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Genuine Coherence as Mutual Confirmation Between Content Elements

Abstract. The concepts of coherence and confirmation are closely intertwined: according to a prominent proposal coherence is nothing but mutual confirmation. Accordingly, it should come as no surprise that both are confronted with similar problems. As regards Bayesian confirmation measures these are illustrated by the problem of tacking by conjunction. On the other hand, Bayesian coherence measures face the problem of belief individuation. In this paper we want to outline the benefit of an approach to coherence and confirmation based on content elements. It will be shown that the resulting concepts, called *genuine coherence* and *genuine confirmation*, can be used in order to solve the two mentioned problems. In a final section we present some results on degrees of genuine coherence and genuine confirmation.

Keywords: (Probabilistic measures of) coherence, (Probabilistic measures of) confirmation, Problem of belief individuation, Tacking problem, Content elements.

1. Introduction

The concepts of coherence and confirmation are closely intertwined: according to a well-known proposal coherence is nothing but mutual confirmation (cf. [10,15,38]). Accordingly, it should come as no surprise that both Bayesian models of these concepts are confronted with similar problems. As regards Bayesian confirmation measures, these can be illustrated by the problem of *tacking by conjunction*: if $h \wedge h'$ is a conjunctive hypothesis such that h deductively entails a piece of evidence e, then e can be shown to not only confirm h but also the conjunction $h \wedge h'$. This, so it is argued, is counter-intuitive because h' might not be related to e in any way (cf. [20]). On the other hand, Bayesian coherence measures face the *problem of belief individuation* according to which there are logically equivalent belief sets that are assigned different degrees of coherence by each extant coherence measure. To illustrate: for all existing measures it can

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be shown that there are propositions a, b and c such that the triple containing a, b and c is assigned a different degree of coherence than the pair containing the conjunction $a \wedge b$ and c. This, so it is argued, is counterintuitive given that the information contained in both situations is identical (cf. [31,51]).

In this paper we consider Bayesian models of coherence and confirmation that provide a solution for both these problems. We illustrate this solution for three languages: the language of truthfunctional propositional logic $\mathcal{L}_0$, the language of truthfunctional logic together with an intensional conditional operator $\mathcal{L}_{\rightarrow}$ and the language of 1st order logic $\mathcal{L}_1$. In what follows languages are identified with the sets of their well-formed formulas.

The basic idea underlying these proposals is that both measures of coherence and confirmation should be applied to some well-specified set of content elements of the propositions whose coherence or confirmation is assessed. The resulting concepts will be called *genuine coherence* and *genuine confirmation*. In a further step, we scrutinize the relationship between these qualitative models. We also turn to the question of how to quantify the corresponding degrees of genuine coherence and genuine confirmation against the background of their classical Bayesian counterparts; in the final paragraphs of the paper we present some results on the relationship between these quantitative concepts.

Here are some formal preliminaries that will prove useful throughout the paper: in what follows let $\mathcal{L}$ be a finite propositional language, i.e., the closure of a finite set of atomic propositions $\mathsf{At}(\mathcal{L}) = \{p_1, \ldots, p_n\}$ under the standard formation rules for truthfunctional connectives $\{\neg, \vee, \wedge, \supset, \equiv\}$. Arbitrary propositions of $\mathcal{L}$ will be denoted by lower case letters $a, b, c, \ldots$ (possibly indexed). By $\mathcal{L}_c$ we denote the set of contingent formulae of $\mathcal{L}$, i.e. $a \in \mathcal{L}_c$ if and only if $a \in \mathcal{L}$ and neither $\models a$ nor $\models \neg a$, where $\models$ stands for classical logical inference. In particular, e denotes an evidence statement, h a hypothesis and b a background assumption. By a "belief set" X we mean a finite set of believed propositions $X \subset \mathcal{L}$, together with an implicit epistemic probability distribution Pr relative to which the coherence of X is assessed. Finite sets of propositions of $\mathcal{L}$ will be denoted by upper case letters from the end of the alphabet $X, \ldots$ (possibly indexed). All of our belief sets (or sets of propositions) will be finite (thus we write $X \subset \mathcal{L}$ since our belief sets are proper subsets of $\mathcal{L}$). We restrict our investigation to finite belief sets because infinite sets cause various complications (infinite conjunctions, limit considerations for infinite sums, questions of compactness) that are postponed to another paper.

As usual, $|X|$ stands for the cardinality of a set $X \subset \mathcal{L}$ and 2^X for the power set of X. The deductive closure of $X \subset \mathcal{L}$ is denoted by $\mathsf{Cn}(X)$, i.e., $\mathsf{Cn}(X) = \{a \in \mathcal{L} | \, X \models a\}$, and the conjunction of all members of X by $\bigwedge X$. We extend the notion of deductive inference to sets of sentences as follows: $X \models Y$ if and only if $Y \subseteq \mathsf{Cn}(X)$. Accordingly, two sets X and Y are logically *equivalent*, in short: $L-$equivalent, iff $X \models Y$ and $Y \models X$. Concerning 1st order languages, individual constants will be denoted by α_i for $i \in \mathbb{N}$ (the set of natural numbers), individual variables by x, y, z and n-ary (possibly complex) predicates by upper case letters from the beginning of the alphabet $A^{(n)}, \ldots, R^{(n)}$; we often omit reference to the arity. A probability measure Pr over a language $\mathcal{L}$ is a non-negative, real-valued function such that $\Pr(a) = 1$ if a is a tautology, and Pr is finitely additive, i.e., $\Pr(a \vee b) = \Pr(a) + \Pr(b)$ if a and b are logically incompatible. For 1st order languages Pr is countably additive and, hence (assuming $\mathcal{L}$ has names for all individuals), satisfies the principles of continuity for quantifiers: $\Pr(\forall x A(x)) = \lim_{n\to\infty} \Pr(A(\alpha_1) \wedge \ldots \wedge A(\alpha_n))$ and $\Pr(\exists x A(x)) = \lim_{n\to\infty} \Pr(A(\alpha_1) \vee \ldots \vee A(\alpha_n))$ (cf. [48], fact 6). The set of all probability measures over $\mathcal{L}$ will be denoted by $\mathbf{P}$. $\Pr(a|b) =_{\mathrm{def}} \Pr(a \wedge b)/\Pr(b)$ denotes the conditional probability of a given b under the condition that $\Pr(b) > 0$. A proposition is called *Pr-normal* if $0 < \Pr(a) < 1$.

Given a probability measure Pr over $\mathcal{L}$, a *Bayesian confirmation measure* is a function $\mathbf{conf} : \mathcal{L}^2 \times \mathbf{P} \to \mathbb{R}$ that assigns to each triple $(e, h, \Pr)$ a real number that is supposed to represent the degree of confirmation that e provides for h under probability measure Pr in accordance with the following *relevance principle*:[1]

(R) $\mathbf{conf}(\mathrm{e,h}, \Pr) > / = / < 0$ if and only if $\Pr(\mathrm{h|e}) > / = / < \Pr(h)$.

The measure Pr is assumed to be relativized to a given background knowledge b, i.e., $\Pr(-) = \Pr(-|b)$ and $\Pr(b) = 1$; we also admit the case that b is empty (i.e., a tautology). A *Bayesian coherence measure*, on the other hand, is a function $\mathbf{coh}\colon (2^{\mathcal{L}} \setminus \emptyset) \times \mathbf{P} \to \mathbb{R}$ that assigns to each pair $(X, \Pr)$ a real number that is supposed to represent the degree of coherence of the non-empty set X under probability distribution Pr.[2] In what follows we will often omit reference to the probability distribution (in **conf** and **coh**) to ease readability.

[1]For a recent survey of Bayesian confirmation theory see [7].

[2]A survey of the various Bayesian coherence measures that have been discussed in the recent literature is given by [40].

2. Confirmation and Tacking by Conjunction

According to the qualitative concept of Bayesian confirmation theory, a piece of evidence e confirms a hypothesis h if and only if h's conditional probability given e exceeds its unconditional probability, i.e. $\Pr(h|e) > \Pr(h)$. Equivalently, one could say that h is more probable given e than given $\neg e$.[3] While this is the common ground of the vast majority of Bayesian approaches to confirmation, there are important issues regarding the comparative concept of confirmation (cf. [5,16]) and the adequacy of various quantitative measures of confirmation that have been proposed over the years (cf. [3,9,12,13,39,52]). In the present section, however, we will dispense with a discussion of these issues and focus on the qualitative confirmation model.[4] Now it is a simple theorem of the probability calculus that logical consequences confirm the hypotheses they are entailed by, i.e., if h logically entails e and e and h are Pr-normal, then e confirms h. This seems to accord well with standard scientific practice in which theories are assessed by scrutinizing their consequences in the light of empirical data. However, the following consequence destroys this nice picture: since the conjunctive hypothesis $h \wedge e$ logically entails each of its conjuncts, the conjunct e also confirms $h \wedge e$ for every arbitrary theory h (cf. [20], p. 67). To see why this is problematic, let h be the proposition that the doctrine of Jehovah's witnesses is true and e the proposition that grass is green, then this latter proposition confirms the hypothesis that grass is green *and* that the doctrine of Jehovah's witnesses is true. Given that there is no connection between the contents of h and e this latter confirmation-assessment seems counterintuitive. Slightly more general, the tacking by conjunction problem (and its special case "e confirms $e \wedge h$") can be defined as follows:

Definition 2.1. [*Tacking by conjunction*] (i) If h logically entails e, then e confirms $h \wedge h'$ for any h' provided that e and $h \wedge h'$ are Pr-normal.
(ii) [Special case $h = e$] In particular, if e and $e \wedge h'$ are Pr-normal, then e confirms $e \wedge h'$.

