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CALCULATION-AND-EXPERIMENTAL INVESTIGATION ON NATURAL FREQUENCIES AND OSCILLATION MODES OF PAIRWISE-SHROUDED COOLED TURBINE BLADES

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The paper presents the results of simulations and experimental investigations on natural frequencies and oscillation modes of pairwise-shrouded cooled turbine blades and their rings with allowance for a possible frequency mistuning and the assured contact of mating surfaces of the root and shroud platform halves. As the research object, the high-pressure turbine wheel of an aircraft bypass turbojet engine has been selected. 3D finite element models of the turbine wheel blades/ period were developed as a system with structural rotational symmetry. The contact interaction between blades over the mating surfaces was modeled on basis of solving the stationary problem. The calculation-and-experimental investigations were performed for two operating conditions of the engine – the non-running engine and the steady-state maximum take-off condition. Results have been cited for individual, isolated blades. The results show a good agreement between calculated and experimental data. The frequency mistuning of those blades is found to be considerably dependent on oscillation mode number, but the mistuning value is practically not affected by the operating condition of the engine. It is confirmed that two oscillation modes are excited in the blade pair as a regular system in the presence of the mistuning of their frequencies, which are close to the inphase and antiphase frequencies. The ratio of natural frequencies of those oscillation modes is dependent on oscillation mode of isolated blades. It is shown that the frequency mistuning of paired blades due to excitation of two oscillation (inphase and antiphase) modes causes doubling of the number of resonant states of the turbine wheel what must be considered when developing methods to ensure their vibrational reliability.

Keywords: pairwise-shrouded cooled turbine blades, blade ring, rotational symmetry, natural frequency of vibration, vibration mode, frequency mistuning.

Introduction. The requirement to ensure the high degree of economic feasibility in modern aircraft gas-turbine engines (AGTE) defines the rise in the turbine entry temperature and increase of the level of centrifugal and non-stationary gas forces acting on rotor blades. These loads are the root causes of the increased dynamic stress intensity in the blade assembly of turbine rotor blades that determine the engine lifetime largely. Thus, experience in

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the development of AGTE exhibits that more than half of defects, which are revealed in the development, are of vibration nature [1].

Shrouded blading is the most powerful method of dynamic stress intensity reduction in rotor blades. It along with its primary function (reduction of gas flow via blade-tip clearance and decrease of tip losses) provides the stiffening of the blades or blade assemblies. Also, it is regarded to be the natural structural damping of vibrations.

Pairwise shrouding is of particular interest when there is coupling by the contact surfaces of the blade assembly flanges. The results of experimental-calculated investigations of dynamic strength of the pairwise-shrouded uncooled blades, which were performed at the Pisarenko Institute of Problems of Strength of the National Academy of Sciences of Ukraine together with SE "Ivchenko-Progress" and JSC "Motor Sich," are embodied in many scientific manuscripts and monographs [2, 3, and others]. They allow one to determine the optimal conditions of the coupling between the contact surfaces in terms of vibration reliability assurance for such blades.

Cooled rotor blades (including with pairwise shrouding) find expanding application in the turbine structures considering the above-mentioned tendency to the rise of turbine entry temperature. Therefore, it is crucial to be aware of the mechanisms of variation of the vibration characteristics. The first stage in the solution to this problem, as is known, is the determination of the modal characteristics, i.e., spectrum of natural frequencies and modes of vibrations of the object of investigation under consideration, as the required condition of the determination of critical operating modes. The fundamental results of the solution to this task are given in [4]. Noteworthy is that in its solution it is required to consider the following. Rotor blades irrespective of their structural features are designed as those having similar geometric dimensions and physical and mechanical properties. However, it is known that due to production tolerances for blades, including pairwise-shrouded ones [2], they exhibit some differences or mistuning of the natural vibration frequencies. This fact makes the analysis of the vibrations in the blade assemblies as regular systems more complicated. Thus, the results of the calculated-experimental investigations of pairwise shrouded blades with mistuning of frequencies under various conditions of their coupling by flanges imply the process of excitation in the case of two modes of vibrations, which are close to in- and anti-phase ones [5, 6]. Some issues of the effect of violations in rotary symmetry of the rotor wheel with pairwise-shrouded blades are considered in [7].

