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Peirce's Contributions to Possible-Worlds Semantics

Abstract. A century ago, Charles S. Peirce proposed a logical approach to modalities that came close to possible-worlds semantics. This paper investigates his views on modalities through his diagrammatic logic of Existential Graphs (EGs). The contribution of the GAMMA part of EGs to the study of modalities is examined. Some ramifications of Peirce's remarks are presented and placed into a contemporary perspective. An appendix is included that provides a transcription with commentary of Peirce's unpublished manuscript on modality from 1901.

Keywords: Peirce, modality, possible-worlds semantics, Existential Graphs.

1. Introduction

A century ago, Charles S. Peirce (1839–1914) proposed a logical approach to modalities that came close to possible-worlds semantics. He also contributed to the development of the possible-worlds idea by investigating modalities through the creation of new logical methods.

The plan of the paper is as follows. Section 1 investigates his early views on modality. Section 2 addresses his diagrammatic logic of Existential Graphs (EGs) and evaluates the contribution of its GAMMA part to the logical study of modalities. Section 3 discusses some ramifications of Peirce's remarks and places them into a contemporary perspective. Appendix provides a transcription with commentary of Peirce's unpublished manuscript on modality from 1901. A different version of that manuscript was published in James M. Baldwin's *Dictionary of Philosophy and Psychology* in 1901.

2. Early Views (1867–): Informed, Essential, and Substantial Breadth and Depth

Peirce was concerned with modalities at a very early stage. In 1867, he drew a trichotomy between *informed*, *essential*, and *substantial* breadth and depth

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of signs. He took the act of predication, namely the joining of the object to its subject to be a way of increasing the logical breadth of a sign without diminishing its logical depth. Accordingly, his notions of logical breadth and depth are related to denotation and connotation of logical terms.

> Sir William Hamilton has borrowed from certain late Greek writers the terms breadth and depth, for extension and comprehension respectively. ... "wide" learning is, in ordinary parlance, learning of many things; "deep" learning, much knowledge of some things. I shall, therefore, give the preference to these terms. Extension is also called sphere and circuit; and comprehension, matter and content. (2.394, 1867, *Speculative Grammar: Terms*).[1]

Peirce went on to categorise breadth and depth as informed, essential and substantial. The *informed breadth* of a term means "all the real things of which it is predicable, with logical truth on the whole in a supposed state of information" (2.407, *ibid.*), while the *informed depth* means "all the real characters which can be predicated of it (with logical truth, on the whole) in a supposed state of information" (2.408, *ibid.*).

The *ground* of the object in Peirce's early career referred to the connotation of a symbolic sign, the second in his three-way notion of reference. The first is the direct reference of a symbol to its objects, namely the *denotation* of the symbol, the second, *connotation*, is the reference of the symbol to the common characters, the ground, of its object, and the third is the reference of the symbol to its interpretants through its object, which Peirce termed the *information* of the symbol.

The symbol's direct reference to its object is an example of informed breadth, and its reference to the ground of the object is an example of informed depth. Whatever reference there is to its interpretant is the information concerning the symbol (2.418, *ibid.*). Later, Peirce subsumed the notion of the ground under interpretants.

The informed breadth and depth of a term lie between the two extremes of the states of information of which no fact is known and of which there is perfect knowledge of all there is. Hence two other states of information exist corresponding to these extremities. First, Peirce distinguishes the *essential depth* of a term, by which he means "the really conceivable qualities predicated of it in its definition" (2.410, *ibid.*). The second is the *substantial breadth* of the term, which is "the aggregate of real substances of which alone a term is predicable with absolute truth". The *substantial depth*, in turn, "is the real concrete form which belongs to everything of which a term is predicable with absolute truth" (2.414, *ibid.*). Furthermore, "the depth, like

[1]The reference is to [28] by volume and paragraph number, followed, in the first instance, by year and title of the text.

the breadth, may be certain or doubtful, actual or potential, and there is a comprehensive distinctness corresponding to extensive distinctness" (2.409, *ibid.*). For completeness, the *essential breadth* is all the objects the sign refers to by virtue of the definition of the object offered through the interpretant.

The three aspects of breadth and the three aspects of depth were not differentiated from each other by the distinction between supposed vs. imaginary states of information. One might expect this kind of division in view of the idea that matured only much later, namely that of intensions, or modal statements, as involving multiplicities of different 'possible worlds' in which statements are evaluated. The possible-worlds approach to intensions, while anticipated by medieval writers [15], is routinely taken to have acquired its formal outfit via the inception of the relational theory of *possible-worlds semantics* in the 1950s and the early 1960s.[2]

In his later philosophy, as Peirce moved away from "one-world" pragmatism to "many-world" pragmaticism and scholastic realism, he referred to a "perfect state of information", as it was phrased in MS 664 [November 1910, *The Rationale of Reasoning*], as sets of possible worlds according to which all worlds are linked by equivalence relations.[3] The "perfect state" explains what it means for the representation of all the characters involved in the uttered word to be one that involves no ignorance.

Accordingly, whilst Peirce considered breadth and depth in his early writings merely in terms of an insufficient dichotomy, the dependence of signs on the dynamics of concepts, states of knowledge and information later called for more extensive categorisations of signs, also taking objects and interpretants into account. What is more, he came to study logic, including modal logic, in terms of the diagrammatic approach. In that system, the accessibility relation materialised in 1903.

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[2] A. Bayart, Marcel Guillaume, Jaakko Hintikka, Stig Kanger, George Kelly, Saul Kripke, Carew A. Meredith and Arthur Prior, Richard Montague and D. Kalish were among those who came up with similar and overlapping ideas of a referential multiplicity, accessibility or alternativeness relation between worlds [1, 5, 7, 8, 12, 13, 14, 17, 23, 24]. Let us nore that the role of Kelly's work on the geometry of psychology in this cluster of early works is yet to be documented. He assumes 'contingent schemata', which begins with beliefs about the way the world is, and projecting onto the set of beliefs a conceptual framework for dividing up a cognitive space. His 'construct' could be viewed as the actual world and 'dichotomy' and 'poles' as valuations. Kelly used the 'range of convenience', not as a relation in the mathematical sense, but as a version of accessibility in the modal sense. Jonsson & Tarski [11] used a binary relation, but for a different purpose. See Copeland [3] and Lindström [19] on the history of the development of possible-worlds semantics.

[3] The reference MS is to [29] by manuscript and, if applicable, page number, followed by year and title.

3. Later Views (1895–): Existential Graphs and Modalities

Peirce was a visual interpreter of language:

> I do not think I ever reflect in words: I employ visual diagrams, firstly, because this way of thinking is my natural language of self-communion, and secondly, because I am convinced that it is the best system for the purpose. (MS 619, 1909, *Studies in Meaning*).
>
> Logic may be defined as the science of the laws of the stable establishment of beliefs. Then, exact logic will be that doctrine of the conditions of establishment of stable belief which rests upon perfectly undoubted observations and upon mathematical, that is, upon diagrammatical, or, iconic, thought. (3.429, 1896, *The Regenerated Logic*).

