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ELASTOPLASTIC ANALYSIS OF ULTIMATE BEARING CAPACITY FOR MULTILAYERED THICK-WALLED CYLINDERS UNDER INTERNAL PRESSURE

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The elastoplastic analysis based on the unified strength theory was performed to evaluate the ultimate bearing capacity of double- and multilayered thick-walled cylinders. The theory provides a new concept and method for the analysis of thick-walled cylinders. The solutions derived herein are widely applicable and can quantitatively account for different tension-compression strength values and mean principal stress. The fundamental solutions for single radii, assemblage pressure, and shrink range are derived with the yield condition of the theory. The traditional existing elastoplastic results by the Tresca or von Mises yield criteria can be seen as a particular case of the new solutions that can overcome shortcomings. The strength parameter, tension-compression strength ratio, radii ratio, and combined cylinder layers were taken as major theory variables for the unified solutions. The new solutions are versatile and can be adapted to the existing formulas, to more accurately calculate the structural stress conditions. The strength theory effect due to adopting different yield criteria is quite significant, which cannot be underestimated.

Keywords: mechanical property, elastoplastic analysis, thick-walled cylinder, intermediate principal stress, unified strength theory.

Introduction. Thick-walled cylinders are widely used in mechanical engineering, civil engineering, aerospace, chemical engineering, etc. [1–8]. In mechanical engineering, the shrink fit between a transmission shaft and sleeve, shaft and hub belong to a combined thick-walled cylinder [9–14]. To improve the ultimate bearing capacity, the method of increasing wall thickness is limited when the inner radius of a thick-walled cylinder is fixed. However, two or more thick-walled cylinders are used to form multilayered combined cylinders by means of interference fit, and the stress distribution is more reasonable than that of a single integral thick-walled cylinder [15]. Multilayered combined thick-walled cylinders are mostly designed with equal strength; that is, when the container fails, the inner and outer cylinders are simultaneously damaged [16–18]. Many researchers have studied the optimization design and stress intensity factor for combined thick-walled cylinders by the Tresca yield criterion, but it is not applicable to tensile-compressive anisotropic materials and does not consider the intermediate principal stress. Until now, the limit analysis of multiple thick-walled cylinders rarely reported in the literature. The elastoplastic bearing capacity solutions herein for double-layered and multilayered combined thick-walled cylinders are presented with unified strength theory (UST), which fully considers the influence of the intermediate principal stress and strength difference. In addition, the separate radius, assemblage pressure and shrink range fundamental solutions are derived from the UST.

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1. Theoretical Method.

1.1. Unified Strength Theory. The UST was developed based on orthogonal octahedron of a twin shear element model, which mathematical expression was introduced in [19]

$$F = \sigma_1 - \frac{\alpha}{1+b}(b\sigma_2 + \sigma_3) = \sigma_s, \qquad \sigma_2 \le \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}, \tag{1a}$$

$$F' = \frac{1}{1+b}(\sigma_1 + b\sigma_2) - \alpha\sigma_3 = \sigma_s, \qquad \sigma_2 \ge \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}, \tag{1b}$$

where α and b are defined as $\alpha = \sigma_t / \sigma_c$ and $b = \dfrac{(1+\alpha)\tau_s - \sigma_t}{\sigma_t - \tau_s}$, F and F' are functions of principal stress strength theory, σ_1, σ_2, and σ_3 are the major, intermediate, and minor principal stresses, respectively, σ_t, σ_c, and τ_s are the tensile, compressive and shear yield strengths, respectively, α denotes the tension-compression strength ratio, and b is a preset parameter of different failure criteria, which range is $0 \le b \le 1$.

1.2. Yield Condition. Under axisymmetric plane strain conditions, the tangential stress σ_θ and radial stress σ_r are the principal stresses σ_1 and σ_3, respectively; the axial stress σ_z is the mean principal stress σ_2. If $\sigma_1 \ge \sigma_2 \ge \sigma_3$, the following expression is valid:

$$\sigma_1 = \sigma_\theta > 0, \qquad \sigma_2 = \sigma_z, \qquad \sigma_3 = \sigma_r < 0. \tag{2}$$