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[3] This concept is usually called the *incremental* concept of confirmation as opposed to the concept of *absolute* confirmation according to which a piece of evidence e confirms a hypothesis h if h's conditional probability given e exceeds some threshold $0.5 \leq r < 1$ (cf. [4]). Although the absolute concept of confirmation proved useful when modeling coherence we will not take it into account in the present paper (cf. [38,40]).

[4] But we will dwell into some issues pertaining to quantitative confirmation models in later sections.

A more general form of the tacking problem that dispenses with the assumption of deductive entailment runs as follows: if e confirms a hypothesis h, then e also confirms the conjunction $h \wedge h'$ for any "tacked" irrelevant conjunct h', provided only that $h \wedge h'$ is Pr-normal. According to this version, the data that confirm Darwinian evolution theory would also confirm the conjunction of Darwinian evolution theory and creationism. This seems highly counter-intuitive, too. Referring to [14], the generalized tacking problem can be spelled out as follows:

Definition 2.2. [*Generalized tacking by conjunction*] If e confirms h, and h' is probabilistically irrelevant to h, e and $h \wedge e$, then e confirms $h \wedge h'$ for any Pr-normal $h \wedge h'$.

Existing Bayesian solution proposals for both the deductive and the generalized version of the tacking problem try to shift the focus from the qualitative notion of confirmation to its comparative counterpart. In a nutshell, their main idea is to swallow the bitter pill of the tacking problems but try to soften its negative impact by showing that even though the conjunction $h \wedge h'$ is (qualitatively) confirmed by e, it is *less confirmed* than its 'relevant' part h. These proposals are certainly great achievements in Bayesian confirmation theory, but in our eyes they nevertheless suffer from two drawbacks:

(1) Of course, philosophical intuitions are never unique, and not all instances of the tacking problem are equally counter-intuitive. For some instances of the generalized tacking problem, the "diminished confirmation" strategy may seem appropriate. However, in regard to the special tacking problem (Definition 2.1, (ii)), a broadly held philosophical intuition says that if h is either entirely unrelated to e or is (even worse) an anti-inductive (Goodman-type) projection of e, then $h \wedge e$ cannot be said to be confirmed by e at all (although it is confirmed according to Bayesian measures).

The Bayesian confirmability of Goodman-hypotheses is maybe the deepest aspect of the tacking problem that is not resolved by the diminished confirmation strategy. It can be illustrated as follows: let E, G, B, and O be the predicates "is an emerald", "is green", "is blue" and "has been observed", respectively, and let $e =_{\text{def}} \forall x(Ex \wedge Ox \supset Gx)$ be the evidence proposition that all observed emeralds have been green. Then e confirms the Goodman-type hypothesis h^* "All emeralds are grue, i.e. green if observed and blue if not observed" as well as h "All emeralds are green", since both hypotheses can be represented by conjunctions of

e with a proposition about non-observed emeralds:

$$h =_{\text{def}} \forall x(Ex \supset Gx) \equiv e \wedge \forall x(Ex \wedge \neg Ox \supset Gx)$$
$$h^* =_{\text{def}} \forall x(Ex \supset G^*x) \equiv e \wedge \forall x(Ex \wedge \neg Ox \supset Bx).$$

(2) A second drawback of the strategy of diminished confirmation is that this solution strategy does not work for all but only for certain confirmation measures. In general, proponents of the strategy of diminished confirmation try to establish the following comparative claim

(COMP) If $\mathbf{conf}(e, h) > 0$
and $\mathbf{Irr}(\mathbf{h}')$ holds true, then $0 < \mathbf{conf}\ (e, h \wedge h') < \mathbf{conf}\ (e, h)$.

where "$\mathbf{Irr}(h')$" is an antecedent entailing certain conditions concerning the irrelevance of h' in regard to e, h and/or $h \wedge e$. Whereas [14] invokes the antecedent of our definition 2.2, [22] focus on a slightly weaker condition (the antecedent of Observation 2.1 below) that is sufficient to establish the same comparative confirmation claim. The common drawback of these proposals is that they are *measure-sensitive* in the following sense: there are pairs of measures $(\mathbf{conf}, \mathbf{conf}')$ such that **(COMP)** holds for **conf** but is violated for $\mathbf{conf}'$. Therefore, in order to defend the comparative confirmation claim **(COMP)** as a possible solution of the tacking problem, one also has to defend a subset of measures as proper confirmation measures against their inappropriate counterparts that violate (**COMP**). However, among those measures that are at odds with the comparative confirmation claim are highly prominent ones like the *log-ratio measure of confirmation* $\log[\Pr(h|e)/\Pr(h)]$ that has many prominent adherents (cf. Horwich [24], [25,30,43]). Another example is Mortimer's [32] confirmation measure $\Pr(e|h) - \Pr(e)$.

Observation 2.1. If h' is (probabilistically) irrelevant to e conditional on h (i.e. $\Pr(e|h \wedge h') = \Pr(e|h)$), then h and $h \wedge h'$ are confirmed (or disconfirmed) by e to the same degree according to (i) the log-ratio measure and (ii) Mortimer's confirmation measure.[5]

Proof. For (i) see [22] (revised Theorem 2, p. 510, 512). For (ii) see [3], obs. 1. ■

[5] Note, however, that these are *not* the only confirmation measures that are at odds with (**COMP**); other examples are the measures β_r and γ_r that will be introduced in Section 6 (proof omitted).

In conclusion we think that one should keep apart these two debates, i.e., the one on the tacking problem and the one on the adequacy of various Bayesian confirmation measures. Our envisaged solution should therefore be as robust as possible so that it holds for the vast majority (if not all) of the existing confirmation measures (see also Section 6).[6] However, before we dwell into the details of our solution, the next section will show that this problem is intimately connected with a fundamental problem of Bayesian accounts of coherence.

3. Coherence and the Problem of Belief Individuation

The concept of coherence is notorious for its elusiveness. According to a prominent paraphrase, "coherence is a matter of how well a body of beliefs 'hangs together', how well its component beliefs fit together, agree or dovetail with each other" ([2], p. 93). Recent years have seen the upshot of a number of formal models that aim for an explication of coherence in terms of probability theory. There are models that explicate coherence in terms of the deviation from probabilistic independence [44,50], the relative set-theoretic overlap [19,29,34] or the degree of mutual confirmation [10,15]. In this paper, we will focus on this latter family of coherence measures for the following reasons: first, some authors have given conclusive reasons why the proposed measures based on the idea of relative overlap should *not* be considered proper explications of the concept of coherence (cf. [27]). Second, as mentioned above, the later sections will be devoted to assessing the relationship between genuine confirmation and genuine coherence. For this purpose it is preferable to concentrate on coherence measures that are built on the idea of mutual confirmation.[7]

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[6]Crupi and Tentori [8] extend the debate to cases involving a disconfirmed hypothesis h considering whether irrelevant conjuncts can also be 'tacked' in cases of disconfirmation. However, we will not dwell into this debate in this paper. One of the reasons is that we think that Crupi and Tentori's proposal which adding 'tacked' hypotheses should result in a conjunctive hypothesis which is at least as strongly disconfirmed than the original hypothesis is not uncontroversial. A plausible alternative proposal (discussed in Sect. 5 of [8]) is to stipulate that adding irrelevant conjuncts will usually be accompanied by a decrease in the absolute value of the degree of disconfirmation, i.e., if $\mathbf{conf}(e, h) < 0$ and $h_1, \ldots, h_n$ are all irrelevant to h, e, and $h \wedge e$, then $\mathbf{conf}(e, h) \leq \mathbf{conf}(e, h \wedge h_1 \wedge \ldots \wedge h_n)$. We think that a thorough discussion of these conflicting intuitions would deserve a paper of its own.

[7]Notice, however, that also the deviation-based measures of coherence and the overlap-based measures are faced by the problem of belief individuation (cf. [31]).

The main idea underlying the mutual confirmation approaches to measuring coherence is that a set of propositions or pieces of information is coherent if each relevant part of the set confirms all other relevant parts of the set under consideration. Obviously, there is an important void within this formulation which is to spell out what the 'relevant parts' of the given set are. In this regard, Douven and Meijs [10] discuss a number of possible proposals of which the most promising is the model of *any-any coherence* according to which the relevant parts are all non-empty, non-overlapping subsets of the given set. Given this basic idea, we can get two notions of qualitative coherence:

DEFINITION 3.1. [*Qualitative full coherence*] A set of propositions $X \subset \mathcal{L}$ is *fully coherent* (relative to probability distribution $\Pr \in \mathbf{P}$) iff $|X| > 1$ and for all non-empty, non-overlapping subsets $X', X'' \subset X$, $\Pr(\bigwedge X' | \bigwedge X'') > \Pr(\bigwedge X')$.

