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VIBRATIONAL STRESSES IN THE LAST-STAGE BLADES OF A POWERFUL STEAM TURBINE UNDER KINEMATIC EXCITATION OF OSCILLATIONS. PART 1. INVESTIGATION OF CYCLIC-SYMMETRIC SYSTEMS

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The influence of kinematic excitation of rotor vibrations of a powerful steam turbine without and with disturbance of blade vibration frequencies on the extension of their trouble-free operation is evaluated. The results of determining the maximum equivalent stresses of the blades under the condition of power and kinematic excitation of stationary oscillations are presented. A system with cyclic symmetry is considered. The three-dimensional finite element models of the disk–blade system and the corresponding mathematical support for calculating stationary harmonic oscillations are used. Computational studies to determine the maximum equivalent stresses of the blades were carried out under the condition of simultaneous action of power excitation of oscillations from the steam flow with a frequency of the forcing force of 2100 Hz (with the number of guide blades of 42) and kinematic excitation due to rotor vibration on sliding bearings with a frequency of 50 Hz. The load from the steam flow on each blade was set to be linearly variable from zero at the root to 1 kPa and 5 kPa at the apex, as well as a uniformly distributed 2.5 kPa along the blade, acting normally at points on their working surface. The kinematic excitation was set as an ellipse describing the motion of the disk center in its plane. It is assumed that the physical and mechanical properties of the blade material are preserved after their repair and surface treatment. The change in the maximum equivalent stresses for different variants of blade loading in a cyclic-symmetric disk–blade system under kinematic excitation of oscillations is evaluated. The obtained results are compared with the data for the system with all damaged blades after restorative repair in their lower part under the condition of kinematic excitation of vibrations and without repair. These results confirm the practicality of assessing the stress state of the last stage blades of a powerful steam turbine, considering the disk–blade system's kinematic excitation when determining their operation's reliability.

***Keywords*:** disk–blade system, forced oscillations, three-dimensional finite element model, amplitude-frequency characteristics, erosion damage, restorative repair.

Introduction. Blades are one of the most stressed elements of powerful turbine units. While the blades of high and medium-pressure cylinders operate at high temperatures and steam flow pressures, the last stages of low-pressure cylinders operate at relatively low temperatures and steam flow pressures with increased humidity. After prolonged operation, this causes droplet and steam erosion of the blade surface at the inlet and outlet edges. This is especially evident in the blades (1.2 m long) of the last stages of 1000 MW turbines at nuclear power plants [1, 2].

The edges of such blades have a sawtooth appearance after operating for about 180 thousand hours. For the further operation of the turbine, they are repaired or replaced. In practice, there are cases when erosion damage to the surface is observed on almost all blades of the last stage or several adjacent ones. The same repair for all blades does not break the cyclic symmetry of the system while repairing a few or one blades leads to its violation. When carrying

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out a vital repair, it is important to assess the change in their vibration stress to ensure the reliability of further operation of the blades.

Papers [1–3] investigated the effect of erosion damage on changes in the material structure, its mechanical properties, and stresses on the blades of the last stage of a 1000 MW steam turbine. They considered the effect of repair measures such as removal of damage zones and surface treatment on the stress-strain state of individual blades and determined the permissible change in the chord of the blades in the places of their surface treatment.

A computational assessment of the effect of changing the cross-sections of blades with their identical restorative treatment on the maximum equivalent stresses in the disk–blade system for a conditional load from a steam flow was considered in [4]. If the cyclic symmetry of the system is not broken, the permissible values of the blade chord in the processing area during the restorative repair were established.

The results of computational studies of changes in the maximum stresses in blades during the restorative repair of one damaged blade, which violates the system's cyclic symmetry, were considered in [5]. The peculiarities of stress changes in all blades compared to the stresses for a cyclicly symmetric system under the same treatment were determined.

The above works assumed that the disk's center did not change its position and ignored the kinematic excitation of its vibrations. Experimental and computational studies of the vibration of steam turbine rotors on sliding bearings (e.g., [6–8]) have shown that rotor precession occurs under the influence of residual unbalance of elastic rotors and forces of the oil layer of sliding bearings.

