J Supercomput (2009) 49: 219–233
DOI 10.1007/s11227-008-0232-y

Embedding fault-free cycles in crossed cubes with conditional link faults

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Published online: 30 July 2008

Abstract The crossed cube, which is a variation of the hypercube, possesses some properties that are superior to those of the hypercube. In this paper, we show that with the assumption of each node incident with at least two fault-free links, an n-dimensional crossed cube with up to $2n-5$ link faults can embed, with dilation one, fault-free cycles of lengths ranging from 4 to 2^n. The assumption is meaningful, for its occurrence probability is very close to 1, and the result is optimal with respect to the number of link faults tolerated. Consequently, it is very probable that algorithms executable on rings of lengths ranging from 4 to 2^n can be applied to an n-dimensional crossed cube with up to $2n-5$ link faults.

Keywords Conditional fault · Crossed cube · Fault-tolerant embedding · Pancycle

1 Introduction

The architecture of an interconnection network (network for short) is usually represented by a graph G, where the vertices (edges) of G represent the nodes (links) of the network. We usually use $V(G)$ ($E(G)$) to denote the vertex set (edge set) of G. Throughout this paper, vertex and node, edge and link, and graph and network are used interchangeably. An *embedding* of one (guest) graph G into another (host) graph H is a one-to-one mapping f from the node set of G to the node set of H. An edge of G corresponds to a path of H under f. The *dilation* of f is the maximal

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length of the paths in H that are the images of edges in G under f. The *pancycle* problem on a graph G is to determine whether or not G contains cycles of lengths ranging from four[1] to $|V(G)|$, and to construct them if they exist. If all these cycles exist in G, then G is called *pancyclic*. A pancyclic graph can embed cycles of lengths ranging from four to $|V(G)|$ with dilation 1.

The hypercube is a popular interconnection network with many attractive properties such as regularity, symmetry, small diameter, strong connectivity, recursiveness, flexible partition, and relatively low link complexity [22]. On the other hand, the crossed cube [7, 8], which was derived by changing some connections of the hypercube, is superior to the hypercube in diameter and mean distance. An n-dimensional crossed cube has a diameter equal to $\lceil (n+1)/2 \rceil$, which is about one half of the diameter of an n-dimensional hypercube (throughout this section, we use n to denote the dimension of the crossed cube and the hypercube). The diameter of a network represents a worst-case lower bound on the time required for performing some fundamental operations such as one-to-one routing, broadcasting, data aggregation, semigroup computation, etc.

Previous results on the crossed cube can be found in the literature [5, 10, 14, 16, 27]. In [16], a $(2^n - 1)$-node complete binary tree was embedded into the crossed cube with dilation 1. The dilation will go up to 2 if the same tree is embedded into the hypercube [25]. In [18], the connectivity of the crossed cube was shown to be n [18]. In [5], the n-wide diameter of the crossed cube was shown to be $\lceil n/2 \rceil + 2$. The same diameter for the hypercube is $n + 1$ [22]. Since processor faults or link faults may occur, it is meaningful in practice to consider faulty networks. A network with a high degree of fault tolerance can work properly when a limited number of processors or links are damaged.

There were two commonly used fault models: *random faults* [14, 27] and *conditional faults* [13]. The former assumed that faults might happen anywhere without any restriction, while the latter assumed that the fault distribution must obey some constraint, e.g., that each node is incident with at least one fault-free node [9] or that each node is incident with at least two fault-free links [4]. In [5], with the assumption of random faults, the fault diameter of the crossed cube was shown to be $\lceil n/2 \rceil + 2$. The *fault diameter* of a network W is the maximal diameter in W with at most $\kappa(W) - 1$ nodes removed, where $\kappa(W)$ is the node connectivity of W. The fault diameter for the hypercube is $n + 1$ [18]. Also, the crossed cube was shown to be $(n-2)$-Hamiltonian [14], $(n-3)$-Hamiltonian connected [14], and $(n-2)$-fault tolerant pancyclic [27]. Since the hypercube is bipartite, it is not pancyclic, not Hamiltonian connected, and not 1-node-Hamiltonian (see [11]). All these results reveal that when faults happen, the crossed cube is superior to the hypercube in fault diameter and Hamiltonicity.

On the other hand, with an assumption that each node is incident with at least one fault-free node, connectivities and fault diameters were computed on some networks [9, 20]. With another assumption that each node is incident with at least two fault-free links, Hamiltonian properties were investigated on some networks [3, 4, 12, 15, 23]. This assumption is meaningful, as its occurrence probability is very close to 1.

[1] Some studies, e.g., [1, 2, 19] defined the pancycle problem with cycle lengths ranging from three to $|V(G)|$. Here, we follow the definition of [27].

In [15], with the same assumption, the authors showed that a crossed cube with up to $2n-5$ link faults contained a fault-free Hamiltonian cycle. In this paper, we extend the work of [15] by showing that fault-free cycles of lengths ranging from 4 to 2^n can be embedded with dilation one into a crossed cube with up to $2n-5$ link faults. In other words, when each node is incident with at least two links, the crossed cube remains pancyclic, even if up to $2n-5$ links are damaged. The result is optimal with respect to the number of link faults tolerated.

In the next section, the structure of the crossed cube is reviewed. Some necessary definitions, notations and fundamental results are also introduced. In Sect. 3, fault-free cycles of all possible lengths in a crossed cube with up to $2n-5$ link faults are constructed. Finally, in Sect. 4, this paper concludes with some remarks.

2 Preliminaries

We use CQ_n to denote an n-dimensional crossed cube. Each node of CQ_n is uniquely identified with an n-bit sequence. CQ_1 and CQ_2 are the same as a one-dimensional hypercube and a two-dimensional hypercube, respectively. For $n \geq 3$, CQ_n can be obtained by joining two CQ_{n-1}'s, denoted by CQ^0_{n-1} and CQ^1_{n-1}, with 2^{n-1} links, where each node of $CQ^0_{n-1}(CQ^1_{n-1})$ is preceded with a bit 0 (1). A node $u = 0u_{n-2}u_{n-3}\dots u_0 \in CQ^0_{n-1}$ is connected to a node $v = 1v_{n-2}v_{n-3}\dots v_0 \in CQ^0_{n-1}$ if and only if $(u_{2i+1}u_{2i}, v_{2i+1}v_{2i}) \in \{(00,00),(10,10),(01,11),(11,01)\}$ for all $0 \leq i \leq \lfloor (n-1)/2 \rfloor - 1$ and $u_{n-2} = v_{n-2}$ if n is even. Formally, CQ_n can be defined as follows, where $u_{2i+1}u_{2i} \sim v_{2i+1}v_{2i}$ denotes $(u_{2i+1}u_{2i}, v_{2i+1}v_{2i}) \in \{(00,00),(10,10),(01,11),(11,01)\}$.