We also introduce a qualitative notion of partial coherence, in which not all but only some non-empty, non-overlapping subsets confirm each other. This notion of qualitative partial coherence should be consistent with the quantitative notion of coherence as the average of the degrees of confirmation between subsets of the set X (see below) in the sense that a positive degree of coherence counts as partial qualitative coherence. In this average, disconfirmation relations can be compensated by confirmation relations between the other propositions of the set, but this cannot be modeled in a qualitative way. To illustrate the point, imagine that $X' = \{a, b, c\}$ so that a confirms b and c to a minute degree, then it might nonetheless be the case that $b \wedge c$ disconfirms a so that the average of all these confirmation degrees and, thus, the assigned degree of coherence is negative. No qualitative definition of partial coherence which abstracts from degrees of confirmation can account for this. We therefore prefer to explicate qualitative partial coherence by a (strong) sufficient condition which entails a positive degree of coherence and a (weak) necessary condition, which is entailed by a positive degree of coherence:

DEFINITION 3.2. [*Qualitative partial coherence*] A set of propositions $X \subset \mathcal{L}$ is *partially coherent* (relative to probability distribution $\Pr \in \mathbf{P}$)

[sufficient condition:] if (i) $|X| > 1$ and for some non-empty, non-overlapping subsets X', X'', $\Pr(\bigwedge X' | \bigwedge X'') > \Pr(\bigwedge X')$, and (ii) for all such subsets $\Pr(\bigwedge X' | \bigwedge X'') \geq \Pr(\bigwedge X')$

[necessary condition:] only if (i) holds.

The quantitative notion of coherence along the lines of Douven and Meijs [10] is defined as the average of mutual confirmation relations between non-empty, non-overlapping subsets.[8] We hereby assume that "**conf**" is one of the standard Bayesian confirmation measures that have been proposed in the literature; some of these measures are mentioned in Section 6.

DEFINITION 3.3. [*Quantitative degree of coherence*] Let **conf** be a measure of confirmation, then the *quantitative degree of coherence* of a set of propositions $X \subset \mathcal{L}$ (relative to probability distribution $\Pr \in \mathbf{P}$) is defined as follows:

$$\mathbf{coh}(X) = \frac{\sum\{\mathbf{conf}(\bigwedge X', \bigwedge X'') :\ \emptyset \neq X' \subset X, \emptyset \neq X'' \subset X, X' \cap X'' = \emptyset\}}{|\{\emptyset \neq X' \subset X, \emptyset \neq X'' \subset X, X' \cap X'' = \emptyset\}|},$$

provided that $|X| > 1$; otherwise $\mathbf{coh}(X) =_{\text{def}} 0$.

Given that all numbers assigned by confirmation measures are positive in cases of confirmation and negative in cases of disconfirmation, zero is also the threshold separating coherence and incoherence. The sufficient qualitative condition for partial coherence guarantees that the average of mutual confirmation relations between non-empty, non-overlapping subsets will exceed this threshold; the necessary condition follows from the average being above this threshold. Thus we obtain:

OBSERVATION 3.1. If the sufficient condition of Definition 3.2 holds for a set of propositions X, then according to Definition 3.3, $\mathbf{coh}(X) > 0$. Vice versa, if $\mathbf{coh}(X) > 0$, then the necessary condition of Definition 3.2 holds for X.

We now turn to the problem of belief individuation. To illustrate the problem, let X be a set containing three propositions a, b and c, then in order to satisfy Definition 3.1 for full coherence we have to check $\Pr(a|b) > \Pr(a)$, $\Pr(a|c) > \Pr(a)$, $\Pr(a|b \wedge c) > \Pr(a)$, $\Pr(a \wedge b|c) > \Pr(a \wedge b)$, etc. Now assume that the set X is neither fully nor partially coherent, because all three proposition are probabilistically independent from the remainder propositions (and likewise for their conjunctions). Then it seems reasonable to assume that all sets that are L(ogically)- equivalent to X should likewise be assessed neither fully nor partially coherent. However, this is not at all true. For example, the L-equivalent set X' containing only the conjunction $a \wedge b \wedge c$ and a is fully coherent: since $a \wedge b \wedge c$ logically entails a (and not vice

[8]An investigation into the various structural properties of these measures is given in [41, 42].

versa), the conditional probability $\Pr(a|a \wedge b \wedge c)$ exceeds $\Pr(a)$ and accordingly the conditional probability $\Pr(a \wedge b \wedge c|a)$ also exceeds $\Pr(a \wedge b \wedge c)$. Hence, given that the singletons $\{a \wedge b \wedge c\}$ and $\{a\}$ are the only non-empty, non-overlapping subsets to be considered, X' is wrongly assessed as fully coherent according to Definition 3.1. Since all mutual confirmation measures in line with Douven and Meijs' recipe comply with Definition 3.1 X' would be wrongly assessed as coherent by each of these measures. This is an instance of the problem of belief individuation as highlighted by Moretti and Akiba [31].

While Moretti and Akiba call this the problem of belief individuation, other authors spoke of the problem of conjunctive decomposition [17,18, 45]. The problem is so fundamental that it undermines all qualitative and quantitative coherence measures that are based on mutual confirmation. To give another example, according to Definition 3.1, the singleton belief set $\{p\}$ is neither fully nor partially coherent and its quantitative degree of coherence is zero. But for every partition (ie., set of mutually disjoint and jointly exhaustive propositions) $q_1, \ldots q_n$, it holds that:

Observation 3.2. $\{p\}$ is logically equivalent to $\{p \vee q_1, \ldots, p \vee q_n\}$.

However, while $\{p\}$ is not coherent, the set $\{p \vee q_1, \ldots, p \vee q_n\}$ will be fully coherent if we only assume that the q_i are equi-probable and p-independent (i.e. $\Pr(q_i) = \Pr(q_i|p) = 1/n$ for all $i \in \{1, \ldots, n\}$). Even more, the quantitative degree of coherence of the latter set will typically be high, if n is large enough, since

$$\Pr(p \vee q_i|p \vee q_j) = \frac{\Pr(p)}{\Pr(p \wedge \neg q_i) + \Pr(q_i)} = \frac{\Pr(p)}{\Pr(p) \cdot (1 - \frac{1}{n}) + \frac{1}{n}},$$

which is close to 1 if n is large enough; so $\Pr(p \vee q_i|p \vee q_j) > \Pr(p \vee q_i)$. To give an example: if no solution to the problem of conjunctive decomposition can be found, then I can turn my belief "Elvis lives" into a coherent system of beliefs by rephrasing it as "Elvis lives or ticket 1 wins the lottery, Elvis lives or ticket 2 wins the lottery, ..., Elvis wins or ticket $1,000,000$ wins the lottery", conditional on the background knowledge that the lottery has $1,000,000$ tickets.[9] This result is certainly not what we want to have.

[9] If this background knowledge is not assumed, the two-elements set of beliefs {"Elvis lives.", "One of the $1,000,000$ tickets wins the lottery."} − which is likewise not coherent since the two propositions do not probabilistically support each other − can equivalently be transformed into the mentioned highly coherent set {"Elvis lives or ticket 1 wins the lottery.",...,"Elvis lives or ticket 1,000,000 wins the lottery."}.

The problem of belief individuation, or conjunctive decomposition, is not a new problem that does not only arise in the context of coherence measures, but was discussed, many years before, in endeavors to explicate the notion of unification. The simple view according to which a set of beliefs is unified if many of these beliefs (e.g. Tycho Brahe's astronomical observations) follow from (or are explained by) a few basic beliefs (Newton's laws) is undermined by the so-called conjunction paradox: any set of (totally unconnected) statements $a_1, \ldots, a_n$ has a "trivial unification" by means of their conjunction $a_1 \wedge \ldots \wedge a_n$. Obviously this is not meant by unification, since the conjunction is just another way to assert the original set. The conjunction paradox was first mentioned by Hempel ([23], p. 273, fn. 6). All accounts of unification had to offer some solution to the conjunction paradox. Friedman ([17], p. 16f.), for example, proposed to solve the conjunction problem by assuming a primitive concept of "independent acceptability" (which, however, was unsatisfying in several technical respects; cf. [26]). Another solution (similar to that proposed in Section 4 of this paper) was developed by Schurz and Lambert [47].

At this point a side remark on the relation between coherence and unification is appropriate. According to [47], unification is "coherence minus circularity". For example, in the belief set $\{p \equiv q, p, q\}$, p is deductively supported by the subset $\{p \equiv q, q\}$ and q by $\{p \equiv q, p\}$, and while both support relations count for coherence, only one counts for unification.

The tacking problems for confirmation measures and the problem of belief individuation are intimately related. To see this, let e be a piece of evidence that is probabilistically irrelevant for a hypothesis h; then it follows from the tacking argument above that although e does not confirm h, it nonetheless confirms the conjunction $h \wedge e$ (given that both h and e are Pr-normal). Similarly, probabilistic independence entails that the set $\{h, e\}$ is assessed neither coherent nor incoherent by any mutual confirmation measure. On the other hand, given the symmetry of incremental confirmation, the set $\{h \wedge e, e\}$, although being equivalent to the former set, is assessed coherent.