If the bearing load from the weight of the rotors and their residual unbalance corresponds to the design values. The rotor center trajectories in the bearings correspond to the shape of an ellipse centered on the moving balance curve [6–8]. The maximum permissible values of the oscillation range of the rotor center in bearings of powerful units during operation, according to regulatory recommendations, should be less than 260 μm. It should be noted that the trajectories of rotor motion differ significantly from elliptical ones when the stability of their motion in sliding bearings is disturbed and when self-oscillations appear. Components with frequencies usually close to half the rotor rotation frequency appear in the rotor vibration spectrum [6–8]. These forms of rotor motion are affected by the excitation of oscillations from the steam flow acting on the blades.

The effect of kinematic excitation with a frequency of 50 Hz on the oscillations of a cyclic-symmetric disk–blade system is investigated below. The loads from the steam flow on the blades with a frequency of 2100 Hz are considered by the distributed pressure on their surfaces. It is assumed that it varies linearly along the length of the trough (from 0 to 1 and up to 5 kPa at the top) and is evenly distributed (2.5 kPa), which corresponds to an equivalent load under operating conditions [9, 10].

System Model and Basic Design Dependencies. The impeller of the last stage of the turbine has 92 blades fixed in the body of the disk. It is made with a massive shaft and has a constant cross-section. The blades are connected at the periphery by split shelf links of a zigzag design. Operating experience shows that the centrifugal forces acting on the impeller ensure the shelf ties are tightly pressed together. The presence of a lumpy damper wire placed in the blade holes needs to be taken into account [3, 4].

The influence of kinematic and power excitation on the change in the maximum equivalent stresses in the blades during oscillations is determined using a three-dimensional finite element model of the disk–blade system (Fig. 1). The latter consists of more than 60 thousand elements and 175 thousand nodes. For its construction, prismatic and tetrahedral finite elements were used. The kinematic excitation in the form of an ellipse describing the motion of the center of mass of the disk in its plane was set at a rotor speed of 50 Hz.

Let us consider the main calculation dependencies, provided that the oscillations in the disk–blade system arise due to the action of the power load from the steam flow and the kinematic load from the residual rotor unbalance. The finite-element version of the equations of harmonic oscillations of the system under consideration is as follows:

$$([\boldsymbol{K}] + i\omega[\boldsymbol{C}] - \omega^2[\boldsymbol{M}])\{\boldsymbol{q}(t)\} = \{\boldsymbol{F}(t)\}, \quad (1)$$

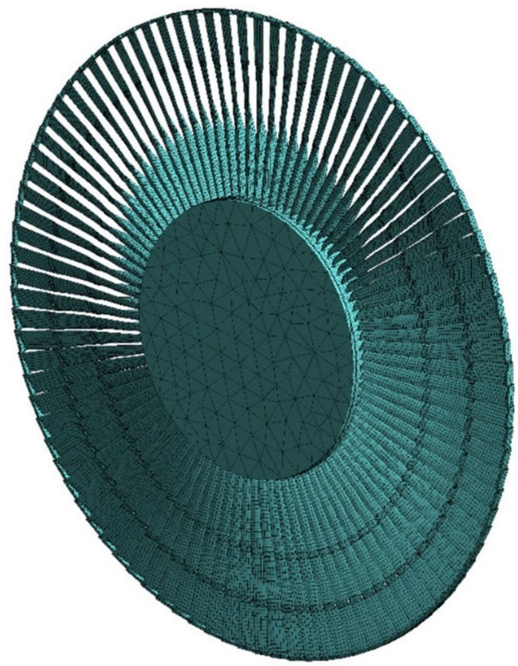

Fig. 1. Finite element model of the disk–blade system.

where $[\boldsymbol{M}]$, $[\boldsymbol{K}]$, and $[\boldsymbol{C}]$ are the global mass, stiffness, and damping matrices, respectively, $\{\boldsymbol{q}\}$ is the vector of nodal displacements, $\{\boldsymbol{F}\}$ is the global vector of nodal loads, ω is the circular frequency of the harmonic load, and $i = \sqrt{-1}$.

Under the condition of kinematic loading, the boundary conditions imposed on the displacement of the system can be written as follows:

$$\{\boldsymbol{q}_m(t)\} = 0, \ \ \{\boldsymbol{q}_j(t)\} = Q_{0j}e^{i\omega t}, \tag{2}$$

where Q_{0j} is the amplitude of kinematic displacements along the *j*th generalized coordinate of the finite element model.

