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A Dynamic-Logical Perspective on Quantum Behavior

Abstract. In this paper we show how recent concepts from Dynamic Logic, and in particular from Dynamic Epistemic logic, can be used to model and interpret quantum behavior. Our main thesis is that all the non-classical properties of quantum systems are explainable in terms of the *non-classical flow of quantum information*. We give a logical analysis of quantum measurements (formalized using modal operators) as triggers for quantum information flow, and we compare them with other logical operators previously used to model various forms of classical information flow: the "test" operator from Dynamic Logic, the "announcement" operator from Dynamic Epistemic Logic and the "revision" operator from Belief Revision theory. The main points stressed in our investigation are the following: (1) The *perspective and the techniques of "logical dynamics"* are useful for understanding quantum information flow. (2) Quantum mechanics does not require any modification of the classical laws of "static" propositional logic, but only a *non-classical dynamics of information*. (3) The main such non-classical feature is that, in a quantum world, *all information-gathering actions have some ontic side-effects*. (4) This ontic impact can affect in its turn the flow of information, leading to *non-classical epistemic side-effects* (e.g. a type of *non-monotonicity*) and to states of "objectively imperfect information". (5) Moreover, *the ontic impact is non-local*: an information-gathering action on one part of a quantum system can have ontic side-effects on other, far-away parts of the system.

Keywords: Dynamic Quantum Logic, Philosophy of Quantum Information, Dynamic Epistemic Logic, Logical Dynamics.

1. Introduction

The received knowledge in the "Quantum Logic" (QL) tradition is that a logical understanding of the foundations of Quantum Mechanics (QM) would necessarily require giving up some of the classical logical principles, such as Distributivity (of Conjunction over Disjunction, and vice-versa) or even Bivalence. In this paper we argue against this claim and in favor of a different approach to the logical foundations of QM, approach belonging to the *Dynamic Logic* tradition. Our philosophical conclusions are based on, and motivated by, the more formal quantum-logical investigations in our previous papers [6, 7, 8].

The views expressed in this paper fit well with the recent trend towards a *"dynamification" of logic* (mostly, but not exclusively, pursued by the "Dutch school" in modal logic, see e.g. [17]): one of the main tenets of this trend is to take a fresh look at various non-classical "propositional" logics as being all about *actions*, rather than about propositions. *Action-based reasoning* has been very important for the study of classical information flow in various contexts. In Philosophical Logic, this topic covers a broad list of subjects ranging from Action Logic, Arrow Logic and Game Logic to semantic games, dialogue logic, Belief Revision and Dynamic Epistemic Logic. In Computer Science, this topic covers for instance Hoare logic, Dynamic Logic, Linear Logic, labeled transition systems, Petri nets, Process Algebra, Automata Theory, Game Semantics, coalgebras etc. What all these approaches have in common is the idea that *information systems are dynamic systems*: a "state" of the system is characterized by its potential changes, i.e. by the actions that can be performed on the state. Within quantum logic, this *dynamic turn* found its way in [24, 25, 28] and this line of research was further developed by the Brussels school in quantum logic in [3, 20, 21, 22, 23, 35].

Using the technical work in [6] (where we gave a *complete dynamic-logical characterization of quantum systems*) and [7, 8], we argue in this paper that the non-classicality of QM is due to the *non-classical features of information flow* in quantum systems. Hence, any logic aiming for a real understanding of quantum behavior should accommodate a *non-classical "logical dynamics" of information*, rather than having non-classical logical laws governing static information (such as in non-Boolean, non-distributive, partial or fuzzy logics etc.). The fundamental action in this logical dynamics is the *quantum test* (corresponding to a successful yes-no measurement performed on a quantum system). We analyze this action as a form of information update, and compare it with other dynamic-informational operators in logic: the classical "test" operator in Dynamic logic, the announcement operator in Dynamic-Epistemic logic and the belief revision operator. Quantum tests share some common features with all these operators, but there are also important differences, which make apparent the non-classical nature of quantum information flow: far from being "purely epistemic" actions (as all the others above), quantum tests exhibit an inextricable blending of *ontic* and *epistemic* features.

First, we observe that *any epistemic action used to extract information from a quantum system may have ontic side-effects*. That is to say, in contrast to classical informational actions, quantum tests typically change the ontic state of the "observed" system. We can formulate this point as a slogan: in a quantum universe "*there is no information change without*

changing the world". Moreover, epistemic actions gathering only *local* information about *one part* of a system may even have "non-local" ontic effects on *another, remote part* of the system. We use this latter feature of quantum information flow to model and understand the notion of *entanglement* in QM.

Note that when we use the notion of "epistemic action" in this paper, we refer to any action by which an "observer" system (or the "environment") can gather information about an "observed" system. These are just any actions through which some information about the observed system is transferred to the observer. Note that, in this sense, an "epistemic action" is *not* necessarily a subjective notion, in the sense of happening only in the mind of a subject.

Second, the above mentioned ontic impact of epistemic actions in a quantum world leads in its turn to *non-classical epistemic effects*. Because of the ontic side-effects, quantum information *does not necessarily accumulate* when a series of successive epistemic actions are performed. Indeed, the ontic impact of a quantum test might change the system in such a way that it renders obsolete the results of a previous test: the property "learned to be true" by the previous test might now become false due to the second test/action. This fundamental *non-monotonicity of quantum information gathering* is unlike the monotonic behavior induced by simple classical learning of "factual" information, which keeps accumulating through various learning stages. On the other hand, there are formal similarities between this phenomenon and the non-monotonic logical behavior encountered in Belief Revision theory and in the Dynamic-Epistemic Logic of higher-order information (information about other agents' information). In both these logics, epistemic/doxastic actions can change the "state of the system". But note that what is changed here is actually not an ontic state, but only an information state (a "theory" or a "belief"). Beliefs are being revised but this has no effect at all on the "ontic facts" (i.e. the non-epistemic atomic propositions, expressing factual information). Further on, we make this analogy formal and use it to show the fruitfulness of the dynamic-epistemic logical formalism. But we certainly do *not* claim that all these information gathering actions are of the same type or that they have the same properties. On the contrary, we can use the fundamental non-monotonicity of quantum learning to give a very simple explanation of the specifically quantum, and highly non-classical, phenomenon of *incompatibility of observables* (such as position and momentum) in QM.

In this paper we subscribe to a realistic stance, and view all (*pure*) quantum states of a system as *ontic states*. We certainly do not identify the ontic state of a system with the state of information held by an external observer.