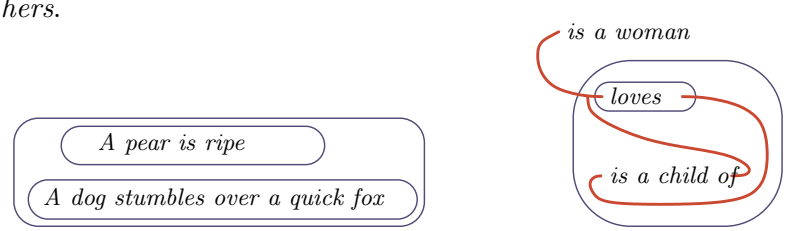
The exact iconic logic was his Existential Graphs (EGs). He developed its ALPHA, BETA and GAMMA parts. The theory of the ALPHA part is isomorphic to the theory of propositional logic. With BETA, Peirce devised a first-order ('first-intentional') diagrammatic counterpart for his 1885 algebra of logic.

3.1. Alpha and Beta Graphs

The ALPHA part of EGs consists of (i) the *sheet of assertion* (SA) on which graph-instances are scribed, (ii) *juxtapositions*, which are placements of graph-instances on the same SA, and (iii) *cuts*, which are thin, simple, non-overlapping closed curves enclosing graph-instances. Its extension, the BETA system, has in addition (iv) *lines of identities* (LI), which are either dots or thick continuous lines composed of a set of contiguous dots, attached to invisible hooks on the peripheries of *spots*, which are bounded regions of the surface of the SA (predicate terms).

Think of an SA as an interpreted structure. If nothing is scribed on an SA, it represents tautology. Juxtaposition is an iconic analogue to Boolean conjunction. A cut is an iconic analogue to Boolean negation. An LI is a diagrammatic analogue to existential quantification, identity and predication.

Below left is an ALPHA graph for *A pear is ripe or a dog stumbles a quick fox*. Below right a BETA graph is scribed for *Some woman loves all children of hers*.



For details concerning the theory of EGs, see [31, 35, 36].

3.2. From Alpha and Beta to Gamma Graphs

3.2.1. Broken Cut

Not nearly all can be expressed by BETA, such as *Aristotle has all the virtues of a philosopher*, because of quantification over properties. Nor can the assertion: *A certain institution will pay every dollar it has borrowed or shall borrow with a borrowed dollar; and the payment of a dollar cannot balance debts of more than one dollar. Nonetheless, there will be some dollars borrowed that never will be repaid.* According to Peirce, the modal words *will* and *shall* "consists in a predicted endless future that never can become a positive fact" (MS 462: 6, 1903, *Lowell Lectures of 1903. 2nd Draught of 3rd Lecture*). Moreover, with BETA graphs only one is "unable to reason about abstractions. It cannot reason for example about qualities nor about relations as subjects to be reasoned about. It cannot reason about ideas" (MS 467: 4, 1903, *Lowell Lectures of 1903. Lecture 4*).

The diagrammatic theory of modalities used a *broken cut* (1903), which scribed around a graph φ denies the necessity of that graph, in other words, is read as *possibly, not* φ.

possibly, not φ — necessarily φ — possibly φ — possible that necessarily φ

These graphs correspond, respectively, to the following expressions of symbolic modal logic: $\Diamond\neg\varphi$, $\neg\Diamond\neg\varphi := \Box\varphi$, $\Diamond\neg\neg\varphi := \Diamond\varphi$ and $\Diamond\neg\Diamond\neg\varphi := \Diamond\Box\varphi$.

3.2.2. Proofs

Proofs are by way of transformations on graphs. For ALPHA they are: (i) *Erasure/Addition of Double Cuts*; (ii) *Insertion*: Any graph may be *inserted* on the *negative area* of the cut (negative = is included within an odd number of cuts); (iii) *Erasure*: Any graph may be *erased* from the *positive* area of the cut (positive = is included within zero or even number of cuts); (iv) *Iteration/Deiteration*: Any copy of a graph may be added to/deleted from the same area or to/from the area enclosed within that area.

The following two transformation rules go with the broken cut:

1. *Opening the Cuts*: Any continuous cut on a positive area may be opened into a broken cut.

2. *Closing the Cuts*: Any broken cut on a negative area may be closed into a continuous cut.

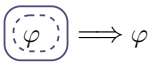
Broken cuts are counted along with continuous cuts to determine the polarity of the area.

We may rewrite these as rules that state what may be erased from and what may be inserted on positive and negative areas:

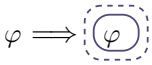
1. *Positive areas*: one may erase  and insert  .

2. *Negative areas*: one may erase  and insert  and  .

Erasure from a positive area corresponds to the elimination of necessity:



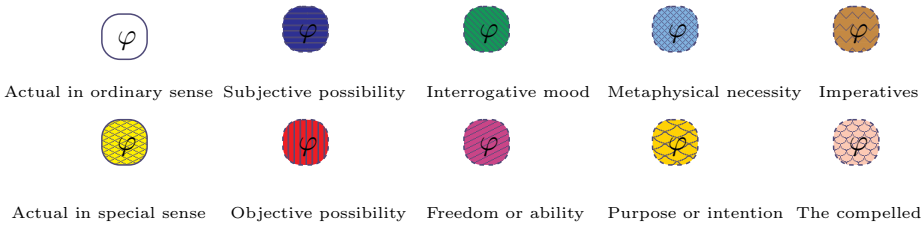
Insertion to a positive area corresponds to the introduction of possibility:



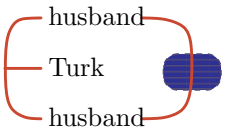
By changing the permissions of which graphs, if any, may be iterated/deiterated across broken cuts gives rise to different modal systems [36]. In MS 478: 158 [1903, *Syllabus. Archegetic Rules of Transformation*] Peirce noted that "The Rule of Iteration and Deiteration does not apply to the broken cut", and so the modal system that he had in mind was weaker than S5.

3.2.3. Tinctures

After a few years, Peirce extended the idea of broken cuts to 'tinctured' EGs. The aim was to capture different kinds of modalities by virtue of the fact that the *verso* of the SA on the area of the broken cut represents possibility, while the reversal of the SA's *verso*, namely the *recto*, represents actuality. Among the possible tinctures are:



The following is Peirce's example of a tinctured graph for *There is a Turk who is the husband of two different persons*:



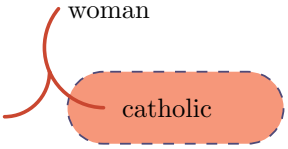
The tinctured area captures that it would be contrary to what is known by the one who scribes the graphs that the two individuals are identical. In other words, the graph may be read such that, *As far as is known, a Turk exists who is the husband of two different persons.*

With tinctures, Peirce anticipated not only alethic modal logic, but also erotetic logic, deontic logic, belief-desire-intention logic, and the logic of imperatives.