Under the condition of simultaneous action of kinematic and power loads, the right-hand side of equation (1) is formed, taking into account the boundary conditions (2), according to which the matrices $[\boldsymbol{M}]$, $[\boldsymbol{K}]$, and $[\boldsymbol{C}]$. Thus, the vector of nodal loads has components that take into account the stationary power and kinematic loads:

$$\{\boldsymbol{F}(t)\} = \{\boldsymbol{F}_0(t)\} - \sum_j [\boldsymbol{H}_{mj}]Q_{0j}e^{i\omega t} \ \ (m \neq j), \tag{3}$$

where $[\boldsymbol{H}]$ is the dynamic stiffness matrix of the system, $[\boldsymbol{H}] = [\boldsymbol{K}] + i\omega[\boldsymbol{C}] - \omega^2[\boldsymbol{M}]$, and $\{\boldsymbol{F}_0(t)\}$ is the vector of the nodal force load.

Thus, from the solution of equation (1), taking into account the boundary conditions (2), we obtain the system response to the kinematic and power excitation (3) of stationary oscillations:

$$\{\boldsymbol{q}(t)\} = [\boldsymbol{H}]^{-1}\left(\{\boldsymbol{F}_0(t)\} - \textstyle\sum_j [\boldsymbol{H}_{mj}]Q_{0j}e^{i\omega t}\right) \ \ (m \neq j).$$

Blade Stresses of a Cyclic-Symmetric System at Light Load. Steam turbine blade systems are designed to be cyclic-symmetric. Therefore, we first determine the effect of kinematic excitation on the stress-strain state of cyclic-symmetric systems. The analysis of its features for such systems with a fixed center is based on computational studies in [4, 5].

Their stresses in a cyclic-symmetric system are the same with the same blade load. It should be noted that, according to the number of guide vanes in the steam inlet system, the excitation frequency of the working blades' vibration was 2100 Hz, while the vibration stress of all the blades was the same.

Taking into account that the loads on the blades of the last stage are insignificant at the startup modes of the turbine (or at idle), we first consider the effect of kinematic excitation on the stress-strain state of the blades for three variants of the intensity of the uniformly distributed load from the steam flow: 2.5×10^{-5}, 2.5×10^{-6}, and 2.5×10^{-7} kPa, which can be taken as close to zero. Therefore, the blade stress at the specified load was determined only at 50 Hz without and with kinematic excitation. In the investigated disk–blade system, two areas of stress increase were identified (in the lower parts of the blades and the upper parts). The maximum stress value in the lower parts is generally 2-3 times higher than the stress in the upper parts without and without consideration of kinematic excitation. It has been established that when the latter is considered, the maximum equivalent stresses in the lower parts of intact blades decrease by 4.2%, increase by 1.01 times, and decrease by 5.8%, respectively, for each of the three variants with reduced blade loading. In the system with equally damaged blades in the lower part and treated blades with a chord of 150 mm in the material sampling area, when taking into account kinematic excitation, the maximum stresses in their lower part decrease by 88.8, 67.8, 75.6%, respectively, for the three variants of the intensity of uniformly distributed loading mentioned above.

The determined changes in blade stress at a frequency of 50 Hz, taking into account the kinematic excitation of the disk–blade system, can be explained by the shift of the resonant stress peak from 54 to 47 Hz and the change in the shape of the amplitude-frequency characteristics generated by the influence of this excitation for the three above-mentioned options for reducing the force load on the blades (Fig. 2).

