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SCIENTIFIC AND TECHNICAL SECTION

DETERMINATION OF THE CRACK SIZE CORRESPONDING TO THE ENDURANCE LIMIT OF METALS AND ALLOYS IN THE PRESENCE OF A STRESS CONCENTRATION

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Based on the assumption that there exists a proportional relationship between the quantity $\sqrt{area}$, which characterizes the stress raiser size, and the size of crack at the endurance limit of a specimen with stress raiser, a procedure for the calculation of the size of such a crack is proposed. The size of non-propagating cracks at the endurance limit of specimens with stress raisers has been calculated for a number of structural materials using the proposed procedure and known data on endurance limit and defects of different size and shape, which are evaluated with the parameter $\sqrt{area}$. The effect of stress gradient and concentration on the crack size at the endurance limit of various materials is considered. It has been shown that the crack size at the endurance limit of specimens with defects firstly increases with increasing stress gradient and theoretical stress concentration factor and, secondly, always remains larger than the size of non-propagating crack, which is observed at the endurance limit of a smooth specimen.

Keywords: endurance limit, crack size at the endurance limit, stress concentration, stress gradient.

When analyzing and generalizing experimental data on the estimation of the size of short cracks at the endurance limit of metals and alloys, the comparison of the crack size in smooth specimens with that in specimens with stress raisers is crucial.

The data presented in publications are limited and contradictory. For example, in [1] it is assumed that the size of fatigue cracks, which corresponds to the endurance limit of smooth specimens and specimens with stress raisers, is the same; in [2–4], the dependence of the threshold stress intensity factor for small cracks on the crack size is assumed; in [5–7], it is shown that the crack size at the endurance limit of specimens with stress raisers decreases with increasing theoretical stress concentration factor. However, the question of what this crack size as compared to that at the endurance limit of a smooth specimen remains open.

In [4, 8–11, et al.], relationships for the threshold stress intensity factor ΔK_{th} and the endurance limit σ_R, which take into account the size of small defects (cracks), are proposed on the assumption that the value of endurance limit is determined according to the non-propagation conditions of fatigue cracks, which appeared near small defects, by taking into account the dependence of the threshold stress intensity factor for small cracks on the crack size and generalizing a large body of experimental data obtained in the study of the dependence of endurance limits on initiated defects of different size and shape.

The value of the square root of the projection of defect or crack area onto a plane perpendicular to maximum normal stresses, $\sqrt{area}$, was adopted as a defect size characteristic.

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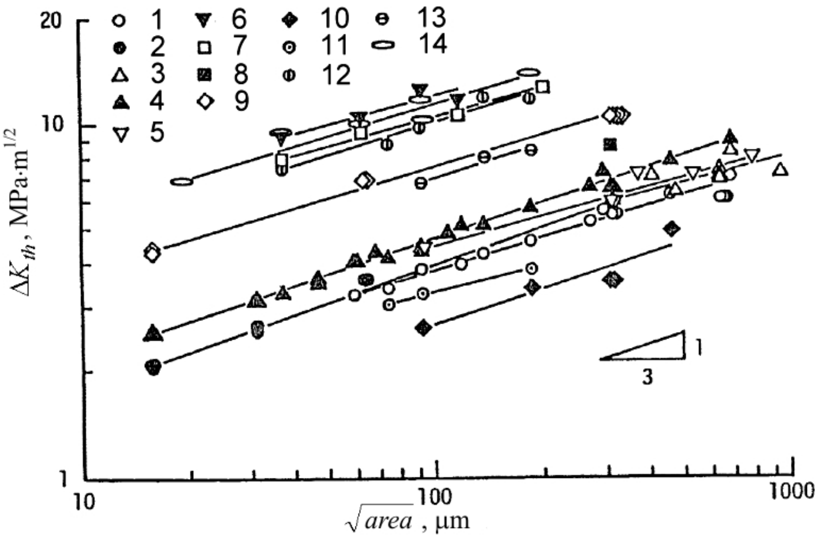


Fig. 1. Dependence of ΔK_{th} on $\sqrt{area}$ for specimens with different defects and cracks. [The investigated materials (*1–14*) and defect types are listed in Table 1.]

Figure 1 shows the dependence of ΔK_{th} on $\sqrt{area}$, obtained in [11] as a result of a generalization of the investigation of numerous materials under circular bending and tension–compression. The values of ΔK_{th} were calculated from the following relation:

$$\Delta K_{th} = 0.65 \Delta\sigma_R \sqrt{\pi\sqrt{area}}. \tag{1}$$

In this study, the following defects were analyzed: very small drill-holes 40–500 μm in diameter and over 40 μm deep [12–18], very shallow notches 5–300 μm deep [19–29], very shallow semicircular cracks 30–260 μm deep [30], defect 72 μm in size generated by Vickers indentation [13].

The data presented in Fig. 1 indicate that the dependence of ΔK_{th} on $\sqrt{area}$ in the range of values $\sqrt{area} \le 1000$ μm for the investigated materials is described in logarithmic coordinates by the linear dependence:

$$\Delta K_{th} \propto (\sqrt{area})^{1/3}. \tag{2}$$

Moreover, for the specimens with very short flat cracks of length $2a$, the dependence

$$\Delta K_{th} \propto (2a)^{1/3} \tag{3}$$

exists.

The simultaneous solution of Eqs. (2) and (3) yields:

$$\sqrt{area} \propto (2a). \tag{4}$$

In [4], the important conclusion was drawn that the specimens with small fatigue cracks and defects in the form of drill holes and notches at the same value of the parameter $\sqrt{area}$ have the same endurance limit. That is at the same endurance limit:

$$(\sqrt{area})_c = (\sqrt{area})_{dh} = (\sqrt{area})_n, \tag{5}$$

where $(\sqrt{area})_c$, $(\sqrt{area})_{dh}$, and $(\sqrt{area})_n$ are values for fatigue cracks and small defects in the form of drill holes, and notches, respectively.

