EFFECT OF THE CONTACT SURFACES ORIENTATION IN THE SHROUDED FLANGES AND LEVEL OF VIBRATION EXCITATION IN THE ROTOR BLADES ON THEIR VIBRATION STRESS STATE

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The authors present the results of the numerical experiments on the determination of the effect of shrouding coupling parameters in the rotor blades on their forced vibrations using an assembly of two blades as an example. The task is solved in the variation of the inclination angle of the shrouded flanges as the parameter of their contact interaction and amplitude of the kinematic excitation of vibrations caused by the harmonic displacement of the root section of the assembly. The displacement amplitude in the plane of the assembly vibrations and stress intensity in the blade airfoil is accepted as the characteristic of the forced blade vibrations. The finite element models of the blade assembly, which are based on the linear 8- and 4-nodal contact finite elements, are used to perform the investigation. The authors have obtained the dependencies of the selected characteristics of forced vibrations within the blade airfoil on the inclination angle of the contact surfaces of their shrouded flanges to the plane of rotation of the rotor wheel and level of kinematic excitation of the assembly vibrations. It is implied that the variation in the angle has a considerable effect on the stress state in the blades under investigation and it results in the occurrence of the local minimum of their forced vibration characteristics. Here the increase in the amplitude of the kinematic displacement of the root section in the assembly causes the increase of rigidity of the shrouded blade coupling and reduction in the triggered energy scatter.

Introduction. Production of turbine machines of various designations, primarily of aircraft gas-turbine engines, is one of the high-tech and energy consumption fields in modern machine building. The necessity to reduce the material consumption and enhance their technical and economical parameters imposes more severe requirements on such highly-loaded structural elements as rotor blades at both stages: design and long-term operation. The first case involves the engine lifetime increase, while the second one is based on the assurance of its functional performance within the wide range of variation in the engine modes, which are, as a rule, accompanied by different types of defects. 40% of all the expenses are spent in the process of turbine machine development. Furthermore, most of the defects detected in the engine operation are of vibration origin, a large number of them are in the turbine rotor wheel blade assemblies. Therefore, their vibration reliability assurance is crucial in the design of rotor blades.

One of the most efficient methods of the blade vibration stress level reduction is the structural damping of vibration, namely, reciprocal displacements of the contact surfaces of shrouded [1–4] and blade–disc [1, 5] joints of blades. Moreover, it can increase due to the presence of underplatform dampers [6, 7] and damping coatings for the

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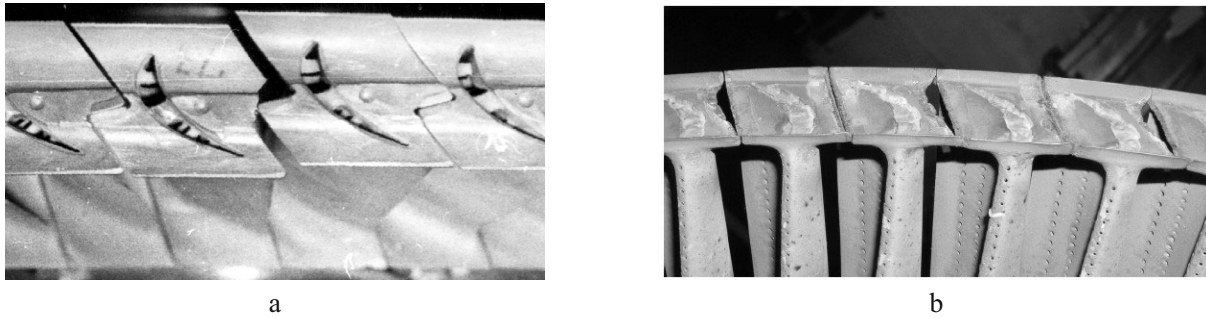
a   b

Fig. 1. Examples of fracture of the contact surfaces of similar (a) and straight (b) shrouded flanges of the rotor blades.

blade airfoil [8]. Such sources of energy loss cause the reduction of the susceptibility of the assembly blades to the induction of critical vibrations.

Concerning the shrouded coupling of the blades, there are different types of flanges (similar and straight), and there are various shrouding methods (annular, package, pairwise). The basic disadvantage of the use of the shrouded blades is the increased stress state in the flanges. It can lead to the occurrence of cracks and their subsequent fracture (Fig. 1). Among the reasons are the peculiar features of the interaction between the contact surfaces of the shrouded flanges, firstly due to its nonlinear nature [9, 10].