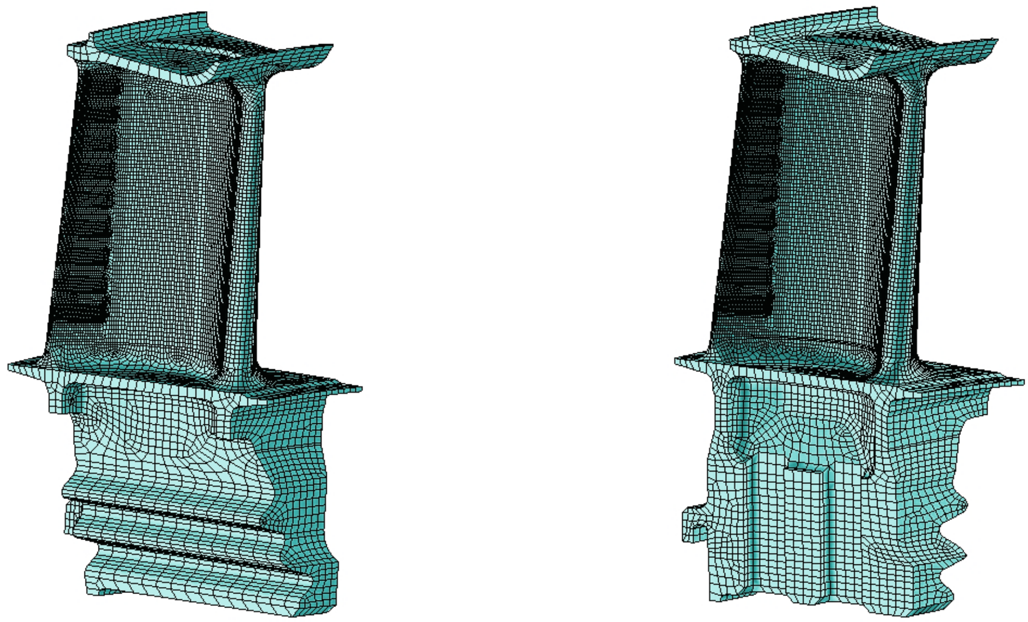
The goal of this paper lies in the determination of the mechanisms of formation of the spectrum of natural frequencies and modes of vibrations of cooled paired shrouded blades and their assemblies considering the possible mistuning of the vibration frequencies at the guaranteed clamping between the contacting surfaces of the halves of the tail and shrouded flanges.

Object of Investigation and Its Modeling. A rotor wheel of the high-pressure turbine (HPT) of bypass engine with the assembly of 82 isolated ($N = 82$) or 41 pairwise-shrouded ($N_s = 41$) blades, i.e., $N_s = N/2$ was chosen as an object of investigation.

Two cooled blades (made of heat-resistant material) shown in Fig. 1 ($i = 1, 2$) are placed in each of the disk grooves. There are the right ($i = 1$) and left ($i = 2$) blades. The blades with split tale are designed so that in the process of their placement in the disk groove there is a pre-gap over the contact surfaces of the shrouded flanges and tails. In the undeformed state the paired blades exhibit different deviations of the longitudinal axes from the radial direction. Based on their bending in the field of centrifugal forces under all the operation engine regimes), tension is ensured over the specified contact surfaces.

Considering the complex structural shapes of the blades an accurate solution to the specified task is possible only based on three-dimensional modeling. Therefore, the calculated 3D blade models have been developed, their finite element meshes are illustrated in Fig. 1. Quadratic three-dimensional elements and their modifications [8] are used for the mesh creation.

In operation under centrifugal forces, there is the clamping between the paired blade tail contact surfaces and the disk by force, which assumes their rigid coupling with the disk. Modeling of the contact interaction between the blades by the shrouded flanges and halves of the tail is accomplished using the solution to the static task. The results of this task were used to determine the contact zones of the specified surfaces. To solve this task, the contact elements "surface-surface" were employed allowing one to consider various conditions of the contact between the shrouded flanges, namely: tension, gap, adhesion, slippage, friction, shear, etc., occurring in the process of operation considering their possible variation.



a b

Fig. 1. Paired shrouded right (a) and left (b) cooled rotor blades of HPT with the deposited finite element mesh.

Figure 2 illustrates the period with two paired shrouded blades and its finite element mesh for the rotor wheel as a system with the structural rotary. It means that the order of symmetry of the rotor wheel equals N_s, i.e., it decreases twice as compared with the case for the period with an individual blade.