TABLE 1. Investigated Materials and Defect Types that are Assessed Using the Parameter $\sqrt{area}$

Material (Fig. 1)	Defect	Material (Fig. 1)	Defect
S10C steel (A) [9, 14, 17–21] (1)	Notch Drill hole	S50C steel (H+A) HV 319 [28, 31] (8)	Notch
S30C steel (A) [22] (2)	Notch	S50C steel (H+A) HV 375–378 [28, 31] (9)	Ditto
S35C steel (A) [22–24] (3)	Notch Drill hole	Bronze 7/3 Brass [18, 29] (10)	Notch Drill hole
S45C steel (A) [9, 14, 18, 25–27] (4)	Ditto	2017–T4 aluminum alloy [18] (11)	Drill hole
S50C steel (A) [28, 30] (5)	Notch Crack	SUS603 steel [4] (12)	Ditto
S45C steel (H) [15, 16] (6)	Drill hole	YUS170 steel [4] (13)	Ditto
S45C steel (H+A) [15, 16] (7)	Ditto	Martensitic steel [13] (14)	Indentation Notch Indentation

Note: (A) annealing, (H) hardening, (H+A) hardening + annealing.

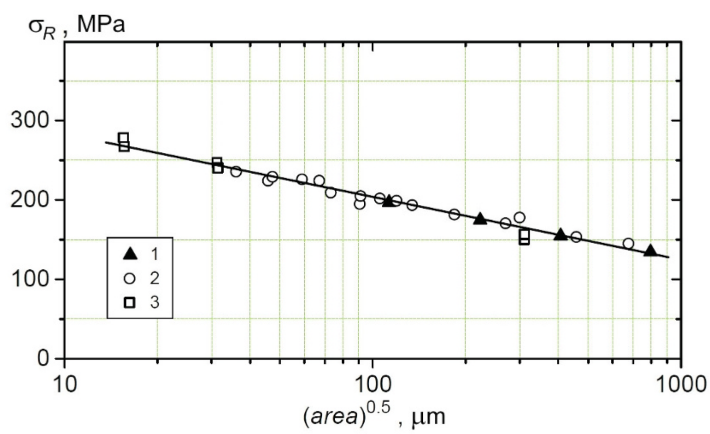


Fig. 2. Comparison of the endurance limit σ_R with the value of $\sqrt{area}$ for specimens with small fatigue cracks and defects: (*1*) crack, (*2*) drill hole, (*3*) notch.

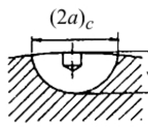
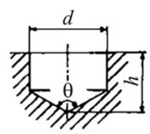
In Fig. 2, a plot of the endurance limit σ_R vs $\sqrt{area}$ was constructed for specimens with fatigue cracks and defects in the form of drill holes and notches, which supports the above conclusion.

This conclusion is also substantiated by the data listed in Table 2 [4].

The above conclusion and the proportional relationship between the parameter $\sqrt{area}$ and the crack size $2a$, which is described by Eq. (4), allow one to calculate the size of non-propagating cracks at the endurance limit using experimental data for a wide range of metals and alloys, obtained in studies by Murakami [4, 8–16], on the threshold stress intensity factor ΔK_{th}, the endurance limit σ_R and defects of different size and shape, which are evaluated with the parameter $\sqrt{area}$, on the one hand, and on the geometry of fatigue cracks at the endurance limit under relevant test conditions (semicircular, semielliptical cracks, etc.), on the other hand. Let us consider some examples.

In the study [8], the effect of small defects (drill holes 40–200 μm in diameter and depth on the endurance limit of S10C and S45C carbon steels under the circular bending of specimens 10 and 6 mm in diameter, respectively,

TABLE 2. Comparison of the Endurance Limits of Specimens with Small Cracks and Small Defects at the Same Values of $\sqrt{area}$ (Carbon Steel)

Crack and drill hole		$\sqrt{area}$, µm	σ_R, MPa
	$2a$ = 213 µm b/a = 0.7	112	200.9
	d = 200 µm h = 95 µm θ = 120°	115	200.9