Definition 1 [7] The node set of CQ_n is $\{v_{n-1}v_{n-2}\dots v_0 \mid v_i \in \{0,1\}$ for all $0 \leq i \leq n-1\}$. Two nodes $u = u_{n-1}u_{n-2}\dots u_0$ and $v = v_{n-1}v_{n-2}\dots v_0$ of CQ_n are adjacent if and only if there exists $0 \leq d \leq n-1$, satisfying the following four conditions:

(1) $u_{2i+1}u_{2i} \sim v_{2i+1}v_{2i}$ for all $0 \leq i \leq \lfloor d/2 \rfloor - 1$, if $d \geq 2$;
(2) $u_{d-1} = v_{d-1}$, if d is odd;
(3) $u_d = \bar{v}_d$ ($\bar{v}_d$ is the complement of v_d);
(4) $u_{n-1}u_{n-2}\dots u_{d+1} = v_{n-1}v_{n-2}\dots v_{d+1}$, if $d < n-1$.

The link (u, v) is referred to as a d-*link*. When $d \geq 2$, it connects CQ^0_d with CQ^1_d. When $d = 1$, it has $u_1 = \bar{v}_1$ and $u_i = v_i$ for $i \in \{0, 1, \dots, n-1\} - \{1\}$; when $d = 0$, it has $u_0 = \bar{v}_0$ and $u_i = v_i$ for $i \in \{1, 2, \dots, n-1\}$. Each node of CQ_n is incident with n links, which are 0-link, 1-link, ..., $(n-1)$-link, respectively. Figure 1 shows the structures of CQ_3 and CQ_4, where $(0001, 0000)$, $(0001, 0011)$, $(0001, 0111)$ and $(0001, 1011)$ are the 0-link, 1-link, 2-link, and 3-link, respectively, incident with the node 0001.

A path (cycle) in a graph G is called a *Hamiltonian path* (*cycle*) if it contains every vertex of G exactly once. The number of edges incident with a vertex v in G is called the *degree* of v. Throughout this paper, we use $G - V'(G - E')$ to denote the graph that results by removing V' (E') from G, where $V' \subseteq V(G)(E' \subseteq E(G))$. We also let

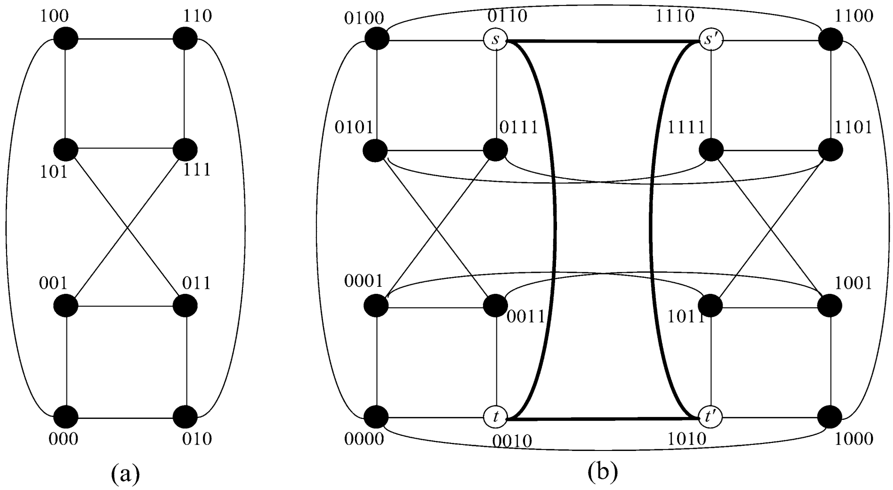

Fig. 1 CQ_n. (**a**) $n = 3$. (**b**) $n = 4$

$P_{x,y}$ denote a path from node x to node y, $\delta(G)$ denote the minimum node degree of G, and $u^{(d)}$ denote the node of CQ_n that is connected to u by a d-link. When $x = y$, $P_{x,y}$ is a cycle. A cycle of length l is referred to as an *l-cycle*.

Lemma 1 [14] *Suppose that $E' \subset E(CQ_n)$ and $|E'| \le n-2$, where $n \ge 3$. Then there exists a Hamiltonian cycle in $CQ_n - E'$.*

Lemma 2 [14] *Suppose that $E' \subset E(CQ_n)$, $V' \subset V(CQ_n)$, and $|E'| + |V'| \le n-3$, where $n \ge 3$. Then for every two distinct nodes u, v in $V(CQ_n) - V'$, there exists a Hamiltonian path between u and v in $CQ_n - V' - E'$.*

Lemma 3 [15] *Suppose that $E' \subset E(CQ_n)$, $|E'| \le 2n-5$, and $\delta(CQ_n - E') \ge 2$, where $n \ge 3$. Then there is a Hamiltonian cycle in $CQ_n - E'$.*

Lemma 4 [27] *Suppose that $E' \subset E(CQ_n)$ and $|E'| \le n-2$, where $n \ge 3$. Then for $4 \le l \le 2^n$, there is an l-cycle in $CQ_n - E'$.*

Lemma 5 [15] *Suppose that u, v, x, and y are four distinct nodes of CQ_n, where $n \ge 4$. Then there are a $P_{u,v}$ and a $P_{x,y}$ such that $V(P_{u,v}) \cap V(P_{x,y}) = \varnothing$ and $V(P_{u,v}) \cup V(P_{x,y}) = V(CQ_n)$.*

Lemma 6 [17] *Suppose that (s,t) is a d-link of CQ_n, where d is odd or $d = n-2$ and $n \ge 2$. Then $(s^{(n-1)}, t^{(n-1)})$ is also a d-link of CQ_n.*

Clearly, when $d \ne n-1$, the four nodes $s, s^{(n-1)}, t^{(n-1)}$ and t mentioned in Lemma 6 form a 4-cycle. In particular, when $d = n-2$, this 4-cycle is referred to as a *crossed* 4-*cycle*. For example, refer to Fig. 1, where the four nodes 0110, 1110, 1010, and 0010 form a crossed 4-cycle. It is not difficult to see that each node of CQ_n