Calculated-Experimental Determination of the Spectrum of Natural Frequencies and Modes of Vibrations of Blades and Their Systems. The calculated determination of the spectrum of natural frequencies and modes of vibrations of the selected objects of investigation (individual blades, their pairs, and assemblies) was performed using the developed 3D finite element models via the solution to the standard task by the eigenvalues:

$$([K]-p^2[M])\{U\}=0,$$

where $[K]$ and $[M]$ are the quadratic matrices of the elastic and inertia characteristics of the vibration system, p is the natural vibration frequency, and $\{U\}$ is the column-vector of the displacement amplitude characterizing the mode of the vibration system.

The calculations were performed applying ANSYS.

The experimental determination of the natural frequencies and modes of vibration of rotor blades using the vibration test bench was the required condition for the development of rotor blades in compliance with the existing standards. Here the experiments are performed up to the excitation frequency comprising 103% of the maximum frequency of the possible driving forces caused by the circumferential irregularity of the gas flow due to the combustion chamber nozzles, struts, etc. For the considered rotor wheel the number of blades in the nozzle assembly is 29, while fuel nozzle – 16, which necessitates the testing with the excitation frequencies up to 9560 Hz.

Calculated-Experimental Investigation Results. A set of calculated-experimental investigations on the determination of the spectrum of natural frequencies and modes of vibrations of the isolated blades, their pairs and assembly as a system with structural rotary was performed. The investigations were performed for two operation regimes: 1 – shut-down engine (frequency of rotor spinning $n=0$) and 2 – maximum take-off mode.

Let us consider the results of the investigations performed complying with the selected objects.

Isolated Blades. The results of the calculated-experimental determination of natural frequencies $p_{ij}^{(i)}$ of the considered blades for the first three modes of their vibrations are listed in Table 1, where j is the number of the

TABLE 1. Calculated-Experimental Determination of the Natural Vibration Frequencies of the Individual Blades

Vibration mode		Blade	Calculation				Experiment
Number	Type	i	Regime 1		Regime 2		Regime 1
j			$p_{ij}^{(i)}$, Hz	$\Delta p_j^{(i)}$, %	$p_{ij}^{(i)}$, Hz	$\Delta p_j^{(i)}$, %	$p_{ij}^{(i)}$, Hz
1	First flexural	1	2307	2.04	2231	1.75	2140–2350
		2	2354		2270		2150–2460
2	Torsional	1	6441	4.74	5746	4.84	6095
		2	6748		6024		–
3	Second flexural	1	9257	0.44	8342	0.44	8750
		2	9298		8379		–

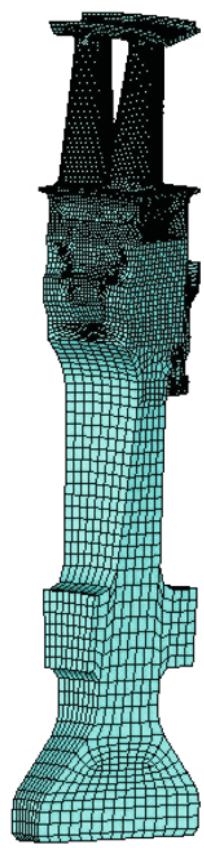


Fig. 2. Finite element calculated 3D model of the HPT rotor wheel period

blade vibration mode. Notice that due to the complex geometry of the blade airfoil portion, which is caused by the natural wrap and asymmetry in the sections, the blade vibration mode is characterized by the interrelation between flexural, longitudinal, and torsional strains. However, as is accepted in the analysis of the blade vibrations [9], let us classify them according to the type of the dominant strain, i.e., flexural, torsional, etc.

The analysis of the test data of the paired blades was performed in the isolated state using the piezoelectric vibration test bench. 226 right and 298 left blades were tested using the first flexural mode of vibrations. As is seen from the obtained values of the frequencies of the first flexural mode of vibrations, there is a large scatter in the values of up to 20%. This is explained by the specified complex geometry of the cooled blades and special features of their material microstructure. The calculated values of the natural frequencies of the first flexural mode of vibrations of isolated blades are within the range of the experimentally determined ones, which corroborates the justification of the developed finite element models.