A simple empirical correlation was introduced by Yu et al. [20]: $\sigma_z = \frac{m}{2}(\sigma_r + \sigma_\theta)$, where m is an empirical constant with $0 \le m \le 1$. It was assumed that, $m = 2\nu$ (ν is Poisson's ratio) in the elastic zone and $m \to 1$ in the plastic zone. Using Eq. (2), the intermediate principal stress σ_2 can be given as $\sigma_2 = \frac{m}{2}(\sigma_1 + \sigma_3)$. Because $\alpha \le 1$, the condition $\sigma_2 \le \dfrac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$ should be adopted. Then, Eq. (1) can be reduced to

$$\sigma_\theta - \frac{\alpha}{1+b}\left(bm\frac{\sigma_r + \sigma_\theta}{2} + \sigma_r\right) = \sigma_s. \tag{3}$$

Equation (3) can be rewritten as follows:

$$\frac{2+2b-\alpha bm}{2+2b}\sigma_\theta - \frac{\alpha bm + 2\alpha}{2+2b}\sigma_r = \sigma_s, \tag{4}$$

where Eq. (4) is the yield condition of a thick-walled cylinder based on the UST.

2. Fundamental Solutions of Double-Layered Cylinder. A thick-walled cylinder with inner radius r_i and outer radius r_o bears inner pressure p_1 and outer pressure p_2. The elastic stresses can be derived as in [19]:

$$\begin{aligned} \sigma_r &= \frac{r_i^2 r_o^2 (p_2 - p_1)}{r_o^2 - r_i^2}\frac{1}{r^2} + \frac{r_i^2 p_1 - r_o^2 p_2}{r_o^2 - r_i^2}, \\ \sigma_\theta &= -\frac{r_i^2 r_o^2 (p_2 - p_1)}{r_o^2 - r_i^2}\frac{1}{r^2} + \frac{r_i^2 p_1 - r_o^2 p_2}{r_o^2 - r_i^2}, \end{aligned} \tag{5}$$

where σ_r and σ_θ are radial and tangential stresses, respectively.

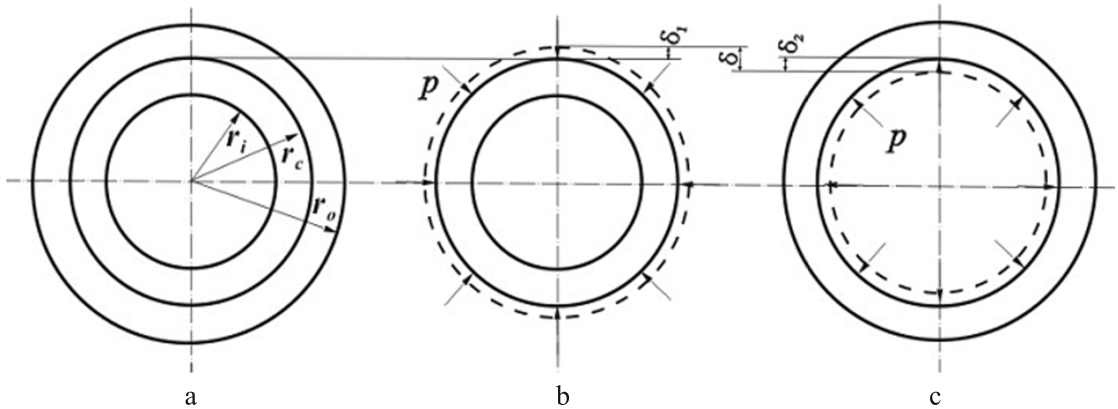

Fig. 1. Double-layered thick-walled cylinder: (a) double-layered cylinder; (b) inner cylinder; (c) outer cylinder.

2.1. Assemblage Pressure. Figure 1a shows a double-layered cylinder made of the same material with an inner radius r_i, outer radius r_o, and separate radius r_c. Figure 1b and 1c shows a combined cylinder composed of the inner cylinder (with inner radius r_i and outer radius $r_c + \delta_1$) and outer cylinder (with inner radius $r_c - \delta_2$ and outer radius r_o), respectively. It is assumed that the material of both cylinders is identical. When assembling, the outer cylinder needs to be heated to increase it inner radius and place it over the inner one. Upon cooling, a certain contact (assemblage) pressure between both cylinders is generated.

As shown in Fig. 1b and 1c, the shrink range δ in the assemblage zone is determined as

$$\delta = \delta_2 - \delta_1. \tag{6}$$

According to theory of elasticity, the assemblage pressure equation takes the following form [15]:

$$p = \frac{E\delta}{r_c} \frac{(r_c^2 - r_i^2)(r_o^2)}{2r_c^2 (r_o^2 - r_i^2)}, \tag{7}$$

where E is the cylinder material's elastic modulus under plane strain conditions.