4. Content Elements

The major conclusion from the last section is that for a reasonable concept of coherence that is robust under mere re-formulations of the belief set, we need a natural method of conjunctive decomposition, or representation. Such a method takes a set of believed propositions of any linguistic format and transforms it into a logically equivalent conjunction of logically “smallest” semantically contained elementary proposition. We call these elementary conjuncts *content elements*. This method will not only help us to cure the notion of coherence from its defects but also to solve the tacking by conjunction problem that besets the Bayesian confirmation account. In this section we motivate and define such a method. Before we start we emphasize that our notions of genuine confirmation and genuine coherence do not depend on the details of the proposed method; they merely depend on the existence of some natural conjunctive decomposition method. In other words, our account is open to improvements of our representation method.

Our guiding idea is that a functioning notion of coherence should assign the same (degree of) coherence to all L-equivalent formulations of a belief set. One sees quickly that some “too modest” notions of conjunctive decomposition would violate this idea. For example, one could propose to split all conjunctive propositions into their conjuncts before the coherence of a belief set is assessed. According to this proposal, the belief set $\{a \wedge b, b \wedge c, a \wedge c\}$ (which has a high “artificial” mutual confirmation among its subsets) has to be decomposed into the set of conjuncts $\{a, b, c\}$. Given that a, b and c are mutually probabilistically independent, this is exactly the right result. However, instead of conjunctions “$x \wedge y$” one may also use the L-equivalent formulations “$\neg(\neg x \vee \neg y)$” and represent the set L-equivalently as $\{\neg(\neg a \vee \neg b), \neg(\neg b \vee \neg c), \neg(\neg a \vee \neg c)\}$. We do not want to say that by this “move” the original set has now become coherent.

So our method of decomposition into content elements must be logically deeper in a way that applies to all possible L-equivalent formulations of a given belief set. If we accept this idea, we have to accept that adding a logical consequence to a belief set does not change the coherence of this set. For example, the two sets $\{p, p \supset q\}$ and $\{p, p \supset q, q\}$ have the same degree of coherence, because they are L-equivalent. Therefore, we have to acknowledge that the resulting notion of coherence is based on the idealization of *logical omniscience*: the coherence of a belief system is assessed under the assumption that a believer knows all L-consequences of her beliefs. This idealization is justified insofar as it is the only possibility to obtain a functioning and yet simple notion of coherence. (Otherwise we would have to relativize L-equivalence to those inferences that are “mastered” by the believer, which would cause various complications.)

We first define the representation method for $\mathcal{L}_0$, which is the language of truthfunctional propositional logic. A *literal* is an atomic sentence or its negation, abbreviated as $\pm p_i$, with “$\pm$” for “unnegated” or “negated” (Carnap [4], p. 67, called them “basic statements”). The set of literals includes also the two propositional constants “$\top$” for “Verum”, which represents the tautology and "$\bot$" ($=_{\text{def}} \neg\top$) for "Falsum", which represents the contradiction. A *clause* is a disjunction of literals that are non-repetitive in their atomic propositions. Literals within a clause are assumed to be uniquely ordered, so that no two distinct clauses are L-equivalent (it follows that $p_i \vee \neg p_i$ is not a clause; the tautological clause is represented by $\top$). $\text{CL}(X)$ denotes the set of clauses following from a set of propositions X. Uniqueness of clauses modulo L-equivalence means that

Observation 4.1. $\forall c, c' \in \text{CL}(X)$: if $\models c \equiv c'$, then $c = c'$.

It is a well known fact that every $\mathcal{L}_0$-sentence is L-equivalent with a set (or nonrepetitive conjunction) of clauses. So it is natural to decompose a given (set of) statements into a conjunction of clauses.

Not every clause c following from a statement counts as a content element of it. Here comes the crucial semantic restriction: a content element of x (or X) is a clause that follows *relevantly* from x (or X), i.e., that doesn't contain irrelevant disjunctive weakenings that can be eliminated or replaced by any other formula, salva validitate. For example, $p \vee q$ is a clause that follows relevantly from $p \vee (q \wedge r)$, but $p \vee q \vee \underline{s}$ is not, since the underlined formula part s can be eliminated or replaced by any other formula s', salva validitate of the inference, which means that $p \vee (q \wedge r) \models p \vee q \vee s'$ is valid for arbitrary s'. In particular, $p \vee q$ is not a content element of p.

This semantic constraint of "relevance" is crucial for making the idea of conjunctive representation by content elements work. Without this constraints all kinds of inadequate consequences would be regained. Concerning coherence, for example, the singleton set $\{p\}$ could be made coherent by adding all disjunctive weakenings $p \vee q_1, p \vee q_1 \vee q_2, \ldots$ to it. Or, if p and q are probabilistically independent, the set $\{p, q\}$ could be made coherent by adding the irrelevant consequences $\neg p \vee q$ and $\neg q \vee p$ to it, producing the L-equivalent set $\{p, \neg p \vee q, q, \neg q \vee p\}$. Gemes [18] and Schurz [45] demonstrated how irrelevant disjunctive weakenings are a major cause of paradoxes in deontic logic and philosophy of science; Gemes [18] called them "tackings by disjunction".

Obviously, clause c' follows relevantly from a clause c iff no clause c'' that is L-stronger than c follows from it. Therefore a simple definition of content elements within the language of propositional logic is possible by characterizing them as maximally strong entailed clauses.

Definition 4.1. [*Content element*] $c \in \mathcal{L}_0$ is a *content element* of a set of propositions $X \subset \mathcal{L}$ iff (i) $c \in \text{CL}(X)$ and (ii) there exists no $c' \in \text{CL}(X)$ distinct from c such that $c' \models c$.

CE(X) denotes the set of X's content elements.
A *content part* of X is a non-empty conjunction of content elements of X.
CP(X) denotes the set of X's content parts.

Note that we don't need to require $c \not\models c'$ in condition (ii) since clauses are unique modulo L-equivalence.

The method of decomposition into clauses, and in particular the decomposition into strongest clauses, have been proved as highly useful in computational logics and Artificial Intelligence. Content elements within propositional logics are also called *prime implicates* (cf. [1]; the basic idea goes back to an old paper of Quine [36]). Schurz [45] and Schurz and Weingartner [49] spoke of *relevant (consequence) elements* and generalized this notion to 1st order logic. The notion of a *content part* has been introduced by Gemes [18], who defines content parts, however, in a slightly different way than we do.

Three important facts about content elements are:

OBSERVATION 4.2 **(Facts about content elements)**.

(a) *Content-preservation*: For every set of propositions X, CE(X) is L-equivalent with X; so no information gets lost by the representation of sets of propositions by content elements. For propositional logic this is a straightforward consequence of the two facts that for every X, X is L-equivalent with CL(X) and CE(X) implies CL(X) since every clause in CL(X) is either identical with or implied by a clause in CE(X).

(b) *Invariance under L-equivalence*: Following from (a), every two L-equivalent sets of propositions X and X' have the same content elements, i.e. CE(X) = CE(X'). Therefore, every property of a set of propositions X explicated in terms of X's content elements (such as the properties of genuine confirmation and genuine coherence) is invariant under L-equivalent transformations of X.

(c) The content elements of a logically consistent set of propositions X are *pairwise* logically independent in the sense that for all $c, c' \in$ CE(X), neither $c \models c'$ nor $c \models \neg c'$ can hold. In other words, there are no entailment relations (i.e., one-premise inferences) between content elements. There are, however, non-trivial inference relations between the content elements of a set of propositions having more than one premises. For example the set $\{p \vee q, \neg q \vee r\}$ non-trivially L-implies the content element$p \vee r$; thus, CE($\{p \vee q, \neg q \vee r\}$) = $\{p \vee q, \neg q \vee r, p \vee r\}$.

Here are some examples of decomposing sets of propositions into their content elements:

(i) $\text{CE}(\{p \wedge q\}) = \text{CE}(\{\neg(\neg p \vee \neg q\})) = \{p, q\}$

(ii) $\text{CE}(\{p \vee (q \wedge r)\}) = \{p \vee q, p \vee r\}$

(iii) $\text{CE}(\{p \vee \neg q, p \vee q\}) = \text{CE}(\{p\}) = \{p\}$

(iv) $\text{CE}(\{p \supset q, q \supset \neg r\}) = \{\neg p \vee q, \neg q \vee \neg r, \neg p \vee \neg r\}$

(v) $\text{CE}(\{p, q\}) = \text{CE}(\{p, p \supset q, q\}) = \text{CE}(\{p, p \supset q, q \supset p, q\}) = \{p, q\}$

(vi) $\text{CE}(p \vee \neg p) = \{\top\}$, $\text{CE}(p \wedge \neg p) = \{\bot\}$

The examples show that the method of decomposition works as intended. Against result (v) one may object that the belief set $\{p, p \supset q, q \supset p, q\}$ is much more coherent than the belief set $\{p, q\}$ insofar as the two conditionals $p \supset q$ and $q \supset p$ allow one to derive p from q and vice versa. The latter assertion is right, but the impression that this fact would make the belief set coherent is illusionary, as soon as one recognizes that "$p \supset q$" is merely the material implication which says no more than "either not p or q". If the set $\{p, p \supset q, q \supset p, q\}$ would really count as coherent, it would be a child's play to make two independent beliefs p and q coherent, simply by adding to them the irrelevant disjunctive weakenings $p \vee \neg q$ and $q \vee \neg p$. For example, I could make the belief set {"Snow is white.", "Grass is green."} coherent by expanding it by the two irrelevant consequences "Snow is white or grass is not green." and "Grass is green or snow is not white." This is obviously not what we have in mind when calling a belief set coherent.