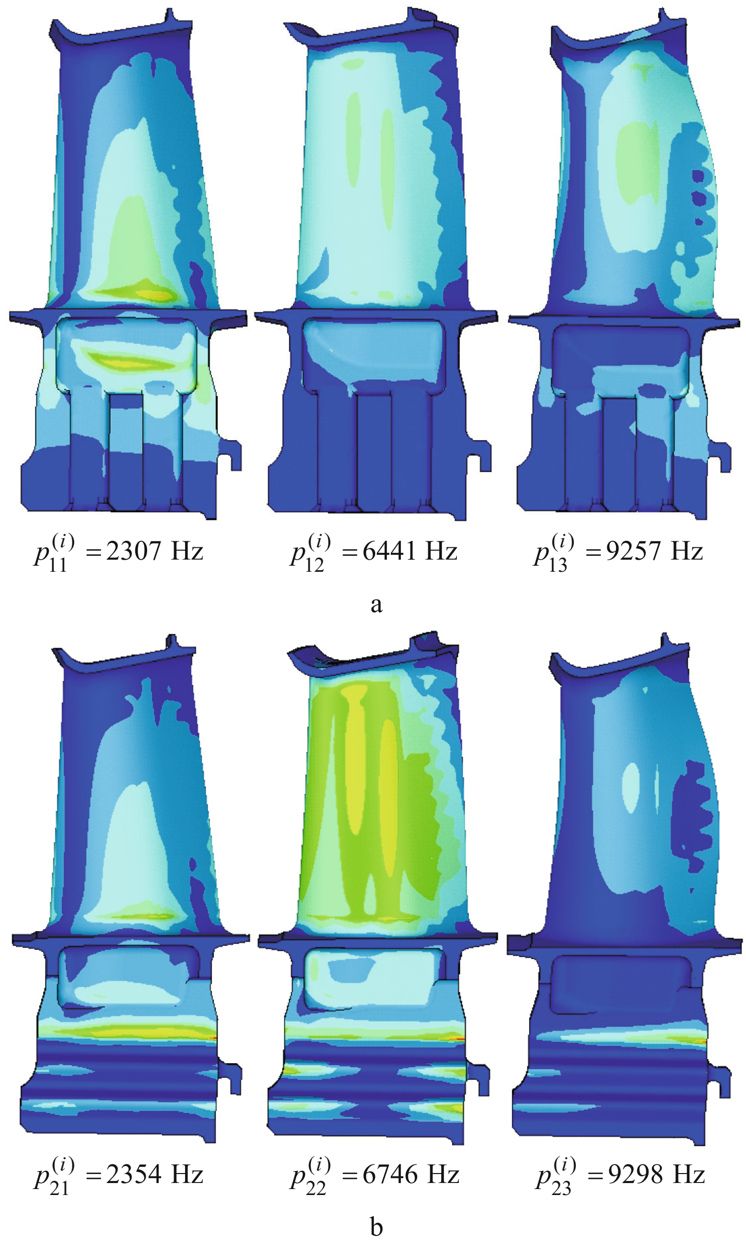
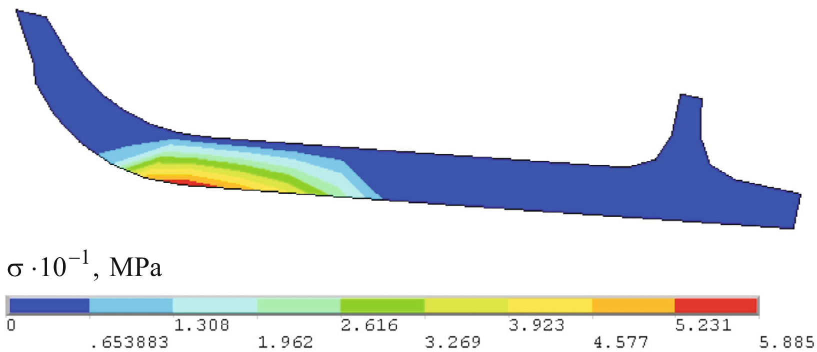


Fig. 3. Distribution of strain intensity for some natural modes of vibrations of right (a) and left (b) cooled blades for engine operation regime 1.

The comparison of the calculated and experimental values of the natural frequencies of the right blade, which correspond to the higher mode of vibrations, implies their significant difference that is explained by the results of testing using these vibration modes as compared to the first flexural mode, as well as by the possible effect of the material microstructure features.