3.2.4. Quantification

As implied by the preceding example, by extending the modal graphs with identity lines, Peirce faced the issue of connecting actual individuals on the *recto* with the possibile individuals on the *verso* [32] of an SA.

For instance, the assertions *something is a woman* and *something is other than any possible catholic* are composed as follows:



The result is a GAMMA graph for *There is a woman who is not and could not be identical with any possible catholic.* In symbols, $\exists x(W(x) \wedge \forall y(\Diamond C(y) \rightarrow x \neq y)$.

3.3. Accessibility and Extensions

A moment's reflection on Peirce's tinctured EGs reveals how they are interpreted in terms of possible-worlds semantics: by varying the tinctures instead of the books of sheets we get other modality types and classes of accessibility relations. Broken cuts overturn the enclosed area; not to negate the assertion but to expose other SAs to view.

Peirce's EGs thus marked the beginning of not only the development of modern possible-worlds semantics for propositional logic, but also for multimodal and quantified modal logics [10].

The GAMMA part played a key role in anticipating, and to some extent even in contributing to, the later development of the semantics of modal

notions. To begin with, some remarks suggest that something like an accessibility relation between "states of information" (4.517, 1903, *The Gamma Part of Existential Graphs*) was what Peirce had in mind. For what he needed was a differentiation between states to relativise possibility and necessity to "the time of learning that graph be true" (4.518, *ibid*).

Sometimes, he referred to such relations as "selectives", drawn as short lines between the states. Sometimes these relations were even crossed to designate a particular state. (4.518, *ibid.*; MS 467). Roberts [35] and Zeman [36] were the first to note the use of a special sign Peirce had in connecting states of information (special spots or predicates that have been abstracted from ordinary spots) with an arrow-like pointer or relation between two states:

Peirce's intended meaning with this relation was that one state of information (B) follows another (A). A preferred interpretation of such a precedence notation is as an accessibility relation.

Peirce also studied *epistemic* aspects of diagrams semantically. Often, a modal proposition is "about the universe of facts that one is in a state of information sufficient to know" (4.520, 1903, *The Gamma Part of Existential Graphs*; cf. [9]). He recognised that propositions have to be evaluated against *conceivable* states of information: "Suppose, however, we wish to assert that there is a conceivable state of information of which it would not be true that, in that state, the knower would not be in condition to know that g is true" (4.520). As noted, he resorted to a special cross mark sign in his GAMMA graphs to distinguish a particular state of information from the one to which it refers.

The selectives to which these marks refer have, Peirce remarked, "the additional peculiarity of having a definite order of succession", and are thus "of great use in cleaning up the confused doctrine of *modal propositions* as well as the subject of logical breath and depth" (5.518, 1903, *Consequences of Critical Common-Sensism*). He was thus thinking about logical depth in terms of modal depth of nested modalities. In EGs this means the succession of the states of information by means of the cross marking.

About the nesting of knowledge, or a version of the KK-thesis (that knowing entails knowing that one knows) Peirce noted that there are "peculiar and interesting little rules, owing to the fact that what one knows, one has the means of knowing that one knows" (4.521, 1903). He refuted the straightforward rule that "whatever one knows, one knows that one knows, which is manifestly false" (4.521). It is immediately after these remarks —

probably the only place in his writings — that he introduced the arrow-like notation signifying the fact that one state of information follows another.

Furthermore, the definite order of succession between states of information bears resemblance to indexical notions of time and location. Nowadays, such notions are customarily couched in two- or multidimensional modal semantics.

Originally developed in the BETA part of EGs, identity in GAMMA is a continuous line of connection or a fracture between spots, and as such anticipated the later *cross-world identification* in the semantics for modal predicate logic.

For Peirce, the notion of identification means that the interpreter has to meet with a singular or proper name many times in several contexts, or else he fails to be fully acquainted with it. The first round of interpretation connected with a name starts with a selective (the outermost occurrence of the name), which is then presented to the interpreter repeatedly, on different occasions and in different contexts (4.568, 1905, *Prolegomena to an Apology for Pragmaticism*).

In the light of the evidence aduced, it is justified to conclude that the GAMMA part comprised many of the key elements of modern possible-worlds semantics. It should also be noted that Peirce also referred to the projected DELTA part. It is not known what it was to be about, but in all probability it was meant to deal with modal predicate logic and to repair sopme of the shortcomings of GAMMA. "The better exposition of 1903 divided the system into three parts, distinguished as the Alpha, the Beta, and the Gamma, parts; a division I shall here adhere to, although I shall now have to add a *Delta* part in order to deal with modals" (MS 500: 3, 1911, *A letter to Allan Douglas Risteen, 'A Diagrammatic Syntax'*, 19 pages preserved).

In places he also mentions the possibility of extending the graphical system beyond assertions, lamenting that he came to confine his sheets to representations of assertions, and suggesting that other sheets would do as well, including non-declarative moods such as interrogatives and imperatives using tinctures, and even feelings and emotions (MS 500).

A noteworthy codicil to EGs, not clearly either BETA or GAMMA, is found in *Logic Notebook* (MS 339: 340r, 1909). Here, Peirce reads various diagrams that look like BETA graphs as "p is true under some circumstances", "p is true under all circumstances", "p is true some times and q is true some times", "p and q are sometimes true", "p is true under some circumstances and q under others" and so on. The line of identity is taken to quantify various circumstances or points of time, not just individuals. This is yet another anticipation of possible-worlds semantics for expressing various modalities,

including temporal ones. Despite its curious title, in one two-page (341r) and two one-page sketches (342r–343r, 1909) entitled *Studies of Modal, Temporal, and Other Logical Forms which relate to Special Universes*, Peirce took no further reflection on temporal aspects of graphs. Peter Øhrstrøm [27] has studied elements of temporal logic in GAMMA graphs.

4. Diagrammatic Modalities and States of Affairs

Gamma graphs amounted to a gamut of ideas. While one compartment comprised quantified modal logics, the other exhibited elements of higher-order type-theoretic logics. Also, by using the metalogical principle of abstraction Peirce reasoned about graphs using graphs (MS 464: 28), thus capturing the notion of "graphs of graphs" [35].[4] He also analysed collections and non-declaratives by them.

The modal character of GAMMA was nevertheless dominant, since "the gamma part of the system deals with what can logically be asserted of meanings" (MS 462: 34). The concept of meaning that he held until 1903 was the "one-world" application of the pragmatic maxim. Thereafter, he signalled interest in what was to be the logical counterpart of meaning: *intensions*.