Equation (7) implies that the assemblage pressure p can be determined if the shrink range δ and separate radius r_c are given; thereby, the assemblage stress generated in the inner and outer cylinders can be obtained.

2.2. Separate Radius and Assemblage Shrink Range. The optimal solution of the separate radius r_c and shrink range δ for assemblage containers made of the same material can be established by utilizing the inner wall of the inner cylinder and inner wall of the outer cylinder simultaneously to satisfy the yielding condition. It is assumed that the inner cylinder bears inner pressure p_1 and assemblage pressure p at $r = r_c$. To ensure that the inner and outer cylinders yield at the same time, the inner cylinder at $r = r_i$ should have the same pressure as that of the outer cylinder at $r = r_c$, where the stress at the interface can be expressed from Eq. (4) as

$$\left(\frac{2 + 2b - \alpha bm}{2 + 2b} \sigma_\theta - \frac{\alpha bm + 2\alpha}{2 + 2b} \sigma_r \right).$$

(1) An inner cylinder comes under internal pressure p_1 at the inner wall and external pressure q at the outer wall. Then, the external pressure q is defined as

$$q = p + \sigma_r|_{r=r_c}, \tag{8}$$

where $\sigma_r|_{r=r_c}$ denotes the radial stress at $r = r_c$ under the internal pressure p_1, which is formulated as follows:

$$\sigma_r\Big|_{r=r_c} = \frac{r_i^2(r_c^2 - r_o^2)}{r_c^2(r_o^2 - r_i^2)} p_1. \tag{9}$$

(2) An outer cylinder bears inner pressure q at the inner wall and zero pressure at the outer wall, i.e., $\sigma_r|_{r=r_o} = 0$. For the inner cylinder, the following equation obtained from Eq. (5) at $r = r_i$ is given as

$$\left(\frac{2+2b-\alpha bm}{2+2b}\sigma_\theta - \frac{\alpha bm + 2\alpha}{2+2b}\sigma_r\right)_{r=r_i} = \frac{2+2b-\alpha bm}{2+2b}\frac{[(r_i^2 + r_c^2)p_1 - 2r_c^2 q]}{r_c^2 - r_i^2} + \frac{\alpha bm + 2\alpha}{2+2b} p_1. \tag{10}$$

For the outer cylinder, the stress component is deduced by setting $r = r_c$ in Eq. (5) as

$$\left(\frac{2+2b-\alpha bm}{2+2b}\sigma_\theta - \frac{\alpha bm + 2\alpha}{2+2b}\sigma_r\right)_{r=r_c} = \frac{2+2b-\alpha bm}{2+2b}\frac{r_o^2 + r_c^2}{r_o^2 - r_c^2} q + \frac{\alpha bm + 2\alpha}{2+2b} q. \tag{11}$$

By integrating Eqs. (10) and (11), the following equation can be manipulated as

$$q = \frac{[(2+2b+2\alpha)r_c^2 + (2+2b-2\alpha bm - 2\alpha)r_i^2](r_o^2 - r_c^2)}{(6+6b-2\alpha bm + 2\alpha)r_o^2 r_c^2 - (2+2b-2\alpha bm - 2\alpha)r_i^2 r_c^2 - (2+2b+2\alpha)(r_o^2 r_i^2 + r_c^4)} p_1, \tag{12}$$

where k_1, k_2, and k_3 are defined as $k_1 = 2+2b+2\alpha$, $k_2 = 2+2b-2\alpha bm - 2\alpha$, and $k_3 = 6+6b-2\alpha bm + 2\alpha$, respectively. Then, Eq. (12) can be simplified as

$$q = \frac{[k_1 r_c^2 + k_2 r_i^2](r_o^2 - r_c^2)}{k_3 r_o^2 r_c^2 - k_2 (r_i^2 r_c^2 - k_1 (r_o^2 r_i^2 + r_c^4)} p_1. \tag{13}$$

By substituting Eq. (13) into $\left(\frac{2+2b-\alpha bm}{2+2b}\sigma_\theta - \frac{\alpha bm + 2\alpha}{2+2b}\sigma_r\right)$, the function of r_c is determined as

$$f(r_c) = \frac{(k_1 r_o^2 + k_3 r_c^2)(k_1 r_c^2 + k_2 r_i^2}{(2+2b)[k_3 r_o^2 r_c^2 - k_2 r_c^2 r_i^2 - k_1 (r_o^2 r_i^2 + r_c^4)]} p_1. \tag{14}$$