The information state and the ontic state of the system are different but not totally unrelated: as mentioned, in QM the ontic states can be changed by epistemic actions. Indeed, we deem this simple fact responsible for some of the confusion in the literature on the interpretation of QM: it may seem like a small and "logical" step to conclude (wrongly, in our view) that the "states" of QM are *nothing but* informational, epistemic states. We consider this step as granted *only* in the case of the so-called *"mixed" states*, for which we indeed accept a purely epistemic interpretation. But we reject the generalization of this interpretation to *all* quantum states: in our view, there can be no epistemics without an ontic basis, no "knowledge" without things to be known, no "information" without an underlying "formation", no software without hardware. We identify this "hard" ontic core with the notion of *(pure) quantum state*.

Our use of the dynamic-epistemic logical (DEL) formalism has several advantages over other logical tools for our comparative philosophical analysis. In contrast to e.g. the standard AGM theory of belief revision, the DEL approach is a *modal approach, based on a "possible world" semantics, and thus grounded in a realistic ontology.* This is due to the fact that the "actual world" plays a central role in modal logic: the actual state is the "point of evaluation" of a formula. In a Kripke-style semantics for epistemic logic, it makes no sense to talk about "epistemic states" in the absence of any "real" states: the epistemic state is simply a derived feature of the actual, real state of the world. The distinction between ontic and epistemic states has of course been made before, but the possible-world-based approach of Modal Logic incorporates it in an essential way in its semantics. In this sense, DEL can be used for modeling realistic situations in philosophy of science. Moreover, our previous work shows that DEL's inherently realistic stance can also be useful as a logical tool in other philosophical contexts, by e.g. applying it to older debates in epistemology about negative introspection and weak notions of "knowledge" (see [13, 5]).

The DEL approach has another advantage over other logical tools, in the sense that the above-mentioned inherent realism applies to "changes of state", and not only to states: this approach allows us to model "true dynamics" in a very specific (and realistic) sense.[1] For instance, as mentioned before, in Belief Revision theory, beliefs change, but their object remains the same. Classical AGM theory is about an unchanged state of the world, while in its alternative "belief update" version (the KM theory of Katsuno

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[1]Note that our use of the notion "true dynamics" in this paper refers to any real change of state, and as such it is clearly different from the standard use of "dynamics" in physics.

and Mendelzon) the world is affected only by "objective", non-epistemic changes, having nothing to do with our belief revision. In other words, Belief Revision theory cannot capture the way in which our changes of information may affect the state of the world. In this sense, traditional belief revision is "static" in a fundamental sense. In contrast, DEL can express and explore the sense in which the world itself is changed by our epistemic actions. This distinction between "statics" and "dynamics" in Logic is useful to model both classical and quantum information flow. Moreover, it was used to give elegant solutions [10] to some well-known problems in Belief Revision theory such as for instance the necessary failure of the so-called "Ramsey test".

As mentioned above, DEL comes with an "action-based methodology". In this sense, the mentioned realism also applies to "actions" (including "epistemic actions"), and not just to unstructured information changes. The DEL approach can capture, analyze and classify specific types of information changes due to concrete actions. Several types of classical information flow that have been studied in the literature are due to various concrete actions: truthful public announcements, private exchanges of messages, lying, information obtained from unreliable sources etc. For quantum information change the actions vary from unitary evolutions (corresponding to the so-called "quantum gates" in quantum computation) to various types of measurements. In this paper we restrict our attention to the so-called "ideal measurements of the first kind" [33], which perturb the observed system *as minimally as possible*. Among these, we consider only "quantum tests", corresponding to *successful yes-no* measurements, usually represented by *projectors* in the standard Hilbert-space formalism of QM. While DEL techniques are known to be useful in the context of knowledge-updates, they are actually rather new in the context of Quantum Logic (being first introduced in our joint papers on dynamic quantum logic [6, 7, 8, 9]) and Belief Revision [10, 11, 12, 13, 14, 16]. Focussing further mainly on the quantum case, we first introduce the necessary background in section 2, explaining the logic of quantum actions in section 3. In section 4 we compare quantum tests with other forms of information update, in particular with the "test" operator in Propositional Dynamic logic, the announcement operator in DEL and the AGM belief revision operator. In the last section, we summarize the conclusions of our investigation into the dynamic-informational roots of quantum "non-classicality".

2. Background

Semantic Information   We distinguish between two fundamentally different approaches to information: the *syntactic* (or *quantitative*) and

the *semantic* (or *qualitative*) approach. The first looks at information in terms of the patterns occurring in a string of bits; there is no concern with meaning, no semantics, no "aboutness". Instead, the main issue is measuring information, and in the classical approach of Shannon's theory of information, such a quantitative measure is given in terms of Shannon entropy (which for quantum information has to be replaced with von Neumann entropy). This approach is the standard one in the study of physical information, and in particular in Quantum Information Theory. In contrast, the semantic, qualitative approach to information is the standard one in Logic and Computer Science. Semantic information can be true or false, and it is always "about" something: propositions have a meaning, an "information content". The main issue in this approach is to find the formal laws governing information flow of a specific type, so that we can analyze it, reason about it, check its correctness etc.

Our approach is logic-based, so we are mainly concerned with *qualitative* (semantic) information. Unsurprisingly, our approach to quantum information intersects with the similarly qualitative approach traditionally known as "quantum logic" (discussed below). For other qualitative conceptions of quantum information, see e.g. [39].

Quantum Logic Quantum logic (QL) originated with the work of J. von Neumann [38] and the paper of G. Birkhoff and J. von Neumann [18]. Their claim that *distributivity of conjunction over disjunction (and vice versa) is "the weakest link" in logic* has fueled a whole tradition of research, based on the idea that in dealing with the logical foundations of quantum physics, we should give up some basic principles of classical propositional logic. The reason for this claim lies in the observation that *the "experimental" (or testable) properties* of a quantum system correspond to the *closed linear subspaces of a Hilbert space*; in particular, the actual "state" of a system is given by an atomic such property, i.e. a one-dimensional subspace (a "ray"). The lattice of closed linear subspaces (with inclusion as the order) does not form a Boolean algebra (and indeed it is not closed under complementation or unions, though it is closed under intersections): thus quantum disjunction (defined as the closed subspace generated by the union) and quantum "negation" (orthocomplement) will be non-classical. The syntax of QL is build up from a set of basic formulas p (denoting some basic testable properties):

$$\varphi \quad ::= \quad \bot \quad | \; p \quad | \sim \varphi \quad | \; \varphi \wedge \varphi \quad | \; \varphi \sqcup \varphi$$