Let us consider the influence of the operation mode on the spectrum of the natural frequencies of the blade vibrations. The analysis of the data in Table 1 demonstrates that for all modes of blade vibrations within the second mode as compared with the first mode, their natural frequencies decrease up to 4% for the first flexural and up to 11–12% – for the other vibration modes. This derives from the fact that within the operating engine regime 2 the natural frequencies of the blade vibrations depend significantly on the centrifugal forces and temperatures acting on them. Thus, with an increase in the rotor spinning speed, which defines the level of centrifugal forces, there is an



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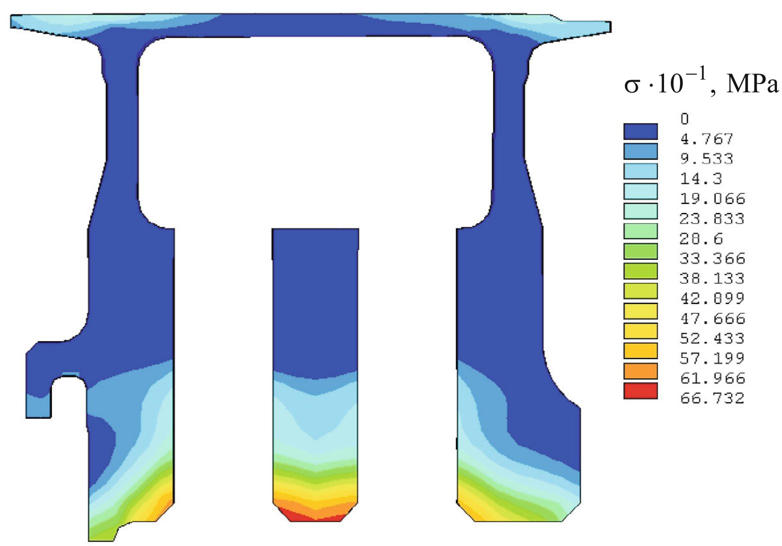

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Fig. 4. Distribution of stresses on the contact surfaces of the shrouded flanges (a) and halves of tails (b) of the pairwise-shrouded rotor blades within engine operation regime 2.

increase in the blade vibration frequency. Here the temperature rise of the blade within the operating engine modes results in the reduction of the material elastic module, which causes the decrease of the natural frequency of the blade vibrations. Therefore, the specified operational factors have a reverse effect on the variation of the vibration frequency.

As follows from the data in Table 1, there is mistuning in frequencies of the left and right blades, which is determined as $\Delta p_j^{(i)} = \dfrac{|p_{1j}^{(i)} - p_{2j}^{(i)}|}{p_{1j}^{(i)}} \times 100\%$. The analysis of the obtained results shows that the frequency mistuning of the blades depends considerably on the number of the vibration mode but it does not affect the operation engine mode.

From the results of the calculations, the vibration modes of the blades were determined as the distributions of strain intensity, which examples are depicted in Fig. 3. It is evident that the specified distributions of the right and left blades coincide practically, which confirms also the validity of the developed calculated models.

Pair of Blades. The efficient operation of the paired shrouded blades, as it was noted, is observed in the guaranteed tension over the contact surfaces of their joint and shrouded couplings. As the results of the solution to the static task imply [10], which is corroborated by the data obtained from bench testing, the specified surfaces of the paired shrouded blades that are considered in the operation engine regimes are in contact. It is seen from Fig. 4 illustrating the stress distribution of the contact surfaces of the shrouded flanges and halves of tails obtained for

TABLE 2. Calculated-Experimental Determination of the Natural Vibration Frequencies of the Pairwise-Shrouded Blades

Vibration mode		Number of paired blades q	Regime 1		Regime 2	
Number j	Type		$p_j^{(q)}$, Hz	$\Delta p_j^{(p)}$, %	$p_j^{(q)}$, Hz	$\Delta p_j^{(p)}$, %
1	First flexural	1	2320	2.84	2235	2.28
		2	2386		2286	
2	Torsional	1	6450	5.30	5732	4.97
		2	6792		6017	
3	Second flexural	1	9283	2.19	8294	2.01
		2	9486		8461	