Although many scientific papers show the results of the calculated [3–7, 9] and experimental [1, 2, 10–12] investigations on the determination of the effect of the contact interaction between the shrouded flanges on the characteristics of forced vibrations of the rotor blades, the contact interaction effect remains an open issue.

The goal of the paper is the numerical investigation of the effect of the inclination angle of the shrouded flange surfaces relative to the rotation plane of the rotor wheel and amplitude of the harmonic excitation of vibrations on the blade vibration stress state.

Object of Investigation and Its Modeling. The analysis of forced vibrations of blade assemblies was performed with the assumption that they are cyclic-symmetric systems, i.e., all blades have similar geometric and mechanical properties. The investigation of these assemblies is based on considering their period with the corresponding boundary conditions. As a rule, it is expected that it consists of one blade. However, to consider the conditions of interaction between the shrouded flanges over the period of the assembly, it is required to select the assembly with two blades. It should be noted that this assembly is the simplest regular system that allows one to determine the influence of structural and operational mechanical factors on the formation of its vibrations, including the frequency mistuning. The assembly with two blades having a straight flange was selected for the calculated experiment (Fig. 2a). It was predicted that there is no effect of the blade–disk joint and field of centrifugal forces.

The shrouded flanges of the blades interact along the contact surfaces K (Fig. 2b), where α is their inclination angle relative to the rotation plane of the rotor wheel, t_s is the blade spacing, and F_N is the resultant of normal forces acting at the contact surfaces.

The blades are made of heat-resistant alloy ZhS 26-VI with the following mechanical characteristics: elastic modulus of the 1st kind $E = 1.9 \cdot 10^{11}$ Pa, density $\rho = 8750$ kg/m^3, and Poisson's ratio $\mu = 0.3$.

To construct the finite element (FE) model of the rotor blade as in [13, 14], a three-dimensional 8-node finite element and its modifications were used, whereas for modeling of the contact interaction of the shrouded flanges – 4-node contact element that allows one to detect the relative positioning of the respective contact surfaces. The characteristics of the stress state in the blades were determined in nodes A and B (Fig. 2a).

Calculation Procedure for Forced Vibrations. Due to the contact interaction between the shrouded flanges, the object of investigation is the nonlinear vibrating system. The matrix equation for forced vibrations of the FE model for this system has the following form:

$$[M]\{\ddot{u}\} + [D]\{\dot{u}\} + ([K_l] + [K_{nl}])\{u\} = \{F(t)\}, \tag{1}$$
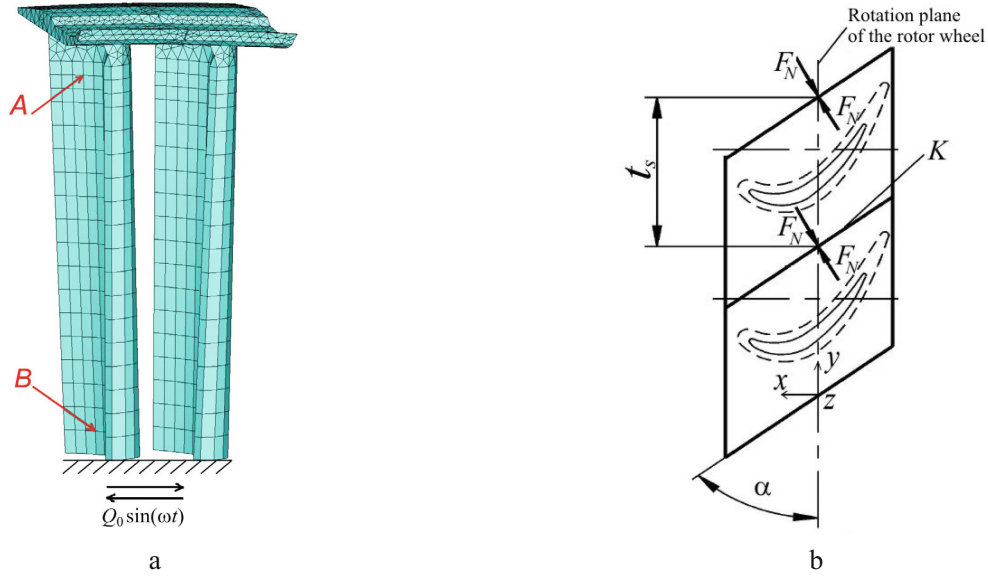
Fig. 2. General view of the FE model of the assembly with two blades (a) and scheme of the interaction between the contact surfaces of shrouded flanges (b).