Here, $\bot$ denotes the "always false" (inconsistent) proposition, $\sim$ the orthocomplement and $\wedge$ the classical conjunction. Note that the falsity of a proposition does not imply the truth of its orthocomplement. The join

(or quantum disjunction) $\sqcup$ differs from the classical disjunction, since the truth of $\varphi \sqcup \psi$ does not imply that either φ is true or ψ is true. Semantically, the quantum join captures *superpositions*: $\varphi \sqcup \psi$ says that the current state of the system is a superposition of (states satisfying) φ and (states satisfying) ψ. As expected, join is not distributive over conjunctions.[2] Finally, "quantum implication" (also called Sasaki hook) is defined as: $\varphi \overset{S}{\to} \psi := \sim \varphi \sqcup (\varphi \wedge \psi)$. The mentioned failure of distributivity has as a consequence that the classical Deduction Theorem fails for the quantum implication.

The Problem of Entanglement "Quantum non-locality" is the name given to one of the most puzzling effects in the microscopic world: the appearance of non-trivial correlations between the results of measurements performed simultaneously on systems that are spatially remote. An intriguing issue arises from the fact that these correlations cannot be explained by any form of "classical communication" or classical information flow between the systems. That is to say, "non-locality" points to what Einstein called the "spooky action at a distance" between the parts of an "entangled" quantum system: this is what is meant by flow of information over a "quantum channel". The issue has a long history, dating back to the early discussions on the EPR thought-experiment in the thirties, followed by J. Bell's argument in [15] against the "locality" assumption used in the EPR-paper [26]. Bell proved that the requirement of "locality", i.e. "that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past" [32], is incompatible with the statistical predictions of quantum mechanics. Taking the point of view of the entangled particles, Bell's result amounts to the fact that these correlations cannot be explained by any "classical strategy" (hidden variable theory) adopted by the particles in advance in order to coordinate their behavior under measurements [32]. The issue has posed interpretative problems to philosophers as well as to physicists, while promising great opportunities to computer scientists: the power of quantum computation is mainly due to the same phenomenon (of non-local correlations between entangled systems).

In QM, entanglement is easily modeled by representing a "compound" system S, made of two subsystems S_1 and S_2, as a *tensor product* $H_1 \otimes H_2$ of the two Hilbert spaces (instead of a Cartesian product, as in classical physics). A global state $s \in S$ is "separated" if it belongs to the Cartesian

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[2]The distributivity law has to be replaced by a weaker version, called Weak Modularity: if ψ follows from φ then ψ is equivalent to $\varphi \sqcup (\psi \wedge \sim \varphi)$. In the framework of traditional quantum logic, this law can be considered as the axiomatic embodiment of "quantum behavior".

product $H_1 \times H_2$, i.e. it is of the form $s = s_1 \otimes s_2 := (s_1, s_2)$; in this case, each of the subsystems S_i is in a well-defined ("pure") local state s_i. When this is not the case, then the global state s is called "entangled" (and the two subsystems are also said to be "entangled" with each other). An entangled state cannot be neatly separated in two local states $s_1 \otimes s_2$, though it can be written as a superposition (linear combination) $\sum_i s_1^i \otimes s_2^i$ of separated states. So an entangled subsystem cannot said to be in any defined (pure) local state. Nevertheless, QM uses a higher-order representation, in terms of density operators called "mixed states", to capture (all the available information about) the "local state" of an entangled subsystem. But finding *a correct, generally-agreed interpretation of mixed states* is still a very much debated *open problem*. One of the most popular views is the so-called "ignorance interpretation" (see e.g. [31]), which takes mixed states as expressing the observer's lack of knowledge about which pure state is the real one. In this paper, we shall propose a different interpretation, in terms of the "objectively imperfect information" (rather than subjective "ignorance") that an entangled subsystem has about its environment.

Entanglement also poses *a problem to the lattice-theoretic approach* of traditional Quantum Logic. It has been convincingly argued [2] that there cannot exist any general lattice-theoretic analogue of tensor product, i.e. any well-behaved such operation on quantum lattices (satisfying a given set of natural constraints). See e.g. [37] for a recent overview on the matter. Finding *a formal logical setting that could naturally accommodate entanglement* can still be regarded as an *open problem*, although possible solutions have been proposed by Abramsky and Coecke [1] and ourselves [7, 8].

Dynamic Logic Propositional Dynamic Logic (PDL) is a logic of actions developed by computer scientists, as an extension of the so-called Hoare Logic, designed to capture important properties of programs, such as correctness [30]. Its syntax has two types, a static one consisting of *propositional formulas*, and a dynamic one consisting of *actions* (or *programs*), the two being defined by mutual induction:

$$\varphi \quad ::= \quad p \quad | \neg\varphi \quad | \varphi \wedge \varphi \quad | [\pi]\varphi$$
$$\pi \quad ::= \quad a \quad | \varphi? \quad | \pi \cup \pi \quad | \pi; \pi \quad | \pi^*$$

Here, $\neg$ is the *classical negation*, $\wedge$ is the *classical conjunction*; the variables p range over a given set of basic atomic propositions. It is convenient to introduce the notation $\perp := p \wedge \neg p$ to denote the "always false" proposition. The action modalities $[\pi]$ are used to build formulas $[\pi]\varphi$, expressing *weakest preconditions*: $[\pi]\varphi$ means that, if action π was performed on the current

state of the system, then after that the output-state would necessarily have property φ. So $[\pi]\varphi$ expresses the *weakest* condition ensuring that (postcondition) φ will be satisfied after performing π. The "test" $\varphi?$ is an abstract action of testing property φ: this action succeeds if and only if the state of the system satisfies φ, in which case the state is left unchanged; otherwise the action fails. As a consequence, a successful test $\varphi?$ indicates that *property φ was true before the test, and will continue to be true after the test.* Observe that the formula $[\varphi?]\psi$ is equivalent in PDL to the classical implication $\varphi \rightarrow \psi$. The variables a range over a given set of basic actions (which may change the state of the system). An action of the form $\pi \cup \pi'$ expresses *non-determinism* (arbitrary choice between actions): do either π or π'. An action of the form $\pi; \pi'$ expresses *temporal succession* (composition) of actions: first do π then do π'. Finally π^* expresses *iteration* of actions: repeat π some finite number of times.

