APPLICATION OF MODIFIED FATIGUE CURVE
FOR EVALUATION OF FATIGUE DAMAGE OF STEELS AT VARIABLE STRESS AMPLITUDES.
PART 2. FATIGUE DAMAGE SUMMARIZATION
UNDER TWO-STAGE SYMMETRIC CYCLIC LOADING

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The linear model for calculating durability at variable stress amplitudes and the input data for such calculation presented in Part 1 were used to summarize fatigue damage under two-stage cyclic loading of specimens made of 45 and 1Kh13 steels. The results confirm the assumption made in the model that the scattering of data at constant stress amplitudes used to construct the fatigue curve can be associated with the scattering of the endurance limits of the tested specimens – as characteristics of their fatigue resistance. For this purpose, the total fatigue damage D (relative durability) of 45 and 1Kh13 steels under two-stage cyclic loading with sequences of high/low and low/high stress amplitudes with different operating times at the first stage were calculated. In the linear summation of fatigue damage, the fatigue curve is used in the proposed form, in which the endurance limit was found by successive approximations to bring the results of two-stage loading to D = 1. The calculation results show that to match the experimental data used to the condition of linear damage D = 1 in the proposed fatigue curve equation, the endurance limit (or damaging stress) should be changed within three times its standard deviation determined in experiments with constant load amplitudes. That is, the use of a modified fatigue curve allows us to determine such parameters of the fatigue curve of the required material, which provides a linear summation of fatigue damage. In addition, according to the parameters of the endurance limit distribution, the probability of the case with the found value of the endurance limit, at which the linear summation of fatigue damage is observed, can be determined. The relationship between the loads at the first stage on the total damage of steels with different cyclic deformation kinetics is also analyzed.

Keywords: fatigue damage summation, initial data scattering, fatigue curve, endurance limit.

Introduction. The peculiarities of the structural behavior of various structural materials during the accumulation of fatigue damage at the stages of fatigue crack initiation and growth have not yet allowed us to propose a universal method for summarizing fatigue damage, although a large number of them have been proposed in the literature [1–4]. Despite the variety of proposals to solve this problem, the main aspects that were solved were to propose methods for taking into account in new models of fatigue damage summation those characteristics of the material that are not present in the linear Palmgren–Miner hypothesis (LPMH) [5, 6], namely,

(i) stresses below the endurance limit, since they can change during cyclic operation even at a constant stress amplitude [7–11];

(ii) difference in the behavior of different materials when changing the amplitude of stresses from low to high and vice versa.

The introduction of parameters sensitive to changes in material properties due to these factors into the calculation models will allow taking into account both the nonlinearity of fatigue damage accumulation and their features at different stages of the fatigue process.

In [12], the use of a fatigue curve is proposed, in the equation of which a damaging stress is introduced, which is part of the endurance limit, and therefore, stresses below this level are taken into account. Considering the influence of the order of change of stresses on the subsequent fatigue damage of the material (D), it is known that $D > 1$ at the transition from lower to higher amplitude (L/H transition) and $D < 1$ at the reverse order of change of stresses (H/L) [13–15]. Methods for introducing parameters that ensure the adjustment of LPMH for its compliance for a given order of stress change have also been proposed [16–18]. At the same time, the peculiarities of the kinetics of cyclic plastic deformation [19, 20] and the different reactions of materials to the order of change in stress amplitudes at the stages of fatigue crack initiation and growth [21, 22] can play a significant role in damage accumulation. The level of influence of these factors on fatigue damage can be assessed by experimental verification of the stress interaction. The most common and simple method of such testing is two-stage cyclic loading with different operating times at the first stage [14, 16, 21, 23].

In this paper, the calculation model proposed in [12] for fatigue damage summarization is tested using experimental data under two-stage cyclic loading given in [24, 25], and the results of its application are analyzed.

Analysis of Fatigue Damage Accumulation under Two-Stage Cyclic Loads. Table 1 presents data on the study of fatigue damage of 1Kh13 steel at varying amplitudes of stresses and cyclic operating times at the first stage in two-stage experiments. The results were obtained under conditions of symmetrical loading with a frequency of 36 Hz in the field of multicycle fatigue in the laboratory at room temperature [24, 25].