The impression that the belief set $\{p, p \supset q, q\}$ is coherent arises from the intuition that "$\supset$" establishes a kind of connection between p and q. This means that we have a conditional "$\rightarrow$" in mind that is stronger than "$\supset$". So the belief set we have in mind is $\{p, p \rightarrow q, q\}$, where "$\rightarrow$" is an intensional (non-truthfunctional) conditional, for example, that of strict implication or a relevant implication. For such a conditional the two inferences $\neg p \models p \rightarrow q$ ("ex falso quodlibet") and $q \models p \rightarrow q$ ("verum ex quodlibet") are invalid, whence $\{p, q\}$, $\{p, p \rightarrow q, q\}$ and $\{p, p \rightarrow q, q \rightarrow p, q\}$ are pairwise logically non-equivalent.

Another possibility to interpret the conditional $p \supset q$ would be to assume that it is backed up by a certain regularity that can be expressed in first order logic. In this case, the above belief sets are interpreted, for example, as $\{F\alpha_1, G\alpha_1\}, \{F\alpha_1, \forall x(Fx \supset Gx), G\alpha_1\}$ and $\{F\alpha_1, \forall x(Fx \supset Gx), \forall x(Gx \supset Fx), G\alpha_1\}$, respectively. Again these belief sets are pairwise logically non-equivalent.

What these consideration tells us is that, although our Definition 4.1 of content elements for the language $\mathcal{L}_0$ of truthfunctional propositional logic is correct, for many interesting applications we need to extend this definition to the language $\mathcal{L}_{\rightarrow}$ of intensional conditional logic and to the language $\mathcal{L}_1$ of first order logic. For this purpose, the general definition of relevant elements as developed in [45,46] comes at help:

DEFINITION 4.2. [*Content element*] e is a *content element* of a set of propositions X iff

(a) e is an "element", i.e., it is not L-equivalent with a conjunction of sentences $x_1 \wedge \ldots \wedge x_n$ $(n \geq 1)$ each of which is shorter than e. Thereby the length of a sentence is defined as the number of its primitive symbols when statements are expressed by means of the following logical bases: $\{\neg, \vee, \wedge\}$ for $\mathcal{L}_0$, $\{\neg, \vee, \wedge, \exists, \forall\}$ for $\mathcal{L}_1$ and $\{\neg, \vee, \wedge, \rightarrow\}$ for $\mathcal{L}_{\rightarrow}$.

(b) e is a relevant logical consequence of X in the sense that $X \models e$ and no predicate in e is replaceable on some of its occurrences by any other predicate (of the same degree) salva validitate of $X \models e$ (where propositional atoms are regarded as 0-ary predicates).

(c) e is the first statement among all statements L-equivalent with e and satisfying (a) and (b), according to a given enumeration of all statements of the underlying language.

Examples: According to clause (a), "$p \wedge p$" is not an element, but "p" is one; "$p \vee (q \wedge r)$" is not an element, but "$p \vee q$" and "$p \vee r$" are elements, "$\forall x(Fx \wedge Gx)$" is not an element, but "$\forall xFx$" and "$\forall xGx$" are elements.

Examples of *irrelevant* consequences according to clause (b) (underlined occurrences are salva validitate replaceable) are: $p \models p \vee \underline{q}$; $p \models \underline{q} \supset p$; $p \models (p \vee \underline{q}) \wedge (p \vee \neg \underline{q})$; $\forall x(Fx \supset Gx) \models \forall x(Fx \supset (Gx \vee \underline{Hx}))$. Examples of *relevant* consequences are: $p \wedge q \models p$; $p \supset q, q \supset r \models p \supset r$; $\forall x(Fx \supset Gx), F\alpha_1 \models G\alpha_1$; $\forall x(Fx \supset Gx), \forall x(Gx \supset Hx) \models \forall x(Fx \supset Hx)$.

Schurz and Weingartner ([49], Lemma 5) prove that in propositional logic, Definition 4.2 of content elements (they call them relevant elements) is *equivalent* with the definition 4.1 in terms of strongest clauses, assuming literals occurring in consecutive disjunctions are ordered according to a fixed enumeration. This latter condition ensures that in propositional logic content elements are unique modulo L-equivalence. For intensional or 1st order logic, this condition is not sufficient. For these languages the uniqueness is explicitly required in condition (c).

In intensional propositional logic, Definition 4.2 splits conditional formulas with complex antecedents and consequents into conjuncts, depending on the specific logical rules for the conditional operator. For example, in a conditional logic which satisfies the two following rules

disjunction of antecedents	$\models (a \vee b) \rightarrow c$	$\equiv$	$(a \rightarrow c) \wedge (b \rightarrow c)$
conjunction of consequents	$\models a \rightarrow (b \wedge c)$	$\equiv$	$(a \rightarrow b) \wedge (a \rightarrow c)$

we have

$$\mathrm{CE}(\{(a \vee b) \rightarrow (c \wedge d)\}) = \{a \rightarrow c, a \rightarrow d, b \rightarrow c, b \rightarrow d\}$$

A similar decomposition is performed in first order logic:

$$\begin{aligned} &\mathrm{CE}(\{\forall x((Fx \vee Gx) \supset (Hx \wedge Qx))\}) \\ &\quad = \{\forall x(Fx \supset Hx), \forall x(Fx \supset Qx), \forall x(Gx \supset Hx), \forall x(Gx \supset Qx)\} \end{aligned}$$

Recall that in $\mathcal{L}_0$ all one-premise inferences (entailments) between clauses are irrelevant; so content elements are automatically strongest content elements. In first order logic this is not so: certain one-premise inferences (entailments) are relevant. This is in particular true for universal instantiations, existential generalizations and existential simplifications, which give us the following decompositions (where $\mathcal{I}$ denotes the set of all individual constants of the language $\mathcal{L}_1$):

(i) $\mathrm{CE}(\{\forall x Fx\}) = \{\forall x Fx\} \cup \{F\alpha_i : \alpha_i \in \mathcal{I}\}$

(ii) $\mathrm{CE}(\{F\alpha_1\}) = \{F\alpha_1, \exists x Fx\}$

(iii) $\mathrm{CE}(\{\exists x(Fx \wedge Gx)\}) = \{\exists x(Fx \wedge Gx), \exists x Fx, \exists x Gx\}$

For applications to truthlikeness, content elements obtained from universal instantiation or existential generalization or simplification are important (cf. [45, 49]). However, for purposes of coherence these non-maximal content elements are problematic, insofar as they can produce artificial coherence. If we would include them into the belief representation, then we would have to say that the singleton set $\{\forall x Fx\}$ has a high degree of coherence (given a suitably inductive probability measure) because its representation contains the infinite set of content elements $F\alpha_1, F\alpha_2, \ldots$. Likewise, we do not want to say that the belief set $\{F\alpha_1\}$ is coherent because $F\alpha_1$ entails $\exists x Fx$, or the belief set $\{\exists x(Fx \wedge Gx)\}$ is coherent because $\exists x(Fx \wedge Gx)$ entails $\exists x Fx$ and $\exists x Gx$. Thus, for the purpose of obtaining an adequate measure of coherence, it is reasonable to confine the conjunctive decomposition of a belief set into its strongest content elements, which we call *basic* content elements.

Definition 4.3. [*Basic content elements*] e is a *basic content element* of a set of propositions X iff e is a content element of X and there exists no content element e' of X that is (properly) L-stronger than e. $\mathrm{BCE}(X)$ is the set of all basic content elements of X.

A *basic content part* of X is a non-empty conjunction of basic content elements of X.

BCP(X) denotes the set of X's basic content parts.

In the next two sections we apply this method of decomposition to the notions of genuine coherence and genuine confirmation.

5. Genuine Coherence: Qualitative and Quantitative

Following from the previous sections we base our notion of genuine coherence on the representation of the given belief set X by X's basic content elements, BCE(X). Since we assume the idealization of logical omniscience, BCE(X) may contain some content elements that are not contained in X but follow from content elements in X. Recall that we assume that the believer knows all consequences of her beliefs; so it is only logical to count the contribution of all basic content elements of her belief set, fundamental or derived, to the overall coherence or degree of coherence of her belief set. Our notion of qualitative full genuine coherence is simply obtained by applying Definition 3.1 to the set of basic content elements of the belief set:

Definition 5.1. [*Qualitative full genuine coherence*] A set of propositions $X \subset \mathcal{L}$ is *fully genuinely coherent* (relative to probability distribution $\Pr \in \mathbf{P}$) iff BCE(X) satisfies Definition 3.1, i.e., iff $|\mathrm{BCE}(X)| > 1$ and for all non-empty, non-overlapping subsets $X', X'' \subset \mathrm{BCE}(X)$, $\Pr(\bigwedge X' | \bigwedge X'') > \Pr(\bigwedge X')$.

