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SCIENTIFIC AND TECHNICAL SECTION

COMPUTATIONAL INVESTIGATION OF THE EFFECT OF REINFORCEMENT SCHEMES AND ANGLES ON THE STRESS-STRAIN STATE AND STRENGTH OF COMPOSITE CYLINDERS UNDER AXISYMMETRIC INTERNAL EXPLOSION. PART 1. EFFECT OF THE DISCRETIZATION SPACINGS OF THE COMPUTATIONAL DOMAIN ON THE ACCURACY OF DETERMINATION OF STRESS-STRAIN STATE AND STRENGTH

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The effect of two-dimensional regular finite-difference mesh spacings on the accuracy of calculation of the dynamic axisymmetric stress-strain state and strength of hollow composite cylinders of finite length, fixed overall dimensions and thickness has been numerically determined by the Wilkins method modified for helical orthotropy and implemented in an application package, created earlier by the authors. The cylinders were made by a ribbon consisting of VMPS glass fibers, impregnated with an ÉDT-10 epoxy binder, on a technological mandrel. Loading is done by exploding a spherical explosive charge in the center of symmetry of a cylinder in an air atmosphere. The obtained results allow one to choose mesh spacings along the radial and axial coordinates, which ensure an acceptable accuracy of determination of the maximum values of hoop stresses and strains, as well as strength functions by maximum stress and strain criteria and by the generalized von Mises criterion.

Keywords: modified two-dimensional Wilkins method, application package, discretization spacings and calculation accuracy, single- and two-layer composite cylinders, internal explosive loading, stress-strain state, strength.

Introduction. Cylindrical shells of finite length, including thick-walled ones, are used in vessels, hulls and protective structures of aviation and space equipment, in containers for the storage and transportation of explosive goods and toxic substances, in fusion chambers, etc. Such structural elements (SE) are made from metallic, multilayer fabric and winding composite materials (CM), metal composites consisting of an inner metallic layer (steel, titanium alloys, etc) and an outer multilayer composite, etc. Experiments show advantages of winding CMs over metallic and fabric CMs [1]. Earlier [1] limitations of experimental methods, in comparison with theoretical ones, were noted.

The effect of reinforcement angle on the stress-strain state (SSS) of winding hollow composite and metal composite cylinders under internal explosive loading with a spherical explosive charge in an air atmosphere was studied theoretically using shell approximations in [2–8] and equations of three-dimensional elastoplasticity in [1, 9–14], and strength was considered in [1–3, 7–12, 14]. In [9, 10, 14], the strength of CMs was estimated by maximum stress and strain criteria, the Hoffman criterion, and the generalized von Mises criterion, in [1, 11] by the

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Ashkenazi strength criterion for materials with the same ultimate tensile and compressive strengths and by the generalized von Mises criterion for materials with different tensile and compressive strengths, in [12] by the Hoffman criterion for the matrix and by the maximum stress criterion for reinforcing elements. It should be noted that in all of these theoretical studies [1–14], with the exception of [9, 10, 13], the external load under the explosion of a spherical explosive charge in air was modeled by an approximate procedure [15] and in studies [9, 10, 13] by the procedure presented in [16], which generalizes numerous experimental data and is widely used in applications and in commercial application packages (AP).

Taking into account the limitations of theoretically studied reinforcement schemes and angles, strength criteria, load modeling mainly by an approximate relation [15] and the disadvantages of shell approximations in comparison with the theory of elasticity of anisotropic media for the solution of problems in question [1], this paper studies the effect of reinforcement schemes and angles on the SSS and strength of single- and two-layer hollow cylinders of finite length made of winding CMs, which are elastic up to fracture, on the basis of equations of three-dimensional elasticity theory, phenomenological strength criteria for anisotropic materials, which are most commonly used in applications, and a procedure for determining the external load [16].