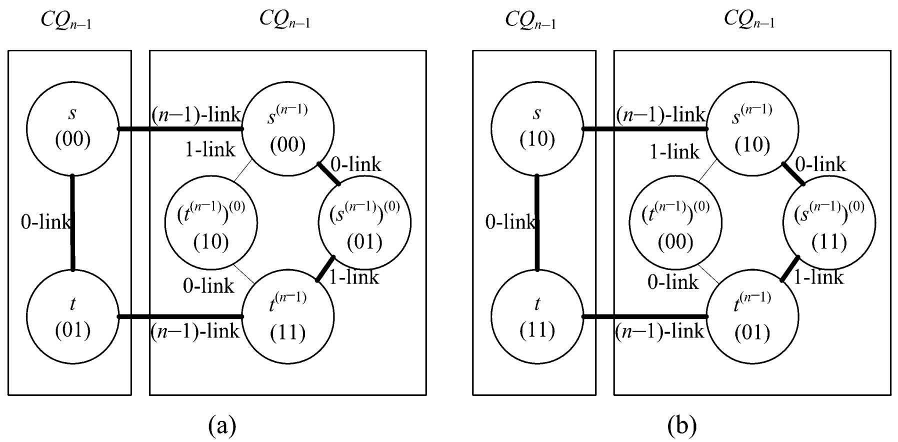

Fig. 2 Two crossed 5-cycles. (**a**) $\{s_1s_0, t_1t_0\} = \{00, 01\}$. (**b**) $\{s_1s_0, t_1t_0\} = \{10, 11\}$

is contained in a unique crossed 4-cycle. Thus, there are 2^{n-2} mutually node-disjoint crossed 4-cycles in CQ_n.

Similarly, when (s, t) is a 0-link of CQ_n, where $n \geq 3$, there are two 5-cycles formed by s, t, $t^{(n-1)}$, $(s^{(n-1)})^{(0)}$, $s^{(n-1)}$, and s, t, $t^{(n-1)}$, $(t^{(n-1)})^{(0)}$, $s^{(n-1)}$, respectively, as explained below. We only need to show $(t^{(n-1)}, (s^{(n-1)})^{(0)})$, $((t^{(n-1)})^{(0)}, s^{(n-1)}) \in E(CQ_n)$. It is not difficult to check that $s^{(n-1)}$ and $t^{(n-1)}$ differ at the rightmost two bits (by the fact that s and t differ at the rightmost bit). So, $(t^{(n-1)}, (s^{(n-1)})^{(0)})$, and $((t^{(n-1)})^{(0)}, s^{(n-1)})$ are two 1-links of CQ_n. Also, notice that $(s^{(n-1)})^{(0)}$ and $(t^{(n-1)})^{(0)}$, which are the two distinct nodes in the two 5-cycles, differ at the rightmost two bits. A 5-cycle thus defined is referred to as a *crossed* 5*-cycle*, if either $(s^{(n-1)})^{(0)}$ or $(t^{(n-1)})^{(0)}$ has the rightmost bit 1.

For example, refer to Fig. 2, where crossed 5-cycles are illustrated with bold lines. Let s_1s_0 and t_1t_0 be the rightmost two bits of s and t, respectively. We have $\{s_1s_0, t_1t_0\} = \{00, 01\}$ or $\{10, 11\}$. Figure 2a shows the crossed 5-cycle with $\{s_1s_0, t_1t_0\} = \{00, 01\}$, and Fig. 2b shows the crossed 5-cycle with $\{s_1s_0, t_1t_0\} = \{10, 11\}$. Since there are a total of 2^{n-1} 0-links, CQ_n contains 2^{n-1} crossed 5-cycles.

Lemma 7 [15] *Suppose that $E' \subset E(CQ_n)$ and $|E'| \leq n-2$, where $n \geq 4$. If $(u, v) \notin E'$ is an $(n-1)$-link or a d-link for some odd d, then there exists a Hamiltonian cycle in $CQ_n - E'$ that contains (u, v).*

Lemma 8 [10] *Suppose that (u, v) is an $(n-1)$-link of CQ_n, where $n \geq 3$. Then for $4 \leq l \leq 2^n$, there is an l-cycle in CQ_n that contains (u, v).*

Lemma 9 *Each link of CQ_n is contained in at most two crossed* 5*-cycles.*

Proof Consider an arbitrary d-link (x, y) of CQ_n, where $0 \leq d \leq n-1$. Recall that a crossed 5-cycle can be expressed as s, t, $t^{(n-1)}$, $(s^{(n-1)})^{(0)}$, $s^{(n-1)}$ or s, t, $t^{(n-1)}$, $(t^{(n-1)})^{(0)}$, $s^{(n-1)}$, depending on which of $(s^{(n-1)})^{(0)}$ and $(t^{(n-1)})^{(0)}$ has the rightmost bit 1. For convenience, we use z_1, z_2, z_3, z_4, z_5 to represent a crossed 5-cycle, where z_4 is the distinct node with the rightmost bit 1. Moreover, (z_1, z_2) and (z_4, z_5) are two 0-links, (z_2, z_3) and (z_5, z_1) are two $(n-1)$-links, and (z_3, z_4) is a 1-link. It is easy to see that the rightmost bits of z_1, z_2, z_3, z_4 and z_5 are 0, 1, 1, 1 and 0, respectively.

When $d \notin \{0, 1, n-1\}$, (x, y) is not contained in any crossed 5-cycle. Let x_0 and y_0 be the rightmost bits of x and y, respectively. When $d = 0$, we have $x_0 \neq y_0$. Without loss of generality, we assume $x_0 = 0$ and $y_0 = 1$. Then (x, y) is contained in two crossed 5-cycles with $(z_1, z_2) = (x, y)$ and $(z_4, z_5) = (y, x)$, respectively. When $d = 1$, we have $x_0 = y_0$. If $x_0 = y_0 = 1$, then (x, y) is contained in two crossed 5-cycles with $(z_3, z_4) = (x, y)$ and $(z_3, z_4) = (y, x)$, respectively. If $x_0 = y_0 = 0$, then (x, y) is not contained in any crossed 5-cycle. When $d = n-1$, we have $x_0 = y_0$. If $x_0 = y_0 = 1 (x_0 = y_0 = 0)$, then (x, y) is contained in two crossed 5-cycles with $(z_1, z_5) = (x, y)$ and $(z_1, z_5) = (y, x)((z_2, z_3) = (x, y)$ and $(z_2, z_3) = (y, x))$, respectively. □

Lemma 10 *Suppose that $E' \subset E(CQ_n)$ and $|E'| \leq n-3$, where $n \geq 3$. For every two distinct nodes u, v of CQ_n, there exists a $P_{u,v}$ of length $2^n - 2$ in $CQ_n - E'$.*

Proof When $|E'| \leq n-4$, a node $w \notin \{u, v\}$ of CQ_n is arbitrarily selected. By Lemma 2, there exists a $P_{u,v}$ of length $2^n - 2$ in $CQ_n - \{w\} - E'$. When $|E'| = n-3$, a node $w \notin \{u, v\}$ of CQ_n is selected such that there is a link $(w, z) \in E'$. There exists a $P_{u,v}$ of length $2^n - 2$ in $CQ_n - \{w\} - (E' - \{(w, z)\})$, by Lemma 2 again. □