For the qualitative notion of partial genuine coherence we do not give a both sufficient and necessary condition, for the same reasons as for the corresponding notion of ordinary coherence (recall Definition 3.2). Instead, we only give a sufficient and a necessary condition, in order to make this notion consistent with the quantitative notion of genuine coherence (similarly as we did this for ordinary coherence in Definition 3.2). Thus we define:

Definition 5.2. [*Qualitative partial genuine coherence*] A set of propositions $X \subset \mathcal{L}$ is *partially genuinely coherent* (relative to probability distribution $\Pr \in \mathbf{P}$)

> [sufficient condition:] if (i) $|\mathrm{BCE}(X)| > 1$ and for some non-empty, non-overlapping subsets $X', X'' \subset \mathrm{BCE}(X)$ it holds $\Pr(\bigwedge X' | \bigwedge X'') > \Pr(\bigwedge X')$, and (ii) for all such subsets $\Pr(\bigwedge X' | \bigwedge X'') \geq \Pr(\bigwedge X')$.
>
> [necessary condition:] only if (i) holds.

Note that Definition 5.1 can be rephrased more elegantly by saying that for all pairs c, c' of non-overlapping basic content parts of X, $\Pr(c|c') > \Pr(c)$. Similarly we can express Definition 5.2 more elegantly in terms of content parts.

The quantitative measure of genuine coherence is based on the average degree of confirmation between conjunctions of basic content elements (i.e. basic content parts). Thus, this measure is defined as follows:

DEFINITION 5.3. (*Quantitative degree of genuine coherence*) The degree of genuine coherence of belief set $X \subset \mathcal{L}$ (relative to probability distribution $\Pr \in \mathbf{P}$), $\mathbf{cohg}(X)$, equals $\mathbf{coh}(\mathrm{BCE}(X))$, where $\mathbf{coh}(Y)$ is defined as in Definition 3.3.

Based on the discussion in the last section it is obvious that this notion of genuine coherence solves all the mentioned problems that result from the dependence of simple Bayesian coherence on different formulations of the belief set. In particular, since two L-equivalent belief sets have the same set of basic content elements, their genuine coherence is exactly the same. In other words:

OBSERVATION 5.1. Genuine coherence of a belief set is invariant w.r.t. L-equivalence.

6. Genuine Confirmation

We now turn to the notion of Bayesian confirmation and the tacking by conjunction problem. Recall the most extreme case of the tacking problem (Definition 2.1 (ii)): it amounts to the fact that each piece of evidence e confirms the conjunctive hypothesis $e \wedge h'$ for any arbitrary h'. Given that there may not be any confirmatory relationship between e on the one hand and h' on the other, this seems counter-intuitive.

Clearly the probability increase $\Pr(h' \wedge e|e) > \Pr(h' \wedge e)$ is not a case of genuine confirmation, since e is probabilistically irrelevant to that content-part of $h' \wedge e$ that goes beyond e, namely h' (i.e., $\Pr(h'|e) = \Pr(h')$). e increases $h' \wedge e$'s probability only because e is a content part of $h' \wedge e$ and increases its own probability to 1 ($\Pr(e|e) = 1$). Gemes and Earman (who quotes Gemes; cf. [18], p. 98, and 242, fn. 5) have called this type of non-genuine confirmation mere *content-cutting*.

In order to avoid the problem of "confirmation by mere content-cutting" one ought to require that the confirmation takes place in some 'part' of the hypothesis h that is not already contained in the evidence. Accordingly, in order for e to 'really' confirm the conjunctive hypothesis $e \wedge h'$, e has to confirm h'. This is the idea of *genuine confirmation* (cf. Schurz [46]). This idea implies that an evidence e that raises h's probability can only count as a *genuine* confirmation of h if h's content *transcends* e in the sense that h is not entailed by e.

It follows that the logical entailment of a Pr-normal hypothesis h by e, though being a special case of ordinary confirmation, is not a special case of genuine confirmation, because in our understanding genuine confirmation requires a probability increase of e-transcending content elements of h. In other words, what we call "genuine confirmation" can more explicitly be called "genuine *inductive* confirmation". However, we do not want to take a definite stance in regard to the question whether an appropriate notion of "genuine confirmation" should be restricted to genuine inductive confirmation or should include deductive entailment as a special case. The latter notion, call it "genuine inductive-or-deductive confirmation" can easily be defined as follows: e genuinely confirms h in the inductive-or-deductive sense if either e genuinely confirms h (in the inductive sense) or e entails h. In what follows we mean by "genuine confirmation" always "genuine confirmation in the inductive sense", because it is this notion that we intend to explicate in this section.

For the notion of *full* (qualitative) genuine confirmation, we require that not only some, but all e-transcending basic content elements of the hypothesis h have to be confirmed by the evidence e (in the ordinary Bayesian sense). As a further complication, it is important to require this not only for the (basic) content elements, but also for their conjunctions, i.e., the (basic) content parts, because following from the non-monotonicity of conditional probabilities it may well be that $\Pr(h|e) > \Pr(h)$, $\Pr(h'|e) > \Pr(h')$, but $\Pr(h \wedge h'|e) < \Pr(h \wedge h')$ holds.

However, we restrict the demand of genuine confirmation to those e-transcending parts of h that do not themselves entail e, for the reason that we don't want to introduce effects of "tacking by conjunction" (e confirms $e \wedge h$) into our notion of genuine confirmation. This restriction is not really necessary for full genuine confirmation − here the inclusion of these "e-tacked" content parts would not harm − but it *will* become important for the corresponding notions of qualitative partial genuine confirmation and the quantitative measure of genuine confirmation. So we define:

Definition 6.1 (*Qualitative full genuine confirmation*). *e fully genuinely confirms h* (given probability distribution Pr) iff (i) there exists some

e-transcending $x \in \mathrm{BCP}(h)$ that does not entail e such that $\Pr(x|e) > \Pr(x)$ holds and (ii) this holds for all such $x \in \mathrm{BCP}(h)$.

Thus, e genuinely confirms the conjunctive hypothesis $h \wedge e$ only if $\Pr(h|e) > \Pr(h)$ because h is also a basic content part of $h \wedge e$. Moreover, e fully genuinely confirms a conjunction $h \wedge h'$ only iff e fully genuinely confirms both h and h' (and their conjunction). Thus, not only the special but also the generalized tacking problem are avoided by the notion of full genuine confirmation.

Full genuine confirmation is a rather strong notion. For certain purposes we want to say that, if e genuinely confirms h but is irrelevant to h', then e confirms the conjunction $h \wedge h'$ at least partially. Thus we need a notion of partial genuine confirmation. As for the case of partial genuine coherence, we give only a sufficient and a necessary, but not a sufficient and necessary condition for partial genuine confirmation, in order to make this notion consistent with the quantitative notion of genuine confirmation, in the sense that e partially genuinely confirms h iff e quantitatively genuinely confirms h to a positive degree. We define:

Definition 6.2. [*Partial genuine confirmation*] *e partially genuinely confirms h* (given probability distribution $\Pr \in \mathbf{P}$)

> [sufficient condition:] if (i) there exists some e-transcending $x \in \mathrm{BCP}(h)$ that does not entail e such that $\Pr(x|e) > \Pr(x)$ holds and (ii) $\Pr(x|e) \geq \Pr(x)$ holds for all such $x \in \mathrm{BCP}(h)$.
>
> [necessary condition:] only if (i) holds.

It is important that we confine Definition 6.2 to basic content parts that (are e-transcending and) do not entail e. This prevents that tacking by conjunction counts as a case of partial genuine confirmation. Note that $e \wedge h'$ is a basic content part of itself, and $\Pr(e \wedge h'|e) > \Pr(e \wedge h')$ holds; however, the hypothesis $e \wedge h'$ has no confirmed basic content part that does not entail e. Therefore "$e \wedge h'$" is not even partially genuinely confirmed by e, since the necessary condition of Definition 6.2 is violated.

In this context it is important to note that our concept of genuine confirmation does *not* fall prey to the well-known Popper-Miller objection to inductive confirmation, which runs as follows: every hypothesis h is logically equivalent to the conjunction of $h \vee e$ and $h \vee \neg e$. Now the conjunct $h \vee e$ is already entailed by the evidence e and the remaining conjunct $h \vee \neg e$ is provably *dis*confirmed by e, i.e. $\Pr(h \vee \neg e|e) < \Pr(h \vee \neg e)$ for any $\Pr \in \mathbf{P}$. From this observation Popper and Miller [35] conclude that non-deductive confirmation is impossible. However, obviously neither $h \vee e$ nor $h \vee \neg e$

are content elements of the belief set $\{h\}$. So what Popper and Miller have shown is only that for each hypothesis h that is inductively confirmed by the evidence e there is a logical consequence, to wit $h \vee \neg e$, that is inductively disconfirmed.

So far our discussion of the concept of confirmation focused exclusively on the qualitative side. Now we want to turn to the definition of a measure of genuine confirmation. Since it is based on some average of classical (Bayesian) confirmation degrees, we will first introduce a couple of confirmation measures. The common core of all these measures is the relevance-principle as introduced in Section 1, which is a bridge principle linking the qualitative and the quantitative concept of confirmation. According to this principle, the degree of confirmation that **conf** assigns to a pair (e, h) of a piece of evidence and a hypothesis is positive iff h's posterior probability given e exceeds its prior probability. Accordingly, $\mathbf{conf}(e, h) > 0$ if and only if $\Pr(h|e) > \Pr(h)$, i.e., iff e qualitatively confirms h (relative to probability distribution Pr).