Epistemic Logic in Computer Science Epistemic Logic is a logic that explicitly deals with *knowledge*, i.e. possession of (or access to) information [27]. In addition to classical propositional logic, it contains a modality K for knowledge, such that $K\varphi$ means that φ is "known", i.e. the information φ is in principle accessible to the "observing agent". In computer science, an "agent" is simply a part (sub-system) of the global system, or process, under investigation. This expresses the *distributed, localized* nature of information: not all information is accessible in all parts of the system. Hence we can consider any sub-system as the "observed" one, and the rest of the system as the "observer" or the "environment". "Knowledge" in this sense has nothing to do with consciousness, it is only an "external" notion of knowledge, that we can attribute to subsystems: it simply expresses the potential availability of information at a given location.

Dynamic Epistemic Logic This is a logic that combines dynamic and epistemic logics [4], in order to deal with information changes, such as communication, learning the outcome of an experiment etc. The most basic example is an epistemic version of the PDL-test $\varphi?$, namely the action $\varphi!$ of *learning* the (truth of) proposition φ. This is called *a (truthful, public) announcement that φ*. An announcement $\varphi!$ is an action in which the observer "learns" a true property φ; so a successful such action induces a change in the information-state of the observer. Since the observer's state is an explicit part of the total state of an epistemic system, an announcement induces a change of the total state. As a consequence, the very property that was announced may become false, e.g. when φ is of the form $p \wedge \neg Kp$. So, a successful announcement $\varphi!$ indicates that *property φ was true before*

the announcement, but not necessarily after. However, the ontic state of the observed system is not affected by announcements. In dynamic-epistemic logic, this is expressed by the fact that atomic formulas (corresponding to ontic facts, i.e. properties of the observed system), are left invariant under announcements: communication or observations do not change the world. In other words, Dynamic Epistemic Logic is *based on a classical (non-quantum-mechanical) theory of information flow.*

Belief Revision Another form of information change is *belief revision.* This is the action of revising a previously held belief ("theory") T about the world after receiving some new information φ, that may contradict T. For instance, an experiment is done, which establishes a new fact φ, in contradiction with the predictions of the current theory. T encodes the original information state of the "observer", and the belief revision action $*\varphi$ changes this state to a new information state, given by a revised theory $T * \varphi$. The classical AGM postulates [29] express minimal rationality conditions that belief revision must satisfy: the revised theory must incorporate the new fact ($\varphi \in T * \varphi$); the theory is left the same if it correctly predicted the new fact (i.e. $\varphi \in T$ implies $T * \varphi = T$); otherwise, the theory is "minimally revised" in order to accommodate the new fact (i.e. the revised theory preserves as much as possible of the original theory).

3.   The Logic of Quantum Information Flow

In [6], we showed that Hilbert spaces can be naturally structured as non-classical relational models of Propositional Dynamic Logic. These models, called *Quantum Transition Systems*, consist of a set Σ of *states*, together with a family of basic *transition relations*, which are binary relations between states in Σ. The states are meant to represent (complete descriptions of) *possible states of a physical system*, while the transition relations describe the changes of state induced by *possible actions* that may be performed on the system. In a Hilbert space H, the "states" correspond to *rays (one-dimensional subspaces)* of H, while the actions correspond to specific *linear maps on H*. There are two main types of basic actions: *"quantum tests"* φ? and *"quantum gates"* U. Tests are meant to represent *successful measurements*[3] of some yes/no property ("bit") φ: property φ is tested, the answer is "yes" and as a result of this the state of the (observed) system collapses

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[3]More precisely, they correspond to what has been called "filters" by C. Piron in [34]. A filter is an "ideal measurement of the first kind" (in the sense of [33]) that destroys the system in case of a negative result (i.e. if the system does not exhibit the property).

(to a state satisfying property φ). In a Hilbert space, a test $\varphi?$ corresponds to a *projector* onto (the subspace generated by property) φ. Quantum gates U represent *reversible evolutions* of the observed system. In a Hilbert space, they correspond to *unitary transformations* on H. The language of PDL can be interpreted in a Quantum Transition System, by interpreting tests $\varphi?$ as quantum tests and the basic actions a as quantum gates (and keeping the classical interpretation for all the other operators, in particular keeping Boolean negation and conjunction). The program expressions π can now be interpreted as *quantum programs*. The resulting logic is called the *Logic of Quantum Actions* (LQA).

Bivalence, Boolean negation, testable properties Observe that LQA is a *bivalent, Boolean* logic, in which the propositional connectives satisfy all the classical laws of propositional logic. Dynamic-logic formulas denote possible *properties of quantum states*, which in a Hilbert space H correspond to arbitrary unions of rays. Any such property either holds at a given state or it doesn't, hence the bivalence. The negation $\neg\varphi$ of a given property φ simply expresses the fact that property φ does not hold. However, not all the expressible properties are "testable" (i.e. corresponding to an "experimental" property). In particular, the negation of a testable property might not be testable. Syntactically, we can characterize testable properties as the ones that can be expressed by negation-free formulas of LQA: any formula built without the use of (Boolean) negation denotes a testable property.

Quantum Logic, Measurements, Causation, Superpositions Traditional quantum logic can be re-interpreted inside LQA, by defining the *orthocomplement* $\sim\varphi$ of a property as the *impossibility of a successful test*, i.e. by putting: $\sim\varphi := [\varphi?]\bot$. Quantum join is then definable via de Morgan law, by putting: $\varphi \sqcup \psi :=\sim (\sim\varphi \land \sim\psi)$. As mentioned, this notion captures all possible *superpositions* of (states satisfying) φ and (states satisfying) ψ. The "quantum implication" (Sasaki hook) is simply given by the weakest precondition of a "test": $\varphi \xrightarrow{S} \psi := [\varphi?]\psi$. In other words, we obtain a *dynamic interpretation of the non-classical connectives of quantum logic*.

A *measurement* can be expressed as a complex LQA program, namely a non-deterministic sum ("choice") of quantum tests of mutually orthogonal properties. As observed in [22], the weakest precondition $[\pi]\varphi$ captures a *weak notion of causation*: $[\pi]\varphi$ captures the *(weakest) cause* that induces property φ to be actualized after an action π. In particular, this means that the quantum implication $[\varphi?]\psi$ captures the cause of a property being actualized by a test. The "quantum dual" $\varphi?[\psi] :=\sim [\varphi?]\sim\psi$ of the weakest precondition captures the *(strongest) effect* induced by (a given cause) ψ

after a test $\varphi?$ is performed. This corresponds to what is known in Computer Science as *the strongest post-condition* that is actualized by performing an action $\varphi?$ on a state satisfying (the precondition) ψ.