The logical approach to modalities was one of Peirce's central concerns. He wanted to supersede the mere parallelisms of universality versus necessity and particularity versus possibility, and the idea of modality as involving simultaneous possibilities. He attributed the impetus for the study of GAMMA to Mitchell's *New Algebra of Logic* for rendering the scholastic concepts of modals exact by introducing the technique of the multidimensional logical universe (see Appendix).[5] Peirce perceived the key aspects to do with the affinity of modalities and quantification to be present in Mitchell's contributions to the logic of quantification.

It remains unclear whether Peirce succeeded in implementing his idea of modality based on Mitchell's concept of a multidimensional logical universe in terms of GAMMA graphs in a satisfying manner. As far as his conceptual repertoire was concerned, these graphs did not require any radically new kinds of signs beyond those involved in the ALPHA and BETA parts, although such signs took some new forms, such as the broken cut in addition to the

[4]It is only a minor step from the notion of hypostatic abstraction, which treated relations and predicates themselves as diagrammatic objects of study, to the inception of category theory in mathematics. The germs of a vast conceptual arsenal were already there during Peirce's lifetime.

[5]MS 467, paragraphs marked "omit" by the editors of the *Collected Papers*, vol. 4; cf. MS 1147: 1–2.

continuous cut (MS 467: 20). What was required was the replacement of the sheet of assertion by "a book of separate sheets, tacked together at points" (MS 467: 22). In the same manuscript Peirce remarked on the necessity for a relational arrow-like symbol to express relations between the sheets of assertions via abstraction (MS 467: 58, 60).

Accessibility was the key to the semantics of modal notions, as recognised by a number of inventors of possible-worlds semantics during the 1950s and the early 1960s. The idea was not implemented in GAMMA graphs in full, but Peirce fares much better verbally. The multiple logical dimensions involve aggregates of hypotheses, the false ones being those that a supposed state of information does not exclude. The whole range of possibility then measures the amount of ignorance in the given state (MS 1147: 3).

The arrow-like relational symbol introduced in 1903 was not linked with remarks about modality such as the one that Peirce provided for Baldwin's dictionary in 1901 (see Appendix). Moreover, as the arrow relation is *iconic*, his animadversions concerning the lack of iconicity in modalities appear puzzling: "If that be the case, Modality is not, properly speaking, conceivable at all, but the difference, for example, between possibility and actuality is only recognizable much in the same way as we recognize the difference between a dream and waking experience" (4.553ff, c.1906, *Phaneroscopy*). These animadversions are just the tip of the iceberg in Peirce's attempts to navigate the modal deep.

There are many junctures at which modal concepts forced Peirce to fall back on the book-of-sheets type of analysis. One of them is in relation to *hypotheticals*:

> In a paper which I published in 1880, I gave an imperfect account of the algebra of the copula. I there expressly mentioned the necessity of quantifying the possible case to which a conditional or independential proposition refers. But having at that time no familiarity with the signs of quantification, the algebra of which I developed later, the bulk of the chapter treated of simple consequences *de inesse*. Professor Schröder accepts this first essay as a satisfactory treatment of hypotheticals; and assumes, quite contrary to my doctrine, that the possible cases considered in hypotheticals have no multitudinous universe. This takes away from hypotheticals their most characteristic feature. It is the sole foundation of his section 45, in which he notes various points of contrast, between hypotheticals and categoricals. According to this, hypotheticals are distinguished from categoricals in being more rudimentary and simple assertions; while the usual doctrine of those who maintain that there is a difference between the two forms of assertion is quite the reverse. (2.349, 1895, *Speculative Grammar: Propositions*).

Quantifying the possible cases is a most astute reflection of the idea that there is a universe with a set of possible worlds (states of affairs). In Peirce's

opinion, Ernst Schröder did not advocate this way of viewing hypotheticals, which Peirce thought necessary in order to be able to deal with modalities. He was quite clear in this passage in taking the possible cases to refer to the multitudinous universe, the universe consisting of several possible states of affairs or information.

There is plenty of further evidence that Peirce supported a fairly advanced outlook on modal semantics, especially in relation to expositions that go beyond the idea that necessity and possibility move on the same plane as universality and particularity. He made frequent use of concepts such as 'states of ignorance', 'range of possibility', 'hypothetical states of universe', 'indistinguishable propositions', and 'conceivable states of ignorance'. These bore the traces of a much more modern semantics for modalities:

> The propositions analogous to A are all those propositions which in some conceivable state of ignorance would be indistinguishable from A. Error is to be put out of the question; only ignorance is to be considered. This ignorance will consist in its subject being unable to reject certain potentially hypothetical states of the universe, each absolutely determinate in every respect, but all of which are, in fact, false. The aggregate of these unrejected falsities constitute the "range of possibility," or better, "of ignorance." Were there no ignorance, this aggregate would be reduced to zero. The state of knowledge supposed is, in necessary propositions, usually fictitious, in possible propositions more often the actual state of the speaker. The necessary proposition asserts that, in the assumed state of knowledge, there is no case in the whole range of ignorance in which the proposition is false. In this sense it may be said that an impossibility underlies every necessity. The possible proposition asserts that there is a case in which it is true. (2.382, 1901, *Modality*; DPP: 89).

In the same entry he noted that Friedrich Albert Lange (1828–1875) had said (in *Logische Studien*) the nature of modality to be "put in the clearest light by the logical diagrams". This was likely to have been one of the major sources of inspiration for Peirce to embark on studies developing upon the extensional ALPHA and BETA systems.

Peirce had reasons for not pushing the analysis of modalities further by means that would lead to relational modal models, however. For what was one of the most important and most difficult questions for Peirce not only in logic and mathematics, but also in how they interface with metaphysics, was *continuity*. This *synechistic* doctrine that ensued from the rightly-understood mathematical continuity does not seem to apply to the possible-worlds analysis of modal assertions, as the received relational structures are typically discrete. But for Peirce, modalities *ens rationis* were in a continuous connection with actualities.

Hence, following his suggested study of "the continuity of the Universe of Discourse" (CP 4.561) in his diagrammatic logic, similar considerations

would need to be brought to bear on possible-worlds conception of modal assertions. For example, lines of identities are an ingenious case in point of the kind of continuity of individuals that needs to be taken into account *in actu* as well as across multiple states of information. Given the struggles that Peirce had with the topological notion of continuity in diagrams, he must have foreseen the difficulties that were arising.