The function equation f has a minimum value, where the function must satisfy $df/dr_c = 0$. Then, the separate radius is given as

$$r_c = \sqrt{r_o r_i}. \tag{15}$$

By introducing Eq. (15) into Eq. (13), the following expression is obtained:

$$q = \frac{(k_1 r_o + k_2 r_i)(r_o - r_i)}{k_3 r_o^2 - k_2 r_i^2 - 2k_1 r_o r_i} p_1. \tag{16}$$

Then, the assemblage pressure p is generated as

$$p = q - \sigma_r\Big|_{r=r_c} = \frac{k_1 r_o^3 + (k_2 - k_3)r_o^2 r_i + k_1 r_o r_i^2}{(r_o + r_i)(k_3 r_o^3 - k_2 r_i^2 - 2k_1 r_o r_i)}. \tag{17}$$

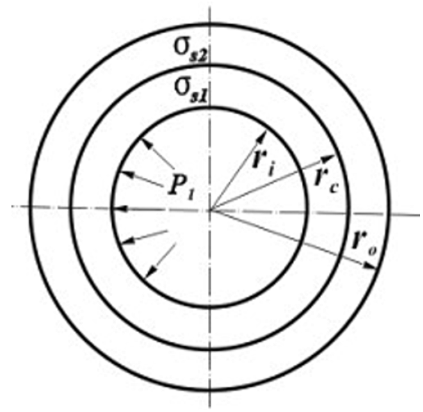

Fig. 2. Double-layered cylinder with different yield limits.

If the combined cylinder yields, by integrating Eq. (16) and using the yield condition with Eq. (10), that is, $\frac{2+2b-\alpha bm}{2+2b}\frac{[(r_i^2+r_c^2)p_1-2r_c^2 q]}{r_c^2-r_i^2}+\frac{\alpha bm+2\alpha}{2+2b}p_1=\sigma_s$, the internal pressure p_1 can be transmuted as

$$p_1=\frac{k_3 r_o^2-k_2 r_i^2-2k_1 r_o r_i^2}{(k_1 r_o+k_2 r_i)^2}(2+2b)\sigma_s. \tag{18}$$

By integrating Eqs. (5), (15), and (17), the assemblage shrink range can be expressed as

$$\delta=\frac{2\sqrt{r_o r_i}}{E}\frac{k_1 r_o^3+(k_2-k_3)r_o^2 r_i+k_1 r_o r_i^2}{k_3 r_o^3-(2k_1+k_3)r_o^2 r_i+(2k_1-k_2)r_o r_i^2-k_2 r_i^3}p_1. \tag{19}$$

3. Elastoplastic Analysis of the Double-Layered Cylinder. An infinitely long cylinder with yield strength σ_{s1} of the inner cylinder and yield strength σ_{s2} of the outer cylinder satisfies $\sigma_{s1}>\sigma_{s2}$. Figure 2 shows a double-layered cylinder with inner radius r_a, outer radius r_b, and separate radius r_c which is subjected to uniform pressure p_1. When both the inner and outer cylinders reach the plastic limit state, the plastic ultimate load of the combined cylinder can be determined.

The stress equilibrium equation for a thick-walled cylinder can be obtained as [15]

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\theta}{r}. \tag{20}$$

By combining Eqs. (4) and (20), the following equation is obtained:

$$\frac{d\sigma_r}{dr}+\frac{2+2b-2\alpha-2\alpha bm}{r(2+2b-\alpha bm)}\sigma_r=\frac{2+2b}{r(2+2b-\alpha bm)}\sigma_s. \tag{21}$$

By integration, the radial stress σ_r is derived as

$$\sigma_r=Cr^{\frac{2\alpha+2\alpha bm-2-2b}{2+2b-\alpha bm}}+\frac{\sigma_s}{1-\alpha}, \tag{22}$$

where C is an unascertained coefficient.