where $[M]$ and $[D]$ are inertia and dissipative matrices, $[K_l]$ and $[K_{nl}]$ are linear and nonlinear components of the stiffness matrix $[K]$, $\{F(t)\}$ is the column-vector of the generalized forces, and $\{u\}$, $\{\dot{u}\}$, and $\{\ddot{u}\}$ are vector-columns of the displacements, rates, and accelerations, respectively.

All the matrices and vector-columns of Eq. (1) have the block form. Considering blocks as elements of the specified components of the given equation regarding the selected object of investigation, their order coincides with the number of blades in the assembly. The order of the matrix blocks is defined by the dimensions of the FE mesh of its model.

The block size $[K_{nl}]$, which is defined by the contact interaction between shrouded flanges of the blades [15], depends on the selected option of their contact interaction, namely, the number of contact nodes with the elements presented as

$$\begin{vmatrix} k_s & 0 & -k_s & 0 \\ 0 & k_n & 0 & -k_n \\ -k_s & 0 & k_s & 0 \\ 0 & -k_n & 0 & k_n \end{vmatrix}, \tag{2}$$

where k is the stiffness factor in qth node of the joint, while n, s are the indices characterizing the normal and tangential components of the stress state characteristics, respectively.

The stiffness factor components k_s should meet the following conditions:

$$\begin{cases} k_s \neq 0, & \text{if} \quad \eta|F_n| > |F_s|, \\ k_s = 0, & \text{if} \quad \eta|F_n| = |F_s|, \end{cases} \tag{3}$$

where F represents the internal forces in the node, and η is the friction coefficient of the contact surfaces of the shrouded coupling.

The solution to Eq. (1) is possible only with the determined nonlinear component $[K_{BN}]$ of the stiffness matrix $[K]$. The Newton–Raphson method [16] is used for this purpose, which is based on the solution to the static nonlinear contact problem described by

$$([K_l] + [K_{BN}])\{u\} = \{F(t_i)\}. \tag{4}$$

207

As a result, the contact surface nodes q and p are determined, which are in contact with each other. The following relation is true for them

$$F_n = k_n (u_n^q - u_n^p - \Delta_n) \neq 0, \qquad F_s = k_s (u_s^q - u_s^p - \Delta_s) \neq 0, \tag{5}$$

where Δ is the parameter of the iteration process in compliance with the Newton–Raphson procedure.

Next, the complete system of nonlinear differential equations (1) is solved with its integration by the Newmark method [16]:

$$\begin{cases} \{\dot{u}\}_{t+\Delta t} = \{\dot{u}\}_t + [(1-\lambda)\{\ddot{u}\}_{t+\Delta t}]\Delta t, \\ \{u\}_{t+\Delta t} = \{u\}_t + \{\dot{u}\}\Delta t + \left[\left(\frac{1}{2} - \beta\right)\{\ddot{u}\}_t + \beta\{\ddot{u}\}_{t+\Delta t}\right]\Delta t^2, \end{cases} \tag{6}$$

where β and λ are the parameters that determine the accuracy and stability of integration, respectively.

The key advantage of this method lies in the determination of the nonlinear component of the stiffness matrix with each time step using the Newton–Raphson method.

Results and Discussion. A set of numerical experiments was performed to determine the effect of the inclination angle α of the contact surfaces of shrouded flanges' surfaces relative to the plane of the rotor wheel rotation and excitation level of vibrations on the vibration stress state of the blades using the developed FE models of the selected assembly of blades and calculation procedure.

The analysis of forced vibrations of the object of investigation was performed in the variation of the inclination angle α as in [13] within the range from 45° to 65°.

The excitation of forced vibrations of the assembly was modeled by the harmonic displacement $Q_0 \sin(\omega t)$ of the end elements of the blade airfoil perpendicular to the plane of the rotor wheel rotation along the Oy axis, namely, its kinematic was considered. The amplitude Q_0 of such displacement varied from of 0.01 to 0.1 mm.