But the presence of classical negation $\neg$ gives us more expressive power. We can of course define the classical disjunction via de Morgan law: $\varphi \vee \psi :=$ $\neg(\neg\varphi \wedge \neg\psi)$. But, more importantly, we can take the classical dual $< \varphi? > \psi$ $:= \neg[\varphi?]\neg\psi$ of the weakest precondition, which corresponds to the *existential modality* in PDL: this expresses the *possibility* of actualizing a property ψ by a successful test of property φ. In particular, we can express the fact that a property φ is *potentially true* at a given state, i.e. *it can be actualized by some measurement* (performed on the current state). Given the axioms of Quantum Mechanics, this is equivalent to saying that the test $\varphi?$ may be successful. So the formula $\Diamond\varphi$ expressing potentiality of φ can be defined as the possibility of testing for φ, i.e. $\Diamond\varphi :=< \varphi? > \top$.

Complete Axiomatization of Single Systems, Orthomodularity, Covering Law   In our paper [6], we expressed all the important qualitative properties of single quantum systems as dynamic-logical properties. In particular, some unnatural postulates of quantum logic of a rather technical nature (e.g. Non-distributivity, Orthomodularity, Piron's "Covering Law") were recovered as natural (although non-classical) properties of quantum logical dynamics. Moreover, we proved an "abstract completeness result" for these axioms, showing that *all* qualitative features of single quantum systems are captured by our axioms.

Compound Systems and Local Information: Quantum Dynamic-Epistemic Logic   In our papers [7, 8], inspired by previous work of Abramsky and Coecke [1, 19], we added *(qualitative) spatial features* to quantum dynamic logic, allowing us to talk about *local properties* of given *subsystems* of a quantum system. Essentially, the setting is still given in terms of a Quantum Transition System, but now the unitary actions are of various types, depending on their location. Consider a compound system S made of two subsystems S_1 and S_2. The quantum transition system[4] includes now, besides the "global" actions U (affecting the whole system), *i-local unitary actions* U_i (for $i = 1, 2$) performed only on subsystem S_i. We also need to distinguish a *special separated state* $c = c_1 \otimes c_2 = (c_1, c_2) \in H_1 \times H_2 \subseteq H_1 \otimes H_2$, designated by a constant symbol c.

This logic can be used to give an analysis of the dynamic-informational aspects of *compound systems*. One can internalize in the logical language

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[4]associated to the tensor product $H_1 \otimes H_2$ of the Hilbert spaces H_i associated to each subsystem.

the notion of "local state of an entangled subsystem S_i" (usually described in QM as a "mixed state", and represented as a density operator), by defining an equivalence relation $\simeq_i$ on global states: e.g. for $i = 1$, we put $s \simeq_1 s'$ iff there exists a 2-local unitary action U_2 such that $s' = U_2(s)$. Intuitively, this says that two possible states s and s' of the compound system S are *"indistinguishable" from the point of view of subsystem S_1* iff s' can be obtained from s by performing a unitary action only on the environment S_2. We can now define *the local state s_i of subsystem S_i* in global state s simply as the $\simeq_i$-equivalence class $s_i := \{s' : s \simeq_i s'\}$ of s. This represents the set of global states that are "possible" according to subsystem S_i, i.e. that are consistent with all the information available at location i. In this sense, we could understand the local state of an entangled subsystem S_i as an *"epistemic" (or "informational") state*: it encodes all the information that subsystem S_i "has" about the global system S. It is thus natural to introduce an "epistemic" operator K_i as the Kripke modality associated to the indistinguishability relation $\simeq_i$: for every property $\varphi \subseteq S$ and every component $i \in \{1, 2\}$, we define the property

$$K_i\varphi := \{s : t \in \varphi \text{ for all } t \simeq_i s\} = \{s : s_i \subseteq \varphi\}.$$

K_i has the formal properties of a "knowledge" operator, satisfying the axioms of the modal system $S5$, so one could loosely read $K_i\varphi$ as saying that *subsystem S_i "knows"* φ. But this way of speaking only refers to the above-mentioned "external" notion of "knowledge", as potential local information (not as "subjective" knowledge by an actual "observer"). So a better reading of $K_i\varphi$ is: *the information that (the global system satisfies) φ is potentially available at location i*. In particular, for a *separated* state (e.g. the special state $c = (c_1, c_2)$), the "local state of subsystem S_1" (e.g. $\{s : s \simeq_1 c\} = \{(c_1, d) : d \in H_2\}$) *does* in fact correspond to a *pure* local state of S_1 (here, the state c_1). So a separated subsystem *can* be said to be in a well-defined ontic state.

Before investigating non-locality, we first need to define what is a *local property*: for $i = 1, 2$, a property φ is *i-local* (i.e. it is a property of the separated subsystem S_i) if it entails separation of the global system S and if it can only hold when it is "known" to subsystem S_i (i.e. φ is logically equivalent to $K_i\varphi$). Local measurements on subsystem S_i are quantum tests of the form $\varphi_i?$, where φ_i is any testable *i*-local property. It is convenient to have *local propositional variables p_i* (ranging over testable local properties) in our syntax. We obtain a *quantum semantics for a Dynamic Epistemic Logic* of the form:

$$\varphi \quad ::= \quad \sigma \quad | \ \sigma_i \quad | \ \neg\varphi \quad | \ \varphi \wedge \varphi \quad | \ [\pi]\varphi \quad | \ K_i\varphi$$
$$\pi \quad ::= \quad \alpha \quad | \ \alpha_i \quad | \ \varphi? \quad | \ \pi \cup \pi \quad | \ \pi;\pi \quad | \ \pi^*$$

where σ is either a propositional variable p or the constant state c, and similarly α is either an action variable a or a constant action symbol (denoting some special "quantum gate", from a given list of unitary evolutions).[5]

Entanglement, Non-locality, Bell States, Teleportation Instead of defining entanglement as usually done in QM (in a quantitative manner, using linear combinations and tensor product), we can take as its definition the very existence of non-local ontic effects of local measurements, or equivalently, the existence of an "informational correlation" (without communication) between the two subsystems. We start with the second formulation, in terms of static information. It is easy to see that a global state s is *separated* (or, more precisely, subsystem S_1 is separated in the state s) iff there exists s' such that $s \simeq_2 s' \simeq_1 c$ (where c is the special constant). Otherwise, the state is *entangled*. Using our modal operators, we obtain an *"epistemic" characterization of entanglement*: s is entangled iff it satisfies the sentence $K_2K_1\neg c$. So *two subsystems are entangled if they have some specific (nontrivial) "knowledge" about each other (prior to any communication).*