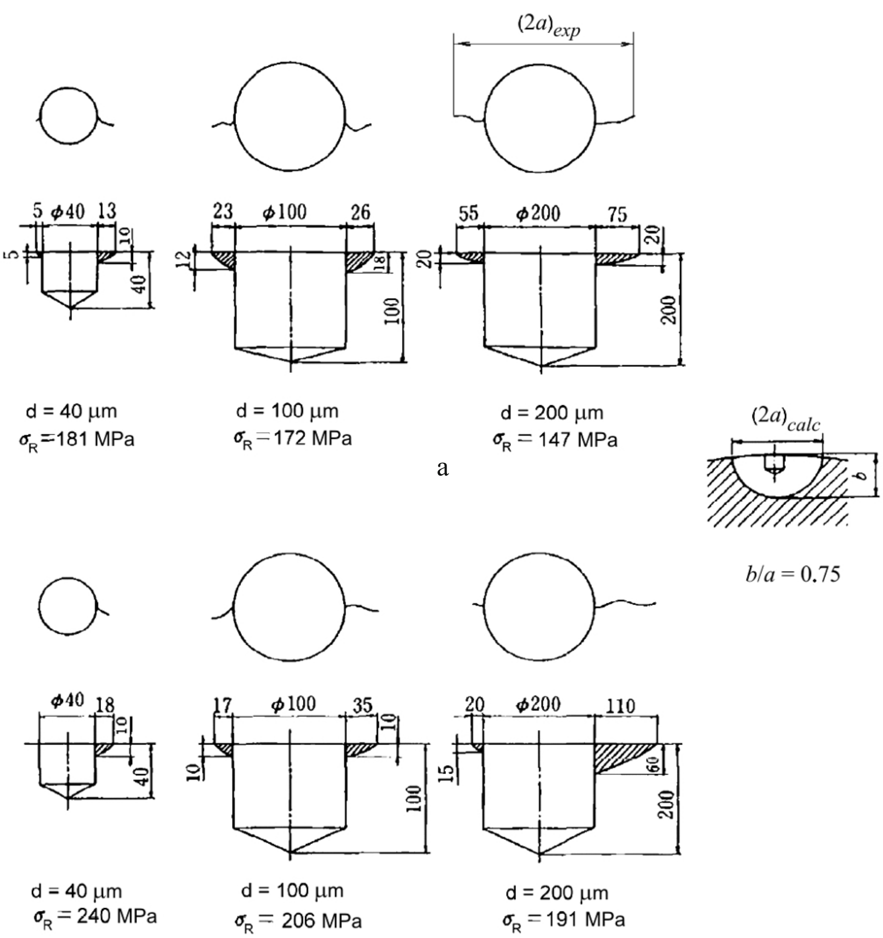


Fig. 3. Size of non-propagating cracks originating from drill holes at the endurance limit: (a) S10C steel, (b) S45C steel [8].

was investigated. It was noted that at the endurance limit, which is defined as maximum nominal stresses at a number of cycles to fracture of 10^7, non-propagating cracks were observed in all specimens (Fig. 3). The size of non-propagating surface crack is denoted by $(2a)_{exp}$. During a circular bending fatigue test of cylindrical specimens, cracks of semielliptic geometry with the ratio of the ellipse semiaxes $b/a = 0.75$ generally nucleate and propagate [32].

TABLE 3. Comparison of Calculated Crack Size Values with Experimental Ones

Carbon steel	σ_R, MPa	d/h, μm/μm	$(2a)_{exp}$, μm	$(2a)_{calc}$, μm	$\frac{(2a)_{calc}-(2a)_{exp}}{(2a)_{calc}}$, %
0.13% C	181	40/40	58	68.2	15.0
	172	100/100	149	171.4	13.0
	147	200/200	330	340.9	3.2
0.46% C	240	40/40	58	68.2	1.5
	206	100/100	152	171.4	11.3
	191	200/200	330	340.9	3.2

Note: d and h are the diameter and depth of drill hole, respectively.

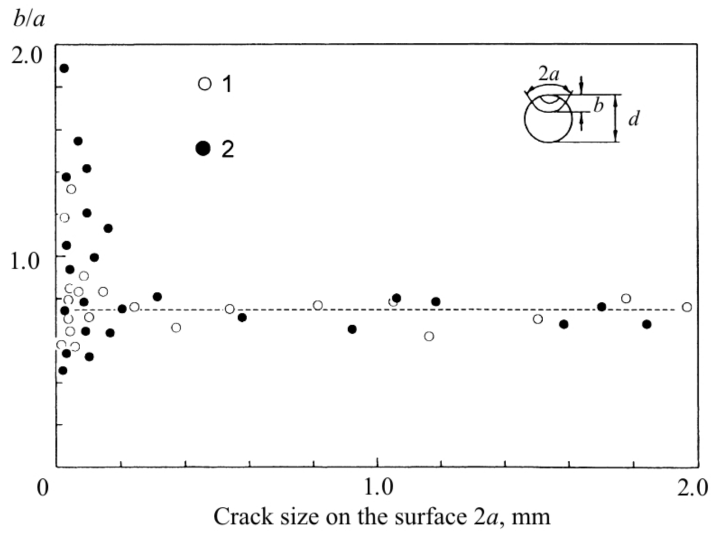


Fig. 4. Dependence of the ratio of the geometric crack dimensions, b/a, on the crack length on the surface for fine-grained (*1*) and coarse-grained (*2*) 0.43 C steel.

The data on the size of non-propagating cracks, $(2a)_{exp}$, presented in Fig. 3 allow one to assume, in view of relations (4) and (5), that $(\sqrt{area})_{dh}$ for a defect as a drill hole and $(\sqrt{area})_c$ for a surface crack of the size $(2a)_{calc}$ are equal:

$$(\sqrt{area})_{dh} = (\sqrt{area})_c.$$

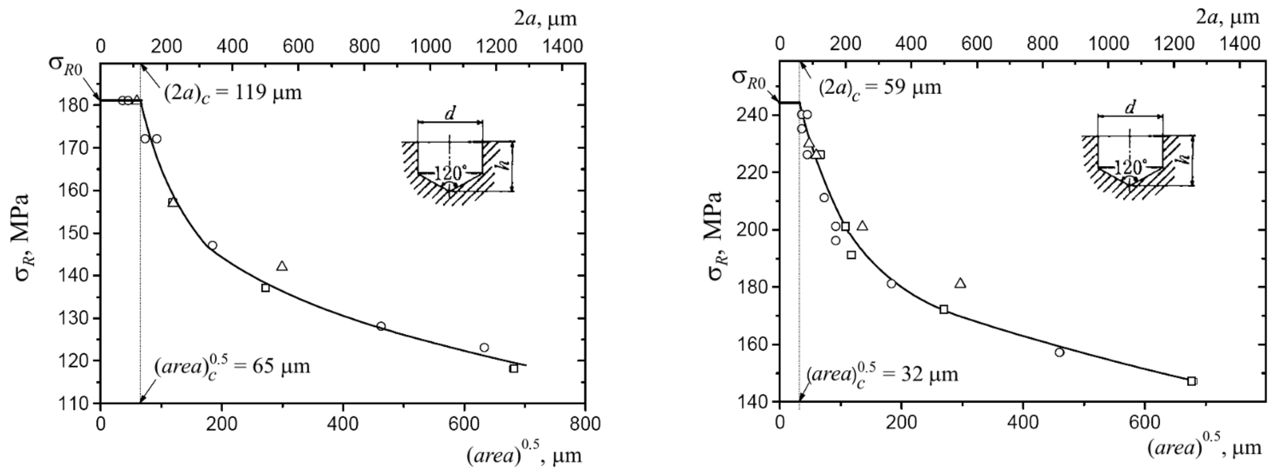
For a semielliptical crack with the ratio of the ellipse semiaxes $b/a = 0.75$, we shall have