The numerical experiments were performed assuming that the assembly consisted of the blades without frequency mistuning. It means that in the kinematic excitation there will be only in-phase vibrations.

According to the developed procedure for the calculations, the authors obtained the relations of displacements (node A in Fig. 2a) and stress intensities of the blades (node B in Fig. 2a), according to the following equation:

$$\sigma_j = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1}, \tag{7}$$

where σ_j ($j = 1, 2, 3$) are the principal stresses.

Using the results of the harmonic analysis for the portions of these relations characterizing the stationary regime of vibrations, the amplitude-and-frequency characteristics (AFC) were derived for the parameters of forced vibrations of the blade assembly depending on the inclination angle of the contact surfaces of shrouded flanges (Fig. 3). In Fig. 3 and further: $\overline{A}_{\alpha Q_0}$ and $\overline{\sigma}_{\alpha Q_0}$ are values of the displacement amplitude along the Oy axis and stress intensity in the blade airfoil, respectively, normalized by their maximum values at $\alpha = 45°$ and $Q_0 = 0.1$ mm.

The data analysis implies that, firstly, the resonance vibrations of the blade assembly for the specified values are within the narrow range of the variation of the excitation frequency in the direction of relative displacements of the contact surfaces of shrouded flanges. As a result, as seen from the numerical model of the elastic-dissipative coupling between the shrouded blades [17], there is a decline in the natural frequency of in-phase mode of vibrations of the assembly as a regular system. The increase of the contact area and the maximum values of pressure on the contact surfaces of shrouded flanges with an increase in α corroborates this phenomenon. It is in concert with the static contact problem solution [13] presented in Fig. 4.

