognition. The feature of the triangle normal can also play a good role in recognition, and gains 90.4%, 88.7% and 89.2% for neutral, expression and overall faces under FLDA ranking scheme, which indicates this feature is also propitious to face recognition. The feature of triangle area gets the worst recognition performance among the three types of face features, because the triangle patches on the facial mesh are so small that one cannot describe the face available. If combining few triangle patches as a unit, maybe we can get a little better results.

Usually, a different type of face feature provides different discriminative information and also has respective discriminative capability. The single type of feature can provide limited discriminative information and local applicability. Face recognition based on a single feature usually cannot achieve satisfactory performance. So it is reasonable to fuse more types of face features to promote the recognition rate. The features-fusion strategy can synthesize all of the features, reserve more original face properties, and get satisfactory results. Thus, in this paper we collect all the three types of features to form a combined feature pool, and all the descriptors in the pool are ranked together. The method of fusing different features is outstanding, and it gets the best recognition results among all the experiments in this paper, 96.2%, 93.7% and 95.3% for neutral, expression and overall faces under FLDA ranking scheme.

In addition, we should mention that the sparse representation framework is applicable to 3D face recognition. We also test the PCA and LDA algorithms on the same faces. Comparing the results among PCA, LDA and our algorithm, PCA and LDA only get 61.7% and 71.9% for overall tested samples, while our algorithm achieves over 80% for every type of face features and the best 95.3% for overall tested faces. It can be concluded that the algorithm based on the sparse representation is superior to PCA and LDA algorithms, and it is feasible to use it to address the 3D face recognition task.

4.4 Comparative study

In order to test the validity of our algorithm, we conduct another experiment on FRGC v2.0 database by collecting all the three types of features and under the FLDA ranking scheme. We select 350 individuals from the subset ‘Spring2004range’ of FRGC v2.0, and each of them also has six scans, three neutral faces and three expressional faces. We sort them as six groups according to expressions. In each time, we choose one group for test, and the left for training. We conduct our algorithm for six times, and get the final average recognition scores as Table 2.

In Table 2, we compare the rank-one recognition rates of our method to several state-of-the-art results. Compared with the existing methods, our method has a little better performance than others. The improvement mainly relies on the validity of the sparse representation framework and is further enhanced by our FLDA feature

Table 2 Comparison of rank-one identification rate

Methods	Database	Individuals	Accuracy
Ours	FRGC v2.0	350	95.04%
Chang et al. [3]	FRGC v2.0	400	92.3%
Mahoor et al. [8]	FRGC v2.0	370	93.7%
Li et al. [9]	FRGC v2.0 + GavabDB	120	94.68%

ranking scheme. In addition, different data normalization methods also affect recognition performance. Finally, we should point out that similar work is proposed by Li et al. [9], but our data normalization method, the extracted 3D features and the features ranking scheme are different from theirs.

5 Conclusion and future work

In this paper, we extend the sparse representation framework to 3D face recognition, and propose a novel ranking scheme based on Fisher linear discriminant analysis (FLDA) to guarantee the feasibility of the sparse representation framework. Three types of facial geometrical features, triangle area, triangle normal and geodesic distance, are extracted to describe the 3D faces. The experiments, tested on BJUT-3D and FRGC v2.0 databases, demonstrate the effectiveness of the extracted features for 3D face recognition, and the validity of the proposed 3D face recognition algorithm based on sparse representation framework. We also conclude that the proposed FLDA ranking scheme is necessary for our algorithm and plays a key role in the sparse representation framework. In the future, we will expand the experiments on other 3D face database and validate our algorithm. Another possible future work includes incorporating other face features into the facial descriptor and investigating their effectiveness for face recognition.

Acknowledgements This work is supported by the National Natural Science Foundation of China (Nos. 60825203, 60973057, U0935004), and 973 Program (2011CB302703).

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