Lemma 11 *Suppose that $E' \subset E(CQ_n)$ and $|E'| \leq n-2$, where $n \geq 4$. If $(u, v) \notin E'$ is an $(n-1)$-link, then there exists a $(2^n - 1)$-cycle in $CQ_n - E'$ that contains (u, v).*

Proof Partition E' into E_0, E_1 and E_c, where $E_0 = E' \cap E(CQ^0_{n-1})$, $E_1 = E' \cap E(CQ^1_{n-1})$, and $E_c = E' \cap \{(x, y) | x \in V(CQ^0_{n-1})$ and $y \in V(CQ^1_{n-1})\}$. Without loss of generality, assume that $u \in V(CQ^0_{n-1})$, $v \in V(CQ^1_{n-1})$, and $|E_0| \geq |E_1|$. When $n = 4$, this lemma can be easily verified with the aid of a computer program (see [24]) that performs an exhaustive search on CQ_4. For $n \geq 5$, three cases are discussed below.

Case 1. $|E_0| \leq n-3$. When $|E_0| \leq n-4$, select $(s, s^{(n-1)}) \notin E'$ so that $(s, s^{(n-1)}) \neq (u, v)$. Without loss of generality, assume $s \in V(CQ^0_{n-1})$. By Lemma 2, there exists a Hamiltonian path between s and u in $CQ^0_{n-1} - E_0$. By Lemma 10, there exists a $P_{v,s^{(n-1)}}$ of length $2^{n-1} - 2$ in $CQ^1_{n-1} - E_1$. The desired $(2^n - 1)$-cycle in $CQ_n - E'$ can be constructed as the bold cycle of Fig. 3a. When $|E_0| = n-3$, the bold cycle of Fig. 3a remains valid after a modification. The Hamiltonian path in $CQ^0_{n-1} - E_0$ is not available again. Instead, a Hamiltonian cycle in $CQ^0_{n-1} - E_0$ is constructed by Lemma 1. Moreover, since $|E_c| \leq 1$, there exists $(s, s^{(n-1)}) \notin E'$ such that (s, u) is a link of the Hamiltonian cycle.

Case 2. $|E_0| = n-2$. Arbitrarily select a link $(x, y) \in E_0$. By Lemma 1, there exists a Hamiltonian cycle in $CQ^0_{n-1} - (E_0 - \{(x, y)\})$. If the Hamiltonian cycle does not contain (x, y), then select $(s, s^{(n-1)}) \notin E'$ such that (s, u) is a link of the Hamiltonian cycle. By Lemma 10, there exists a $P_{v,s^{(n-1)}}$ of length $2^{n-1} - 2$ in $CQ^1_{n-1} - E_1$. The desired $(2^n - 1)$-cycle in $CQ_n - E'$ can be obtained as shown in Fig. 3a.

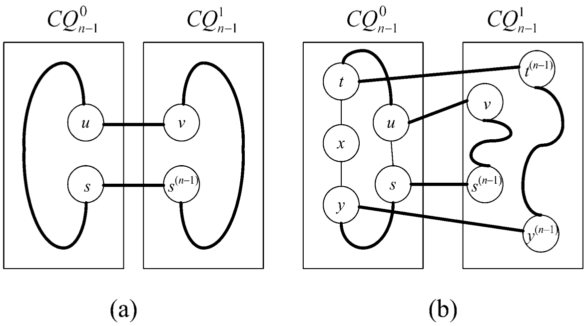

Fig. 3 $(2^n - 1)$-cycles in $CQ_n - E'$ that contain (u, v). (**a**) $|E_0| \leq n - 3$. (**b**) $|E_0| = n - 2$ and $u \notin \{x, y\}$

If the Hamiltonian cycle contains (x, y), then two situations, $u \in \{x, y\}$ or $u \notin \{x, y\}$, are discussed. When $u \in \{x, y\}$, we assume $u = x$ without loss of generality. By Lemma 10, there exists a $P_{v,y^{(n-1)}}$ of length $2^{n-1} - 2$ in $CQ^1_{n-1} - E_1$. The desired $(2^n - 1)$-cycle in $CQ_n - E'$ can be obtained as shown in Fig. 3a (replacing s with y). When $u \notin \{x, y\}$, select a node $s \notin \{x, y\}$ such that (s, u) is a link of the Hamiltonian cycle. Also, select a node $t \notin \{u, s\}$ such that (t, x) or (t, y) is a link of the Hamiltonian cycle. By Lemma 5, there exist a $P_{v,s^{(n-1)}}$ and a $P_{t^{(n-1)},y^{(n-1)}}$ satisfying $V(P_{v,s^{(n-1)}}) \cap V(P_{t^{(n-1)},y^{(n-1)}}) = \varnothing$ and $V(P_{v,s^{(n-1)}}) \cup V(P_{t^{(n-1)},y^{(n-1)}}) = V(CQ^1_{n-1})$. The desired $(2^n - 1)$-cycle in $CQ_n - E'$ can be constructed as the bold cycle of Fig. 3b. □

3 Fault-free cycles of all possible lengths

It was shown in [27] that a CQ_n with $n - 2$ random link faults can embed cycles of lengths ranging from 4 to 2^n with dilation one (no 3-cycle in CQ_n [14]). In this section, we show that when each node is incident with at least two fault-free links, CQ_n can embed cycles of the same lengths with dilation one, even if there are up to $2n - 5$ link faults.

Theorem 1 *Suppose that $E' \subset E(CQ_n)$, $|E'| \leq 2n - 5$, and $\delta(CQ_n - E') \geq 2$, where $n \geq 3$. Then there are cycles of lengths ranging from 4 to 2^n in $CQ_n - E'$.*

Proof For $n = 3$, the correctness of the theorem can be assured by Lemma 4. Thus, we assume $n \geq 4$. We prove by induction that there are cycles of lengths ranging from 4 to 2^n in $CQ_n - E'$ that each contain at least two $(n - 1)$-links. When $n = 4$, cycles of lengths ranging from 4 to 16 containing at least two 3-links each can be constructed in $CQ_4 - E'$, which can be verified with the aid of a computer program (see [24]). We assume that it also holds for $n = k \geq 4$, i.e., there are cycles of lengths ranging from 4 to 2^k in $CQ_k - E'$ that each contain at least two $(k - 1)$-links. In the rest of the proof, the situation of $n = k + 1$ is discussed.