Secondly, the resonance amplitudes of displacement $A_{\alpha 0.1}^{(R)}$ are practically identical for all values of α (diagram of the relative resonance amplitudes in Fig. 5). The data on the determination of the logarithmic vibration decrement δ for the assembly of the blades using the resonance curve method validates this fact (Table 1). They correlate with the maximum amplitudes of the blades' displacement (Fig. 3a).
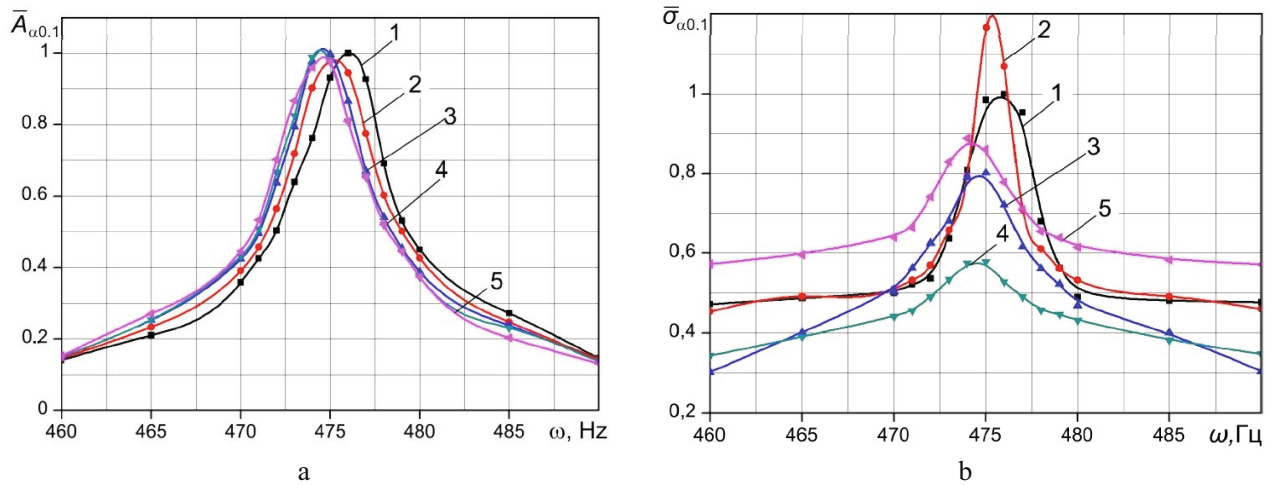
Fig. 3. Amplitude and frequency characteristics of displacements along the *Oy* axis (a) and stress intensity (b) of the blade assembly with the amplitude of the harmonic displacement $Q_0 = 0.1$ mm and inclination angle α equal to 45° (*1*), 50° (*2*), 55° (*3*), 60° (*4*), and 65° (*5*).
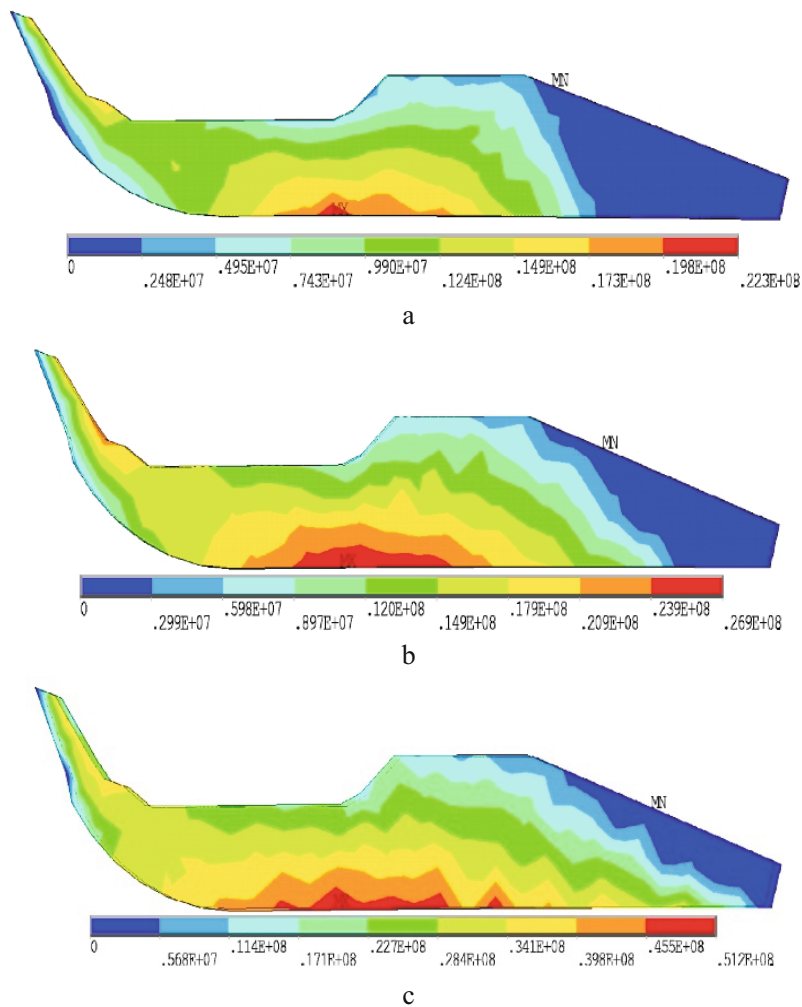

Fig. 4. Pressure distribution over the contact surfaces of the straight shrouded flanges at $\alpha = 50°$ (a), 55° (b), and 65° (c).

TABLE 1. Logarithmic Decrement of the Blade Assembly Vibrations in the Variation of the Inclination Angle of the Contact Surfaces of Shrouded Flanges and Amplitude of the Kinematic Displacement of the Root Section

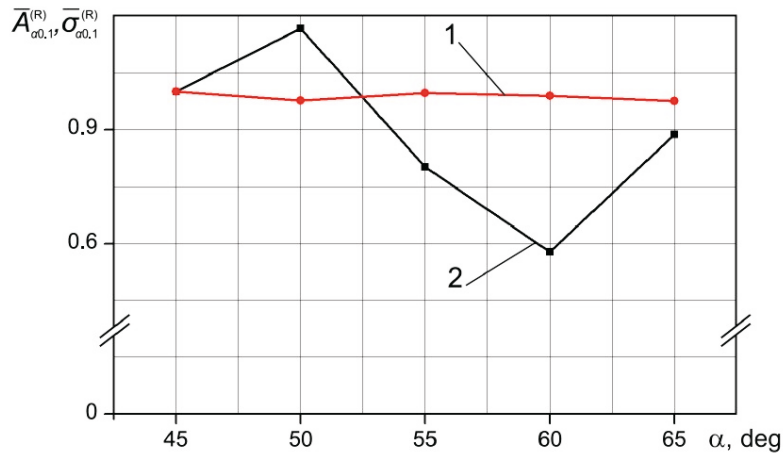| α, deg	δ, %, at Q_0, mm			
	0.01	0.05	0.07	0.10
45	–	–	–	2.61
50	–	–	–	2.75
55	5.31	2.56	2.67	2.59
60	–	–	–	2.60
65	–	–	–	2.75