Let us present some considerations associated with the use of an AP developed earlier [17] for calculations. A transtropic material, which is geometrically nonlinear in the general case and elastic up to fracture, is adopted in the AP in two-dimensional problems for winding composite layers. To estimate the strength, phenomenological theories of the strength of orthotropic solid are employed, which are most commonly used in applications: maximum stress and strain criteria, the Hoffman criterion and the generalized von Mises criterion, which simulate the initial fracture of the CM both with the same and with different ultimate tensile and compressive strengths. After fracture in the case of continuing calculations, the elastic and strength properties of the material are considered to be invariable. The possibility of assessing the fracture mode is not envisaged in the AP. Note that the CM fractures completely when the strength of both the base and the reinforcing elements deteriorates.

It follows that the composite SEs under consideration can be exactly studied with the aid of APs only up to fracture initiation. After fracture, the properties of real CM change. It is known that during initial fracture, the base degrades [18–20], cracks appear, and the continuity of the composite is broken. Taking into account that the reinforcing elements in the winding CM continue to function, as before fracture, in the first half-period of radial vibrations of the cylinder (in tension) with small fractures of the base, before the occurrence of compressive stresses (second half-period), APs can be roughly used in applied research to calculate SSS and for the comparative evaluation of strength functions using different fracture models. The closer the strength function values after fracture to unity (from above), the smaller the change in the mechanical properties of the CM and the more accurate the determination of SSS and the comparative evaluation of strength functions. Under compression, the fracture of the base causes a decrease in the stiffness and strength of the material and leads to the fact that the reinforcing elements do not practically function. As follows from publications, it does not appear at present to be possible to experimentally estimate the change in the elastic and strength characteristics of CM after initial fracture.

Then the main attention is given to the initial fraction of CM, and it is assumed that the SEs under consideration are used only to contain a single explosion. It follows that it is most important to ensure strength in the most loaded first half-period of radial vibrations, when reinforcement allows one to ensure strength in some cases, even after the initial fracture of the base. The loading parameters for a given material which led to fracture in the vicinity of the first quarter of the period of radial vibration and ensured the functioning of the SE material at a strength function value of about unity in the determination of SSS and in the comparative evaluation of strength functions after fracture were mainly adopted in calculations.

In what follows, we examine the effect of the element dimensions along the radial and axial coordinates on the accuracy of determination of SSS and strength. A single-layer and a two-layer hollow cylinder, made of a winding CM, with the inner radius $R_1 = 0.15$ m, thickness $H = 0.04$ m, and length $L = 0.6$ m have been investigated. The layers of the two-layer cylinder are symmetrically reinforced and have the same thickness. For the single-layer cylinder, the reinforcement of the layer is defined by the reinforcement angle α and is denoted by $[\alpha]$ and for the two-layer symmetrically reinforced cylinder by $[\alpha; -\alpha]$, the first angle relating to the internal layer and the second angle to the external layer. The spherical explosive change is placed on the axis in the central cross-section of the cylinder in an air atmosphere. The load was determined by the procedure presented in [16] and corresponded to the explosive charge mass in TNT equivalent $M_{TNT} = 0.05$ kg. The cylinders were made by winding a ribbon consisting of VMPS glass fibers, impregnated with an ÉDT-10 epoxy binder, on a technological mandrel. The properties of the material and notations correspond to [9]: $\rho = 1880$ kg/m^3, $E' = 37{,}900$ MPa, $E =$ 8900 MPa, $G' = 2840$ MPa, $\nu' = 0.28$, $\nu = 0.36$, $\sigma'_t = 1831$ MPa, $\sigma'_c = 634$ MPa, $\sigma_t = 36.75$ MPa, $\sigma_c = 193$ MPa, $\tau' = 63.1$ MPa, $\varepsilon'_t = 4.83\%$, $\varepsilon'_c = 1.67\%$, $\varepsilon_t = 0.41\%$, $\varepsilon_c = 2.14\%$, $\gamma' = 1.11\%$, and $\gamma = 0.74\%$. As in [9], the prime denotes the reinforcement direction, the unprimed quantities represent the isotropy plane, the subscripts c and t denote compression and tension, respectively. The ultimate shear strength τ in the isotropy plane is determined subject to the invariance condition for the generalized von Mises criterion [1, 9], from which it follows that $\tau = \sqrt{\sigma_c \sigma_t / 3} = 48.6$ MPa. A two-dimensional dynamic axisymmetric problem is solved.