To begin with, we partition E' into E_0, E_1, and E_c, where $E_0 = E' \cap E(CQ^0_k)$, $E_1 = E' \cap E(CQ^1_k)$, and $E_c = E' \cap \{(x, y) | x \in V(CQ^0_k) \text{ and } y \in V(CQ^1_k)\}$. Without loss of generality, we assume $|E_0| \geq |E_1|$. Since $|E_0| + |E_1| + |E_c| \leq 2k - 3$, we have

$|E_1| \le k-2$. Recall that there are 2^{k-1} mutually node-disjoint crossed 4-cycles and 2^k crossed 5-cycles in CQ_{k+1}. They each contain two k-links, and by Lemma 9, each link of CQ_{k+1} is contained in at most two crossed 5-cycles. Since $2^{k-1} > 2k-3$ ($2^k > 2 \times (2k-3)$) for $k \ge 4$, there are crossed 4-cycles (crossed 5-cycles) in $CQ_{k+1} - E'$. According to Lemma 3, there are 2^{k+1}-cycles in $CQ_{k+1} - E'$. In the following, l-cycles for $6 \le l < 2^{k+1}$ in $CQ_{k+1} - E'$ containing at least two k-links each are constructed with two cases.

Case 1. $6 \le l \le 2^k + 2$. Define $S = \{(u,v) | (u,v)$ is a $(k-1)$-link of CQ_{k+1}, $(u,v) \notin E'$, and $\{(u,u^{(k)}), (v,v^{(k)}), (u^{(k)}, v^{(k)})\} \cap E' \ne \varnothing\}$. Recall that $(u^{(k)}, v^{(k)})$ is a $(k-1)$-link of CQ_{k+1} by Lemma 6, and $(u,v), (u,u^{(k)}), (v,v^{(k)}), (u^{(k)}, v^{(k)})$ constitute a crossed 4-cycle in CQ_{k+1}. Suppose that $(s,t) \notin E'$ is a $(k-1)$-link of CQ_{k+1}. Then $\{(s,s^{(k)}), (t,t^{(k)}), (s^{(k)}, t^{(k)})\} \cap E' \ne \varnothing$ if and only if $(s,t) \in S$. So, if $(s,t) \notin S$, then $(s,s^{(k)}), (t,t^{(k)}), (s^{(k)}, t^{(k)}) \notin E'$.

Let $S_0 = S \cap E(CQ_k^0)$ and $S_1 = S \cap E(CQ_k^1)$. Suppose $(s,t) \in S_1$. Then $(s,s^{(k)}) \in E'$ or $(t,t^{(k)}) \in E'$ or $(s^{(k)}, t^{(k)}) \in E'$, where $(s,s^{(k)}), (t,t^{(k)}) \in E_c$ and $(s^{(k)}, t^{(k)}) \in E_0$. It means that each link $(s,t) \in S_1$ induces at least one link ($(s,s^{(k)})$ or $(t,t^{(k)})$ or $(s^{(k)}, t^{(k)})$) in $E_0 \cup E_c$. Similarly, each link $(s,t) \in S_0$ induces at least one link in $E_1 \cup E_c$. Since no two distinct links in S_1 induce the same link in $E_0 \cup E_c$, we have $|S_1| \le |E_0 \cup E_c| = |E_0| + |E_c|$, which further implies $|S_1 \cup E_1| = |S_1| + |E_1| \le |E_0| + |E_1| + |E_c| \le 2k-3$. Since $|E_1| \le k-2$ and each node in CQ_k^1 is incident with at most one link in S_1, we have $\delta(CQ_k^1 - (S_1 \cup E_1)) \ge 1$. Two subcases are discussed below.

Case 1.1. $|S_1 \cup E_1| \le 2k-4$. When $\delta(CQ_k^1 - (S_1 \cup E_1)) = 1$, there is exactly one node of degree one in $CQ_k^1 - (S_1 \cup E_1)$; for otherwise, $|S_1 \cup E_1| \ge 2k-3$, which is a contradiction. Suppose that x is the node of degree one in $CQ_k^1 - (S_1 \cup E_1)$. Then, $(x, x^{(k-1)}) \in S_1$. By the induction hypothesis, there are cycles of lengths ranging from 4 to 2^k in $CQ_k^1 - (S_1 \cup E_1 - \{(x, x^{(k-1)})\})$ containing at least two $(k-1)$-links each. Let C denote any of these cycles, and $(s,t) \ne (x, x^{(k-1)})$ be a $(k-1)$-link of C. Since $(s,t) \notin S_1$, we have $(s,s^{(k)}), (t,t^{(k)}), (s^{(k)}, t^{(k)}) \notin E'$. A $(|C|+2)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be constructed as the bold cycle of Fig. 4a, where $|C|$ is the length of C.

Next, we assume $\delta(CQ_k^1 - (S_1 \cup E_1)) \ge 2$. If $|S_1 \cup E_1| = 2k-4$, then select a link $(y, y^{(k-1)}) \in S_1$, and by the induction hypothesis, there are cycles of lengths

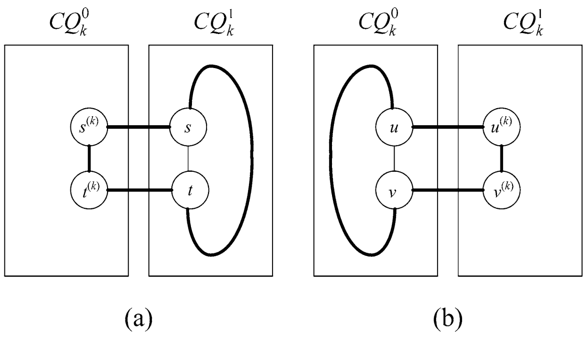

Fig. 4 l-cycles for $6 \le l \le 2^k + 2$ in $CQ_{k+1} - E'$. (**a**) $|S_1 \cup E_1| \le 2k-4$. (**b**) $|S_1 \cup E_1| = 2k-3$, $|E_1| > 0$, and there is no $(k-1)$-link in E_1

ranging from 4 to 2^k in $CQ_k^1 - (S_1 \cup E_1 - \{(y, y^{(k-1)})\})$ containing at least two $(k-1)$-links each. If $|S_1 \cup E_1| \leq 2k-5$, then by the induction hypothesis, there are cycles of lengths ranging from 4 to 2^k in $CQ_k^1 - (S_1 \cup E_1)$ containing at least two $(k-1)$-links each. For both, l-cycles for $6 \leq l \leq 2^k + 2$ in $CQ_{k+1} - E'$ containing two k-links each can be obtained, which is similar to the situation when $\delta(CQ_k^1 - (S_1 \cup E_1)) = 1$.