5. Conclusions

Peirce's explorations on modalities brought him close to instigating possible-worlds semantics and even contributing to its development. His manifold ideas, especially in relation to diagrammatic approach to modalities are of considerable contemporary interest and provide rich sources for further development.[6]

Appendix: Peirce's Entry on 'Modality'

Peirce's struggles with modality came together in his attempt to provide a definition of it for James M. Baldwin's (1861–1934) *Dictionary of Philosophy and Psychology* in 1901. MS 1147 contains a 12-page draft in the midst of numerous other alphabetised draft entries intended for the same dictionary. The final version that appeared was a considerably modified and shortened version of that draft. A likely reason why Peirce made so many changes and omissions is that the editors preferred a survey to the somewhat more original ideas that Peirce first put forward.[7]

[6]Peirce also conceived of the meaning of modalities being liked with *dialogic* or *game-theoretic* activities akin to semantic evaluation of possible-worlds models as a piecemeal exploration of conceivable states of information [6, 31]. Moreover, possible-worlds semantics and *extensive forms of games* are two sides of the same conceptual insight. Extensive-form games were first presented in [26] and soon generalised in [16, 18, 22]. [16] and [22] also contributed to the development of modal logic. Yet another connection is provided by *automata theory*, which has multiple contacts with games. Automaton is a self-operational system that transmits information by state transformations, with relative notions of memory and a differentiation between deterministic and non-deterministic choices. By rewriting an automaton system as a tree we are close to the extensive game built up of states and transformations between states. The automaton itself is a player. Together with Dana Scott, Michael Rabin became the initiator of the theory of automata by formulating non-deterministic finite state machines that entertain choices. The invention was preceded by [33] on game strategies that may be computed by non-human beings. Technically, automata and games connect via monadic second-order logic and with its subfragments, such as modal μ-calculus expressing fixed points and modal temporal logics expressing tenses. These inventions are curiously linked with the contemporaneous discovery of possible-worlds semantics.

[7]The acronym DPP refers to [30], the entry 'Modality' that actually appeared in Baldwin's *Dictionary*.

What follows is a diplomatic transcription of Peirce's draft definition in its original order and composition, interspersed with comments. I will compare this definition not only with Peirce's general views on modalities, but also with some of the more current logical theories of modality. The entry was written after the inception of the ALPHA and BETA graphs (1896–) and slightly before attempts to inject more expressivity through the GAMMA systems from 1903 onwards.

> **Modality**
>
> [*Ger. Modalität*]
>
> The qualification of a predication on the one hand, or of a truth on the other, in respect to possibility and necessity. Although the assertoriness, with its metaphysical correspondent, actuality, is the absence of modality, yet it has to be considered from the point of view of modality.
>
> The doctrine of modality, even more, if possible, than other topics of logic, remains to this day an arena of dispute. The simplest point of view, from which considerable insight is gained in regard to others, consists in regarding the doctrine as a part of prioristic analytic, that is, in considering what distinctions of this nature are needed for the purposes of deductive inference, without any regard either to the usages of language or to methodeutic principles. Here Professor Mitchell's idea of a multidimensional logical universe, which is one of several fecund conceptions contained in his paper *On a New Algebra of Logic (Studies in Logic by Members of the Johns Hopkins University.* Boston: Little, Brown & Co. 1883. p. 72), serves to render [p. 2] the view of common sense, which was partly developed in the scholastic doctrine of modals, exact.

The opening sentence of this entry sets the theme in a clear-cut manner. Peirce was concerned not only with what possible and necessary predicates mean, but also with the meaning of complete statements involving modalities. Even though sympathetic to what was later to become the field of pragmatics, he thought that such semantics had to be general so that it could be established independently of concerns about the use of language or individual methodologies.

Oscar Howard Mitchell (1851–1889) was one of the contributors to the book *Studies in Logic* edited by Peirce. Peirce acknowledges him in several places for making fundamental inventions in logic. Indeed, the development of the logic of quantifiers dates back to Mitchell's exposition in *On a New Algebra of Logic*, although Peirce made several amendments later on, such as to the notion of scope and the order of quantifiers. He also settled the question of what kinds of deductive inferences are allowed in the logic of quantification. The concept of multidimensional logical universes is attributed to Mitchell as one of his major inventions.

Peirce went on to elaborate on this connection.

> A logical universe of two or more dimensions must not be confounded with two or more logical universes. When we consider, in addition to the usual limited universe of individual subjects, also a limited universe of marks, we have two logical universes. That which is contained in the one is not contained in the other. But if, in addition to the universe of subjects, we conceive each of these as enduring through more or less time, so that on the one hand, each subject exists

> through part or all of time, and on the other hand, in each instant of time there exist a part or all of the subjects, we are considering a logical universe of two dimensions, and the same terms have their place in both. The word *dimension* is here applied with perfect propriety; for were we to restrict it to cases in which measurement could be applied, we should be forced to abandon its use in topical geometry, to which no mathematician (and it is a mathematical word) would consent.

One way of looking at the distinction Peirce makes between logical universes in two or more dimensions and two or more logical universes is, in modern terms, that of the distinction between *quantified* modal logic and *many-sorted* logic. In predicate modal logic, possible worlds contain domains that may be independent of other domains, and the question is how the concept of an individual endures in different worlds such as through time, or as objects of knowledge or belief. In many-sorted logic, variables are specified in relation to individuals belonging to some class of individuals, including specifications that delineate between first-order and higher-order objects. The definition of a quantifier does not include all individuals of a totality of a logical universe.

A caveat here is that Peirce was at pains to explicate the possible-worlds conception in full. In places other than this, he comes close to the notion of an accessibility relation between the multitudinous "states of information" in relation to the GAMMA graphs he started to develop soon after DPP appeared (4.522, 1903; MS 467: 58, 60). In the end, he hesitates to connect this with the idea of dimensionality of logical universes. The "limited logical universe of marks" sorting the universe of discourse into two or more 'sorts', 'types' or 'characters', so that the elements of these sorts do not overlap. According to Peirce (2.453ff, 1893), some logicians hold the idea of the limited universe of marks to be "extra-logical", but this is only because it does not fall within the scope of their own studies.

Another "extra-logical" matter during Peirce's era was time, but as this definition indicates, this is seen as serving as the basic inspiration for introducing the concept of dimension to logical universes in the first place. The term 'mark' was later amended to 'qualities', and when the GAMMA part was invented, he noted that qualities gave rise to logical possibility. In this sense, many-sortedness and multi-dimensionality converge. It was only much later that steps were taken to show that modal languages could be viewed as many-sorted ones on relational structures [34]. These steps also facilitated the first representation theorems on Peirce's algebraic logic, although they use methods that go beyond the resources that were available in his time, namely the completeness proof for dynamic modal logic. Neither completeness nor the dynamics of logic were notions articulated in Peirce's writings on relational algebra, although they are implicit in it. Furthermore, the question of whether the notions of quality and modality are of the same nature was a subject of fruitful explorations.

Peirce also makes the observation that the notion of dimension does not imply that the geometry of logical space is metric. If we have dimension, we already have a topological space (topical geometry, topology, topics) that is not subject to measurement. Around 1900 (3.569, *Infinitesimals*), he expressed his displeasure

with the state of topical geometry, as no one had yet developed a logical way of reasoning about it.

> But for the purposes of modality, it is not time which is to be considered, [p. 3] but an aggregate of hypotheses, each completely determinate in respect to every question concerning every object in the universe, but each false, which a supposed (and commonly fictitious) state of information does not exclude. This is called the *whole range of possibility*, which measures the ignorance in the supposed state of information. All the scholastic logics rightly teach that a necessary proposition is one which is in a certain respect universal, while a possible proposition is one which is in a certain respect particular; but they fail to define that respect.