Then the maximum values of the tensile hoop stresses $\sigma^{ext}_{\varphi\,\max}$ and strains $\varepsilon^{ext}_{\varphi\,\max}$ of the external surface of the cylinder are numerically determined along its entire length and the strength functions by the maximum stress criterion $\Phi_{\sigma\,\max}$, the maximum strain criterion $\Phi_{\varepsilon\,\max}$, and the generalized von Mises criterion $\Phi_{M\,\max}$ throughout the volume. Calculations show that in all cases presented below, $\varepsilon^{ext}_{\varphi\,\max}$, $\sigma^{ext}_{\varphi\,\max}$, $\Phi_{M\,\max}$, $\Phi_{\sigma\,\max}$, and $\Phi_{\varepsilon\,\max}$ differ in the central cross-section of the cylinder, where the explosive charge is situated.

Single-Layer Cylinder. ***Effect of the Element Size along the Radial Coordinate on the Accuracy of Determination of the Maximum Tensile Hoop Stresses and Strains of the External Surface of the Cylinder and Strength Functions by Maximum Stress and Strain Criteria and by the Generalized von Mises Criterion.*** A single-layer cylinder $[\alpha]$ was chosen for numerical calculation. The first half-period of radial vibrations and three reinforcement angles $\alpha = 0$, 45, and 90° are analyzed.

The results of a calculation of the effect of the finite-difference mesh spacing along the radial coordinate, dr, on the accuracy of determination of $\varepsilon^{ext}_{\varphi\,\max}$, $\sigma^{ext}_{\varphi\,\max}$, $\Phi_{M\,\max}$, $\Phi_{\sigma\,\max}$, and $\Phi_{\varepsilon\,\max}$ are listed in Table 1. A square mesh $(dx/dr = 1)$ was chosen for all calculations.

Analysis of data shows the following.

1. The accuracy of determination of $\varepsilon^{ext}_{\varphi\,\max}$ depends only slightly on the mesh spacing along the radial coordinate and on the reinforcement angle. Comparison of the results of calculations with the spacings $dr = 4$ and 0.5 mm shows that at $\alpha = 0$, the discrepancy is 0.57%; at $\alpha = 45$ and 90°, it is 0.49 and 0.5%; with the spacings $dr = 1$ and 0.5 mm, it is 0.11, 0.08, and 0.12%, respectively. The values of $\varepsilon^{ext}_{\varphi\,\max}$ increase with decreasing dr for all reinforcement angles. Changing the mesh spacing from 4 to 2 mm, e.g., for $\alpha = 90°$, results in a 0.25% increase in the accuracy of $\varepsilon^{ext}_{\varphi\,\max}$ values.

2. As a result of calculations, a weak dependence of $\sigma^{ext}_{\varphi\,\max}$ on the mesh spacing along the radial coordinate has been established. If the results of calculations with the spacings $dr = 4$ and 0.5 mm are compared, at $\alpha = 0$ the discrepancy is 1.95%; at $\alpha = 45$ and 90°, it is 1.78 and 1.82%; with spacings of 1 and 0.5 mm, it is 0.14, 0.26, and 0.39%, respectively.

3. The accuracy of determination of the strength function maximum by the maximum strain criterion depends not only on the mesh spacing along the radial coordinate, but also on the reinforcement angle. If the results of calculations with the spacings $dr = 4$ and 0.5 mm, at $\alpha = 0$ the discrepancy is 1.96%; at $\alpha = 45$ and 90°, it is 25.63 and 13.76%; with spacings of 1 and 0.5 mm, it is 0.29, 8.86, and 3.21%, respectively. At $\alpha = 0$, the least influence of the mesh spacing along the radial coordinate is observed. For all reinforcement angles, the value of the strength function is not larger than 1 and increases with decreasing spacing along the radial coordinate.