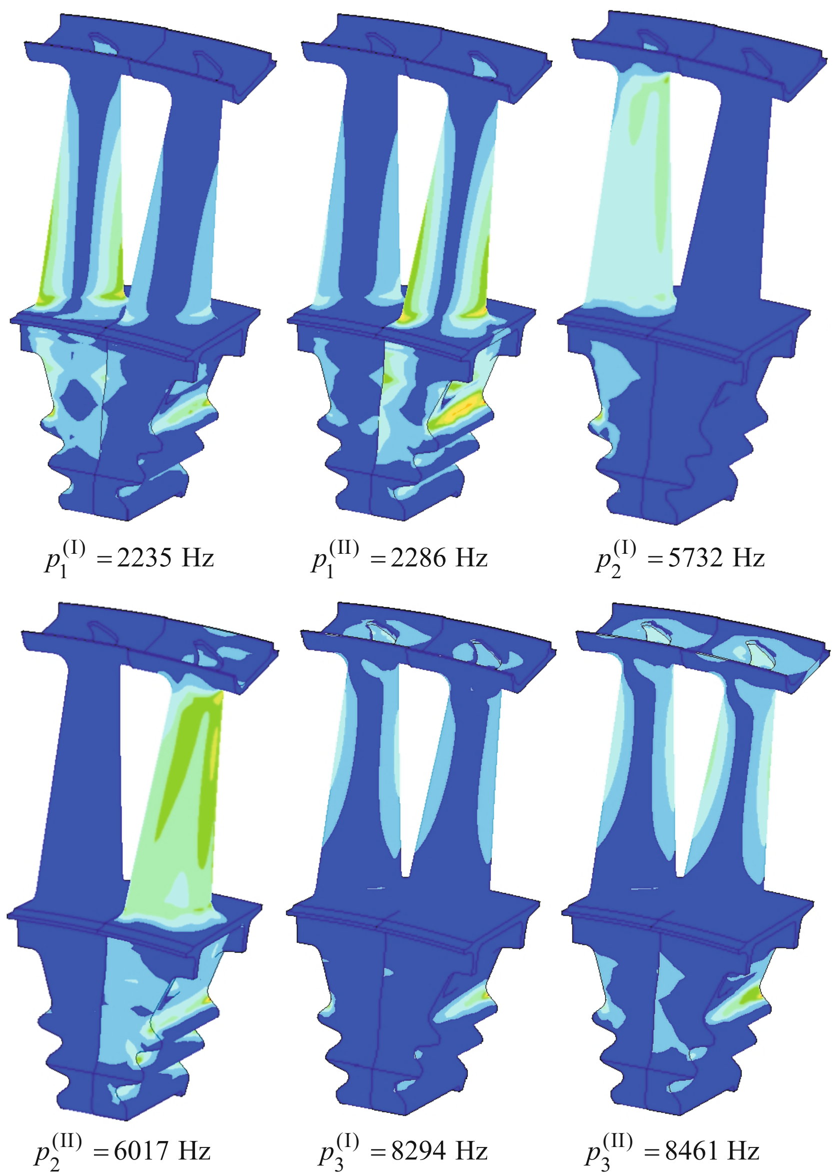

Fig. 5. Distribution of strain intensity of the pair of cooled blades for some natural modes of their vibration within engine operation regime 2.

TABLE 3. Calculated Natural Frequencies $p_{1j}^{(q)}$ for the Vibration Mode of the Rotor Wheel with One Nodal Diameter ($m = 1$)

Vibration mode: Number j	Vibration mode: Type	Displacement of blades	$p_{1j}^{(q)}$, Hz	$\Delta p_{1j}^{(p)}$, %
1	First flexural	Inphase	2075/2044	12.00/9.59
		Antiphase	2324/2240	
2	Torsional	Inphase	5884/5273	7.61/6.71
		Antiphase	6332/5627	
3	Second flexural	Inphase	8394/7922	9.26/3.79
		Antiphase	9171/8222	

Note. Here and in Table 4: above the line are data for regime 1, below the line – for regime 2.