Here Peirce dismisses the use of time for modal purposes. He changed his mind, as in a couple of other entries written at a later date he did conceive of time as certainly not "extra-logical" and, in fact, amenable to analysis by the GAMMA graphs [27]. In its place, however, he introduced two other concepts for the treatment of modality: the (whole) *range of possibility* and the *measure of ignorance* in a given state of information. It was by means of these concepts that he attempted to transgress the boundaries of modalities beyond the mere parallelism of the necessary versus the universal and the possible versus the particular advocated by the scholastics. This is the parallelism that, incidentally, was known not only to the scholastic logicians, but also to the logicians of the late 11th century [15]. The logicians that Peirce probably had in mind were John Duns Scotus and William Ockham. In the entry that actually appeared, he held that the parallelism account of modality by the scholastics could be considered the simplest account.

> If, disregarding the usages of language, we define a universal proposition as one which ~~possesses the qualifications requisite for~~ [asserts only what it needs to assert in order to serve as] the major premise of a direct syllogism, and a particular proposition as one which precisely denies a universal proposition; then a simple universal proposition will not assert the existence of the subject, but ~~Any~~ 'All A is B' must ~~for~~ [in its] analytic ~~purposes~~ [sense] be equivalent to 'Whatever A there may be is B' or 'An A that is not B does not exist,' while a particular proposition must assert the existence of the subject, and so must assert something which no simple universal [p. 4] proposition asserts.

Here Peirce digresses to criticise the simple dualism between universal and particular propositions: he notes the peculiarity of asserting existence with regard to a particular proposition as a sheer consequence of duality by which a universal proposition expresses the non-existence of exceptions. The purpose of this digression becomes clearer as we proceed.

> A universal proposition is merely a hypothetical proposition in which the object of an indefinite pronoun in the consequent is identified with that of a similar pronoun in the antecedent. Thus, 'If anything is a man, something is mortal' is hypothetical, but 'If anything is a man, that same thing is mortal' is universal. So a particular proposition is merely a copulative proposition to which such an identification is ~~made~~ [added]. Thus, 'There was a patriarch and there was a translated man' is copulative; but 'There was a patriarch, and *he* was translated' is particular. ~~A universal proposition asserts that a certain description of object (as,~~

~~an immortal man) *does not exist* in the universe of logical extension; a particular proposition asserts that a certain description of object (as, a translated patriarch) *does exist* in the universe.~~

The difference between hypotheticals and universals is, according to Peirce, that in the latter, there is a connection between the object applied to the pronoun in the consequent of the proposition and in its antecedent. Likewise, the difference between a copulative and a particular proposition is that in the latter, the connection obtains between the two parts of the conjunction.

> In both these respects, unless we are to be guided by the usages of language, the necessary proposition ought, for analytic purposes, to be understood as analogous to the universal, the compossible to the particular proposition. Thus, ~~'A man without sin must be happy'~~, ['If there is a man in the moon, there must have been a woman'], in order to be best adapted for a logical form, should [p. 5] not be understood as asserting that a man ~~without sin is possible~~ [in the moon is possible], but only that in whatever hypothetic state of the universe the contemplated state of information ~~allows~~ [may allow] in which there should be a man in the moon, in that very ~~state~~ [hypothetic] state there would have been a woman; or in other words in the state of information supposed, ~~it would be known that the~~ the existence of a man in the moon without the previous existence of a woman is absolutely excluded.

Peirce now draws the connection between *necessary* and *universal* on the one hand, and between *possible* and *particular* on the other, in terms of what hypothetical states of the universe are allowed by the contemplated state of information. This is the added element to the treatment of modalities that the scholastic philosophers lacked in the idea of modality as merely analogous to universal and particular quantifiers (propositions). In Peirce's example 'If there is a man in the moon, there must have been a woman', the object of a particular proposition in the antecedent ('a man') exists in the hypothetic state of information, in which case the object of the consequent proposition ('a woman') also exists in that state. (The phrase "previous existence" in the last sentence is slightly mysterious.) The "contemplated state of information" coincides with that of the state of information in which the proposition is evaluated, that is, the actual or designated state. We do not yet find any forthright statement that hypothetic states are to be connected with the contemplated state relationally, however.

> Furthermore, ~~'If a man in the moon is possible'~~ [If there be any allowable hypothetic state in which there is a man in the moon], then there is an allowable hypothetic state in which there had been a woman in the moon' is merely hypothetical; but 'If there be any allowable state in which there is a man in the moon, then in that very hypothetic state there had been a woman in the moon' is a necessary proposition. So 'Foreknowledge is an ~~allowable~~ [admissible] hypothesis, and so is freewill,' is merely copulative, but 'There is an ~~allowable~~ [admissible] hypothesis in which there is foreknowledge, that same hypothesis supposing freewill,' is an assertion of compossibility.

If the hypothetic states of information in which the extant objects applying to the indefinites 'a man' and 'a woman' coincide, we are dealing with a necessary propo-

sition, because there is a link between the two indefinites given by the existence within the same hypothetical state of information. A proposition is merely hypothetical when its translation into modal terminology does not assert the existence of such a link. (The fact that the two hypothetical states of information happen together may be purely coincidental.) Similar remarks apply to the distinction of the copulative versus compossible propositions.

> [p. 6] The reason why the range of possibility is confined to *false* hypothetical states of the universe, neglecting the true one, is that in the first place, the hypothetic states excluded, that is, really known not to exist, ~~the true state~~ never can embrace the one true state of the universe; and in the second place, were "excluded" to be taken in the sense of "believed not to exist," we should be taking into account, not merely *ignorance*, but also *error*. Now the doctrine of ignorance and the doctrine of error ought to be kept distinct, and investigated one at the time. The doctrine of ignorance is the simpler; and in prioristic analytic there is no occasion for considering the supposition of erroneous premisses, further than negation covers the ground.

Here Peirce comes back to the notion of ignorance, and contrasts it with the notion of error. Apart from in this drafted entry, he perceived substantial differences between ignorance and error in other connections. His general claim is that states of information can accommodate ignorance but not error. For instance, in 2.382 [1901], which consists of the part of the contribution to the *Dictionary* that actually appeared, he expounded the difference between 'must' and 'may' in the sentences 'A must be true' and 'A may be true' in terms of propositions analogous to A, so that in the former, the meaning of the modality 'must' in the sentence is that all propositions analogous to A are true, and that in the latter, the meaning of 'may' in the sentence is that some proposition analogous to A is true. The notion of being an analogous proposition is, in turn, explicated in terms of the class of propositions that "in some conceivable state of information would be indistinguishable from A" (2.382, 1901; DPP: 89). The range of information or ignorance so defined dictates when necessary and particular propositions are true. The necessary proposition is true "in the assumed state of knowledge" (2.382) if there is no case in the whole range of ignorance in which the proposition is false. The particular proposition is true if there is a case in the range of ignorance in which the proposition is true. Peirce comes close to the idea of possible-worlds semantics and its indistinguishability relation between states. Once more, necessity is defined as the non-existence of exceptions, and possibility as its duality, in other words that there is a state of information within the whole range of ignorance.