We can also encode the first formulation above, in terms of measurements (saying that *entanglement corresponds to non-local ontic side-effects of local measurements*), as follows: s is entangled iff we have

$$s \not\simeq_2 \varphi_1?(s)$$

for every testable 1-local property φ_1. In other words: *two systems are entangled if every local measurement of the first system changes the other.*

In fact, QM postulates a stronger property of entanglement than the simple existence of non-local side effects of local measurements. Namely, it asserts that the non-local ontic impact is *deterministic*: there exists a deterministic correlation between the results of a local measurement on S_1 and the subsequent ontic state of the environment S_2. Formally: for every global state s, there exists a deterministic program π_s, satisfying

$$p_1?(s) \simeq_2 \pi_s(p_1)\,.$$

Dually, for every deterministic program π sending 1-local states into 2-local states, there exists a corresponding global state s such that $\pi_s = \pi$.

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[5]This logic is *new* in this form, being only available on the web (as Lecture Notes of our ESSLLI'06 course) at http://www.vub.ac.be/CLWF/SS/ESSLLI.html

In [7, 8], we call the state s "*entangled according to π*", and we denote it by $\overline{\pi}$. So we have:

$$p_1?(\overline{\pi}) \simeq_2 \pi(p_1).$$

Abramsky and Coecke [1, 19], using a very different formalism, take (an equivalent version of) this equation as the main axiom of their system for entanglement, and use it to explain Teleportation. In [7, 8] we followed them, taking this equation as one of our axioms for a weaker version ("purely dynamic", i.e. without the K_i operator) of the above logic. We used this to characterize various *specific forms of entanglement*, such as *Bell states*, and to study multi-partite quantum information flow, giving a *formal logical analysis of various protocols* known from quantum computing (Teleportation, Super-dense Coding, Quantum Secret Sharing, etc.). The power of this axiom comes from the fact that it gives a *complete logical characterization of entanglement, in terms of the dynamics of quantum information*. The existence of such a characterization has important conceptual-philosophical and computational consequences, which we discuss in the next section.

4. Comparison of Quantum Tests with Other Forms of Information Update

We can use dynamic-epistemic logic to analyze a quantum test as a form of information update. As a convenient way to talk about localized information, we speak of "knowledge" and use the formalism of epistemic logic, to describe the information (about the "observed system") that is potentially accessible (in principle) to the "observer". As in the Computer Science discussions of epistemic logic, we make a conventional distinction between the "observed" system and its "observer", or "environment".[6] In this context, quantum test operators φ? share some common features with other forms of "test" operators in logic: PDL tests, announcements $\varphi!$ and belief revision $*\varphi$. All these are informational actions, by which the state of the "observer" is updated with some new information φ about the "observed" system: a test is performed, a measurement is done, a statement is announced, a theory is revised. In the remainder of this section we present a comparative analysis of these informational actions, using the overview in Table 1.

As in belief revision, a testable property is *actualized by its successful quantum test*: the observed system will always satisfy the tested property φ *after* the action φ? of quantum testing is successfully performed. As the third

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[6]Any separated subsystem can serve as the "observed" one, and the rest of the total system can serve as the "observer"; the roles are symmetric: the observer is itself observed.

row of Table 1 shows, this is *not* the case for truthful public announcements. As a counter-example we refer to the Moore sentence $p \wedge \neg Kp$, which is true before it is being announced but false afterwards (see Section 2).

TABLE 1:

PDL Tests φ?	Public Announcements φ!	Belief Revision with φ	Quantum Tests φ?
No ontic change	No ontic change	No ontic change	Minimal ontic change
φ true before	φ true before		φ potentially true before
φ true after		φ accepted after	φ true after
	$K(\varphi$ true before) is true after	$B\varphi$ is true after	$K\varphi$ is true after
no epistemic change		minimal epistemic change	epistemic change

Unlike the case of PDL tests and announcements, a successful quantum test does not require that the tested property was true before the action. But, as in the case of PDL tests and of announcements φ!, a successful quantum test *carries some information about the original state* of the observed system (before the action): the property φ was "potentially true" in the original system, in the sense that the original state was not orthogonal to (the subspace corresponding to the property) φ. In our logic LQA, this means that the original state satisfied $\Diamond \varphi$. For a successful PDL tests φ?, the system has to satisfy φ before the action and similarly a successful announcement φ! can only happen if φ is true beforehand. These features are expressed in the second row of Table 1.

Quantum tests have one more feature in common with AGM belief revision operators: namely, the intuition that the change affecting the original state is *the minimal possible such change*, that can accommodate the tested property φ. In QM, the output-state of a measurement φ? is *the minimal modification of the original state that has the tested property φ*.[7]

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[7]In a Hilbert space, the new state vector after a successful measurement of φ is the projection $P_\varphi(s)$ of the original state vector s onto the subspace (corresponding to) φ, i.e. the vector in the subspace φ that is the closest (in the sense of angular distance) to s.

Similarly, in AGM Belief Revision, the revised theory $T * \varphi$ is understood to be the minimal modification of the original theory that contains (i.e. accepts) the new information φ. (In Table 1 these features are expressed by the use of the word "minimal" in the first and last rows.)

Some of the *epistemic effects of quantum tests* are similar to the ones induced by other forms of information update: new information becomes accessible to the observer. Indeed, the fourth row of Table 1 shows that after a successful quantum test φ?, we have not only that φ is the case (i.e. the observed system satisfies the property), but also that $K\varphi$ is the case: this means that the information (encoded by) φ has become potential knowledge of the "environment".

The first row of Table 1 expresses the fact that tests are "real interactions" between the two subsystems, having an *ontic* (and not only epistemic) impact. Unlike all the other above-mentioned forms of test, quantum tests induce real changes in the observed system.[8]

Another interesting feature, shown in the last row of Table 1, is that *the ontic effects of quantum testing have non-classical epistemic consequences*, leading to a *non-monotonic dynamics of information*. Indeed, while adding a new bit of information (about the observed system) to the "knowledge" state of the observer, a quantum test *may also increase his "uncertainty", by making obsolete some of the previously acquired information*. To explain this non-monotonic feature of quantum learning, notice that any acquired quantum information is entirely due to past interactions between the observed system and its environment, and all such interactions can be regarded as measurements, or "tests". But a new test, as an ontic action, changes the observed system, so that the properties induced by previous tests may not hold anymore. We use this non-monotonic feature of quantum information to give a simple explanation to the well-known quantum phenomenon of "incompatible" tests: testing a property P typically induces an ontic change affecting another ("incompatible") property Q, thus making obsolete the information that was previously obtained by testing for property Q. Note that the "uncertainty" we are dealing with here is of a special type. In classical epistemic logic and game theory, such uncertainty was called "imperfect information". To stress that there is no subjective element present here, and that the quantum "uncertainty" is entirely due to an objective lack of access of the environment to the relevant information (since past information has been overridden by new interactions), we call this *"objectively imperfect information"*.