TABLE 4. Calculated Natural Frequencies $p_{2j}^{(q)}$ for the Vibration Mode of the Rotor Wheel with Two Nodal Diameters ($m = 2$)

Vibration mode: Number j	Vibration mode: Type	Displacement of blades	$p_{2j}^{(q)}$, Гц	$\Delta p_{2j}^{(p)}$, %
1	First flexural	Inphase	2068/2035	12.38/10.07
		Antiphase	2324/2240	
2	Torsional	Inphase	5664/5120	11.42/9.82
		Antiphase	6311/5623	
3	Second flexural	Inphase	8537/7858	7.50/4.75
		Antiphase	9177/8231	

regime 2 based on the solution to the static task. Here in the shrouded coupling, there is a low value of the contact surfaces of the tail halves. Moreover, it is pertinent to note that stresses on the contact surfaces of the paired blades are nonuniformly distributed, which fact has to be considered in the analysis of their vibrations.

The computational experiments were performed to determine the spectrum of the natural frequencies and modes of vibration of the paired blades, which boundary conditions and condition of consistency of their displacements over the contacting surfaces were set using the solutions to the static task. The calculations were conducted in the assumption that the paired blades have similar natural frequencies of vibrations as in the isolated state (Table 1),.e. i., a pair of blades deviated from the strict regularity.

Table 2 lists the results of calculations on the determination of the natural frequencies of vibrations $p_j^{(q)}$. As is apparent, irrespective of the vibration mode of the blade in their isolate state, their pair is characterized by two modes of vibrations ($q = \text{I, II}$), the first ($q = \text{I}$) being close to in-phase displacements, while the second ($q = \text{II}$) is

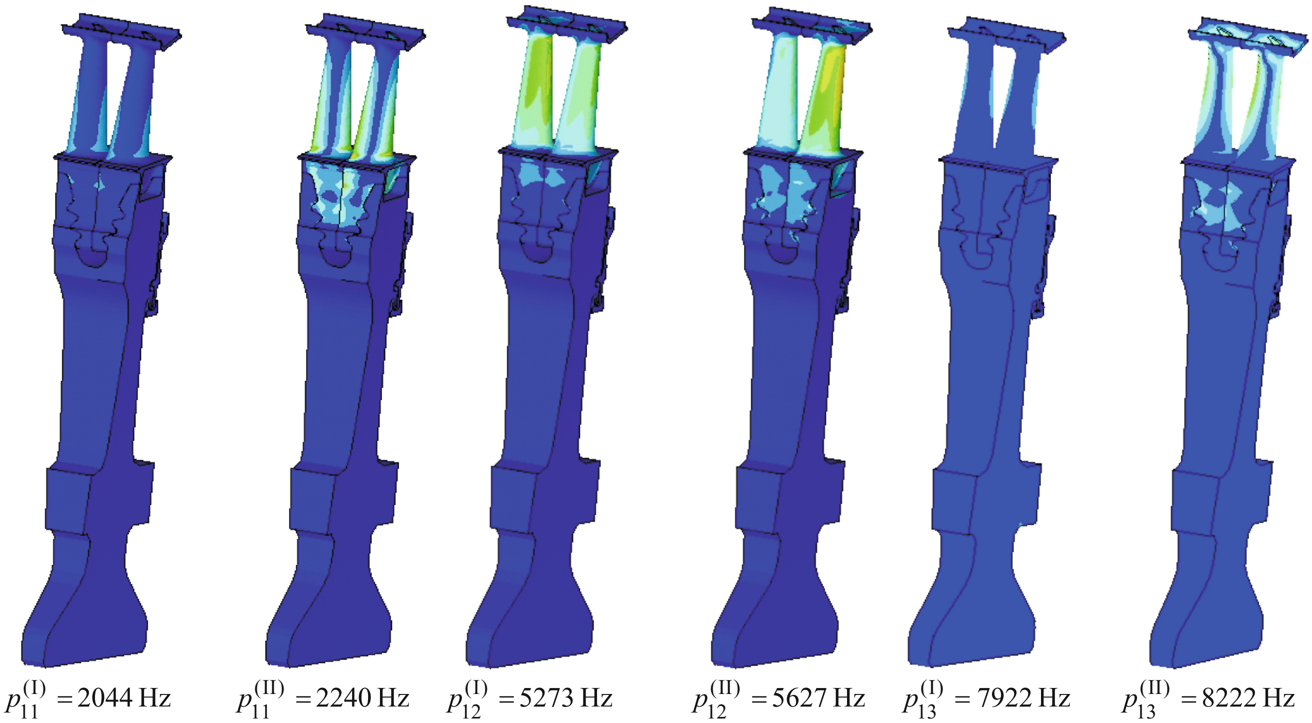

Fig. 6. Distribution of the strain intensity in the blades for the natural vibration frequency of the rotor wheel with one nodal diameter ($m = 1$) within engine operation regime 2.

close to anti-phase displacements, which confirms the test data of the beam model of the pair of shrouded blades [6]. Examples of the obtained modes of vibrations of the pair of blades are illustrated in Fig. 5. As in the case of the blade in the isolated state, natural vibration frequencies of the pair of the blades drop within the take-off mode, which is explained by the same reason as in the above example.