TABLE 1. Results of Two-Stage Cyclic Tests of 1Kh13 Steel

σ_1 / σ_2, MPa	No.	$(N_1 \cdot 10^{-3} / N_2 \cdot 10^{-3})$, cycles	Sum D via LPMH	Sum D via [24, 25]	Coefficient b in dependences (2), (3)	$\Delta\sigma_r$ at $D = 1$, MPa
			The L/H order of stress amplitudes' variation			
280/290	1	2160/2445	2.09	1.45	1.3709	11.2710
	2	615/1376	1.02	1.3	1.0843	8.9147
	3	385/801	0.6	1.17	-1.2489	-10.268
	4	115/543	0.37	1.21	-1.86	-15.2922
280/300	5	700/1198	1.99	1.21	1.45	11.9213
	6	415/420	0.74	1.04	-0.519	-4.2670
	7	3000/692	1.83	1.35	1.153	9.4795
280/310	8	803/874	3.4	1.13	2.498	20.5376
	9	540/342	1.39	0.74	0.647	5.3194
	10	2600/433	2.25	0.85	1.563	12.8504
290/300	11	675/900	1.78	0.68	1.247	10.2523
	12	740/610	1.38	1	0.755	6.2073
	13	3030/584	2.78	1.1	1.99	16.3610
			The H/L order of stress amplitudes' variation			
310.0/280.0	1	100/740	0.55	0.7	-2.2858	-18.7929
	2	90/4041	1.38	0.8	0.5269	4.3319
310.0/290.0	3	698/1552	3.51	0.86	2.525	20.7595
300/280	4	141/460	0.33	0.65	-2.1927	-18.0275
	5	101/2787	0.87	0.7	-0.2503	-2.0578
	6	50/991	0.33	0.77	-2.0832	-17.1272
	7	430/230	0.71	0.9	-0.6114	-5.0267
	8	106/11258	0.46	0.95	-1.792	-14.7331
	9	245/5736	1.86	0.74	1.0507	8.6384
	10	53/1402	0.44	0.73	-1.5097	-12.4121
300/290	11	93/1601	1.14	0.89	0.3701	3.0428
	12	86/760	0.6	0.88	-0.8689	-7.1437

Notes. 1. The slash separates the stress amplitudes and cyclic operating time at the first (numerator) and second (denominator) stages. 2. The last two columns show the results of the calculation by the model [12], which will be explained below.

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The results presented in Table 1 show a significant scattering of the obtained total damage calculated by LPMH for both orders of change in stress amplitudes. For the L/H mode, the scattering range is $D = (0.37–3.4)$, and for the H/L mode, it is $D = (0.33–3.51)$. That is, regardless of the order of application of stress amplitudes, the ranges of scattering of total damage have close values, although for the H/V mode, most of the values of $D > 1$ (except for three results) and for the L/H mode $D < 1$ (except for four results), which statistically confirms the known regularity from the literature sources.

Another observation follows from the data in the column of Table 1, where the results of fatigue damage summation proposed in [24, 25] using the magnitude and kinetics of inelastic deformations of 1Kh13 steel are presented: given that inelastic deformations reflect the individual fatigue resistance capabilities of each individual specimen, a significant decrease in total damage and their approximation to unity is observed. That is, our proposed consideration of the individual fatigue properties of each studied material sample through the assessment of the scattering of their endurance limit [12] is also aimed at a more perfect assessment of the initial material data in subsequent calculations.

Several specimens were tested for the L/H loading mode at 280/290 MPa, which makes it possible to analyze the effect of the operating time at the first stage (N) on the total damage. According to these data, an increase in the total durability with an increase in the number of load cycles at the first stage can be traced (Fig. 1a), both according to the linear hypothesis and according to calculations by the method [24, 25]. Other results from the same mode with some scatter show the same trend.

For the mode of loading H/L at stress amplitudes of 300/280 MPa, where there is also a sufficient number of tested specimens, there is a significant scatter of results, and a tendency to increase or decrease the total durability with an increase in the number of load cycles at the first stage is not observed (Fig. 1b).