4. The accuracy of determination of the maximum values of strength functions by the maximum stress criterion, as by the maximum strain criterion, depends not only on the mesh spacing along the radial coordinate, but also on the reinforcement angle. If the results of calculation with the spacings $dr = 4$ and 0.5 mm are compared, then at $\alpha = 0$, the discrepancy is 9.91%; at $\alpha = 45$ and 90°, it is 11.42 and 11.2%; with spacings of 1 and 0.5 mm, it is

TABLE 1. Effect of dr on SSS and Strength at Three α Values

Calculated quantity	dr, mm	α, deg		
		0	45	90
$\varepsilon^{ext}_{\varphi\max}$	4	0.2815	0.2630	0.1596
	2	0.2822	0.2636	0.1600
	1	0.2828	0.2641	0.1602
	0.5	0.2831	0.2643	0.1604
$\sigma^{ext}_{\varphi\max}$, MPa	4	28.28	38.17	65.44
	2	27.82	38.52	64.89
	1	27.78	38.76	64.52
	0.5	27.74	38.86	64.27
$\Phi_{M\ \max}$	4	1.323	0.9556	0.8021
	2	1.413	1.0230	0.8281
	1	1.497	1.0880	0.8593
	0.5	1.556	1.1170	0.8922
$\Phi_{\sigma\ \max}$	4	0.9604	0.6959	0.6891
	2	1.0060	0.7292	0.7229
	1	1.0410	0.7588	0.7532
	0.5	1.0660	0.7856	0.7760
$\Phi_{\varepsilon\ \max}$	4	0.9399	0.5260	0.5178
	2	0.9495	0.5617	0.5517
	1	0.9559	0.6446	0.5811
	0.5	0.9587	0.7073	0.6004

2.35, 3.41, and 2.94%, respectively. The strength function values for all reinforcement angles increase with decreasing mesh spacing along the radial coordinate. In this case, the strength function value is larger than 1 only at $\alpha = 0$ and a mesh spacing along the radial coordinate of less than about 2.3 mm.

5. The accuracy of determination of the strength function maximum by the generalized von Mises criterion also depends not only on the mesh spacing along the radial coordinate, but also on the reinforcement angle. If the results of calculations with the spacings $dr = 4$ and 0.5 mm are compared, then at $\alpha = 0$, the discrepancy is 14.97%; at $\alpha = 45$ and 90°, it is 14.45 and 10.1%; with spacings of 1 and 0.5 mm, it is 3.79, 2.6, and 3.69%, respectively. In this case, the strength function increases with decreasing spacing along the radial coordinate for all α values, in which case the strength function value is larger than 1 only at $\alpha = 0$ and 45°. At $\alpha = 0$, an exceeding of the function is observed for all considered spacings along the radial coordinate and at $\alpha = 45°$, at a mesh spacing along the radial coordinate of less than about 2.6 mm.

Effect of the Element Size along the Axial Coordinate on the Accuracy of Determination of the Maximum Tensile Hoop Stresses and Strains of the External Surface of the Cylinder and Strength Functions by Maximum Stress and Strain Criteria and by the Generalized von Mises Criterion. The results of calculation of the effect of the relative mesh spacing along the axial coordinate, dx/dr, on the accuracy of determination of $\varepsilon^{ext}_{\varphi\max}$, $\sigma^{ext}_{\varphi\max}$, $\Phi_{M\max}$, $\Phi_{\sigma\max}$, and $\Phi_{\varepsilon\max}$ for three reinforcement angles are listed in Table 2; it was adopted that $dr = 0.5$ mm.

Analysis of data chows the following.

1. When meshes with $dx/dr = 1$ and 2 are used, the quantity $\varepsilon^{ext}_{\varphi\max}$ does not change.

2. At $dx/dr = 1$ and 2, the maximum difference between the $\sigma^{ext}_{\varphi\max}$ values for all reinforcement angles is under 0.1%.