Now a straightforward idea for how to calculate the *degree* of confirmation that e provides for h is simply either to take the difference or the (logarithm of the) ratio between $\Pr(h|e)$ on the one hand and $\Pr(h)$ on the other. Other prominent confirmation measures are based on the observation that for Pr-normal h and e, the condition (i) $\Pr(h|e) > \Pr(h)$ is equivalent to each of the following ones: (ii) $\Pr(h|e) > \Pr(h|\neg e)$, (iii) $\Pr(e|h) > \Pr(e)$ and (iv) $\Pr(e|h) > \Pr(e|\neg h)$ (which does not mean that the corresponding quantitative measures are ordinally equivalent). The following list contains many prominent measures.[10] They come in two versions: the left-hand side presents the difference-based version of the measure, and the right-hand side the log-ratio-based version.

(α)	$\alpha_d(e,h) = \Pr(h|e) - \Pr(h)$	$\alpha_r(e,h) = \log[\Pr(h|e)/\Pr(h)]$
(β)	$\beta_d(e,h) = \Pr(h|e) - \Pr(h|\neg e)$	$\beta_r(e,h) = \log[\Pr(h|e)/\Pr(h|\neg e)]$
(γ)	$\gamma_d(e,h) = \Pr(e|h) - \Pr(e)$	$\gamma_r(e,h) = \log[\Pr(e|h)/\Pr(e)]$
(δ)	$\delta_d(e,h) = \Pr(e|h) - \Pr(e|\neg h)$	$\delta_r(e,h) = \log[\Pr(e|h)/\Pr(e|\neg h)]$

[10] Measure α_d has been supported (among others) by Earman [11]; a prominent defender of measure α_r is Milne [30]. β_d is the measure proposed by Christensen [6], its ratio-based counterpart β_r is (to the best of our knowledge) so far undefended. γ_d is Mortimer's [32] measure, whereas γ_r is the measure defended by Kuipers [28]. Measure δ_d is most prominently discussed by Nozick [33] while δ_r is the well-known log-likelihood measure endorsed, among others, by Good [21].

Further prominent measures are the following:[11]

(ε)	$\varepsilon(e,h) = \Pr(h \wedge e) - \Pr(h) \cdot \Pr(e)$
(ζ)	$\zeta(e,h) = \begin{cases} \alpha_d(e,h)/\Pr(\neg h), & \text{if } \Pr(h\|e) > \Pr(h) \\ \alpha_d(e,h)/\Pr(h), & \text{if } \Pr(h\|e) \leq \Pr(h) \end{cases}$
(η)	$\eta(e,h) = 1 - [\Pr(\neg e\|h)/\Pr(\neg e)]$

We finally define our quantitative notion of genuine confirmation by averaging the degrees of confirmation by e for all those basic content parts x of h that transcend e but do not entail e:

Definition 6.3. (Quantitative degree of genuine confirmation) The *degree of genuine confirmation* that e provides for h (given probability distribution $\Pr \in \mathbf{P}$), $\mathbf{confg}(e,h)$, is defined as follows:

$$\mathbf{confg}(e,h) = \frac{\sum\{\mathbf{conf}(e,x) : \ x \in \mathrm{BCP}(h) \text{ and } e \not\models x \text{ and } x \not\models e\}}{|\{x \in \mathrm{BCP}(h) : e \not\models x \text{ and } x \not\models e\}|},$$

provided that $\{x \in \mathrm{BCP}(h) : e \not\models x \text{ and } x \not\models e\} \neq \emptyset$; otherwise $\mathbf{confg}(e,h) =_{\mathrm{def}} 0$.

Is is easy to see that:

Observation 6.1. If the sufficient condition of Definition 6.2 is satisfied than $\mathbf{confg}(e,h) > 0$. If $\mathbf{confg}(e,h) > 0$, then the necessary condition of Definition 6.2 is satisfied.

For the special tacking problem Observation 6.1 implies that the degree of genuine confirmation that e conveys to $e \wedge h'$ (for Pr-normal e and $e \wedge h'$) is zero, since $e \wedge h'$'s only basic content part that is logically independent of e is h'.

Now let us return to the generalized tacking problem. At the end of Section 2 we saw that classical Bayesian solution proposals tried to solve the problem by arguing as follows: although the conjunctive hypothesis $h \wedge h'$ is confirmed by e if h is, even if h' is irrelevant to h, e and $h \wedge e$, it is so only to a lower degree, i.e. $\mathbf{conf}(e, h \wedge h') < \mathbf{conf}(e,h)$. Our main concern with these solution proposals was that they are measure-sensitive in the sense that the inequality only holds for a subset of (prominent) confirmation measures. Accordingly, we would like to have a robust analysis of the tacking problem that holds for the vast majority (if not all) of the existing

[11] ε is the (mutual) relevance measure that is due to Carnap [4], ζ is known as the 'relative distance measure of confirmation' and has been introduced by Crupi et al. [9] while η has been defended by Rips [37].

confirmation measures. That this is the case for genuine confirmation is the content of the following observation:

Observation 6.2. If e confirms h and h' is probabilistically irrelevant to h, e and $h \wedge e$, then $\mathbf{confg}(e, h \wedge h') < \mathbf{confg}(e, h)$ for each of the above measures $\alpha_d - \delta_d$, $\alpha_r - \delta_r$, ε, ζ and η provided that h and h' are basic content elements of $h \wedge h'$ and e, h, h' and $h \wedge h'$ are Pr-normal.

Proof. In what follows, where "$x(-)$" denotes one of the Bayesian confirmation measures , "$xg(-)$" denotes its genuine counterpart (obtained from "$x(-)$" by replaing in Definition 6.3 "$\mathbf{conf}(e, h)$" by this measure).

If h' is probabilistically irrelevant to h, e and $h \wedge e$, then $\Pr(e|h \wedge h') = \Pr(e|h)$. Now let us first see what the degree of (partial) genuine confirmation that e provides for the conjunctive hypothesis $h \wedge h'$ amounts to. Given our assumption that h and h' are basic content elements of $h \wedge h'$, we get:

$$\mathbf{confg}(e, h \wedge h') = \frac{\mathbf{conf}(e, h) + \mathbf{conf}(e, h') + \mathbf{conf}(e, h \wedge h')}{3}$$

Now given the assumption that h' is irrelevant to e, we conclude that $\mathbf{conf}(e, h') = 0$. Hence, $\mathbf{confg}(e, h \wedge h') = 1/3 \cdot [\mathbf{conf}(e, h) + \mathbf{conf}(e, h \wedge h')]$. This holds true for all considered confirmation measures. For measures $\alpha_d, \beta_d, \delta_d, \delta_r$, ε and ζ, [3] has shown that $\mathbf{conf}(e, h \wedge h') < \mathbf{conf}(e, h)$. Hence, for all these measures we get: $\mathbf{confg}(e, h \wedge h') < \mathbf{conf}(e, h)$, and since $\mathbf{conf}(e, h) = \mathbf{confg}(e, h)$ because h is a basic content element of itself, we obtain the result $\mathbf{confg}(e, h \wedge h') < \mathbf{confg}(e, h)$.

For α_r and the equivalent γ_r it is straightforward to show that $\alpha_r(e, h \wedge h') = \alpha_r(e, h)$; nonetheless, we still obtain $\alpha_r g(e, h \wedge h') = 2/3 \cdot \alpha_r(e, h) < \alpha_r(e, h) = \alpha_r g(e, h)$ for these two measures. The same holds for γ_d, for which the above equality $\Pr(e|h \wedge h') = \Pr(e|h)$ can be utilized. To see that also $\beta_r(e, h \wedge h') = \beta_r(e, h)$ one only needs to observe that $\Pr(h \wedge h'|e) = \Pr(h|e) \cdot \Pr(h')$ and $\Pr(h \wedge h'|\neg e) = \Pr(h|\neg e) \cdot \Pr(h')$, since h' is irrelevant to $h \wedge e$. Hence, $\beta_r g(e, h \wedge h') < \beta_r g(e, h)$.

Finally, we have to prove that Observation 6.2 holds also for η. This proof is straightforward given the observation that $\Pr(\neg e|h \wedge h') = \Pr(\neg e|h)$. ■

Thus, our solution to the generalized tacking problem is twofold: we argued that in order for e to fully genuinely confirm the conjunctive hypothesis $h \wedge h'$, e has to confirm *both h and h'* (and their conjunction). Accordingly, if h' is an irrelevant conjunct, $h \wedge h'$ is *not* fully genuinely confirmed; it is merely *partially* genuinely confirmed. Moreover, we were able to show that given the assumptions in Definition 2.2, the degree of partial genuine confirmation that e provides for h exceeds the degree of partial genuine confirmation that e provides for $h \wedge h'$ for all considered measures. Thus adding irrelevant conjuncts to confirmed hypotheses produces a similar diminishing effect for the degree of partial genuine confirmation as for certain measures of ordinary Bayesian confirmation; however, this diminishing effect is far more robust in regard to the choice of particular measures than it is in former Bayesian solution proposals.