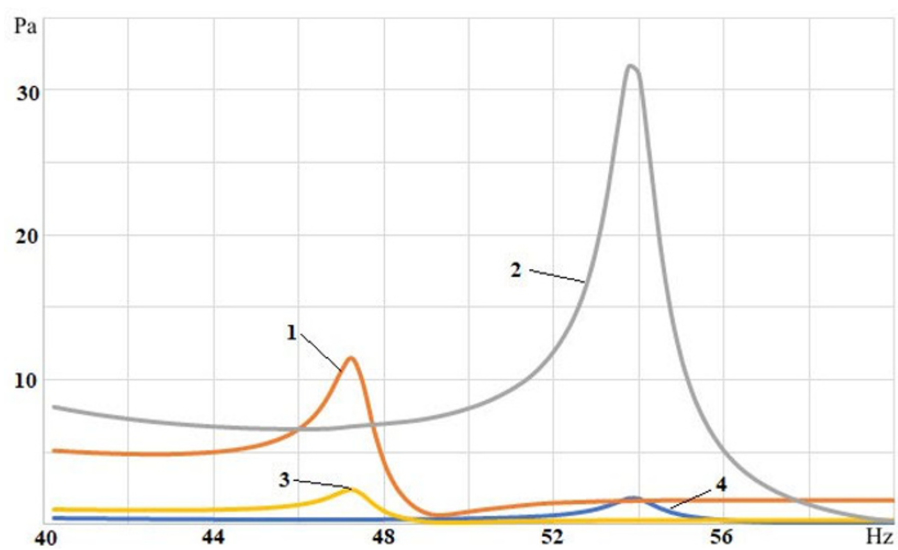

Fig. 2. Amplitude-frequency characteristics of the maximum stresses in the lower part of damaged blades under reduced power load in the frequency range of 40–60 Hz: (*1*, *2*) load of 2.5×10^{-5} kPa with and without kinematic excitation of oscillations and (*3*, *4*) load of 2.5×10^{-6} kPa with and without kinematic excitation of oscillations.

Blade Stresses of a Cyclic-Symmetric System under Operating Loads. The blade stresses are considered for three variants of power load simulating the effect of steam flow at reduced and rated power of the unit. The effect of kinematic excitation of the blown disk vibrations on the maximum equivalent blade stresses in a cyclic-symmetric system without damage and with the same damage in the lower part is evaluated. The movement of the disk center was set to an elliptical trajectory with a rotation frequency of 50 Hz, which corresponds to the stable behavior of the rotor in sliding bearings. The maximum movement of the disk center was set according to the normatively permissible values. With the same blade load, their maximum equivalent stresses in a cyclic-symmetric system are also the same.

By the number of guide vanes in the steam inlet system, the excitation frequency of oscillations of all working vanes was assumed to be 2100 Hz, and their power load was the same.

Let us consider the stress of undamaged blades of a disk–blade system. Loading of the blades' working surface with a uniformly distributed pressure, linearly varying from zero at the root to 1 kPa at the apex, corresponds to the unit's power, which is reduced several times compared to the nominal power. The results of the blade stress assessment at a rotor speed of 50 Hz indicate the presence of two areas of stress increase: in the lower part and the upper part, both without and with kinematic excitation. The obtained results show that the maximum equivalent stresses in the lower part of the blades are greater than in the upper part by 5.18 times without taking into account and by 12.57 times without considering kinematic excitation.

When the blade load was increased from zero at the root to 5 kPa at the apex, which corresponds to the equivalent action on the blades at the unit's rated power, it was determined that the maximum equivalent stresses in the lower part of the blades were 7.24 times higher than those in the upper part without taking into account kinematic excitation and 14.06 times higher with it.

The following was obtained with a uniform distribution of the load on the blades with an intensity of 2.5 kPa, the resultant of which corresponds to a linearly varying load from zero to 5 kPa. The maximum stresses in the lower part of the blades are 3.41 times higher than those in the upper part without considering kinematic excitation and 4.65 times higher with it.

The results of the assessment of the maximum equivalent stresses in undamaged blades at a steam flow excitation frequency of 2100 Hz indicate the presence of three zones of stress increase: the lower, middle, and upper parts. The maximum stresses in their middle part are lower than in the lower and upper parts. In the upper part, they are higher than in the lower part, both without and with consideration of kinematic excitation.

When the blades are loaded from zero to 1 kPa at the apex, the maximum stresses in the upper part are 6.05 times higher than in the lower part, without considering kinematic excitation, and 5.03 times higher with it. At the same time, with a force loading intensity of zero to 5 kPa, the maximum stresses in the upper part are 6.86 and 5.84 times higher.

Under uniformly distributed blade loading with an intensity of 2.5 kPa, the maximum equivalent stresses in the upper part of the blades are 1.92 times higher than in the lower part, without considering kinematic excitation, and 3.69 times higher with it.

It should be noted that at a frequency of 50 Hz, the increase in the maximum stresses due to kinematic excitation in the lower part of the blades without damage is close to 3.76, 3.73, and 1.58 times, which corresponds to a load of zero to 1 and 5 kPa, as well as to a uniformly distributed 2.5 kPa. In contrast, at a frequency of 2100 Hz, there is a decrease in the maximum stresses in their upper part due to kinematic excitation. Thus, in blades without damage at this frequency, considering the kinematic excitation, the maximum stresses decrease by 29, 26, and 28%, respectively, for the three variants of blade force loading considered above.