With the stress boundary situation, $\sigma_r = -p_1$ and $\sigma_s = \sigma_{s1}$ at $r = r_i$, the constant C can be expressed as

$$C = \left(-p_l - \frac{\sigma_{s1}}{1-\alpha}\right)\left(\frac{1}{r_i}\right)^{\frac{2\alpha+2\alpha bm-2-2b}{2+2b-\alpha bm}}, \tag{23}$$

where p_l is the plastic limit internal pressure. Then, Eq. (22) takes the following form:

$$\sigma_r = \left(-p_l - \frac{\sigma_{s1}}{1-\alpha}\right)\left(\frac{r}{r_i}\right)^{\frac{2\alpha+2\alpha bm-2-2b}{2+2b-\alpha bm}} + \frac{\sigma_{s1}}{1-\alpha}. \tag{24}$$

With the outer boundary condition, $\sigma_r = -q$ at $r = r_c$, the following formula is given by

$$p_l - q = \left(-p_l - \frac{\sigma_{s1}}{1-\alpha}\right)\left[\left(\frac{r_c}{r_i}\right)^{\frac{2\alpha+2\alpha bm-2-2b}{2+2b-\alpha bm}} - 1\right]. \tag{25}$$

The plastic limit pressure q of the outer cylinder takes the following form [21]:

$$q = \left(\frac{\sigma_{s2}}{1-\alpha}\right)\left[\left(\frac{r_c}{r_o}\right)^{\frac{2\alpha+2\alpha bm-2-2b}{2+2b-\alpha bm}} - 1\right]. \tag{26}$$

After substituting Eq. (26) into Eq. (25), p_l is taken as

$$p_l = \frac{\sigma_{s1}}{1-\alpha}\left[\left(\frac{r_i}{r_c}\right)^{\frac{2\alpha+2\alpha bm-2-2b}{2+2b-\alpha bm}} - 1\right] + \left(\frac{r_i}{r_c}\right)^{\frac{2\alpha+2\alpha bm-2-2b}{2+2b-\alpha bm}} \frac{\sigma_{s2}}{1-\alpha}\left[\left(\frac{r_c}{r_o}\right)^{\frac{2\alpha+2\alpha bm-2-2b}{2+2b-\alpha bm}} - 1\right]$$

$$= p_{l1} + \left(\frac{r_i}{r_c}\right)^{\frac{2\alpha+2\alpha bm-2-2b}{2+2b-\alpha bm}} p_{l2}. \tag{27}$$

It can be seen from Eq. (27) that the plastic allowable pressure of a combined cylinder with different materials is not a simple superposition of the plastic ultimate bearing capacity of a two-layer cylinder. This result is different from the analysis result based on the Tresca yield. The plastic allowable bearing capacity of the combined cylinder with the same material can be obtained by setting $\sigma_{s1} = \sigma_{s2} = \sigma_s$ in the above equation as follows:

$$p_l = \frac{\sigma_s}{1-\alpha}\left[\left(\frac{r_i}{r_o}\right)^{\frac{2\alpha+2\alpha bm-2-2b}{2+2b-\alpha bm}} - 1\right], \tag{28}$$

where m is an empirical coefficient, which value is presented by theory and experiment [20]. For simplicity, the general approximation is $m = 1$, then Eq. (28) is reduce to:

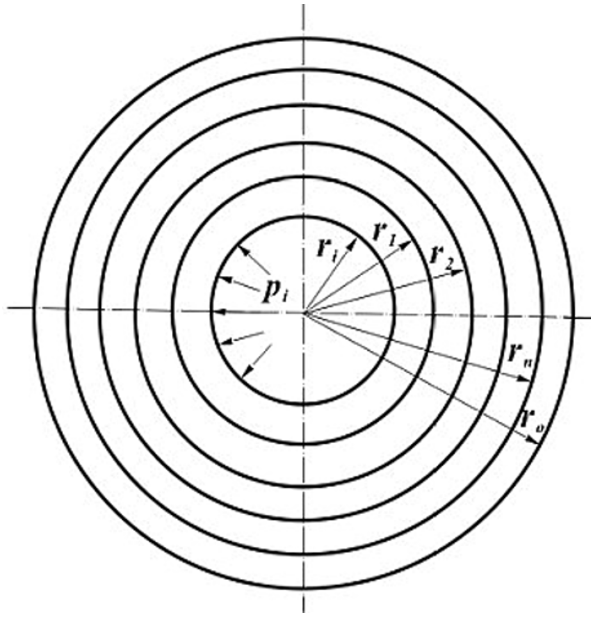

Fig. 3. Multilayered thick-walled cylinder.

$$p_l = \frac{\sigma_s}{1-\alpha}\left[\left(\frac{r_i}{r_o}\right)^{\frac{(2+2b)(\alpha-1)}{2+2b-\alpha b}} - 1\right], \tag{29}$$

where the degradation formula in Eq. (29) herein is the same as the corresponding result in [21].