3. The element size along the axial coordinate and the reinforcement angle affect only slightly the accuracy of determination of the strength function value by the maximum strain criterion. The results of calculations at

TABLE 2. Effect of dx/dr on SSS and Strength at Three α Values

Calculated quantity	dx/dr	α, deg		
		0	45	90
$\varepsilon_{\varphi\max}^{нар}$	1	0.283	0.2643	0.1604
	2	0.283	0.2643	0.1604
$\sigma_{\varphi\max}^{нар}$, МПа	1	27.74	38.86	64.27
	2	27.76	38.85	64.33
$\Phi_{M\ \max}$	1	1.556	1.117	0.8922
	2	1.525	1.098	0.8674
$\Phi_{\sigma\ \max}$	1	1.066	0.7856	0.7760
	2	1.054	0.7660	0.7585
$\Phi_{\varepsilon\ \max}$	1	0.9587	0.7073	0.6004
	2	0.9587	0.6914	0.5834

$dx/dr = 1$ and 2 for $\alpha = 0$ coincide; for $\alpha = 45°$, the discrepancy between them is 2.24% and for $\alpha = 90°$, 2.83%. For all calculated values, the strength function is not larger than 1.

4. The element size along the axial coordinate and the reinforcement angle affect only slightly the accuracy of determination of the strength function value by the maximum stress criterion. The discrepancy between the results at $dx/dr = 1$ and 2 for $\alpha = 0$ is 1.13% and for $\alpha = 45$ and 90°, 2.49 and 2.26%. For a square mesh, all calculated quantities for each α have the largest values.

5. The element size along the axial coordinate and the reinforcement angle affect only slightly the accuracy of determination of the strength function value by the generalized von Mises criterion. The discrepancy between the results at $dx/dr = 1$ an 2 for $\alpha = 0$ is 1.99% and for $\alpha = 45$ and 90°, 1.7 and 2.78%. For a square mesh, all calculated quantities for each α have the largest values.

Two-Layer Cylinder. A two-layer cylinder with symmetrical reinforcement [45°; −45°] and a thickness of the layers of 0.02 m was chosen for numerical calculation. The first half-period of radial vibrations is analyzed. The results of calculations for a single-layer $[\alpha]$ and a corresponding two-layer $[\alpha; -\alpha]$ cylinder at $\alpha = 0$ and 90° completely coincide (Tables 1 and 2).

Effect of the Element Size along the Radial Coordinate on the Accuracy of Determination of the Maximum Tensile Hoop Stresses and Strains of the External Surface of the Cylinder and Strength Functions by Maximum Stress and Strain Criteria and by the Generalized von Mises Criterion. The results of a calculation of the effect of the mesh spacing along the radial coordinate on the accuracy of determination of $\varepsilon_{\varphi\max}^{ext}$, $\sigma_{\varphi\max}^{ext}$, $\Phi_{M\max}$, $\Phi_{\sigma\max}$, and $\Phi_{\varepsilon\max}$ are listed in Table 3. A square mesh ($dx/dr = 1$) was chosen for numerical calculation.

Analysis of data shows the following.

1. The accuracy of determination of $\varepsilon_{\varphi\max}^{ext}$ for a two-layer cylinder with the symmetrical reinforcement of the layers depends only slightly on the mesh spacing along the radial coordinate. If the results of calculations at $dr = 4$ and 0.5 mm are compared, then the discrepancy is 0.54%; at 1 and 0.5 mm, it is 0.08%. When dr is decreased, the values of $\varepsilon_{\varphi\max}^{ext}$ increase.

2. The maximum tensile hoop stresses of the external surface of the cylinder depend only slightly on the mesh spacing. If the results of calculations at $dr = 4$ and 0.5 mm are compared, then the discrepancy is 2.4%; at 1 and 0.5 mm, it is 0.75%.

3. The accuracy of determination of strength function maxima by the maximum strain criterion depends on the mesh spacing. If the results of calculations at $dr = 4$ and 0.5 mm are compared, then the discrepancy is 14.54%; at 1 and 0.5 mm, it is 4.26%. For all calculations, the strength function value is not larger than 1 and increases with decreasing mesh spacing.