Let us consider the change in the stress-strain state of the system under study, with all blades damaged in the lower part equally treated when kinematic excitation is considered. Vibrations of such a system without the influence of kinematic excitation were considered in [4]. The presence of machining in their lower part at the outlet edges, which corresponds to the minimum permissible value of the chord in this region, does not violate the system's cyclic symmetry.

The analysis of the maximum equivalent stresses in the disk–blade system with equally damaged blades at a frequency of 50 Hz shows that the stresses in their lower part, without taking into account kinematic excitation, are 5.82, 5.51, and 4.71 times higher than those in the upper part, respectively, according to the three variants of blade loading. Considering the kinematic excitation, the excess of these stresses in the lower part of the blades relative to the upper part is 9.11, 9.07, and 7.52 times, respectively.

At a frequency of 2100 Hz, the highest maximum stresses are observed in the upper part of the blades, as in the system with blades without damage. The maximum stresses in this part of the blades are higher than the corresponding values in the lower part by 5.62, 3.42, and 4.08, without considering the kinematic excitation.

Considering the latter, the excess of these stresses in the upper part of the blades is 5.96, 2.47, and 2.98 times, respectively. In the middle part of the blades, the maximum stresses are less than those in the upper and lower parts. Notably, at a frequency of 50 Hz, the stresses in the lower part of the blades are maximum.

Based on the results obtained, we note that a greater effect of kinematic excitation on the maximum equivalent blade stresses is observed at the rotor speed. Thus, at a frequency of 50 Hz, the maximum equivalent stresses in the lower part of the blades with damage from kinematic excitation increase by 2.1, 2.13, and 2.01 times, and without damage – by 3.76, 3.72, and 1.58 times. At a frequency of 2100 Hz, the maximum stresses in the upper part of the blades with damage from kinematic excitation increase by 1.25, 1.31, and 1.37 times. Note that in blades without damage at this frequency, considering kinematic excitation, the maximum stresses decrease by 29, 26, and 28%.

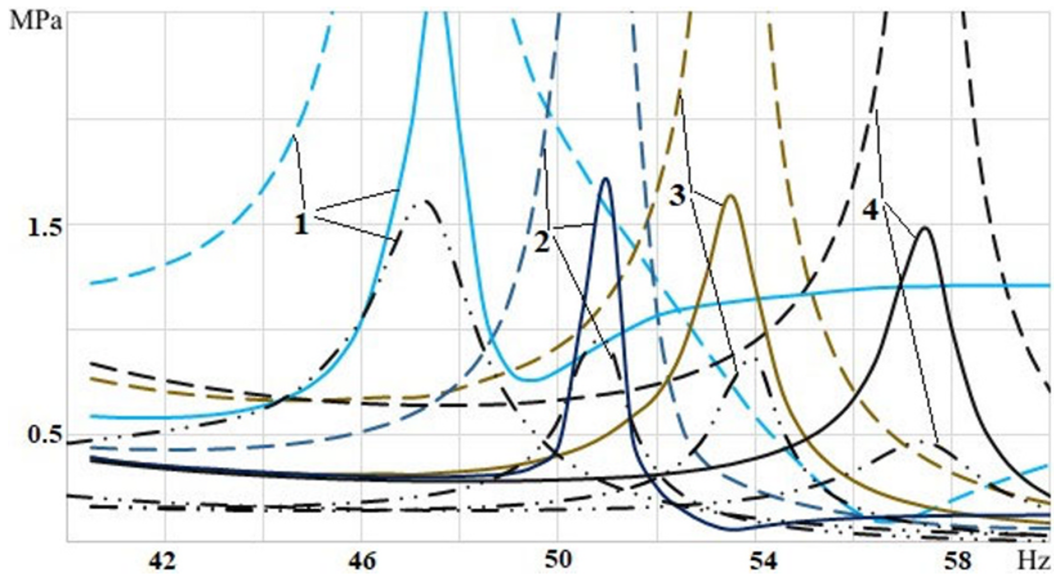

Fig. 3. Amplitude-frequency characteristics of the maximum stresses in the blades in the frequency range of 40–60 Hz for three types of force loading: from 0 at the root to 1 kPa in the upper part of the blade (dashed-dotted lines) and from 0 to 5 kPa (dashed lines) and a uniform load of 2.5 kPa (solid lines). (*1*, *2*) a system with all damaged and undamaged blades under kinematic excitation of vibrations, and (*3*, *4*) with no account of the kinematic excitation of vibrations.