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[8]The "minimal revision" is in this case not a belief revision, but a "revision of reality"!

Moving on to *compound quantum systems*, we encounter an apparently new source of non-classicality: *non-local correlations*. But non-locality can in fact be understood as *a more realistic physical embodiment of the above principle that "epistemic actions have ontic side-effects"*. Indeed, in the setting of compound systems, the "external observer" (until now only a purely informational, meta-physical entity) is reintegrated in the physical reality: the global system S splits into an "observer subsystem" S_1 and an "observed subsystem" S_2. Given our *spatial interpretation of "knowledge"*, the "epistemic state" comprises all the information that is potentially available at location 1: the "information state" (i.e. the $\simeq_1$-equivalence class representing the "local state") of S_1. An "epistemic action" is a learning action (i.e. a *test*) that changes this epistemic state: a *local test on S_1*. Its "ontic effect" is the change induced in the "observed" subsystem S_2. "Non-locality" is then just another name for the ontic impact of epistemic actions!

Our logical formalism offers us more insight into this phenomenon. Consider such a compound system S (composed of two subsystems S_1 and S_2). A test performed on S is *local* if it can yield only "local information", i.e. its possible results provide information only about one of the subsystems, say S_1. In our formalism in the previous section, we represented such measurements as tests $\varphi_1?$ of 1-local properties (i.e. properties of S_1). So in this case, the action can be thought of as a measurement of subsystem S_1. But this is just a way of speaking, describing the *informational* content of the action; in reality, *every action should always be thought of as being performed on the global system*[9] S. As in general, this epistemic action may have ontic side-effects, changing the state of S. But there is no reason to expect this change itself to be "local", i.e. to affect only the S_1-part of the system! Indeed, there does not have to be any simple, direct relation between the informational content of an epistemic action and its ontic effect: even if the first is "local", the second may as well be non-local. The state s of system S may happen to be such that there simply is no way to extract information about S_1 without changing S_2. If this is the case, we call the state *entangled*. This, in a nutshell, is our formal definition of entanglement, as encoded in the inequation from the previous section:

$$s \not\simeq_2 \varphi_1?(s)\,.$$

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[9]In this view, the global system is the *only* object of observation or manipulation: *all* actions are always actions performed on the whole universe! One could argue that, at a conceptual level, there still is a potential tension between this view and Relativity Theory. But note that this view does not entail in any way instant (or super-luminal) flow of information between subsystems!

A consequence of this definition is that, whenever the global system is in an entangled state, we cannot really talk about "the ontic state" of subsystem S_1: this is not a fully determined, observationally meaningful concept, since there is no way to observe the "state" of S_1 in isolation from S_2. One can of course perform local measurements on S_1, but the information gathered in this way cannot be thought of as defining a "local state", in the ontic sense: we already saw that, ontically speaking, these measurements are not "local" actions at all, since they act in fact on the whole system S. In QM, this is expressed by the fact that only a separated system can be in a "pure state". In our interpretation, *the only ontic states are the pure states.* So an entangled subsystem S_i has *no ontic state*! In a sense, S_i is not yet an actually existing subsystem, but only a *virtual* one: it is just a location i at which local tests can be performed, resulting in an actual subsystem S_i.

However, we can always meaningfully talk about the "state" of S_i in a *weak, informational, "purely epistemic" sense*: from an informational point of view, local measurements at location i are indeed "local", since they only provide local information (i.e. about S_i). The sum of all the information that could in principle be gained by such "informationally-local" actions at i forms the "information potential" at location i: this can be said to define a weak, purely epistemic notion of "state" of the (virtual) subsystem S_i, represented in our formalism as the $\simeq_i$-equivalence class of the global state s. This notion encodes *all the information that subsystem S_i has about the global system S*: so, in a sense, this is the "information state" of S_i, comprising all that can be "known" (by tests performable) at location i about the ontic state of the global system.

We can compare this "qualitative" representation of the state of an entangled subsystem with the more "quantitative" representation used in Quantum Mechanics. The standard formalism of QM represents the "local state" of an entangled subsystem S_i as a *"mixed state"*, i.e. a density operator obtained by taking the partial trace of (the operator representing) the global state of S. One can easily check that, in fact, these two representations are *equivalent*: the mixed states of subsystem S_i and the $\simeq_i$-equivalence classes on S are just two ways to package the same information. So our representation can be understood as giving *an "informational" interpretation of mixed states as "epistemic" states*[10], comprising *the limited information that an entangled subsystem has about the state of the global system.*

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[10]We stress again that we take the word "epistemic" in the purely "external" or metaphorical sense used in Computer Science (as explained above). This is not the "subjective" sense of "knowledge actually possessed by an actual observer", but it is about the information that is *potentially available at a given location i*: what "could be known" at i.

Note the subtle difference between this view and the standard "ignorance interpretation", which understands the mixed state of a (sub)system S_1 as expressing an external observer's ignorance about the real ontic state of *the same* (sub)system S_1 (assumed to always exist as an actual entity). If consistently pursued, the "ignorance" view is a "no collapse" interpretation, implying that entangled subsystems are always in some (pure) ontic state, even before any measurement! In contrast, our interpretation takes at face value the standard QM picture, in which entangled subsystems are not in any well-defined (pure) ontic state, before they are "collapsed" into one by a local measurement. Nevertheless, there are some similarities between the two views: in our interpretation, the mixed state of subsystem S_1 expresses the "objective ignorance" of S_1 (seen itself as a "potential observer", or rather a "location at which potential observations can be made") about the real ontic state *of the global system S* (or equivalently, of the "observed" subsystem S_2). This "ignorance" is in fact "objectively imperfect information": even in principle, there is only a *limited* amount of information about S that can be gained by doing local tests at location 1. The differences between the two interpretations become striking when applied to *pure* states. In the "ignorance" view, a pure state of subsystem S_1 correspondents to "perfect knowledge" (maximum information): the external observer knows precisely the true ontic state of S_1. But in our view, a pure state of S_1 corresponds to "minimal knowledge"! Indeed, S_1 is in a pure state when it is separated from S_2, in which case S_1 (as a virtual "observer") has *no non-trivial information* about S_2: for all that S_1 "knows", S_2 can be in *any* separated state.