It can be seen from the above equation that the structural plastic ultimate capacity with the same material is related to the radius ratio r_i/r_o, strength parameter b, and tension-compression ratio α and is independent of the separate radius r_c and assemblage pressure p. That is, a composite cylinder of the same material can be considered a single-layered cylinder of the same thickness when solving for the plastic limit bearing capacity.

4. Elastic Analysis of the Multilayered Thick-Walled Cylinder. The stress distribution of multilayered cylinder related to n cylinders is more rational than that of single-layered cylinder. Figure 3 demonstrates a multilayered cylinder set with inner radii and outer radii (r_i, r_1), (r_1, r_2), (r_2, r_3), ..., (r_n, r_o) in sequence with the same material that bears an evenly supported pressure p_i.

By assuming a multilayered thick-walled cylinder subjected to internal pressures $q_1, q_2, \ldots, q_n$ between the layers, the following expression can be obtained when the inner surface of each layer yields:

$$\begin{cases} p_e - q_1 = \sigma_s \dfrac{r_1^2 - r_i^2}{r_1^2 + r_i^2 + \dfrac{\alpha}{1+b} r_1^2 - \alpha r_i^2}, \\ q_1 - q_2 = \sigma_s \dfrac{r_2^2 - r_1^2}{r_2^2 + r_1^2 + \dfrac{\alpha}{1+b} r_2^2 - \alpha r_1^2}, \\ \ldots \ \ldots \ \ldots \ \ldots \ \ldots \ \ldots \ \ldots \ \ldots \ \ldots \\ q_n = \sigma_s \dfrac{r_o^2 - r_n^2}{r_o^2 + r_n^2 + \dfrac{\alpha}{1+b} r_o^2 - \alpha r_n^2}. \end{cases} \tag{30}$$

By adding the above n equations, we can obtain

$$p_e = \sigma_s \left[\frac{r_1^2 - r_i^2}{r_1^2 + r_i^2 + \frac{\alpha}{1+b} r_1^2 - \alpha r_i^2} + \frac{r_2^2 - r_1^2}{r_2^2 + r_1^2 + \frac{\alpha}{1+b} r_2^2 - \alpha r_1^2} + \ldots + \frac{r_o^2 - r_n^2}{r_o^2 + r_n^2 + \frac{\alpha}{1+b} r_o^2 - \alpha r_n^2} \right]. \tag{31}$$

If the elastic ultimate pressure can have a maximum value, where the pressure must satisfy $\frac{\partial p_e}{\partial r_1} = \frac{\partial p_e}{\partial r_2} = \ldots = \frac{\partial p_e}{\partial r_n} = 0$. Then, the following equation is derived as

$$\frac{r_1}{r_i} = \frac{r_2}{r_1} = \frac{r_3}{r_2} = \cdots = \frac{r_o}{r_n} = \left(\frac{r_o}{r_i} \right)^{\frac{1}{n+1}}. \tag{32}$$

As shown in Eq. (32), when the radius of each layer satisfies $\left(\frac{r_o}{r_i} \right)^{\frac{1}{n+1}}$, the maximum value of p_e can be established as

$$p_e = \sigma_s n \frac{1 - \left(\frac{r_i}{r_o} \right)^{\frac{2}{n+1}}}{\left(1 + \frac{\alpha}{1+b} \right) + (1-\alpha) \left(\frac{r_i}{r_o} \right)^{\frac{2}{n+1}}}. \tag{33}$$

Equation (33) can be reduced to the Tresca criterion solution with $\alpha = 1$ and $b = 0$ [15].

5. Results and Discussion.

5.1. Degradation Validation of Solutions. The parameter b exhibits the impact extent that the intermediate principal stress can induce the failure of multiple cylinder. With a specific value of b, the UST can come down to various existing failure criteria. For instance, the UST simplifies to the Tresca criterion with $\alpha = 1$ and $b = 0$. The von Mises criterion can be estimated with $\alpha = 1$ and $b = 1/3$. When the parameter α varies between 0 and 1, the Mohr–Coulomb failure criterion is established with $b = 0$.

The separate radius r_c can be determined by Eq. (15) from the known internal and external radii, and this conclusion is the same as the corresponding result obtained by the Tresca criterion. Equations (17) and (19) are the unified solutions of the assemblage pressure p and shrink range δ with consideration of different tension-compression strength characteristics. It can be seen from Eq. (19) that the shrink range δ is related to the internal radius r_i, external radius r_o, and material characteristics. Under the establishment of the above equations, the Tresca criterion solutions are specified from Eqs. (17) and (19) with $\alpha = 1$ and $b = 0$ [15].