Fig. 5. Diagrams of the relative resonance amplitudes of displacements (*1*) and stress intensity (*2*) of the blade assembly depending on the inclination angle α of the contact surfaces of the shrouded flanges at $Q_0 = 0.1$ mm.

Thirdly, there is a significant (namely, two-fold) difference between the resonance amplitudes of the airfoil stress intensity $\sigma_{\alpha\,0.1}^{(R)}$ as is seen from the data in Fig. 5). The minimum values were observed in a quite narrow range of the variation of the inclination angle of the contact surfaces (in our case, $\alpha = 60°$), which is qualitatively consistent with the results obtained earlier [10].

The significant difference in the formation of AFC of the selected parameters of forced vibrations can be related to the fact that the blade displacements were determined only over the axis Oy, while the vibration stress state was determined by the formula (7) describing their volumetric stress state. Based on this it can be concluded that the excitation mode of vibrations of blades is characterized by their flexural-torsional coupling.

The calculations of the forced vibration characteristics were performed in the variation of the amplitude of the kinematic displacement Q_0 of the root section in the assembly. Using the results of the calculated experiments the AFC of the selected parameters of forced vibrations were obtained for various values Q_0 (Fig. 6). The diagrams of the resonance values for these characteristics were plotted for the more detailed analysis (Fig. 7). Moreover, the logarithmic decrement of vibrations was determined using the obtained AFC for displacements (Table 1).

The analysis of the data implies that, firstly, with the increase of the amplitude of the kinematic displacement Q_0 of the root section there is a linear rise of the resonance amplitudes $A_{55Q_0}^{(R)}$ in the selected range of its values.