The above changes in the maximum stresses in the blades of a cyclic-symmetric system with and without damage in the lower part can be explained by the influence of the factors taken into account on the system's dynamic characteristics. Figures 3 and 4 show the calculated curves of the dependence of the maximum equivalent stresses in the regions of rotational speed (Fig. 3) and oscillation excitation (Fig. 4) of the disk–blade system under the load corresponding to the reduced and rated power of the stage. When considering the kinematic excitation, the increase in stresses in damaged blades is associated with a resonant increase in this frequency region. In blades without damage, in the absence of kinematic excitation of oscillations, a resonant increase in stresses occurs at frequencies exceeding 2100 Hz. When taking into account the kinematic excitation of vibrations, the resonant stress values shift to lower frequencies, which explains their decrease in the upper part of the blades without damage at the frequency of excitation from the steam flow (Fig. 5). The effect of damage in the lower part of the blades on these characteristics with reduced loads (from zero at the root to 1 kPa at the apex) was also considered in [4].

The results show that the maximum stresses significantly depend on the kinematic excitation of the system oscillations. The resonant stress increases in the blades, taking into account the kinematic excitation, shift to the region of vibration frequencies less than 50 Hz (Fig. 3). In the region of the excitation frequency from steam flow (Fig. 4), taking into account the kinematic excitation of vibrations in damaged blades, the position of the resonant curves almost does not change, and the stress values at a frequency of 2100 Hz increase, but less than at a frequency of 50 Hz.