The result of the scatter of data in Fig. 1b can be explained by the interpretation of Morrow [26]: higher stresses (strains) at the first stage of loading cause the formation of a rougher relief of the material surface, which, in turn, contributes to the formation of more zones of microcrack nucleation. In this regard, the subsequent nucleation of the main crack, which will lead to destruction, is associated with a random process of coalescence of microcracks, which causes a significant dispersion of total damage after loading at the second stage.
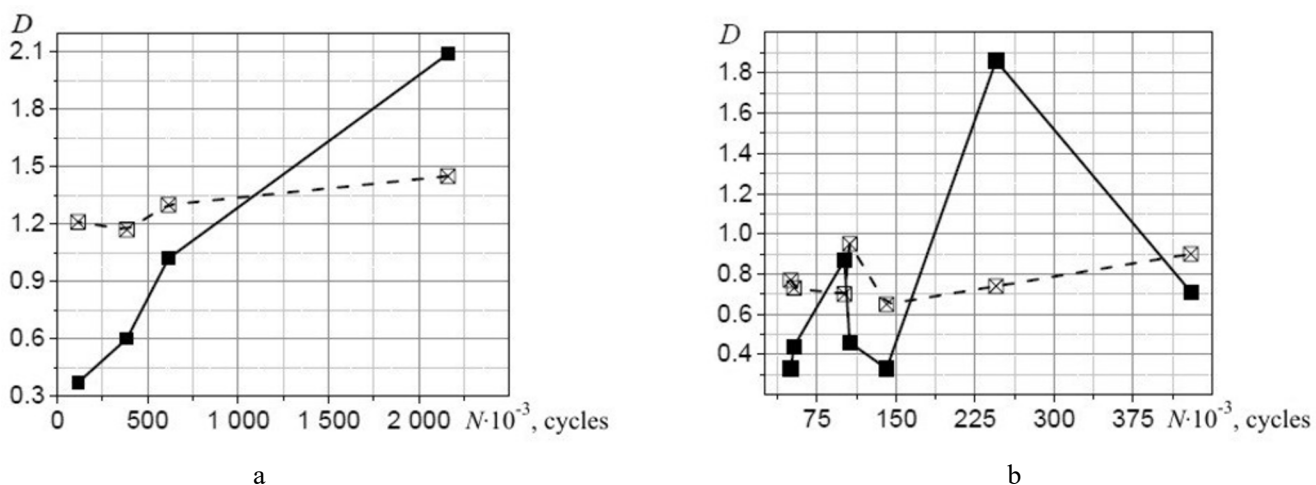

a   b

Fig. 1. Experimental points connected by straight line segments as a function of the change in total fatigue damage as a function of the number of load cycles at the first stage for load conditions of 280/290 MPa (a) and 300/280 MPa (b). Here and in Fig. 2: (*1*) summation of damage according to LPMH, (*2*) according to the method [24, 25].

More data for such analysis are available for 45 steel [24, 25]. These data are shown in Fig. 2 in the same coordinates as for 1Kh13 steel for the L/H and H/L loading modes. As seen, it is difficult to determine the influence

of the load at the first stage on fatigue damage when summing up according to the LPMH for the L/H loading mode, since there is a significant scatter of calculation results (Fig. 2a and 2b). The summation of damage by the method in [24, 25] shows a slight strengthening with an increase in load cycles at the first stage.

Summing up the fatigue damage by LPMH in the H/L loading mode demonstrates an increase in the strengthening of 45 steel with an increase in the number of loading cycles at the first stage (Fig. 2c and 2d), and for the case of 250/230 MPa, the strengthening is more intense, which can be explained by the significant cyclic strengthening of this steel in the kinetic diagram of deformation at high stress amplitudes [25]. Total damage according to the method [24, 25] gives more stable results close to $D = 1$, while for the 250/215 MPa regime, $D < 1$, and for the 250/230 MPa regime, $D > 1$, which can also be attributed to the kinetics of cyclic deformation of 45 steel at different stress amplitudes: higher stresses lead to greater cyclic strengthening, and low stresses have a kinetics of cyclic deformation close to stabilized [25].
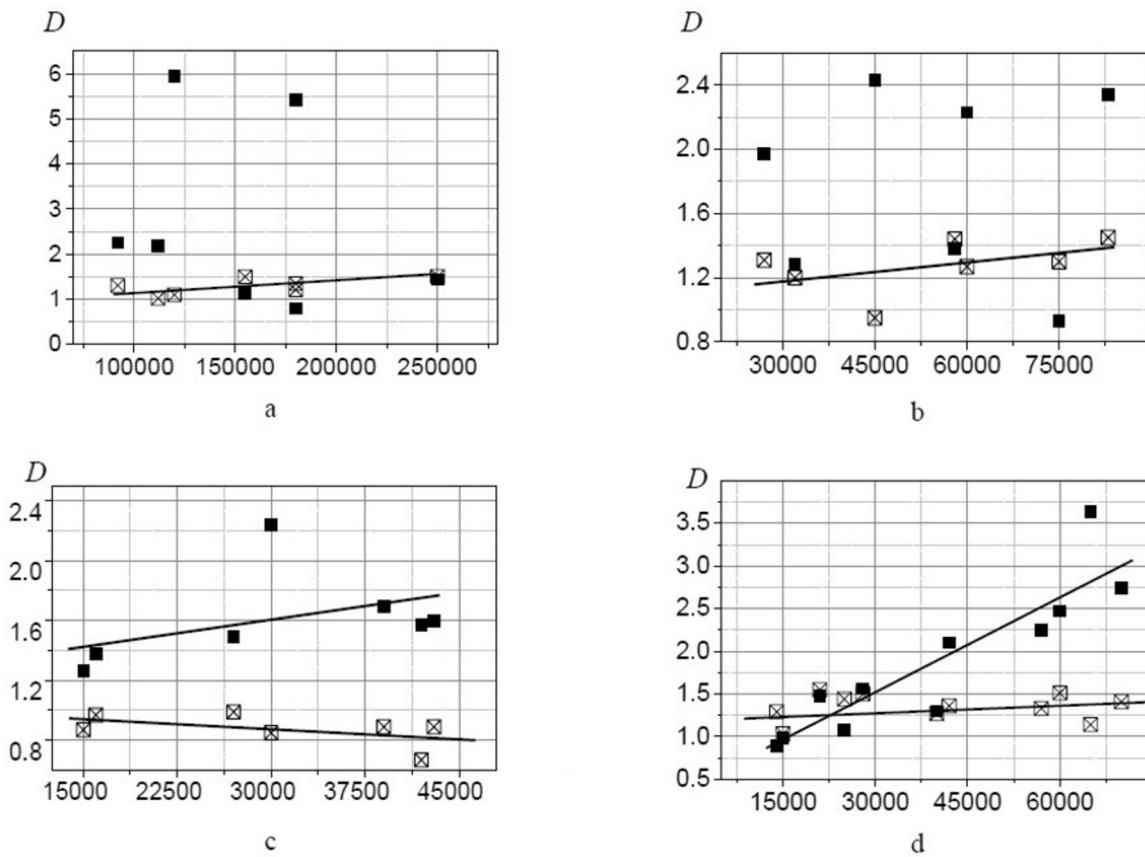