Secondly, the energy scatter level in the blade assembly decreases more than twice with an increase in Q_0 up to 0.05 mm, while at $Q_0 \geq 0.05$ mm it does not practically change. It correlates well with the conclusions [18] since under such conditions of vibrations there is a decrease in the relative displacements of the contact surfaces of the shrouded flanges, which reduces their effectiveness as a structural damper.
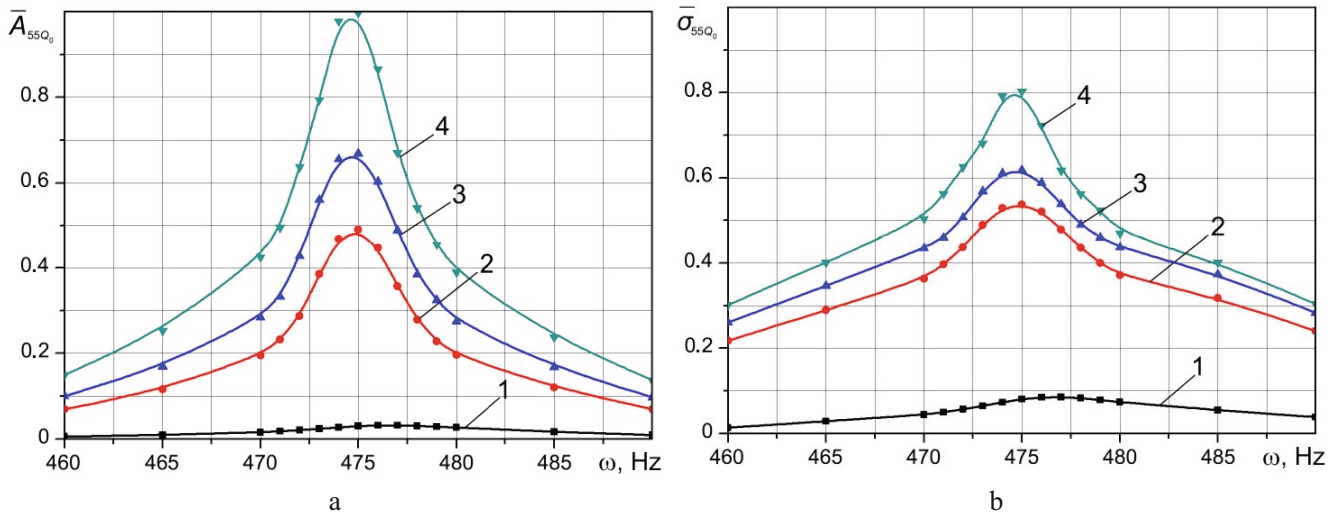
Fig. 6. Amplitude and frequency characteristics of displacements over the Oy axis (a) and stress intensity (b) of the blade assembly at the inclination angle $\alpha = 55°$ and amplitude of the harmonic displacement Q_0 equal to 0.01 mm (*1*), 0.05 mm (*2*), 0.07 mm (*3*), and 0.1 mm (*4*).
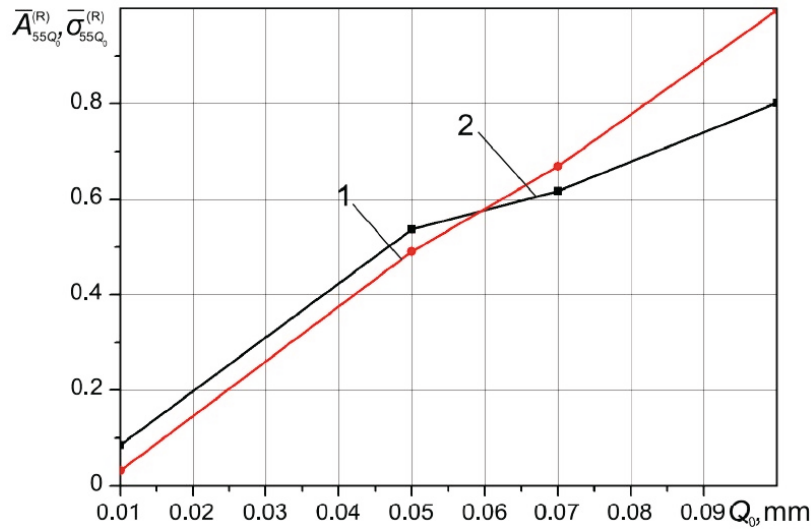

Fig. 7. Diagrams of the relative resonance amplitudes of displacement (*1*) and stress intensity (*2*) of the blade assembly depending on the amplitude of the harmonic displacement Q_0 at the inclination angle $\alpha = 55°$.

Thirdly, the dependence of the change in the stress intensity resonance amplitudes $\sigma_{55Q_0}^{(R)}$ on Q_0 is piecewise-linear. At $Q_0 < 0.05$ mm, the rate of their increase coincides with that for resonance amplitudes $A_{55Q_0}^{(R)}$. This is explained by intensive relative displacements of the contact surfaces of the shrouded flanges, which is consistent with the above conclusion. However, at $Q_0 > 0.05$ mm, the assembly is more similar to a frame structure, which is likely to lead to saturation in growth of stress intensity resonance amplitudes.

CONCLUSIONS

1. The results of numerical experiments indicate that the increase of the inclination angle with the specified level of the kinematic excitation of the blade vibrations has practically no effect on the level of energy scatter under

211

in-phase blade vibrations, and, correspondingly, on their resonance amplitudes of displacements. Considering this orientation of the contact surfaces of the shrouded flanges, the latter rise linearly with an increase in the amplitude of the kinematic excitation of vibrations.

2. The level of the resonance amplitudes of stress intensity in the blade airfoil depends considerably on the orientation of the contact surfaces of the shrouded flanges, as well as the kinematic excitation level. Depending on the inclination angle of these surfaces the scatter of the resonance amplitudes of stress intensity attains the magnitude of two at the specified excitation level of vibrations. With an increase in the amplitude of the kinematic excitation of vibrations for the specified inclination angle, there is an increase in the stress intensity resonance amplitudes, which depends on the amplitude of the vibrations' excitation. In case this rate coincides with the increase (up to certain value) of the displacement resonance amplitudes, it eventually slows down, which is associated with the decrease in the relative displacements of the contact surfaces.

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