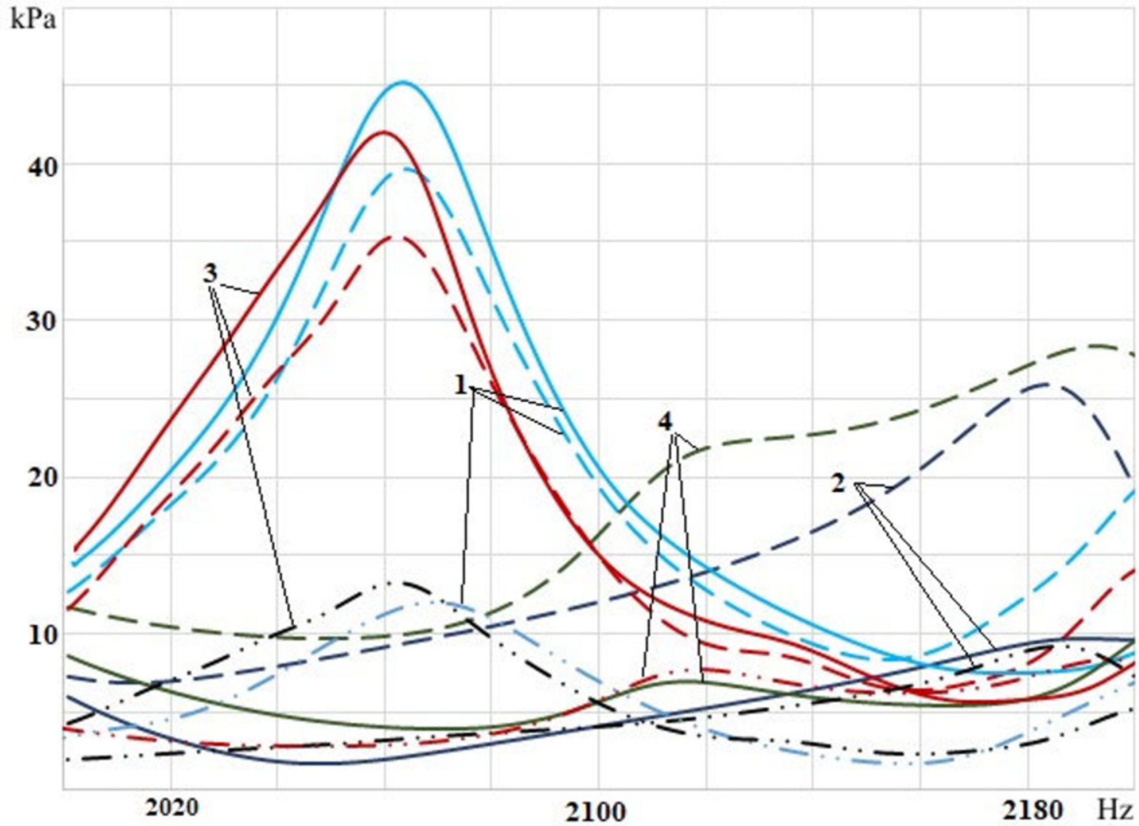

Fig. 4. Amplitude-frequency characteristics of the maximum stresses in the blades in the 2000–2200 Hz frequency range for three types of power load. (The designations are the same as in Fig. 3.)

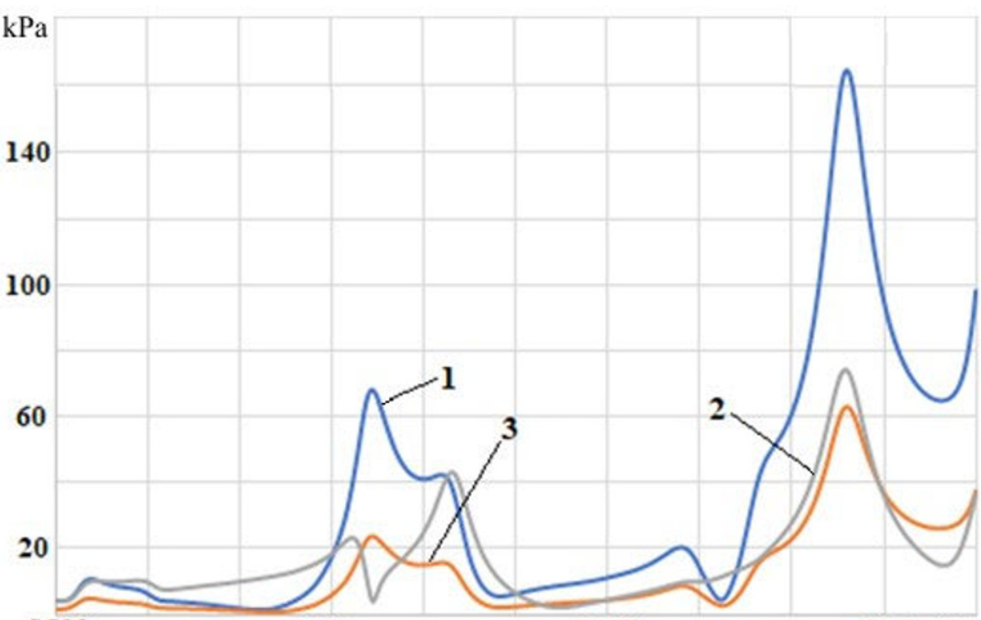

Fig. 5. Amplitude-frequency characteristics of the maximum stresses in the upper part of the blades without damage in the frequency range of 1500–2500 Hz with no account of kinematic excitation: (*1*) load range from 0 to 5 kPa, (*2*) uniform load of 2.5 kPa, and (*3*) load range from 0 to 1 kPa.

Conclusions. Numerical studies of the stressed state of blades of a cyclic-symmetric disk–blade system of a powerful steam turbine have established that in the region of the rotor rotation frequency of 50 Hz, in blades without damage and with damage in the lower part, two areas of increase in maximum stresses are observed: in the lower part and the upper part, both without and with consideration of the kinematic excitation of vibrations from the rotor with residual unbalance. The maximum equivalent stresses in the lower part of the blades under the considered variants of power loading are greater than the corresponding stresses in the upper parts of the blades without damage and with damage by an average of five times without taking into account the kinematic excitation and by 8-11 times with its consideration under operating loads. In the lower part of the blades without damage, the increase in the maximum equivalent stresses at the rotor rotation frequency, taking into account the kinematic excitation of oscillations, reaches about 3.1 times, and with damage and appropriate treatment – up to 2.1 times (on average for the considered forms of blade loading). At the same time, their increase when taking into account the kinematic excitation of vibrations is mainly due to the shift of the second resonant stress peak to the region of rotor rotation frequency for blades without damage or to the region of frequencies less than 48 Hz for blades with damage. At turbine idling conditions, the effect of kinematic excitation is manifested in a decrease in the maximum equivalent blade stresses.