The idea of a set of propositions analogous to a given proposition did have an impact on the history of logic. This impact was instrumental, even though the idea was soon superseded by the fully-fledged development of the relational possible-worlds semantics developed in the 1950s and at the beginning of the 1960s by a number of philosophers and logicians. J. C. C. McKinsey had already used a similar idea to "the range of propositions being analogous to" in his syntactically-driven treatment of modalities [21]. In this work, he defines two sentences as having the same form if one can be transformed into another by substituting new constants

for the non-logical symbols occurring in it. This is accomplished by assuming a given set of substitutions taking sentences of a language into some other sentences of that language. A possible definition is, then, a sentence whose substitution takes it into a true sentence. McKinsey was inspired and influenced by Peirce's writings. Given the nature of semantics in the Peircean sense, namely as the translation of a proposition into another proposition or possibly another language, McKinsey's translation of a modal proposition into its analogues is actually semantic.

Peirce's distinction between syntax and semantics is nonetheless not to be assimilated with the distinction that we are nowadays accustomed to draw. He took syntax to refer in most accounts to the grammatical structure and rules of natural languages. Such rules are not capable of adequately bringing out the logical character of natural-language expressions. He restricted the study of semantics, conceived of as a theory of sign meaning, to the process of translation that takes any sign into another system of signs. For instance, dictionary-type definitions and translations into other languages are semantic constructs. The translation process does not pretend to create an identity between the sign and its semantics, it only approximates its meaning.

McKinsey's work was also a source of stimulation to [2, 4]. These works are among those that link the chain of the development of possible-worlds semantics. In the light of Peirce's influence on subsequent philosophers and logicians concerning the semantics of modality, this chain turns out to be more tightly linked than currently thought.

Three other key terms that Peirce introduced in the previous passage were false hypothetic states of the universe, the notion of the exclusion of states, and negation. A few years later Peirce observed that negation and possibility are similar notions.

> In a necessary proposition, the range of possibility, or ignorance, will not practically be assumed to be less than the speaker's actual ignorance. For though it does [p. 7] not logically follow from what is asserted in the proposition 'A person who should know all that I do and much more beside would know that A is true' that 'I know that A is true,' yet the ~~fact~~ [additional premiss] required to make consequence logical is supplied by the mere fact that the former proposition is asserted.

What he takes up here is the *speaker's ignorance*. The speaker is the agent who asserts the proposition containing modalities, and becomes liable for such assertions. What is interesting is that, in his study of modal propositions, Peirce compares the meaning of a necessary proposition as given by a range of possibility or ignorance with that of the ignorance of the speaker. This idea of the speaker, although long dormant after Peirce, was revived in Arthur N. Prior's system of modal tense logic in the 1950s and 1960s, in which he took a chosen 'date variable' to represent the date on which the proposition under consideration was uttered. Like McKinsey, Prior was deeply inspired by Peirce's philosophy and logic, although he acknowledged him very sparingly in his published writings.

> This point will be made clear by considering that the copulative proposition 'A is true, and I say so' asserts in addition to 'A is true' only a fact that the utterance

> of the latter proposition supplies to the hearer. That the two propositions are not logically identical is shown by the difference between their contradictories, which are, 'A is false' and 'If A is true, I do not say that it is so.'

Peirce makes a note about the information that is added to a performatively-asserted proposition. This added information is enough to render the two propositions 'I say that A is true' and 'A is true' logically distinct. Sentences with performatives do not reduce to sentences stripped of them so as to preserve logical identity. There are no modalities in these sentences, but the point Peirce tried to make was that the speaker has to have sufficient background knowledge, drawn from contextual, collateral and common sources of information, in order to assert sentences that are true, and that involves modalities.

> Thus, necessary propositions practically all involve reference to a range of ignorance greater than the speaker's own or than some contemplated state of knowledge; and the kind of ignorance they assume is one of particular cases, ~~special~~ arbitrary conditions, or special experiences, [p. 8] knowledge of laws of one kind or another being supposed to exist. In other words it is an ignorance of *existences*, such as are stated in particular propositions, with a knowledge of *non-existences* such as are stated in universal propositions.

What is worth noting here, as indeed in the two preceding passages, is the introduction and use of epistemic concepts in order to properly tackle modalities. In particular, Peirce refers to the knowledge that the speaker of propositions involving modal expressions has. Such knowledge naturally goes with the notion of necessary ignorance in unfolding the meaning of necessary propositions. In modern terminology, this could be paraphrased by there being more states accessible from the state in which the proposition is asserted than there are states capturing the speaker's range of ignorance. What is different from this speaker's range is the range of ignorance of particular propositions, which is ignorance of what exists, and also the knowledge that no exceptions exist regarding the meaning of universal propositions. How this ties in with the modal idea of knowledge as the elimination of uncertainty was never completely answered by Peirce, despite the occasional references he made to what could be characterised as an elementary logical investigation of epistemic concepts (4.520). For example, he took what was later termed the KK-thesis to be manifestly false (4.521, *ibid.*). In attempting to explain modalities in terms of a comparison of two different ranges of information (ignorance) — those prompted by the meaning of the modal proposition and those prompted by the speaker's knowledge — Peirce comes close to a logical analysis of epistemic concepts similar to that of modalities.

> A farmer proposes to measure two sides of a triangular field, merely in order to ascertain which is the longer. I tell him that is needless, since the side of adjacent to the smaller angle *must be* the greater. The kind of ~~knowledge that remains~~ [ignorance] determines the kind of necessity. The greater the range of ignorance supposed, the greater is the amount of knowledge the necessary proposition embodies. With the possible proposition it is the other way: the greater the range of ignorance the less the information the possible proposition carries. In practice,

> therefore, even when a possible proposition relating to a greater range of ignorance than his own is all that is relevant, a speaker will, more frequently [p. 9] than not, substitute a statement about what he, in his actual condition, does not know; and thus '*A* may be true,' although, strictly admitting the ~~possibility~~ [case] of A being known to be true, is ~~usually~~ [more commonly] used in the copulative sense of '*A* may be true, and *A* may not be true, for ~~all~~ [ought] I now know.' The contradictory to this proposition is 'I know whether *A* is true or not,' which is seldom said; and indeed, there is no form of possible proposition in ordinary use which precisely denies a form of necessary proposition in ordinary use. When possible propositions do not refer to the speaker's actual ~~state~~ [range] of ignorance, to the expression of which the word *may* is commonly appropriated, they usually refer to a state of information superior to his own, often expressed by 'might', 'might even', or 'still might.' ~~As 'the laws of nature might be what they are; yet still, if only one atom body were known to exist, they could not be verified.' For~~ Example: 'If we could prepare pure chemical bodies, it might [p. 10] still be doubtful whether there was any exact relation between the atomic weights,' or 'Even if ~~it were known that~~ the sum of the angles of a triangle were exactly two right angles at one instant of time, it might not be so at another.' But it is not very often that we are able to say, or have any object in saying, that a definite proposition would not be known to be true in a higher state of knowledge than our own; and were ~~all things known~~ [knowledge perfect] there would, of course, be no ignorance, so that possibility and necessity, in reference to such a state coincide, and are equal empty forms.