Case 1.2. $|S_1 \cup E_1| = 2k-3$. Since $|E_0| + |E_1| + |E_c| \leq 2k-3 = |S_1 \cup E_1| = |S_1| + |E_1|$, we have $|E_0| + |E_c| \leq |S_1|$. Recall that $|S_1| \leq |E_0| + |E_c|$, and each link in S_1 induces at least one link in $E_0 \cup E_c$. So, $|S_1| = |E_0| + |E_c|$, and there is a one-to-one correspondence between S_1 and $E_0 \cup E_c$. Also, notice that the image of each link in S_1 under the one-to-one correspondence is a $(k-1)$-link if it is located in E_0 and is a k-link if it is located in E_c. It follows that all links in E_0 are $(k-1)$-links. Moreover, if $(s, t) \in E_1$ is a $(k-1)$-link, then $(s, s^{(k)}), (t, t^{(k)}), (s^{(k)}, t^{(k)}) \notin E'$ for the following reason. Suppose, conversely, that $(s^{(k)}, t^{(k)}) \in E'$, without loss of generality. The fact that $(s^{(k)}, t^{(k)}) \in E_0$ and $(s, t) \notin S_1$ contradicts the one-to-one correspondence between S_1 and $E_0 \cup E_c$.

When $|E_1| = 0$, arbitrarily pick a crossed 4-cycle in $CQ_{k+1} - E'$, and let $(s, t) \in E(CQ_k^1)$ be the $(k-1)$-link contained in the crossed 4-cycle. By Lemma 8, there are cycles of lengths ranging from 4 to 2^k in CQ_k^1, each of which contains (s, t). Let C denote any of these cycles. A $(|C|+2)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be obtained as shown in Fig. 4a.

Then we assume $|E_1| > 0$. Suppose that $(u, v) \in E_1$ is a $(k-1)$-link. We have $(u, u^{(k)}), (v, v^{(k)}), (u^{(k)}, v^{(k)}) \notin E'$. When $\delta(CQ_k^1 - (S_1 \cup E_1)) = 1$, we have $(x, x^{(k-1)}) \in S_1$, where x is the node of degree one in $CQ_k^1 - (S_1 \cup E_1)$. By the induction hypothesis, there are cycles of lengths ranging from 4 to 2^k in $CQ_k^1 - (S_1 \cup E_1 - \{(x, x^{(k-1)}), (u, v)\})$ that contain at least two $(k-1)$-links each. Let C denote any of these cycles. If (u, v) is contained in C, then a $(|C|+2)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be obtained as shown in Fig. 4a (replacing (s, t) with (u, v)).

If (u, v) is not contained in C, then there exists a $(k-1)$-link $(w, w^{(k-1)}) \neq (x, x^{(k-1)})$ in C. A $(|C|+2)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be obtained as shown in Fig. 4a (replacing (s, t) with $(w, w^{(k-1)})$). When $\delta(CQ_k^1 - (S_1 \cup E_1)) \geq 2$, arbitrarily select a $(k-1)$-link in S_1, and then l-cycles for $6 \leq l \leq 2^k + 2$ in $CQ_{k+1} - E'$ that contain two k-links each can be obtained similarly.

On the other hand, suppose that there is no $(k-1)$-link in E_1. Recall that each link $(s, t) \in S_0$ induces at least one link in $E_1 \cup E_c$. Since the links induced by (s, t) are $(k-1)$-links or k-links, we have $|S_0| \leq |E_c|$, implying $|S_0| + |E_0| \leq |E_c| + |E_0| \leq 2k-4$. Arbitrarily select a $(k-1)$-link, say (u, v), from E_0. By the induction hypothesis, there are cycles of lengths ranging from 4 to 2^k in $CQ_k^0 - (S_0 \cup E_0 - \{(u, v)\})$ that contain at least two $(k-1)$-links each. Let C denote any of these cycles. If (u, v) is contained in C, then a $(|C|+2)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be constructed as the bold cycle of Fig. 4b. If (u, v) is not contained in C, then let $(w, w^{(k-1)})$ be a $(k-1)$-link of C. A $(|C|+2)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be obtained as shown in Fig. 4b (replacing (u, v) with $(w, w^{(k-1)})$).

Case 2. $2^k + 3 \leq l < 2^{k+1}$. Since $\delta(CQ_{k+1} - E') \geq 2$, we have $\delta(CQ_k^0 - E_0) \geq 1$. Three subcases: $|E_0| = 2k-3$, $|E_0| = 2k-4$ and $|E_0| \leq 2k-5$, are discussed below.

Case 2.1. $|E_0| = 2k-3$. We have $|E_1| = |E_c| = 0$. There are at most two nodes of degree one in $CQ_k^0 - E_0$; for otherwise, $|E_0| \geq 2(k-2) + (k-3) + 2 = 3k-5$ (there is no 3-cycle in CQ_k^0), which is a contradiction. Two node-disjoint links, say (u, v) and (x, y), are selected from E_0 such that they are incident with the nodes of degree one, if such nodes exist in $CQ_k^0 - E_0$. Clearly, $\delta(CQ_k^0 - (E_0 - \{(u, v), (x, y)\})) \geq 2$. By Lemma 3, there is a Hamiltonian cycle, denoted by C, in $CQ_k^0 - (E_0 - \{(u, v), (x, y)\})$.

If neither of (u, v) and (x, y) is contained in C, then obtain paths of lengths ranging from 2 to $2^k - 2$ from C. Let $P_{s,t}$ denote any of these paths. By Lemma 2, there is a Hamiltonian path between $s^{(k)}$ and $t^{(k)}$ in $CQ_k^1 - E_1$. A $(|P_{s,t}| + 2^k + 1)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be constructed as the bold cycle of Fig. 5a, where $|P_{s,t}|$ is the length of $P_{s,t}$. If exactly one (assuming (u, v)) of (u, v) and (x, y) is contained in C, then obtain $P_{s,t}$ without containing (u, v), and a $(|P_{s,t}| + 2^k + 1)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be obtained all the same.

If both (u, v) and (x, y) are contained in C, then without loss of generality, assume that u, v, x, y appear clockwise in C. We use $P_{v,x}(P_{y,u})$ to denote the path between v and x (between y and u) in C that does not contain $(u, v)((x, y))$. Notice that $|P_{v,x}| + |P_{y,u}| = 2^k - 2$. We first construct cycles of lengths ranging from $2^k + 3$ to $|P_{v,x}| + 2^k + 1$ in $CQ_{k+1} - E'$ as follows. Obtain subpaths, denoted by $P_{s,t}$, of $P_{v,x}$ whose lengths range from 2 to $|P_{v,x}|$. By Lemma 2, there is a Hamiltonian path between $s^{(k)}$ and $t^{(k)}$ in $CQ_k^1 - E_1$. A $(|P_{s,t}| + 2^k + 1)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be obtained as shown in Fig. 5a.