Let us introduce the concept of mistuning of frequencies of the vibration modes of the pair of blades and denote as $\Delta p_j^{(p)} = \dfrac{|\, p_j^{(\mathrm{I})} - p_j^{(\mathrm{II})} |}{p_j^{(\mathrm{I})}} \times 100\%$. The analysis of the obtained results indicates that mistuning of the frequencies $\Delta p_j^{(p)}$ of the vibration modes of the pair of blades, as well as $\Delta p_j^{(i)}$ of isolated blades (Table 1), depends significantly on the mode number of the blade vibrations j and does not practically depend on the operation engine mode. However, mistuning of frequencies $\Delta p_j^{(p)}$ for each of the considered modes of the blade vibrations increases a little, which complies to the mechanisms of formation of vibrations of the regular systems consisting of two elements [11]. The relation between the natural frequencies of vibrations of the pair of blades depends on their vibration modes in the isolated state.

The obtained results of investigations imply that the number of resonant states of the blades in the presence of mistuning of their frequencies will double.

Rotor Wheel as a Structural Rotary System. In the calculated experiments it was assumed that the assembly consists of identical pairs of the mistuned blades, which are considered above. The calculations of the natural frequencies $p_{mj}^{(q)}$ of the rotor wheel, modes of its vibrations with the number of node diameters $m = 1$ and 2, are given in Tables 3 and 4.

Analyzing the model of vibrations of the rotor wheel, wherein the contact zones in the calculation of vibrations the boundary conditions and conditions of displacement compatibility were set considering the distribution of contact stresses on the contact surface from the results of calculation of static SSS of the considered rotor wheel it is worthwhile to account that only several first natural frequencies of the blade vibrations affect its formation. In this context, the influence of modal characteristics of vibration of individual blades can be rather weak in the formation of some vibration modes of the rotor wheel and can be rather high in the formation of other vibrations modes. In the coupled vibrations of the pairwise-shrouded blades with the disk of rotor wheel, there are more considerable variations in the considered oscillation system as compared to already considered vibrations of individual blades or a system consisting of two blades. Firstly, this is manifested by the increase of the number of modes of natural vibrations of the disk–blades system due to mistuning of frequencies of pairwise-shrouded blades. Figure 6 illustrates the distribution of strain intensity in the blades for the natural mode of vibrations of the rotor wheel with one node diameter ($m = 1$) within mode 2 of engine operation.

As can be seen from data in Tables 2–4, if for a system of two pairwise-shrouded blades, the eigenfrequency mistuning for the studied natural modes of its vibrations does not exceed 5.3%, then for disk–blades system it reaches 12.38%. In this case, practically no changes in the vibration modes of the blades of the considered systems are observed (Figs. 5 and 6).

The comparison of the spectra of the natural frequencies and considered modes of vibrations of the rotor wheel ($m = 1$, 2) shows their close correlation in some cases or some differences depending on the vibration mode. This is explained by the fact that rotary symmetry of the rotor wheel does not necessarily entail a strict rotary symmetry of its vibration modes. This is particularly true for the wheels with warped blades of axisymmetric section [12].

CONCLUSIONS

1. A satisfactory agreement is reached between the calculated and experimental values of the natural vibration frequencies of the blades in their isolated state, which corroborates the accuracy of the developed 3D finite element models, as well as for the pair of blades and period of the rotor wheel, which allows one to consider the structural peculiar features of the objects of investigation and their operating conditions to the fullest extent possible.

2. Mistuning of the frequencies of paired blades in their isolated state, as well as excited modes of vibration of the pair of blades, depends significantly on the vibration mode of the individual blade but the operation engine regime has no practical effect on their values.

3. Frequency mistuning of paired blades due to excitation of two oscillation (in- and anti-phase) modes causes doubling of the number of resonant states of the turbine wheel what must be considered when developing methods to ensure their vibrational reliability.

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