Fig. 2. Experimental points of dependence of the change in the total fatigue damage of 45 steel on the number of load cycles at the first stage for the loading modes L/H 215/250 MPa (a) and 230/250 MPa (b), L/H 250/215 MPa (c) and 250/230 MPa (d).

Application of the Proposed Calculation Model to Determine Fatigue Damage of 45 and 1Kh13 Steels. For the above results of two-stage cyclic loading of steels, the levels of deviation of the endurance limit from its experimentally determined value when summing up fatigue damage to ensure the condition $D = 1D$ in accordance with the model [12] were performed and analyzed. These values are given both in fractions of the root mean square deviation (RMSD) and in absolute stress values in MPa. The average values of the endurance limit and RMSD determined in [12] are $\sigma_{rm} = 271.97$ MPa, $S_{\sigma_r} = 8.2216$ MPa (1Kh13 steel); $\sigma_{rm} = 209.12$ MPa, $S_{\sigma_r} = 14.9552$ MPa (45 steel).

Note that the values $\Delta\sigma_r$ for 1Kh13 steel, i.e., deviations of the calculated and measured endurance limits presented in Table 1, cannot be split into components related to scattering of research results and those induced by processes of steel strengthening and weakening under the studied loading conditions. Therefore, the values of $\Delta\sigma_r$ represent the total value of these two factors.

The proposed calculation model uses a fatigue curve in the form:

$$(\sigma_a - \sigma_d)N_f^k = C, \tag{1}$$

where σ_a is the stress amplitude; σ_d is the amplitude of the damaging stress; N_f is the number of cycles to failure; and C and k are experimental constants. Endurance limit calculations for the condition $D=1$ with fatigue damage summation were performed according to the following scheme.

For the stresses of the first (σ_{1a}) and second (σ_{2a}) loading levels, the durabilities (N_{1f}, N_{2f}, respectively) are based on the dependence (1) and the accepted relation between the endurance limit σ_r and the damaging stress σ_d: $\sigma_d = 0.707\sigma_r$. Additionally, we provided the possibility of varying σ_r by the parts of the SCE through the coefficient b:

$$N_{1f} = \{C/[\sigma_{1a} - 0.707(\sigma_{rm} \pm bS)]\}^{\frac{1}{k}}, \tag{2}$$

$$N_{2f} = \{C/[\sigma_{2a} - 0.707(\sigma_{rm} \pm bS)]\}^{\frac{1}{k}}. \tag{3}$$

When summing the fatigue damage for each experimentally obtained result at two-stage loading by varying the deviation of the endurance limit $\Delta\sigma_{ri} = \pm bS$ from the average value σ_{rm}, we found its value that satisfied the LPMH condition:

$$D = \frac{N_1}{N_{1f}} + \frac{N_2}{N_{2f}} = 1, \tag{4}$$

where N_1 and N_2 are the numbers of operating cycles in the first and second stages, respectively.