5.2. Parametric Studies. Supposing that the multiple cylinder is constituted of the same material. The effects of the tension-compression ratio α (the value is set to 0.2, 0.4, 0.6, 0.8, and 1.0) and the strength theory parameter b (the value is set to 0, 0.25, 0.5, 0.75, and 1.0) on the plastic limit pressure pl are investigated, as shown in Fig. 4. As shown in Fig. 4a, the ratio of p_l/σ_s decreases with increasing α values. The tensile strength and compressive strength impact the failure of thick-walled cylinders. p_l/σ_s has a minimum value where the coefficient satisfies $\alpha = 1$. Figure 4 shows that the ratio of increases with parameter b. The p_l/σ_s value is increased by 33.3% when $b = 1$ compared with that of $b = 0$ for $r_o/r_i = 4.0$ and $\alpha = 1$. The parameter b value represents different strength theories, which have a large impact on the limit solution of the combined cylinder. Figure 4b illuminates that the increasing rate of p_l/σ_s is relatively obvious for different r_o/r_i values.

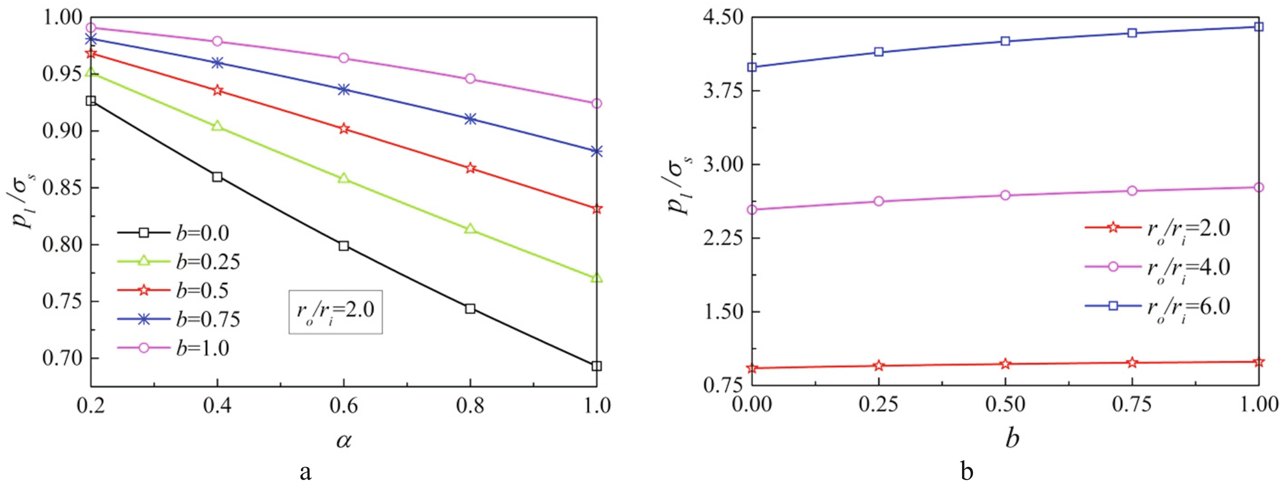

Fig. 4. p_l/σ_s versus the coefficients α and b with different r_o/r_i.