Then we construct cycles of lengths ranging from $|P_{v,x}| + 2^k + 2$ to $2^{k+1} - 1$ in $CQ_{k+1} - E'$ as follows. For each $|P_{v,x}| + 2^k + 2 \leq l \leq 2^{k+1} - 1$, obtain a subpath denoted by $P_{s,t}$, of $P_{v,x}$, and a subpath denoted by $P_{g,h}$, of $P_{y,u}$, such that $|P_{s,t}| + |P_{g,h}| = l - 2^k - 2$. By Lemma 5, there are two paths $P_{s^{(k)},g^{(k)}}$ and $P_{t^{(k)},h^{(k)}}$ in CQ_k^1 satisfying $V(P_{s^{(k)},g^{(k)}}) \cap V(P_{t^{(k)},h^{(k)}}) = \varnothing$ and $V(P_{s^{(k)},g^{(k)}}) \cup V(P_{t^{(k)},h^{(k)}}) = V(CQ_k^1)$. An l-cycle that contains four k-links of $CQ_{k+1} - E'$ can be constructed as the bold cycle of Fig. 5b.

Case 2.2. $|E_0| = 2k-4$. We have $|E_c| + |E_1| \leq 1$. There is at most one node of degree one in $CQ_k^0 - E_0$, for otherwise $|E_0| \geq 2(k-2) + 1$, a contradiction. Select a link, denoted by (u, v), from E_0. If there is a node of degree one in $CQ_k^0 - E_0$, then (u, v) should be incident with it. By Lemma 3, there is a Hamiltonian cycle in $CQ_k^0 - (E_0 - \{(u, v)\})$. There are paths, denoted by $P_{s,t}$, of lengths ranging from 2

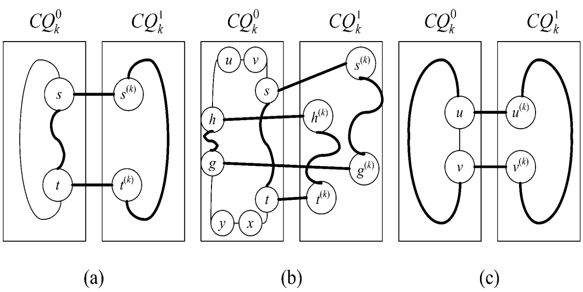

Fig. 5 l-cycles for $2^k + 3 \leq l < 2^{k+1}$ in $CQ_{k+1} - E'$. (**a**) $|E_0| = 2k-3$ and neither of (u, v) and (x, y) is contained in C. (**b**) $|E_0| = 2k-3$ and $|P_{v,x}| + 2^k + 2 \leq l < 2^{k+1}$. (**c**) $|E_0| \leq 2k-5$, $|E_1| \leq k-3$, and $\delta(CQ_k^0 - E_0) = 1$

to $2^k - 2$ in the cycle such that $P_{s,t}$ does not contain (u, v) and $(s, s^{(k)}), (t, t^{(k)}) \notin E_c$. By Lemma 2, there is a Hamiltonian path between $s^{(k)}$ and $t^{(k)}$ in $CQ_k^1 - E_1$. A $(|P_{s,t}| + 2^k + 1)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be obtained as shown in Fig. 5a.

Case 2.3. $|E_0| \leq 2k - 5$. Similarly, there is at most one node of degree one in $CQ_k^0 - E_0$. Two subcases, $|E_1| \leq k - 3$ and $|E_1| = k - 2$, are further discussed below.

Case 2.3.1. $|E_1| \leq k - 3$. When $\delta(CQ_k^0 - E_0) \geq 2$, by Lemma 3, there is a Hamiltonian cycle in $CQ_k^0 - E_0$. There are paths, denoted by $P_{s,t}$, of lengths ranging from 2 to $2^k - 2$ in the cycle such that $(s, s^{(k)}) \notin E_c$ and $(t, t^{(k)}) \notin E_c$. By Lemma 2, there is a Hamiltonian path between $s^{(k)}$ and $t^{(k)}$ in $CQ_k^1 - E_1$. A $(|P_{s,t}| + 2^k + 1)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be obtained as shown in Fig. 5a.

When $\delta(CQ_k^0 - E_0) = 1$, we have $|E_0| \geq k - 1$, which further implies $|E_c| \leq k - 2$. Suppose that u is the node of degree one in $CQ_k^0 - E_0$. Arbitrarily select $(u, v) \in E_0$ such that $(v, v^{(k)}) \notin E_c$. Since $\delta(CQ_{k+1} - E') \geq 2$, we have $(u, u^{(k)}) \notin E_c$. We first construct cycles of lengths ranging from $2^k + 3$ to $2^k + 2^{k-1}$ as follows. By Lemma 3, there is a Hamiltonian cycle in $CQ_k^0 - (E_0 - \{(u, v)\})$. Also, (u, v) is contained in the cycle. Let $P_{s,t}$ be a path in the cycle such that $P_{s,t}$ does not contain (u, v) and $(s, s^{(k)}), (t, t^{(k)}) \notin E_c$. There are $2^k - |P_{s,t}|$ choices for $P_{s,t}$, and we consider $2 \leq |P_{s,t}| \leq 2^{k-1} - 1$ (so, $2^k - |P_{s,t}| > |E_c|$). By Lemma 2, there is a Hamiltonian path between $s^{(k)}$ and $t^{(k)}$ in $CQ_k^1 - E_1$. A $(|P_{s,t}| + 2^k + 1)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be obtained as shown in Fig. 5a.

Then we construct cycles of lengths ranging from $2^k + 2^{k-1} + 1$ to $2^{k+1} - 1$. By the induction hypothesis, there are cycles of lengths ranging from $2^{k-1} + 1$ to $2^k - 1$ in $CQ_k^0 - (E_0 - \{(u, v)\})$. Let C denote any of these cycles. If (u, v) is contained in C, then by Lemma 2, there is a Hamiltonian path between $u^{(k)}$ and $v^{(k)}$ in $CQ_k^1 - E_1$. Otherwise, select a link, say (s, t), of C such that $(s, s^{(k)}), (t, t^{(k)}) \notin E_c$. A $(|C| + 2^k)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be constructed as shown in Fig. 5c (replacing (u, v) with (s, t) if (u, v) is not contained in C).

Case 2.3.2. $|E_1| = k - 2$. We have $|E_0| = k - 2$ or $k - 1$, and $|E_c| \leq 1$. A cycle of length $2^k + 3$ in $CQ_{k+1} - E'$ can be obtained as follows. Let (s, t) be a $(k - 1)$-link of a crossed 4-cycle in $CQ_k^0 - E_0$ such that $(s, s^{(k)}), (t, t^{(k)}) \notin E_c$. By Lemma 6, $(s^{(k)}, t^{(k)})$ is a $(k - 1)$-link of CQ_k^1 (and CQ_{k+1}), and by Lemma 11, there is a $(2^k - 1)$-cycle in $CQ_k^1 - (E_1 - \{(s^{(k)}, t^{(k)})\})$ that contains $(s^{(k)}, t^{(k)})$. A $(2^k + 3)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be constructed as the bold cycle of Fig. 6.