Durability N_{1f} and N_{2f} for the new fatigue curves obtained by varying σ_r were calculated according to dependences (2) and (3); that is, these dependences were substituted into (4) with the same values σ_{ri} and thus represented the same fatigue curve.

As a result, we obtained a set of fatigue curves in the form (1) with values σ_d (σ_r) at which the summation of experimental fatigue damage leads to linear.

Analysis of the Results of Calculations by the Proposed Model. As follows from the calculation model, the experimental total damage and the values of $\Delta\sigma_r$ are interrelated: at < 1, $\Delta\sigma_r < 0$, at $D>1$, $\Delta\sigma_r > 0$. For the studied steels, this relationship is shown in Fig. 3. As seen from the figure, with some approximation and scatter, this dependence can be approximated by two straight segments. In this case, larger values of the total damage according to LPMH correspond to larger values of $\Delta\sigma_r$. This is consistent with the condition that higher values of D reveal specimens with higher endurance limits (according to the scatter of properties), or it is the result of positive scatter and cyclic strengthening of the material in the process of two-stage loading. Accordingly, the reverse statement is also true: low D values indicate a lower endurance limit of the specimen, or together with the processes of weakening under two-stage loading.

Since the determined deviations of the endurance limit $\Delta\sigma_r$ are interpreted as a quantitative assessment of fatigue damage of a single specimen under conditions of loading with variable stress amplitudes, it is advisable to analyze the relationship of the calculated values $\Delta\sigma_r$ with cyclic operating time at the first stage of loading. For this purpose, the entire set of data obtained for the tested steels was processed separately for the L/H and H/L modes. The results are shown in Fig. 4.
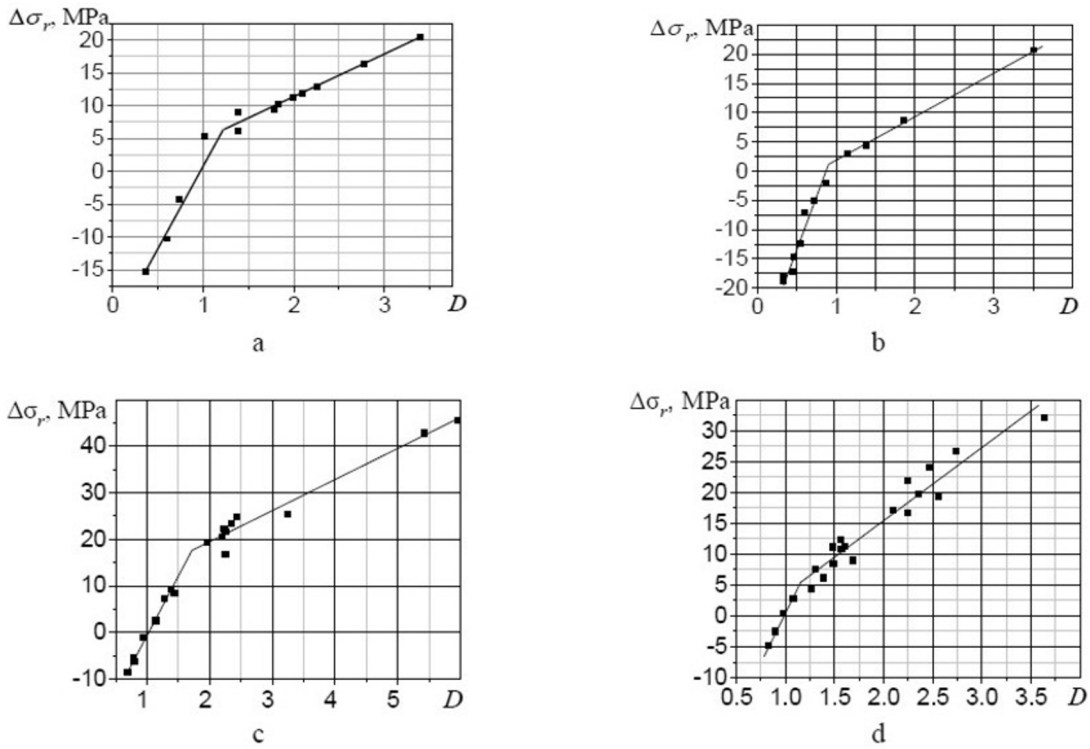
Fig. 3. Correlation between $\Delta\sigma_r$ and total damage of Kh13 (a, b) and 45 (c, d) steels under the loading modes L/H (a, c) and H/L (b, d).
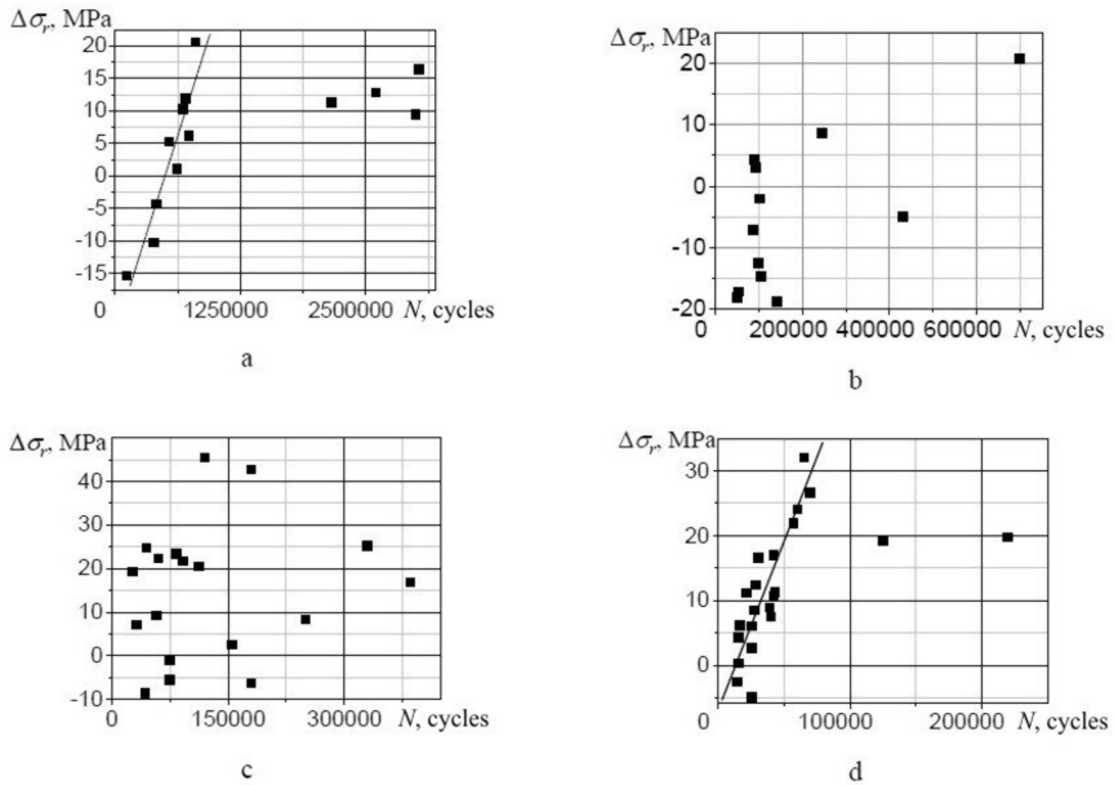

