FATIGUE CRACK GROWTH KINETICS
IN D16AT ALUMINUM ALLOY SPECIMENS
WITH MULTIPLE STRESS CONCENTRATORS

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The fatigue crack growth in D16AT aluminum alloy specimens with multiple stress concentrators in the form of holes is investigated experimentally. A generalized kinetic fatigue crack growth curve is obtained and the Paris equation coefficients are determined for a great number of cracks. It is shown that a general linear, semi-logarithmic relationship exists between these coefficients for aluminum alloys D16AT, 2024-T3, and 7075-T6. The statistical description of the fatigue crack growth based on the Paris equation is specified by the distribution of the exponent m, which is log-normal for aluminum alloys whose numerical characteristics are common to all types of alloys.

Keywords: fatigue cracks, kinetic fatigue crack growth curve, Paris equation, Paris equation coefficients.

Introduction. The problem of multiple site damage (MSD) is one of the main problems in the prediction of the life of aircraft structures with a great number of stress raisers, such as rivet holes [1, 2]. Holes serve as the sources of formation of fatigue cracks that propagate along the riveted joint. These cracks (the MSD cracks) have a relatively small size – their length is limited by the distance between neighboring holes. However, for a sufficiently great number of damages, an accelerated reduction in the residual strength of structures is possible due to the failure of individual ligaments between holes and formation of a large "lead" crack in the riveted joint. Cracks of this kind grow rapidly by the mechanism of coalescence of MSD cracks, broken ligaments and even undamaged holes [3, 4]. In order to predict the limit state related to the formation of large lead cracks, it is necessary to have the information on small-size MSD crack behavior.

The multiple site damage is characterized by two main factors: the number of cycles required for fatigue crack initiation near the hole and the period of this damage propagation prior to fracture of the ligament between the neighboring holes. These factors are of a statistical nature and are to be described in terms of the probabilistic aspect [5]. It is noteworthy that the statistical characteristics of the life (number of cycles) to fatigue crack initiation in aircraft structures have a certainty sufficient for engineering calculations and even are represented in standard documents [6]. The problem of a random growth of fatigue MSD cracks in aluminum alloys requires more extensive studies.

For the computational assessment of the cycle life characteristics of riveted joints in aircraft structures, the numerical simulation by the Monte Carlo method is widely used [6–9]. Here, one of the defining statements is the statistical representation of the crack growth rate (CGR) based on the Paris equation:

$$\frac{da}{dN} = C\Delta K^{m} \tag{1}$$

and its various modifications [10, 11], such as the Walker relation:

$$\frac{da}{dN} = C\left[\frac{\Delta K}{(1-R)^{1-n}}\right]^{m}, \tag{2}$$

where a is the crack length, N is the number of loading cycles, C, m, and n are the material constants, ΔK is the stress intensity factor (SIF) range in the cycle, and R is the load cycle stress ratio.

In [7, 8], it was assumed that the coefficient m in Eq. (1) is a deterministic quantity equal to a mean value for aluminum alloys, and the parameter C is a random quantity distributed according to the log-normal [7] or normal [8] laws. For example, for a 2024-T3 aluminum alloy, it is assumed that $m = 2.218$ [7] and $m = 2.555$ [8]; the mathematical expectation (ME) of the coefficient C is $\mu[C] = 2.47 \cdot 10^{-23}$; its roof-mean-square deviation (RMSD) is $\sigma[C] = 2.47^{-24}$ (the SIF is measured in $\mathrm{Pa \cdot m}^{1/2}$, the CGR in m/cycle) [7] and $\mu[C] = 2.88 \cdot 10^{-7}$, $\sigma[C] = 0.0368$ (the SIF is measured in $\mathrm{MPa \cdot m}^{1/2}$, the CGR in mm/cycle) [8].

In [9], to describe the fatigue crack growth kinetics, the Young model [12, 13]

$$\frac{da}{dt} = X(t)Qa^{b} \tag{3}$$

is used, which represents Eq. (1) in somewhat another form of notation, where Q and b are constants ($b = m/2$), t is the time, or the number of load cycles, and $X(t)$ is a positive random process with the ME equal to unity. In model (3), the process $X(t)$ describes, in a generalized manner, the random variations in crack rates for the deterministic values of the parameters Q and b.

It should be noted that Eq. (3) is commonly used for relatively small-sized cracks. In the numerical simulation of MSD in riveted joints, it is assumed that the value of $\log X(t)$ for the specified value of t is distributed according to the normal law with the ME equal to zero [9].