$$(\sqrt{area})_{dh} = (\sqrt{1/2\,\pi(a)_{calc}\,b})_c = 1.0851\,a_{calc},$$

$$(2a)_{calc} = 1.843(\sqrt{area})_{dh} = 1.843\sqrt{hd - d^2/4\sqrt{3}}.$$

Comparison of the calculated crack size values $(2a)_{calc}$ with the experimental ones $(2a)_{exp}$ (Table 3) for all cases presented in Fig. 3 shows them to differ only slightly. The maximum discrepancy between these values is not over 15%.

It should be noted that the error in the determination of the crack size $(2a)_{calc}$ is primarily due, in our view, to uncertainty in finding the value of b/a, especially at small crack size ($2a \leq 0.2$ mm). This can be confirmed, e.g., by plots of the ratio of the geometrical crack dimensions, b/a, vs surface crack length [32] (Fig. 4). Carbon steel



a b

Fig. 5. Dependence of the endurance limit of S10C (a) and S45C (b) steels on the value of $\sqrt{area}$ and the surface crack size $2a$ proportional to it: (△) $h/d = 0.5$, (○) $h/d = 1.0$, (□) $h/d = 2.0$ ($\sqrt{area} = \sqrt{hd - d^2/4\sqrt{3}}$, $2a = 1.843\sqrt{area}$.)

containing 0.43% C, investigated in the study [32], is close to the materials considered above in mechanical characteristics and grain size.

As is seen from Fig. 4, a considerable scatter of the ratio b/a is observed in the crack size region $2a \le 0.2$ mm. The average value of b/a in the crack size range $2a$ = 0–0.2 mm is 0.95. Taking into account this fact, the error in the determination of crack size (Table 3) can be reduced.

Figure 5 shows, with the use of data presented in [8, 9, 14], the dependence of the endurance limit of S10C and S45C steels on the value of $\sqrt{area}$ and the surface crack size $2a$ proportional to it, calculated by the procedure described above. The values of $\sqrt{area}$ and $2a$ correspond to detects in the form of drill holes with the ratios $h/d = 0.5$, 1.0, 2.0 and $d = 40$–500 μm.

The data presented in Fig. 5 suggest the following. The endurance limit of specimens with defects decreases with increasing defect of the size $\sqrt{area}$ and surface crack size $2a$ proportional to it. The endurance limit of S10C steel with a defect of the size $\sqrt{area} = 37$ μm ($2a = 68$ μm), $\sqrt{area} = 46$ μm ($2a = 85$ μ), and $\sqrt{area} = 60$ μm ($2a = 110$ μm) is equal to that of smooth specimen ($\sigma_R = 181$ MPa). The endurance limit of S45C steel with a defect of the size $\sqrt{area} = 37$ μm ($2a = 68$ μm) and $\sqrt{area} = 46$ μm ($2a = 85$ μm) differs from that of smooth specimen ($\sigma_R = 245$ MPa) by 4.9 MPa.

The critical values $(\sqrt{area})_{cr}$ and $(2a)_{cr}$ at the points of intersection of the horizontal straight line corresponding to the endurance limits of smooth specimen and the plot of σ_R vs $\sqrt{area}(2a)$ for S10C steel are 65 and 119 μm and for S45C steel 32 and 59 μm, respectively. These results indicate that the endurance limits of the above steels are independent of the defect of the size $(\sqrt{area})_{cr} \le 65$ μm and ≤ 32 μm, as well as $(2a)_{cr} \le 119$ μm and ≤ 59 μm, respectively. The critical crack size $(2a)_{cr}$ is in satisfactory agreement with the critical size of non-propagating cracks, which are observed at the endurance limit of smooth specimen (100 μm for S10C steel and 50 μm for S45C steel). Thus, the crack size at the endurance limit of specimens with defects firstly decreases with nominal stresses and, secondly, always remains larger than the size of non-propagating crack, which is observed at the endurance limit of a smooth specimen.

Figure 6 shows, with the use of the results presented in [15, 16, 18], several similar calculated and experimental data for different classes of materials. It can be seen that the presence of small defects (cracks) more strongly affects the reduction in the endurance limit of high-strength materials. Whereas the critical size of a