Fig. 4. Relationship between the calculated values $\Delta\sigma_r$ and the operating time at the first stage for 1Kh13 (a, b) and 45 (c, d) steels under the loading modes L/H (a, c) and H/L (b, d).

From the presented results, it can be seen that when loading 1Kh13 steel in the L/H mode, there is an increase in its hardening with an increase in the number of load cycles at the first stage: small operating times lead to weakening (negative values of $\Delta\sigma_r$), which develops into hardening and reaches a maximum value at a certain operating time. A further increase in the operating time slightly reduces the achieved maximum value of $\Delta\sigma_r$, which indicates a higher fatigue damage of the material at the first stage of loading. Taking into account the fact that the increase in σ_r can be associated with the processes of strengthening the material at the first stage of loading, some decrease in σ_r with an increase in cyclic operating time may indicate the achievement of saturation in strengthening and the emergence of microscopic leaks in the surface layer of the material. This leads to some weakening of the material, which results in a decrease in the calculated value of σ_{ri}. There is a significant scattering of values at the H/L loading mode $\Delta\sigma_r$ values from negative to positive at close values of operating time at the first load stage, which complicates the interpretation of the obtained results.

For 45 steel, the opposite behavior is observed for the studied loading modes: the H/V mode shows a significant scattering of values $\Delta\sigma_r$ and the H/L regime shows a pattern similar to that of 1Kh13 steel in the H/H regime. It should also be noted that the lower values of weakening and higher values of strengthening of 45 steel compared to 1Kh13 steel. This result is explained by the difference in their cyclic inelastic properties. As mentioned above, the first one is cyclic hardening, and the second one is cyclic softening. The same difference can affect the regularities shown in Fig. 4.