7. The Relation Between Genuine Confirmation and Genuine Coherence

In the final part of this paper we will dwell into the relationship between (qualitative) genuine coherence and (qualitative) genuine confirmation. One might be tempted to think that like ordinary coherence of a belief set X is mutual ordinary confirmation between non-overlapping subsets of X, genuine coherence is mutual genuine confirmation between non-overlapping subsets of X. This turns out to be false since the notion of genuine coherence is based on mutual confirmation between non-overlapping subsets not of X itself but of the result of the decomposition of X into basic content elements.

Consider the following example: let $X = \{p \wedge q, r\}$, then X is genuinely coherent if the set $\{p, q, r\}$ is coherent. Accordingly, among others it has to be the case that $\Pr(p|q) > \Pr(p)$, $\Pr(r|q) > \Pr(r)$, $\Pr(p \wedge q|r) > \Pr(p \wedge q)$, etc. In particular, it has also to be the case that $\Pr(p|q \wedge r) > \Pr(p)$. On the other hand, for X to satisfy mutual genuine confirmation it has only to be the case that $\Pr(p|r) > \Pr(p)$, $\Pr(q|r) > \Pr(q)$, and that $\Pr(p \wedge q|r) > \Pr(p \wedge q)$. Thus, the requirement that p's probability is also raised by the conjunction of q and r is *not* contained in an account of mutual genuine confirmation (and so are others).

In conclusion, genuine coherence is not identical with mutual genuine confirmation. We rather have the following facts:

Observation 7.1. Let $X \subset \mathcal{L}$ be a set of propositions, h, e be a hypothesis and the corresponding evidence,

(i) Full genuine coherence of X = existence of at least two non-overlapping basic content parts and mutual confirmation between all basic content parts of X

(ii) Full genuine confirmation of h by e = existence of at least one e-transcending basic content part of h and confirmation of all of h's e-transcending content parts by e.

In both cases the genuine notion has been reduced to the ordinary notion of confirmation applied to conjunctions of content elements, i.e., content parts. Nevertheless we can also establish a direct relation between genuine coherence and genuine confirmation, as follows. Let us define the *complement* x^* of a (basic) content part $x =_{\text{def}} \bigwedge Y$ of some belief set X (for $Y \subset \text{BCE}(X)$) as the conjunction of all content elements of X that are not conjuncts of x, i.e., $x^* = \bigwedge(\text{BCE}(X) \setminus Y)$. Then we get:

Observation 7.2. (i) Full genuine coherence of X is equivalent with (ii) X having at least two non-overlapping content parts and full genuine confirmation of every basic content part of X by its complement.

Proof. (i) $\Rightarrow$ (ii): By (i) X has at least two non-overlapping content parts. Now assume x is a content part of X. Let x^* be its X-complement. We must show that x is fully genuinely confirmed by x^*, which means that there exists an x^*-transcending subconjunction of x and every such subconjunction of x is confirmed by x^*. By (i) this is the case.

(ii)$\Rightarrow$ (i): Assume x and y are two non-overlapping content parts of X, i.e., conjunctions of content elements whose conjuncts are non-overlapping. We must show that y confirms x. By (ii) y fully genuinely confirms y^*, the X-complement of y, which means that y confirms every subconjunction of y^*. But x is a subconjunction of y^*, so x is confirmed by y. ■

A similar connection exists between partial genuine coherence and confirmation. In this case we state only a necessary condition:

Observation 7.3. A necessary condition for partial genuine coherence of X is $|\text{BCE}(X)| \geq 2$ and partial genuine confirmation of at least some basic content part of X by its complement.

Finally we turn to the relation between measures of genuine coherence and genuine confirmation. Also in this case the connection is not straightforward. The degree of genuine coherence is not identical to the simple average of the degrees of mutual genuine confirmation of every content part by its complement, because the notion of genuine confirmation of x by y is itself an average of the degrees of confirmation of x's content parts by y. If we form an average over these averages, then the confirmation of a content part that has only one or a few content subparts would achieve a higher weight in the total average than the confirmation of a content part that has many content subparts. To illustrate, let $X = \text{BCE}(X) = \{x, y, z\}$, then $\mathbf{cohg}(X)$ is the average of 12 degrees of confirmation ($\mathbf{conf}(x, y)$, $\mathbf{conf}(x, z)$, $\mathbf{conf}(y, z)$, $\mathbf{conf}(x, y \wedge z)$, $\mathbf{conf}(y, x \wedge z)$, $\mathbf{conf}(z, y \wedge x)$, and the confirmation degrees for the inverted pairs). All these confirmation degrees are assigned an equal weight in the measure of genuine coherence. On the other hand, there are six basic content parts of X with a non-empty complement $(x, y, z, x\wedge y, y\wedge z, x\wedge z)$. The degree of genuine confirmation of each of these content parts, say c, by its complement, c^*, is the average of the degrees of confirmation of all content subparts of c by c^*. While $\mathbf{confg}(y\wedge z, x)$ is identical with $\mathbf{conf}(y\wedge z, x)$ (because x is a basic content element), $\mathbf{confg}(y, z\wedge x)$ is an average of $\mathbf{conf}(y, x)$, $\mathbf{conf}(y, z)$ and $\mathbf{conf}(y, z \wedge x)$, since $z \wedge x$ has the three content parts x, y and $x \wedge z$. Accordingly, if we would calculate the notion of genuine coherence by the averages of the degrees of genuine confirmation, the degree of confirmation $\mathbf{conf}(y \wedge z, x)$ would count three times as much as the degrees of confirmation $\mathbf{conf}(y, x)$, $\mathbf{conf}(y, z)$ and $\mathbf{conf}(y, z \wedge x)$. We summarize this in the following observation:

Observation 7.4. The degree of genuine coherence is not identical to the average degree of genuine mutual support between content parts and their complements.

Positively, this consideration implies that we can correct this deviation by introducing suitable weights. More precisely, the degree of genuine coherence of a set of propositions X is a weighted average of the degrees of genuine confirmation of the basic content parts x of X by their complements, weighted by the number of content subparts of x. We state this as the final result of this paper. For a set of propositions X, $\mathrm{BCP}^*(X) = \mathrm{BCP}(X)\setminus\{\bigwedge \mathrm{BCE}(X)\}$ is the set of those basic content parts of X that have non-empty complements.

Observation 7.5. For a set of propositions $X \subset \mathcal{L}$ with $|\mathrm{BCP}(X)| > 1$:

$$\mathbf{cohg}(X) = \frac{\sum\{\mathbf{confg}(x^*, x) \cdot |\mathrm{BCP}(x)| :\ x \in \mathrm{BCP}^*(X)\}}{\sum\{|\mathrm{BCP}(x)| :\ x \in \mathrm{BCP}^*(X)\}}$$

Proof. In the definition of the term "$\mathbf{confg}(x^*, x)$", the sum of all $\mathbf{conf}$ (x^*, y) over all content parts y of x is divided by $|\mathrm{BCP}(x)|$. So by multiplying with $|\mathrm{BCP}(x)|$, the resulting expression $\mathbf{confg}(x^*, x) \cdot |\mathrm{BCP}(x)|$ equals the sum of $\mathbf{conf}(x^*, y)$ for all content parts y of x. By summing up the terms "$\mathbf{confg}(x^*, x) \cdot |BCP(x)|$" for all content parts x of $\mathrm{BCP}^*(X)$, we obtain the sum of all terms $\mathbf{conf}(y, x)$ for all ordered pairs of non-overlapping basic content parts of X. This sum is identical with the numerator of the definition of genuine coherence. Moreover, the sum of the cardinalities $|\mathrm{BCP}(x)|$ over all X in $\mathrm{BCP}^*(X)$ is identical with the number of ordered pairs of non-overlapping content parts of X, which is the denumerator of $\mathbf{cohg}(X)$. This proves our claim. ■

8. Conclusion

We have shown that by means of conjunctive decompositions of (sets of) beliefs into content elements it is possible to explicate the concepts of coherence and confirmation in probabilistic terms in a way that avoids two well-known problems: the problem of belief individuation and the tacking problem. We demonstrated this approach for three languages, viz. the language of truthfunctional propositional logic $\mathcal{L}_0$, the language of truthfunctional logic together with an intensional conditional operator $\mathcal{L}_{\rightarrow}$ and the language of 1st order logic $\mathcal{L}_1$. The resulting concepts of genuine coherence and genuine confirmation have been investigated both on a qualitative and a quantitative level. In a further step, we payed closer attention to the relationship between these two concepts. We showed that the direct relationship between the ordinary Bayesian models of the concepts of coherence and confirmation does not transfer to their genuine counterparts, i.e., genuine coherence is *not* identical with genuine mutual support. However, Section 6 contains some results that show that there is nonetheless a close relationship between these refined concepts. Similarly, although the ordinary degree of Bayesian coherence *is* identical with the average degree of Bayesian mutual support, their quantitative genuine counterparts are not related that directly. Nevertheless, we were able to show that by adapting weights it is possible to regain this close link also in a quantitative perspective.

Last but not least we would like to note that although we think that our solution to the problem of belief individuation and the tacking problem is formally sound and intuitively appealing, our employed notions of genuine coherence and genuine confirmation are open to improvements as regards the details of the representation method. We leave the in-depth investigation of the pros and cons of our account and the assessment of its consequences with respect to other problems confronting Bayesian confirmation theory for another paper.

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