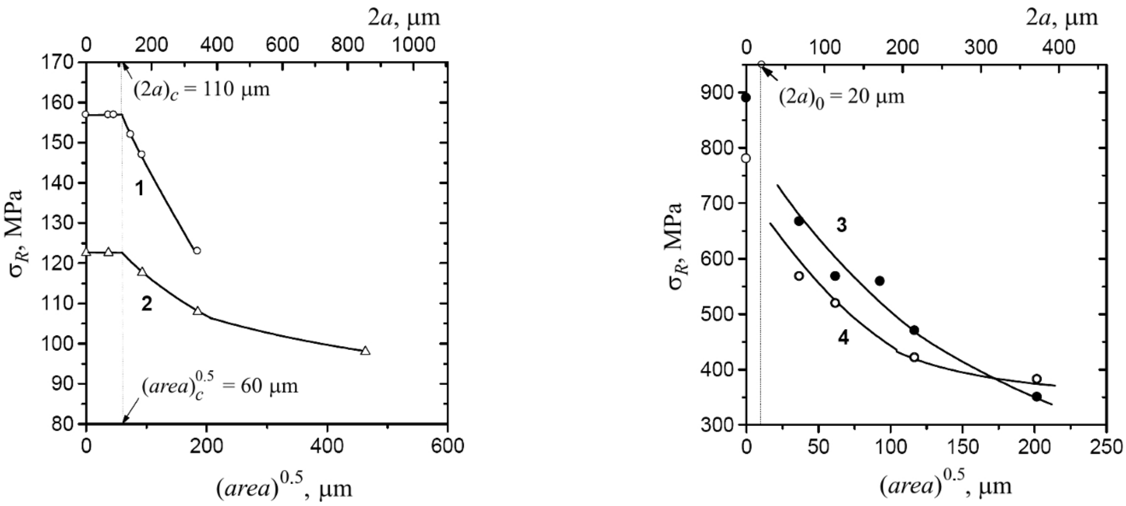
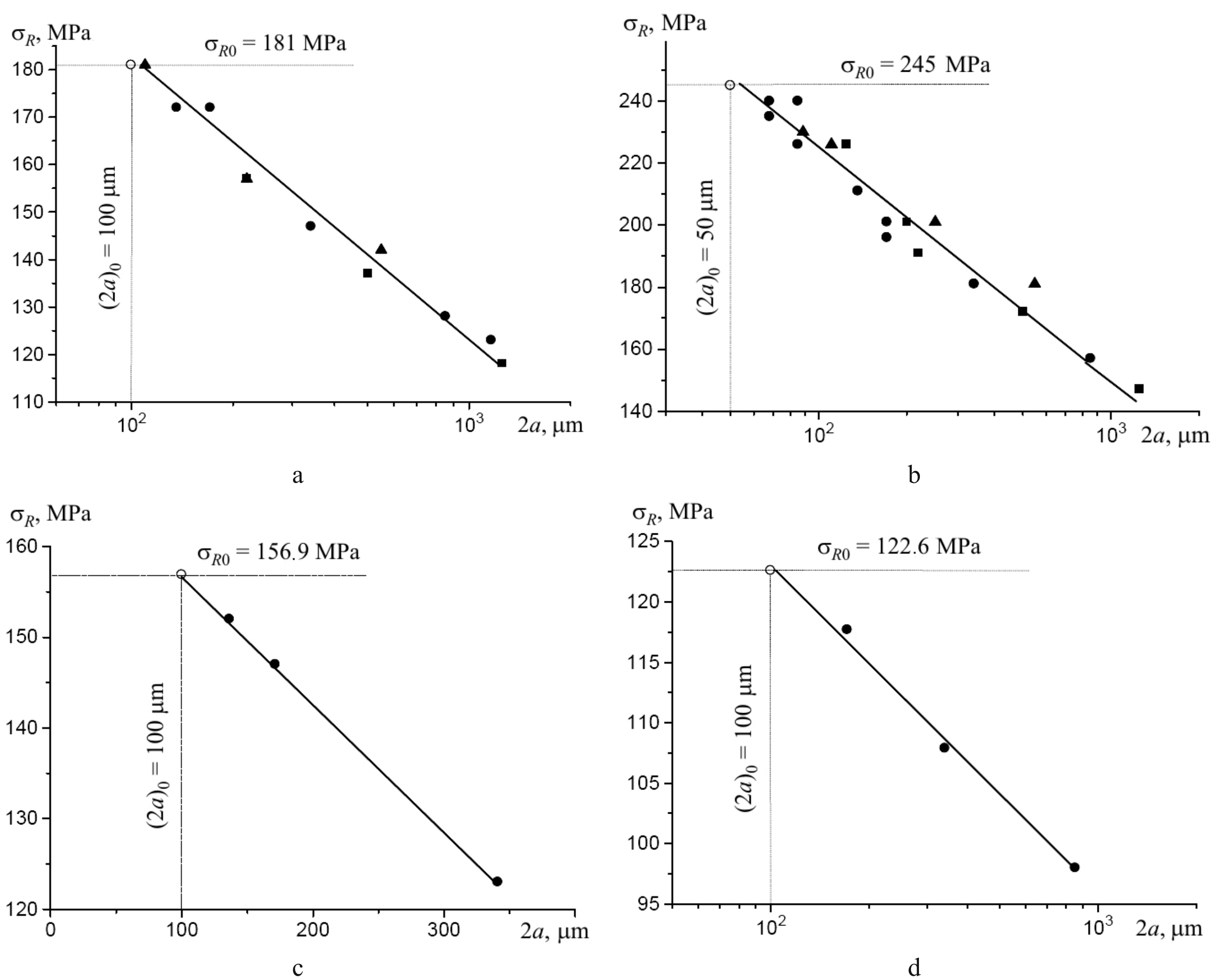


Fig. 6. Dependence of the endurance limit of different materials on the value of $\sqrt{area}$ and the surface crack size $2a$ proportional to it: (*1*) 2017-T4 aluminum alloy, (*2*) 70130 bronze, (*3*) S45C steel (hardening), (*4*) S45C steel (hardening + tempering) ($\sqrt{area} = \sqrt{hd - d^2/4\sqrt{3}}$ and $2a = 1.843\sqrt{area}$.)