Another interesting property of compound quantum systems, logically independent of the above definition of entanglement, has to do with what one might call *global determinism*, according to which "the (state of) the whole determines (the state of) its parts". Indeed, in classical physics, the ontic state of the global system S completely determines the ontic states of its subsystems S_1 and S_2. This cannot happen in Quantum Mechanics, since as we saw entangled subsystems cannot be said to be in a definite ontic state. But a remarkable feature of QM is that one of the ontic side-effects of a local measurement on a subsystem S_1 is to *separate* it from its environment S_2. So, *after a local measurement*, it is meaningful to talk about the ontic states of both subsystems S_1 and S_2. Moreover, *a modified form of global determinism still applies*: if we are given the ontic state of the compound system S *and* the result of a local measurement of the first subsystem S_1, this is enough to completely determine the ontic state of the second subsystem S_2 *after* the measurement. In other words, *the way that any local observation of S_1 changes its environment S_2 is completely encoded in the original state*

of the global system S. The correlation is not an arbitrary function, but is one that *encodes a deterministic quantum program* (i.e. a unitary evolution). This is captured in the equation from the previous section:

$$p_1?(s) \simeq_2 \pi_s(p_1)$$

In fact, the state of an entangled system *is nothing but* an encoding of such a correlation. This is captured in the reverse correspondence, associating to every deterministic program π on 1-local states some global state $\overline{\pi}$ that is "entangled according to π", i.e. it satisfies the other equation above:

$$p_1?(\overline{\pi}) \simeq_2 \pi(p_1)$$

The "essence of entanglement", from an informational and computational perspective, can thus be summarized in a Motto inspired from [1, 19]: *entangled "states" are nothing but "static" encodings of quantum "programs".* More precisely, entangled states *encode virtual information-processing actions* (i.e. potential computations), *via correlations between* (the possible results of) *remote information-gathering actions.* The existence, for *every* deterministic quantum program π on local states, of states $\overline{\pi}$ entangled according to π explains the *power of entanglement, as a computational resource.* As pointed out in [1, 19], the encoding of π into $\overline{\pi}$ is a form of "Quantum Currying", showing that entanglement can in principle form by itself the basis for universal computation. This important feature of quantum information flow is sometimes "explained" by saying that quantum information appears occasionally to flow "backwards in time". We think our formulation above gives a better explanation for this apparent backwards flow of information: entanglement captures *potential properties* (or computations), by encoding the possible effects that future local measurements (on one subsystem) may have on the total system (and thus on other subsystems).

5. Conclusions

In this paper we argued that Dynamic Epistemic Logic can help our understanding of quantum physics. Our main thesis is that the "non-classicality" of QM is only due to the *non-classical "dynamics" of information* in the quantum world, and thus *it does not require any change of the classical logical laws governing "static" information.*

We highlight five important conclusions of our analysis: (1) The *importance of logical dynamics* (including the potential dynamics encoded in

weakest precondition formulas), which reinforces the ideas underlying the current trend towards the "dynamification" of logic. (2) The possibility of a *non-classical logical dynamics*: all the non-classical features of Quantum Logic are captured in our setting as dynamic features, which do not affect the classical laws of ("static") propositional logic. (3) The non-classical nature of quantum logical dynamics is also reflected in the *erosion of the sharp classical separation between "ontic" and "epistemic"*: in the quantum world, information-gathering actions (such as the quantum tests) are always *ontic* actions as well, affecting the observed system. Hence, *there can be no information change without changing the world*. (4) This ontic impact of quantum epistemic actions also leads to *non-classical epistemic effects*, more precisely to the *non-monotonicity* of "learning-through-measurements": unlike a classical test, a quantum test may "erase" (by rendering obsolete) previously acquired information. The resulting "uncertainty" is an objective and necessary effect of quantum information flow, hence the phrase *"objectively imperfect information"*. (5) Another feature of quantum epistemic actions is that *their ontic impact may be non-local*: information-gathering actions performed on one part of a large system may have ontic side-effects on other, far-away parts of the system. This is the "essence" of entanglement, reduced to its abstract dynamic-logical formulation.

Finally, we stress that, while quantum tests do share some formal features with purely epistemic/doxastic actions in Belief Revision theory, we are definitely not trying to explain quantum behavior as a purely epistemic or doxastic artifact. On the contrary, our analysis does start from a realistic stance. In particular, we distinguish our account from the so-called "epistemic view on quantum states" [36], according to which "quantum states are states of incomplete knowledge rather than states of reality". Our view, rooted in the modal approach to epistemic logic, is the opposite of such statements: the distinction between the "real", actual state (the point at which modal formulas are evaluated) and any other "possible" (epistemic or potential, past or future) states lies at the very basis of the possible-world semantics for modal logic. Moreover, the above-mentioned five "conclusions" can be used to reinforce this distinction and to clarify the sense in which a quantum world is "non-classical": essentially, the non-classicality lies in the tight interplay between the "epistemic" (or 'close-by") and the "ontic" (or "far-away") effects of our information-gathering actions. In the classical worlds, these effects could in principle be separated, so that one could have "purely epistemic" or "absolutely local" actions (with no ontic and no remote side-effects). In the quantum world, such a sharp separation is in general impossible, and this gives rise to a different dynamics of information,

one dominated by non-monotonicity, non-locality and "incompatibility" of tests. But this tight interplay between epistemic (close-by) and ontic (far-away) features *presupposes the distinction* between the two. Denying, or just neglecting this distinction (as e.g. in traditional Belief Revision theory, or in the above-mentioned "epistemic view on quantum states") would simply erase all the significant differences between classical and quantum information flow.

In contrast, Dynamic Epistemic Logic provides a comprehensive, unifying perspective, allowing us to compare various forms of epistemic actions across many different settings, and to locate quantum-informational dynamics in the wider landscape of logical dynamics. The DEL formalism gives us a powerful, elegant, transparent way to capture both the "non-classicality" of Quantum Mechanics and its compatibility with classical logic, to investigate its Foundations at a qualitative, conceptual level, and to begin to understand its amazing Computational promises.

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