TABLE 3. Effect of dr on the SSS and Strength of a Two-Layer Cylinder

Calculated quantity	dr, mm	[45°; −45°]
$\varepsilon_{\varphi\ max}^{ext}$	4	0.2595
	2	0.2602
	1	0.2607
	0.5	0.2609
$\sigma_{\varphi\ max}^{ext}$, MPa	4	39.00
	2	39.51
	1	39.66
	0.5	39.96
$\Phi_{M\ max}$	4	0.8996
	2	0.9420
	1	0.9832
	0.5	1.0320
$\Phi_{\sigma\ max}$	4	0.6924
	2	0.7334
	1	0.7590
	0.5	0.7861
$\Phi_{\varepsilon\ max}$	4	0.5271
	2	0.5671
	1	0.5905
	0.5	0.6168

4. The accuracy of determination of the maximum values of strength functions by the maximum stress criterion depends on the mesh spacing. If the results of calculations at $dr = 4$ and 0.5 mm are compared, then the discrepancy is 11.92%; at 1 and 0.5 mm, it is 3.45%. The strength function value increases with decreasing dr.

5. The accuracy of determination of strength function maxima by the generalized von Mises criterion depends on the mesh spacing. If the results of calculations at $dr = 4$ and 0.5 mm, then the discrepancy is 12.83%; at 1 and 0.5 mm, it is 4.73%. The strength function value increases with decreasing mesh spacing; it is larger than 1 only at $dr < 0.8$ mm.

Effect of the Element Size along the Axial Coordinate on the Accuracy of Determination of the Maximum Tensile Hoop Stresses and Strains of the External Surface of the Cylinder and Strength Functions by Maximum Stress and Strain Criteria and by the Generalized von Mises Criterion. The results of a calculation of the effect of the mesh spacing along the axial coordinate on the accuracy of determination of $\varepsilon_{\varphi\max}^{ext}$, $\sigma_{\varphi\max}^{ext}$, $\Phi_{M\max}$, $\Phi_{\sigma\max}$, and $\Phi_{\varepsilon\max}$ are listed in Table 4. We adopted $dr = 0.5$ mm in the calculations.

Analysis of data shows the following.

1. The accuracy of determination of $\varepsilon_{\varphi\max}^{ext}$ at a constant mesh spacing along the radial coordinate is independent of the mesh spacing along the axial coordinate. The difference between the numerical results for $\sigma_{\varphi\max}^{ext}$ at $dx/dr = 1$ and 2 is 0.25%.

2. The element size along the axial coordinate affects only slightly the accuracy of determination of the strength function value by the maximum strain criterion; the discrepancy between the results of calculations at $dx/dr = 1$ and 2 is only 3.15%. In this case, the strength function values for a square mesh are highest.

3. The element size along the axial coordinate affects only slightly the accuracy of determination of the strength function value by the maximum stress criterion. The discrepancy between the results at $dx/dr = 1$ and 2 is 2.48%. When a square mesh is used, the calculated values are highest.

TABLE 4. Effect of dx/dr on the SSS and Strength of a Two-Layer Cylinder

Calculated quantity	dx/dr	[45°; −45°]
$\varepsilon_{\varphi\max}^{ext}$	1	0.2609
	2	0.2609
$\sigma_{\varphi\max}^{ext}$, MPa	1	39.97
	2	40.06
$\Phi_{M\ \max}$	1	1.032
	2	1.005
$\Phi_{\sigma\ \max}$	1	0.7861
	2	0.7666
$\Phi_{\varepsilon\ \max}$	1	0.6168
	2	0.5974

4. The element size along the axial coordinate affects only slightly the accuracy of determination of the strength function value by the generalized von Mises criterion. The discrepancy between the results at $dx/dr = 1$ and 2 is 2.62%. When a square mesh is used, the calculated values are highest.

Conclusion. The obtained results allow the choice of mesh spacings along the radial and axial coordinates, which provides acceptable (for numerical calculations) accuracy of determination of the maximum values of tensile hoop stresses and strains of the external surface of the cylinder and strength functions by the maximum stress and strain criteria and by the generalized von Mises criterion.

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