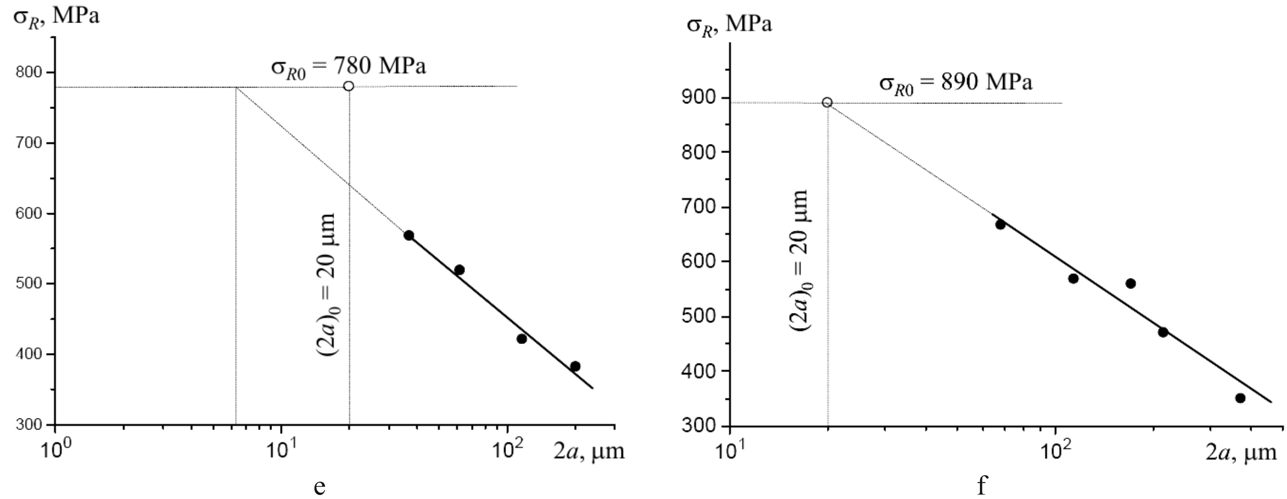


Fig. 7. Dependence of the endurance limit of different materials on the value of $2a$ for S10C steel (a), S45C steel (b), 70/30 bronze (c), 2017-T4 aluminum alloy (d), S45C steel in the hardened state (e), and in hardened and tempered state (f): (▲) $h/d = 0.5$, (●) $h/d = 1.0$, and (■) $h/d = 2.0$.

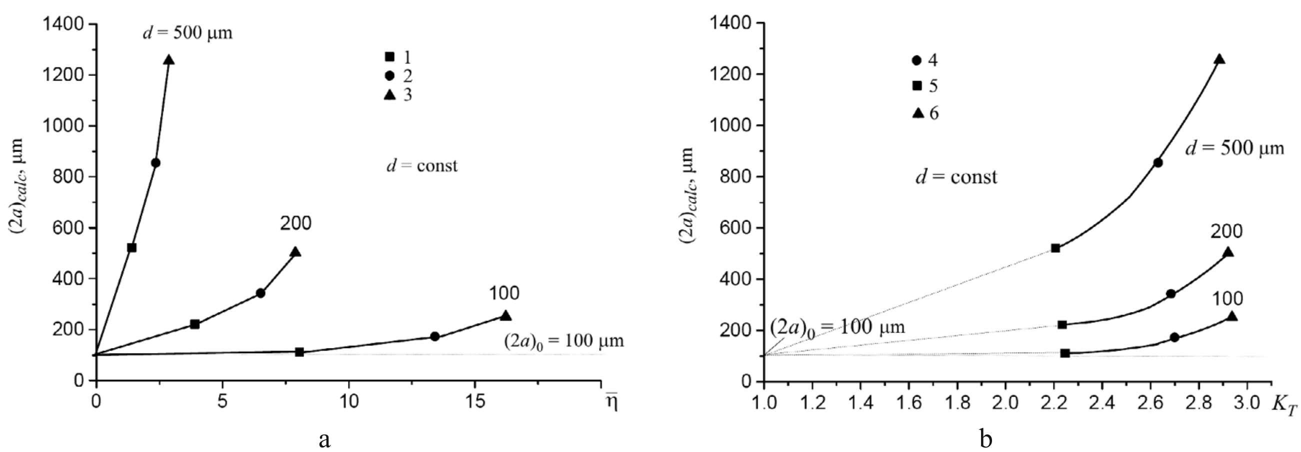


Fig. 8. Dependence of the crack size at the endurance limit of S10C steel on the relative stress gradient (a) and theoretical stress concentration factor (b): (*1*, *4*) $h/d = 0.5$, (*2*, *5*) $h/d = 1.0$, and (*3*, *6*) $h/d = 2$.

non-propagating crack $(2a)_{cr} = 110$ μm for the low-strength 2017-T4 aluminum alloy and 70/30 bronze does not affect the reduction in endurance limit, this crack size greatly reduces it for high-strength S45C steel in the hardened state. The critical crack size for this steel is 20 μm. This conclusion is in good agreement with the known fact that high-strength steels are very sensitive to notches and defects.

In Fig. 7, the results presented in Fig. 5 and 6 are shown in the coordinates endurance limit–logarithm of the crack size corresponding to the endurance limit. As can be seen, the set of experimental data for all investigated materials corresponds to a single straight line. This indicates that the crack size at the endurance limit of specimens with defects firstly always remains larger than the size of non-propagating crack, which corresponds to the endurance limit of smooth specimen, and, secondly, decreases with increasing nominal stresses and tends in the limit to the crack size at the endurance limit of smooth specimen.

Based on the above data, let us consider the effect of stress concentration and gradient on the crack size corresponding to the endurance limit of different materials.