Probabilistic Assessment of Fatigue Damage Summation. Given a sample mean value of the endurance limit σ_{rm} and its standard deviation S_{σ_r} calculated in [12], it becomes possible to estimate the probability of realization in the tests of each of the obtained results of two-stage loads. This is based on the fact that the deviation of the endurance limit determined by the model $\Delta\sigma_{ri}$ from its average value, as demonstrated above, is closely related to the deviations of fatigue damage from the linear one. For quantitative determination of probability characteristics, we will perform the transition to the standard normal probability distribution $\phi(Z)$ using the normalized and centered value Z [27]:

$$Z = (\sigma_{ri} - \sigma_{rm})/S_{\sigma_r}; \tag{5}$$

$$\sigma_{ri} = \sigma_{rm} \pm \Delta\sigma_{ri}, \tag{6}$$

where $\Delta\sigma_{ri}$ is the deviation from σ_{rm} for each of the cases of two-stage loading of 1Kh13 steel given in Table 1.

As seen from the above dependences (5) and (6), the expression in the brackets of formula (5) is the value $\Delta\sigma_{ri}$. According to the table of values of the Laplace integral function [27] and the values determined from experiments Z, the probability values $\phi(Z)$, corresponding to the following limits of change of the endurance limit can be obtained [27]:

(i) probability that the endurance limit exceeds its average value (i.e., $\Delta\sigma_{ri} > 0$ or $\sigma_{ri} > \sigma_{rm}$);
(ii) probability of not exceeding the endurance limit of its certain value $\sigma_{ri} \le (\sigma_{rm} \pm \Delta\sigma_{ri})$;
(iii) probability of exceeding the endurance limit of its certain value $\sigma_{ri} \ge (\sigma_{rm} \pm \Delta\sigma_{ri})$.

Since the listed probability values are interrelated, Table 2 shows the following $P(\sigma_{ri})$ corresponding to the condition of not exceeding the endurance limit of the values $\sigma_{ri} \le \sigma_{rm} \pm \Delta\sigma_{ri}$ values calculated by the model [12] for steel 1Kh13 under the L/H loading mode. As seen from the table, the probability of low values σ_{ri} does not exceed $P(\sigma_{ri}) \le 0.3018$, and most results show that the endurance limit is in the range between the average value σ_{rm} and the value of $\sigma_{ri} > \sigma_{rm}$ (probabilities $[P(\sigma_{ri}) - 0.5]$ for $P(\sigma_{ri}) > 0.5$ in Table 2).

Similar assessments can be performed for other loading options under study.

The presence of the parameters of the normal distribution of the calculated endurance limits according to the model [12] also allows the use of well-known methods for calculating the interval estimation of a random variable and the confidence limits of the interval with a given reliability (confidence probability) [28]. That is, given the required confidence probability of the endurance limit, the lower (left) and upper (right) confidence limits of its

average value are determined $\sigma_{rmin} \leq \sigma_{rm} \leq \sigma_{rmax}$, which are used in equation (1) (through σ_d) to calculate the upper and lower confidence limits of fatigue damage at variable load amplitudes. For one of the commonly accepted confidence levels $\phi(-Z, Z) = 0.95$, the confidence limits of a random variable according to the standard normal distribution table are - $1.96 \leq Z \leq 1.96$, which corresponds to the confidence limits of the endurance limit of $255.86 \leq \sigma_{rm} \leq 288.08$ MPa. For these values of endurance limits, the corresponding confidence limits of fatigue damage of 1Kh13 steel for the studied modes of two-stage loading were calculated. The calculation results are shown in the form of diagrams (Fig. 5).

TABLE 2. Probability of the Case $\sigma_{ri} \leq \sigma_{rm} \pm \Delta\sigma_{ri}$ and Values $P(\sigma_{ri}) - 0.5$ for 1Kh13 Steel Specimens Tested in L/H Mode of Two-Stage Loading