Cycles of lengths ranging from $2^k + 4$ to $2^{k+1} - 1$ can be obtained as follows. When $\delta(CQ_k^0 - E_0) \geq 2$, by the induction hypothesis, there are cycles of lengths ranging from 4 to $2^k - 1$ in $CQ_k^0 - E_0$ that contain at least two $(k - 1)$-links each. Let C denote any of these cycles, and (s, t) be a $(k - 1)$-link of C such that $(s, s^{(k)}), (t, t^{(k)}) \notin E_c$. By Lemma 6, $(s^{(k)}, t^{(k)})$ is a $(k - 1)$-link of CQ_k^1 (and CQ_{k+1}), and by Lemma 7, there is a Hamiltonian cycle in $CQ_k^1 - \{E_1 - (s^{(k)}, t^{(k)})\}$ that contains $(s^{(k)}, t^{(k)})$. A $(|C| + 2^k)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be obtained as shown in Fig. 5c (replacing (u, v) with (s, t)).

When $\delta(CQ_k^0 - E_0) = 1$, assume that u is the node of degree one in $CQ_k^0 - E_0$. Arbitrarily select a d-link $(u, v) \in E_0$ such that d is odd and $(v, v^{(k)}) \notin E_c$. By Lemma 6, $(u^{(k)}, v^{(k)})$ is a d-link of CQ_k^1 (and CQ_{k+1}). By the induction hypothesis, there are

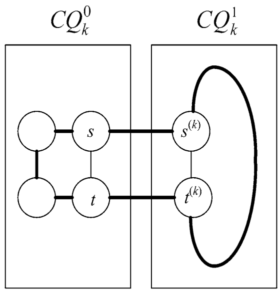

Fig. 6 A (2^k+3)-cycle in $CQ_{k+1} - E'$ when $|E_1| = k-2$

cycles of lengths ranging from 4 to $2^k - 1$ in $CQ_k^0 - (E_0 - \{(u, v)\})$ that each contain at least two $(k-1)$-links. Let C denote any of these cycles. If (u, v) is contained in C, then by Lemma 7, there is a Hamiltonian cycle in $CQ_k^1 - \{E_1 - (u^{(k)}, v^{(k)})\}$ that contains $(u^{(k)}, v^{(k)})$. Otherwise, let (s, t) be a $(k-1)$-link of C such that $(s, s^{(k)}), (t, t^{(k)}) \notin E_c$. A $(|C| + 2^k)$-cycle in $CQ_{k+1} - E'$ that contains two k-links can be obtained as shown in Fig. 5c (replacing (u, v) with (s, t) if (u, v) is not contained in C). □

4 Discussion and conclusion

This paper aims to solve the pancycle problem on a faulty crossed cube. Under the random fault model, the pancycle problem was solved on a CQ_n with up to $n-2$ link faults [27]. In this paper, with the assumption that at least two fault-free links are incident with each node, the pancycle problem was solved on a CQ_n with up to $2n-5$ link faults. The result is optimal, as there are distributions of $2n-4$ link faults in CQ_n that can prevent a fault-free Hamiltonian cycle in the faulty CQ_n (see [15]). In addition, it was indicated in [15] that the probability that the assumption holds is very close to 1. Therefore, the assumption is meaningful in practice, and as a result, algorithms that are executable on rings of lengths ranging from 4 to 2^n can be applied to a CQ_n with up to $2n-5$ link faults, with a very high probability.

With the same assumption, a fault-free Hamiltonian cycle in a faulty n-dimensional hypercube was constructed in [4], in which up to $2n-5$ link faults could be tolerated. The hypercube is highly symmetric, and it can be partitioned at any dimension into two smaller hypercubes. By the aid of this favorable property, a fault-free Hamiltonian cycle could be successfully constructed in a faulty hypercube. On the other hand, the crossed cube is not symmetric, and it can be partitioned into two smaller crossed cubes only at two dimensions. Thus, it is more difficult to construct a fault-free Hamiltonian cycle (and fault-free cycles of all possible lengths) in a faulty crossed cube.

There were three intractable distributions of $2n-5$ link faults over a faulty CQ_n. One occurred when the faulty CQ_n was partitioned into two CQ_{n-1}'s, such that some node in one CQ_{n-1} was incident with only one fault-free link. Another occurred when one CQ_{n-1} had too many ($\geq 2n-6$) link faults. For both, the induction hypothesis could not be applied, and thus some skillful routing methods must be developed to bypass the link faults. The routing methods were illustrated in Fig. 5.

The other occurred when one CQ_{n-1} had $n-3$ link faults and the other CQ_{n-1} had $n-2$ link faults. It was shown in [27] that there is a fault-free Hamiltonian path between every two distinct vertices of a faulty CQ_{n-1} with up to $n-4$ link faults. If such a fault-free Hamiltonian path existed in the two CQ_{n-1}'s, then it would be easy to bypass the link faults. Unfortunately, it did not necessarily exist in both CQ_{n-1}'s. Instead, the induction hypothesis was applied to construct cycles of all possible lengths in the CQ_{n-1} with $n-2$ link faults. The additional constraint (i.e., at least two $(n-2)$-links in each cycle) associated with the induction hypothesis assured the existence of two fault-free $(n-1)$-links, i.e., $(u, u^{(k)})$ and $(v, v^{(k)})$ in Fig. 5c. Then Lemma 7 assured the existence of a fault-free Hamiltonian path between $u^{(k)}$ and $v^{(k)}$ in the other CQ_{n-1}. The routing method was shown in Fig. 5c.

With the assumption that each node was incident with at least one fault-free node, the connectivities of hypercubes [9], k-ary n-cubes [6], cube-connected cycles [20], undirected de Bruijn networks [20], and Kautz networks [20] were computed. Moreover, the fault diameters of hypercubes [18] and star graphs [21] were obtained. One further research topic is to compute the connectivity and fault diameter of the crossed cube under the same assumption.

The *edge-pancycle* problem was defined as follows: Given any edge e of a graph G, the problem is to determine whether or not G contains cycles of lengths ranging from three to $|V(G)|$ that contain e, and to construct them if they exist [1]. Previously, the problem was solved (i.e., constructing cycles of lengths ranging from three to $|V(G)|$ for any e) on recursive circulants [2] and coupled graphs [19]. In addition, cycles of lengths ranging from four to $|V(G)|$ for any e were constructed for crossed cubes [10] and Möbius cubes [26]. Fault-free networks were assumed in these works. To solve the edge-pancycle problem on faulty networks is another topic for further research.

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