Table 4 lists values of theoretical stress intensity factors (K_T) and relative stress gradients ($\overline{\eta}$) for stress raisers in the form of drill holes with different ratio of h/d. The numerical values of K_T and $\overline{\eta}$ were calculated by

TABLE 4. Theoretical Stress Concentration Factors, Relative Stress Gradients, and Crack Sizes at the Endurance Limit for Stress Raisers with the Different Ratio h/d

d/h, μm/μm	$\sqrt{area}$, μm	$(2a)_{calc}$, μm	K_T	η
50/100	59.64	109.9	2.249	8.074
100/200	119.30	219.8	2.237	3.924
250/500	298.20	549.6	2.209	1.427
50/50	46.25	85.2	2.710	26.392
100/100	92.50	170.5	2.702	13.440
200/200	185.00	341.0	2.687	6.532
500/500	462.50	852.4	2.634	2.376
100/50	68.10	125.5	2.946	31.862
200/100	136.20	251.0	2.937	16.225
400/200	272.40	502.0	2.921	7.885
1000/500	681.00	1255.0	2.885	2.869

the finite element method using the ANSYS software product. It also lists surface crack sizes $(2a)_{calc}$, which correspond to the endurance limits of different materials in the presence of such stress raisers.

Figure 8 illustrates the dependence of crack size at the endurance limit of S10C steel on relative stress gradient and theoretical stress concentration factor at the different ratio h/d. It can be seen that the crack size corresponding to the endurance limit of S10C steel specimens with stress raisers decreases with relative stress gradient and theoretical stress concentration factor and tends in the limit (at $K_T = 1$ and $\overline{\eta} = 0$) to the crack size at the endurance limit of a smooth specimen, $(2a)_0$.

Results similar to those presented in Fig. 8 have also been obtained for all other materials investigated in studies [4, 8–10].

CONCLUSIONS

1. At the endurance limit, which is defined as maximum nominal stresses at a number of cycles to fracture of 10^7, non-propagating cracks were observed on all specimens.

2. The value of the square root of the projection of defect area onto a plane perpendicular to maximum normal stresses, $\sqrt{area}$, has been adopted as a defect size characteristic.

3. A proportional relationship between the parameter $\sqrt{area}$ and the crack size $2a$ has been established: $\sqrt{area} \propto 2a$.

4. It has been found that the specimens with small fatigue cracks and defects in the form of drill holes and notches with the same value of the parameter $\sqrt{area}$ have the same endurance limit.

5. A procedure for the calculation of the size of the size of non-propagating cracks at the endurance limit of specimens with stress raisers in the form of drill holes has been proposed. Using experimental data for a wide range of metals and alloys, obtained in studies by Murakami, on the endurance limit σ_R and the size of defects of different size and shape, which are evaluated with the parameter $\sqrt{area}$, on the one hand and on the geometry of fatigue cracks at the endurance limit under relevant test conditions (semicircula, semielliptical cracks, etc) on the other hand, the size of non-propagating cracks at the endurance limit of these materials has been calculated.

6. The effect of the stress concentration and gradient level on the crack size at the endurance limit of different materials has been considered.

7. It has been shown that the crack size at the endurance limit of specimens with defects firstly decreases with nominal stresses and, secondly, is always larger than the size of non-propagating crack at the endurance limit of smooth specimen.

8. It has been found that the crack size corresponding to the endurance limit of specimens with stress raisers in the form of a drill hole decreases with the relative stress gradient and theoretical stress concentration factor and tends in the limit (at $K_T = 1$ and $\overline{\eta} = 0$) to the crack size at the endurance limit of a smooth specimen.