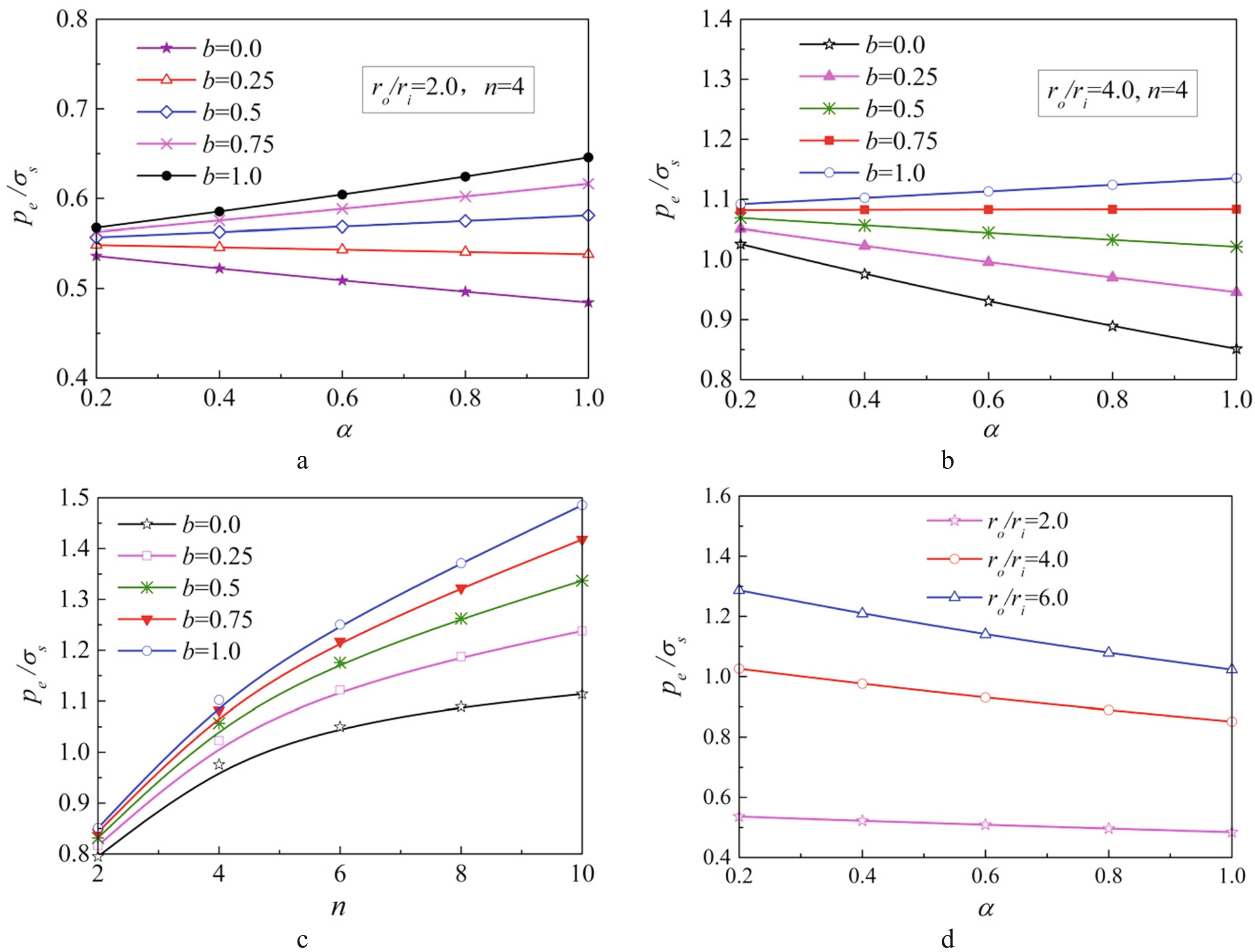

Fig. 5. p_e/σ_s versus the coefficient α, b, n, and r_o/r_i.

Figure 5 shows the elastic limit pressure p_e/σ_s versus the different parameters. As shown in Fig. 5a and b, the ratio of p_e/σ_s improves with increasing b values. The significant differences in the results with various b values are a clear indication that the intermediate principal stress effect should be rationally considered. In observing

Fig. 5c, the value of p_e/σ_s increases with the increase in the number of cylinder layers n. From Fig. 5a–c, p_e/σ_s is taken as the minimum value when $\alpha = 1$. A significant difference with various α is a clear indication that the error might arise if tension-compression ratio is not properly considered. The influence of the tension-compression ratio α can be better applied to the ultimate load analysis of various materials. According to the solution in this paper, the effect of the tensile-compression ratio can be discussed, and test guidance can be given. Moreover, Fig. 5d illustrates that the influence of the radius ratio r_o/r_i on the p_e/σ_s value is obvious with an increasing tension-compression strength ratio.

Conclusions. Considering intermediate principal stress and different tension-compression strengths, the elastoplastic unified solutions of double-layered and multilayered cylinders are derived on the basis of the UST. Moreover, the fundamental solutions of the separate radius, assemblage pressure and shrink range are derived for double-layered thick-walled cylinders. The effects of failure criteria parameter b, tensile-compressive strength ratio α, radius ratio r_o/r_i, and combined cylinder layers n on the ultimate results are significant. Furthermore, structural strength potentialities are fully achieved with the UST. The final solutions have general application and are universal for use. In conclusion, the proposed formulation of allowable limit pressure for multiple cylinder is more consistent with the true results by considering the intermediate principal stress and cylinder layers. According to the solutions in this paper, we can discuss the influence degree of the tension-compression ratio and different strength criteria and solve the related problems of thick-walled cylinder pressure vessels with different criteria.

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