| $\Delta\sigma_{ri}$, MPa	Z	σ_{ri}, MPa	$P(\sigma_{ri})$	$P(\sigma_{ri}) - 0,5$
11.2766	1.3716	283.2366	0.9147	0.4147
1.0932	0.1329	273.0532	0.5529	0.0529
-7.2141	-0.877	264.7459	0.1902	-
-15.2922	-1.8610	256.6598	0.0316	-
11.9276	1.4508	283.8876	0.9266	0.4266
-4.2692	-0.5193	267.6908	0.3018	-
9.4845	1.1536	281.4445	0.8756	0.4756
20.5483	2.4993	292.5083	0.9937	0.4937
5.3222	0.6473	277.2822	0.7390	0.239
12.8571	1.5638	284.8171	0.9399	0.4399
10.2659	1.2486	282.2259	0.8941	0.3941
6.2105	0.7554	278.1705	0.7750	0.275
16.3695	1.9910	288.3295	0.9767	0.4767
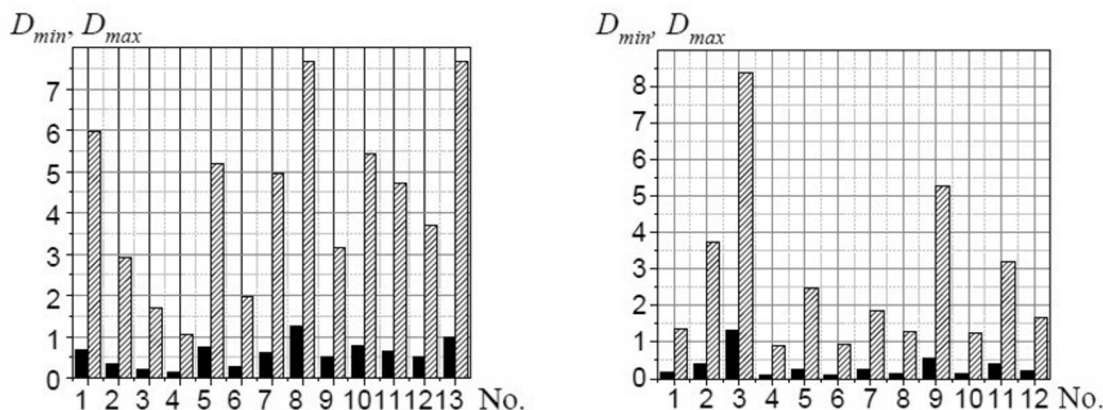
Fig. 5. Confidence limits of fatigue damage of 1Kh13 steel for the studied modes of two-stage loading: ∎ − D_{min}; ▨ − D_{max}, No. – loading modes in accordance with Table 1.

As seen from the diagrams shown in Fig. 5, when the endurance limit is realized in the samples in the interval of its values corresponding to a probability of 95%, the total fatigue damage of 1Kh13 steel in the studied L/H and H/L modes will change in a wide range: from D_{min}=0.1248 to D_{max}= 7.6742 for L/H mode in Nos. 3 and 13 and from D_{min}=0,0993 to D_{max}= 8.3974 for H/L mode in Nos. 6 and 3, respectively. It is clear that the damage D_{min} is associated with the upper confidence limit of the endurance limit, and D_{max} is associated with the lower one. To ensure the durability of the material without destruction, fatigue damage should not exceed D=1. Therefore, using the above calculations, it is necessary to select the endurance limit at the level of the upper confidence limit at which the damage is obtained D_{min}.

Despite the fact that the probability of occurrence in the studied batches of samples with such ($\sigma_r = 288.08$ MPa) or higher endurance limit is low (less than 2.5% in accordance with the selected confidence level of 95% with a two-sided confidence interval for the mean value); in our case, this calculation was used to determine the parameter σ_d (through the endurance limit) of the fatigue curve (1). Since its free term and the regression coefficient were calculated earlier [12], all the parameters of this curve were determined in this way, at which the durability of steel will be ensured for all modes of two-stage loading, except for L/H mode No. 8 and H/L mode No. 3, which fall into the zone of 2.5% error at the accepted level of confidence. If it is necessary to reduce the error, it is necessary to increase the level of confidence, which will increase the upper confidence limit for the endurance limit. increase the value $\Delta\sigma_r$ by which it is necessary to increase the endurance limit in equation (1) to ensure durability at all studied load modes without exception.

We emphasize that the summation of damage in the calculations is carried out according to the LPMH rule. Calculations for 45 steel are omitted since they were performed according to a scheme similar to that for 1Kh13 steel.

CONCLUSIONS

1. Using a modification of the fatigue curve recording, a parameter is proposed to determine the total influence of the scatter of fatigue properties and strengthening-weakening processes of the material at variable amplitudes of cyclic loading, which lead to deviations from the linear summation of fatigue damage.

2. The proposed design parameter in the form of deviation of the endurance limit from its average value to fulfill the condition $D = 1$ closely correlates with the total damage of the studied steels, determined by LPMH under two-stage cyclic loads.

3. For a given alternating mode of cyclic loading and data for the construction of the fatigue curve by varying the endurance limit (damaging stress), its value can be found such that the total damage of the alternating mode is determined by linear summation with the required level of confidence.

4. From the analysis of the relationship between the total fatigue damage under two-stage loading and cyclic operating time at the first stage, it follows that the mutual influence of loading levels depends not only on the number of loading cycles but also on the nature of the kinetics of inelastic cyclic deformation: damage accumulation is different for materials that exhibit cyclic strengthening and weakening.

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