REFERENCES

1. P. Lukáš, L. Kunz, B. Weiss, and R. Stickler, "Non-damaging notches in fatigue," *Fatigue Fract. Eng. Mater. Struct.*, **9**, No. 3, 195–204 (1986).
2. H. Kobayashi and H. Nakazawa, "A stress criterion for fatigue crack propagation in metals," in: Proc. of the Int. Conf. on Mechanical Behaviour of Materials (Aug. 15–20 1971, Kyoto, Japan), Vol. 2, Kyoto (1971), pp. 199–208.
3. H. Kitagawa and S. Takahashi, "Applicability of fracture mechanics to very small cracks, or the cracks in the early stage," in: Proc. of the Second Int. Conf. on Mechanical Behaviour of Materials, Boston, MA (1976), pp. 627–631.
4. Y. Murakami, *Metal Fatigue: Effect of Small Defects and Nonmetallic Inclusions*, Elsevier, Amsterdam (2003).
5. D. G. Shang, W. X. Yao, and D. J. Wang, "A new approach to the determination of fatigue crack initiation size," *Int. J. Fatigue*, **20**, No. 9, 683–687 (1998).
6. M. Makkonen, "Statistical size effect in the fatigue limit of steel," *Int. J. Fatigue*, **23**, No. 5, 395–402 (2001).
7. M. Makkonen, "Notch size effects in the fatigue limit of steel," *Int. J. Fatigue*, **25**, No. 1, 17–26 (2003).
8. Y. Murakami and T. Endo, "Effects of small defects on the fatigue strength of metals," *Int. J. Fatigue*, **2**, 23–30 (1980).
9. Y. Murakami and M. Endo, "Quantitative evaluation of fatigue strength of metals containing various small defects or cracks," *Eng. Fract. Mech.*, **17**, No. 1, 1–15 (1983).
10. Y. Murakami, Y. Tazunoki, and T. Endo, "Existence of coaxing effect and effects of small artificial holes on fatigue strength of an aluminum alloy and 70-30 brass," *Metall. Trans. A*, **15**, 2029–2038 (1984).
11. Y. Murakami and M. Endo, "Effects of hardness and crack geometry on ΔK_{th} of small cracks emanating from small defects," in: K. J. Miller and E. R. De los Rios (Eds.), *The Behaviour of Short Fatigue Cracks*, EGF 1. Mechanical Engineering Publications (1986), pp. 275–293.
12. Y. Murakami and M. Endo, "A geometrical parameter for the quantitative estimation of the effects of small defects on fatigue strength of metals," *Trans. Jpn. Soc. Mech. Eng. Ser. A*, **49** (438), 127–136 (1983).
13. Y. Murakami, M. Abe, and T. Kiyota, "Effects of small defects and inclusions on fatigue strength of maraging steel," *Trans. Jpn. Soc. Mech. Eng. Ser. A*, **53** (492), 1482–1491 (1987).
14. Y. Murakami, S. Fukuda, and T. Endo, "Effect of micro-hole on fatigue strength [1st Report, Effect of micro-hole dia.: 40, 50, 80, 100 and 200 μm on the fatigue strength of 0.13% and 0.46% carbon steels]," *Trans. Jpn. Soc. Mech. Eng. Ser. I*, **44** (388), 4003–4013 (1978).
15. Y. Murakami, H. Kawano, and T. Endo, "Effect of micro-hole on fatigue strength [2nd Report, Effect of micro-hole of 40–200 μm in diameter on the fatigue strength of quenched or quenched and tempered 0.46% carbon steel]," *Trans. Soc. Mech. Eng. Ser. A*, **45** (400), 1479–1486 (1979).
16. Y. Murakami and T. Endo, "The effects of small defects on the fatigue strength of hard steels," in: Proc. of the Int. Conf. Fatigue 81. Materials Experimentation and Design, Warwick University (1981), pp. 431–440.
17. H. Nisitani and M. Kage, "Rotating bending fatigue of electropolished specimens with transverse holes – observation of slip bands and non-propagating cracks near the holes," *Trans. Jpn. Soc. Mech. Eng. Ser. I*, **39** (323), 2005–2012 (1973).
18. Y. Murakami, Y. Tazunoki, and T. Endo, "Existence of coaxing effect and effect of small artificial holes of 40–200 μm diameter on fatigue strength in 2017S-T4 Al alloy and 7:3 brass," *Trans. Jpn. Soc. Mech. Eng. Ser. A*, **47** (424), 1293–1300 (1981).

19. H. Ohba, Y. Murakami, and T. Endo, "Effects of artificial small holes on fatigue strength of notched specimens," *Trans. Jpn. Soc. Mech. Eng. Ser. A*, **49** (444), 901–910 (1983).
20. H. Nisitani and S. Nishida, "The change of surface states and the incipient fatigue cracks in electro-polished low carbon steel (plain and notched specimens) subjected to rotating bending stress," *Trans. Jpn. Soc. Mech. Eng. Ser. I*, **35** (280), 2310–2315 (1969).
21. H. Nisitani and Y. Murakami, "Torsional fatigue and bending fatigue of electropolished low carbon steel specimens," *Trans. Jpn. Soc. Mech. Eng. Ser. I*, **35** (275), 1389–1396 (1969).
22. H. Nisitani, "Correlation between notch sensitivity of a material and its non-propagating crack, under rotating bending stress," in: Proc. of the Int. Conf. Mechanical Behaviour Material, Vol. II, Kyoto (1972), pp. 312–322.
23. H. Nisitani and K. Kawano, "Non-propagating crack and crack strength of shafts with a shoulder fillet subjected to rotary bending," in: Proc. of the 11th Japan Congress on Materials Research – Metallic Materials. Society of Materials Science, Kyoto (1968), pp. 49–51.
24. H. Kobayashi and H. Nakazawa, "The effects of notch depth on the initiation propagation and non-propagation of fatigue cracks," *Trans. Jpn. Soc. Mech. Eng. Ser. I.*, **35** (277), 1856–1863 (1969).
25. H. Nisitani and M. Endo, "Fatigue strength of carbon steel specimen having an extremely shallow notch," *Eng. Fract. Mech.*, **21**, 215–227 (1985).
26. H. Nisitani and M. Endo, "Unifying treatment of notch effects in fatigue," *Trans. Jpn. Soc. Mech. Eng. Ser. I*, **51**, 784–789 (1985).
27. H. Nisitani and M. Endo, "Unified treatment of deep and shallow notches in rotating bending fatigue," in: J. T. Fong, R. P. Wei, R. J. Fields, and R. P. Gangloff (Eds.), *Basic Questions in Fatigue*, ASTM STP 924 (1988), pp. 136–153.
28. H. Nisitani and I. Chishiro, "Non-propagating micro-cracks of plain specimens and fatigue notch sensitivity in annealed or heat-treated 0.5% C steel," *Trans. Jpn. Soc. Mech. Eng. Ser. I*, **40** (329), 41–52 (1974).
29. H. Nisitani and K. Okasaka, "Effects of mean stress on fatigue strength, crack strength and notch radius at branch point under repeated axial stresses," *Trans. Jpn. Soc. Mech. Eng. Ser. I*, **39** (317), 49–59 (1973).
30. H. Kobayashi and H. Nakazawa, "On the alternating stress required to propagate a fatigue crack in carbon steels," *Trans. Jpn. Soc. Mech. Eng. Ser. I*, **36** (291), 1789–1798 (1970).
31. Y. Murakami, H. Kawano, and T. Endo, "Effect of artificial small defects on fatigue strength of metals," *J. Soc. Mater. Sci.*, **29** (325), 988–992 (1980).
32. W. Zhixue, "Short fatigue crack parameters describing the lifetime of unnotched steel specimens," *Int. J. Fatigue.*, **23**, No. 4, 363–369 (2001).