At a blade loading frequency of 2100 Hz from a steam flow with the same forms of distribution, three stress increases are observed in the blades' lower, middle, and upper parts without damage and with the same damage. The calculated stresses in the upper parts of the blades are almost 2–7 times higher than those in the lower parts for different forms of their loading. In blades without damage at this frequency, the maximum equivalent stresses decrease by an average of 27.6% when considering kinematic excitation. Meanwhile, blades with damage increase by 1.31 times (on average, for the considered forms of blade loading). In the region of the steam flow excitation frequency, the position of the resonance curves of damaged blades, considering the kinematic excitation, almost does not change, and the stresses increase, but less than at the rotor speed. In the blades without damage (unlike the other options considered), the maximum stresses decrease with kinematic excitation, which is associated with a significant shift of their resonant value to frequencies less than 2100 Hz compared to the results for the system with damaged blades.

Noteworthy is that the considered kinematic excitation of the disk–blade system is generated by the movements of the disk center along an elliptical trajectory during oscillations of the rotor with residual unbalance in sliding bearings at steady-state operating conditions. Taking into account the above results for a cyclic-symmetric disk–blade system, it is considered expedient to take into account the influence of kinematic excitation from a rotor with residual unbalance when determining the parameters of oscillations and service life indicators and when assessing the reliability of operation of the last stage blades of powerful steam turbines. The maximum equivalent stresses at the frequency of steam flow action are much lower than the stresses at the rotor speed. When assessing service life, it is necessary to consider the significant intensity of service life at high frequencies and the relatively high stresses in the lower and upper parts of the blades at the rotor speed.

Conflict of Interests. All authors declare that they have no conflicts of interest.

REFERENCES

1. V. M. Torop, O. V. Makhnenko, G. Yu. Saprykina, and E. E. Gopkalo, "Results of research on the causes of crack formation in titanium alloy blades of K-1000-60/3000 steam turbines," *Technological Diagnostics and Non-Destructive Testing*, No. 2, 3–15 (2018). https://doi.org/10.15407/tdnk2018.02.01
2. Yu. S. Vorobiov, N. Yu. Ovcharova, A. S. Olkhovskyi, et al., "Vibration featuring of titanium alloy blades with erosive damages," *J Mech Eng*, **21**, No. 4, 13–21 (2018) https://doi.org/10.15407/pmach2018.04.013
3. O. Makhnenko, Yu. Vorobiov, N. Ovcharova, and A. Olkhovskyi, "Vibration of titanium blades of turbomachines for nuclear power plants with erosive damage," in: Proc. of the Second Int. Conf. on Theoretical, Applied, and Experimental Mechanics ICTAEM 2019 (2019), pp. 334–340. https://doi.org/10.1007/978-3-030-21894-2_61

4. N. G. Shulzhenko and A. S. Olkhovskyi, “Vibrational stresses of damaged steam turbine blades after renovation repair,” *J Mech Eng*, **24**, No. 14, 42–52 (2021). https://doi.org/ 10.15407/pmach2021.01.042
5. M. G. Shulzhenko, A. P. Zinkovskyi, and A. S. Olkhovskyi, “Vibration stress of the last-stage blades of a steam turbine after repair of a single blade,” *Strength Mater*, **54**, 565–575 (2022). https://doi.org/ 10.1007/s11223-022-00433-z
6. F. M. Dimentberg and K. S. Kolesnikov (Eds.), *Vibrations in Engineering* [in Russian], Vol. 3: *Vibrations of Machines, Structures and Their Elements*, Mashinostroenie, Moscow (1980).
7. J. Kicinski, *Rotor Dynamics*, IMP PAN, Gdansk, Poland (2006).
8. N. G. Shulzhenko, P. P. Gontarovskii, and B. F. Zaitsev, *Problems of Thermal Strength, Vibration Diagnostics and Service Life of Power Units. Models, Methods, Research Results* [in Russian], LAP LAMBERT Academic Publishing, Saarbrücken, Germany (2011).
9. L. A. Shubenko-Shubin (Ed.), *Strength of Steam Turbines* [in Russian], Mashinostroenie, Moscow (1973).
10. B. M. Troyanovskii, G. A. Filippov, and A. E. Bulkin, *Steam and Gas Turbines of Nuclear Power Plants* [in Russian], Energoatomizdat, Moscow (1985).