Apart from illustrating the relation of knowledge of the agent to necessary and possible propositions, what is conspicuous in the preceding passage is that, despite the initial warnings in the opening sentences of his article, Peirce did not entirely avoid propositions in "ordinary use".

The other example presented here illustrates the difference in meaning between the sentences containing the modality *may* and those containing the modality *might*. The difference is spelled out in terms of the increased state of information exceeding the speaker's own state in the latter types of sentence.

> Usually, a possible proposition expresses a leaning toward belief in the *dictum*; so that if one man says '*A* may be true,' another who considers the hypothesis gratuitous will reply 'Yes, but may be it is false.' If it be said that this remark is not germane to logic, which has nothing to do with mere inclinations, the reply is that the only logical value which any scientific theory has in its first stage, [p. 11] rests upon the hope that out of any large number of similar guesses some finite proportion would be approximately right. Every scientific doctrine has its germ in a pure guess; and science is an idle dream unless man has an instinct for embodying scientific truth in his hypotheses, as a bee has for embodying it in his cell. All my science is nothing but instinct [???]. It is this which ~~gives~~ [entitles] the possible proposition, or question with an inclination, to a place in logic.
>
> When a particular proposition is asserted apodictically, or a universal proposition problematically, there is a distinction between the composite and the divided sense of the modal. The difference in the case of necessary propositions is between asserting that 'In every admissible hypothetic state of the universe some *A or other* is *B*,' which is the composite state, and asserting that 'There is some *A*

> which *same* A is B in all hypothetic states of the universe,' which is the divided sense. In possible propositions, the difference is between ~~saying~~ [asserting] that 'In an admissible hypothetic state of universe every A there may be is B,' which is the composite sense, [p. 12] and asserting that 'Every A there may be is B in some admissible hypothetic state of the universe,' which is the divided sense. It will be seen that the denial of a composite modal is composite, and of a divided modal is divided. In necessary universal and possible particular propositions the distinction between the two senses disappears. [end of the manuscript]

The issue that Peirce takes up in the last paragraph is the distinction between composite and divided senses of the necessary particular proposition and possible universal proposition, which does not apply to the more customary necessary universal propositions or possible particular propositions. The distinction appears in DPP in a revised form, supplemented with the conclusion that "the divided sense asserts more than the composite in necessary particular propositions, and less in possible universals" (DPP: 90). What is interesting is the added qualification in DPP that "in most cases the individuals do not remain identifiable throughout the range of possibility" (DPP: 90). This hints at a modal logic that also takes individuals into account, and addresses the question of what happens when one needs to consider individuals within a range of states of information or possibility. This was one of the main struggles Peirce had with his GAMMA graphs, which he aimed to sort out after having finished writing the articles for the *Dictionary*.

A salient feature missing in the draft entry compared with the one that actually appeared is the added historical review of modality. This review was much longer than Peirce's own explanations, containing an account of the theory of modality put forward by Aristotle, Kant's epistemological and metaphysical renderings, and an examination of German thought as it emerged in Peirce's contemporaries Trendelenburg, Lotze and Sigwart. Peirce's review of their thoughts on modality was lukewarm, and he considers his own logical account of modality to be superior to such informal and inexact musings.

What is also missing in the supplementary review is any mention of Hugh MacColl's (1837–1909) logic of modality, which appeared in a series of papers entitled *The Calculus of Equivalent Statements* in the *Proceedings of the London Mathematical Society* [20]. Peirce did acknowledge MacColl elsewhere, in the part of the entry on *Symbolic Logic* (DPP: 645) that was written by Louis Couturat and Peirce's associate Christine Ladd–Franklin, for his contribution to symbolic logic in terms of propositions that are assigned three truth-values (true, false and undefined). (The rest of the entry appeared under the initials of both Ladd–Franklin and Peirce.)

Because of the shift in focus that the published article finally took due to the addition of historical material, what were perhaps the most interesting aspects of modality around the turn of the 20th century, namely Peirce's contributions to the meaning of modal statements, did not finally appear at all in print (or appeared only in such a condensed form that rendered some of the remarks that remained in the published article almost incomprehensible). For example, the idea of taking the speaker's (or the thinker's) own state of knowledge into account is likely to have left any casual reader of the *Dictionary* bewildered as to why it is needed

at all. Admittedly, the idea is not carried through in the draft version either, but at least an earnest attempt is made to relate it to the information-relative account of modalities in terms of ranges of information and ignorance.

Moreover, the idea of a logical universe in multiple dimensions remains without mention in the published version. This is alleviated to some extent in the other entries that Peirce contributed to the same work, especially in the subentry on 'Dimension' that is to be found within the entry on 'Logic' (this entry was exclusively written by Peirce), in which the role of dimension in the study of modality is rightly and visibly underscored (cf. also Peirce's 1901 article 'Syllogism' in DPP).

Charles G. Morgan [25] has argued that what he calls the information-relative account of modalities is presented by Peirce as a special case of the analogue account. Although it is certainly true that Peirce's view evolved and changed over the years, he was quite clear in trying to render the analogue account into a more formal information-relative account in terms of ranges of possibility and ignorance, not taking it as a special case of the analogue account. What is notable in Morgan's presentation of the information-relative account is its exposition in terms of maximally consistent sets, a technique that was to become instrumental in establishing subsequent metamathematical results for both first-order and modal logic after the 1940s.

* * *

Seen from a wider perspective, Peirce's struggle with modality was as innovative as it was frustrating. In his *Prolegomena* that appeared in *The Monist* in 1906, he made an attempt to get EGs to tackle modal statements. Later he admitted that this account had not been satisfactory, and that it was still necessary to add a DELTA part in order to deal with them. That was never realised, but Peirce envisioned diagrams that would encompass not only declarative propositions, but also commands (imperatives) and questions and answers, or requests for information (interrogatives).

In order to fulfil Peirce's vision, a DELTA part might be reconstructed by transforming the possible-worlds semantics for predicate modal logic into a diagrammatic system of EGs. Such a transformation needs to distance itself from the straightforward rendering of traditional Lewis-type modal logics into diagrammatic logic. For example, it might then be possible to incorporate some of the latest advances of hybrid, dynamic and multi-dimensional modal logics. The reconstruction would also enhance our basic understanding of possible-worlds semantics as well as quantification in modal logic.

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