The models for random fatigue crack growth based on equation (1) are also used to solve quite a number of problems related to the assessment of the operating condition of in-service aircrafts. It is assumed that the parameters m and C in the stable (second) region of the kinetic fatigue crack growth (KFCG) curve are random quantities with a uniform distribution of its own possible values within the specified boundaries. It was determined in [14, 15] based on the experimental data for a 7076-T6 aluminum alloy obtained in [16] that the coefficient C (measured in $\mathrm{m}^{1-m/2} \cdot \mathrm{MPa}^{-m}$) is uniformly distributed in the range of values from $5 \cdot 10^{-11}$ to $5 \cdot 10^{-10}$, and the coefficient m in the range from 3.0 to 4.3. The representation of the coefficient values in the $m - \log C$ coordinates is indicative of the presence of a linear correlation between them (the correlation coefficient is equal to –0.8065):

$$\log C = q - pm, \tag{4}$$

where p and q are the coefficients.

For 7075-T6 and 7075-T651 aluminum alloys, it was assumed in [17, 18] that the coefficient m is uniformly distributed in the range from 3.3 to 4.3 [15] and from 3.2 to 4.6, and the coefficient C was treated as a deterministic quantity equal to $3.8 \cdot 10^{-11}\ \mathrm{m}^{1-m/2} \cdot \mathrm{MPa}^{-m}$ [18] or as a random quantity $\log C$ with a uniform distribution in the range from -10.3 to -9.3 [17].

In some models, other kinds of distribution of the coefficients m and C are used. For example, in the optimization of aircraft inspection schedules, the lognormal distribution of the parameter m with the numerical characteristics $\mu[m] = 2.97$ and $\sigma[m] = 1.05$ for a 7075-T651aluminum alloy is proposed [19].

Thus, in the simulation of the random process of fatigue cracks in aluminum alloys of aircraft structures, the parameters m and C of Eq. (1) are treated as random quantities. In some models it is assumed that the coefficient C is a random quantity, while the exponent m is a deterministic quantity; in others, vice versa, m is a random quantity and the coefficient C is a deterministic quantity. A number of models are based on the assumption that both coefficients are random quantities that have a uniform distribution in the region of their possible values and are
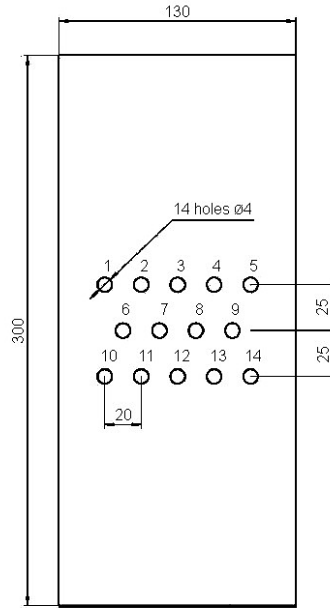
Fig. 1. Test specimen.

interdependent. Common to the models is that the majority of them are based on a very limited amount of experimental data on the coefficients m and C for structural aluminum alloys (taken mainly from [16, 20]). Therefore, the goal of this paper is to determine the coefficients of Paris Eq. (1) for a D16AT aluminum alloy. In this case, specimens with multiple concentrators in the form of holes for rivets are used.

Experimental Investigation Procedure. Consider 1.5 mm-thick plain specimens made of a D16AT sheet aluminum alloy with 14 holes of 4 mm in diameter arranged in three rows (Fig. 1).

Specimens were loaded by cyclic tension ($R = 0$) at a frequency of 11 Hz for three values of the maximum nominal stress in a cycle (80, 100, and 120 MPa) in the net sections along the horizontal line with five holes (three specimens each for the regime of loading). The stresses in the net section along the row with four holes were lower in magnitude and this was taken into account in determining the SIF.

After being drilled, the holes were reamed with a special reamer to remove any burrs. The surface area with the holes was polished on both sides of the specimen in order to facilitate visual observation and recording of cracks.

The occurrence of cracks and their growth were monitored by a digital camera (with a resolution of 960×720 pixels and $\times 20$ magnification) mounted on a special lightweight tripod that was fixed directly on the specimen. The tripod design allows using only one camera, which is moved to one hole or another, to shoot all cracks. The use of this procedure ensures obtaining clear, unfuzzy photographs at the same focal length. Each photo had a sequential number of the hole, the cracks as such and their growth paths. The time of each photo corresponded to a certain value of the number of loading cycles. The crack length was determined from digital images as a distance in pixels between two points – the hole edge and the alternative crack – using the Scale 1.0 program [21].

The SIF range was calculated by the formula

$$\Delta K = Y(a)\Delta\sigma\sqrt{\pi a}, \tag{5}$$

where $\Delta\sigma$ is the nominal stress range per cycle and $Y(a)$ is the geometric correction function.

The parameter $Y(a)$ describes the influence of high stresses at the hole due to the concentration effect on the SIF. For the plate with a hole of radius r, from which a crack of length a emanates, the parameter $Y(a)$ is defined as [22]

$$Y(a) = 1 + 2.36\exp\left[-2.08\left(\frac{a}{r}\right)\right]. \tag{6}$$

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TABLE 1. Values of the Coefficients of Eq. (1) in the Approximation of the Characteristic Regions of the FCG Curve

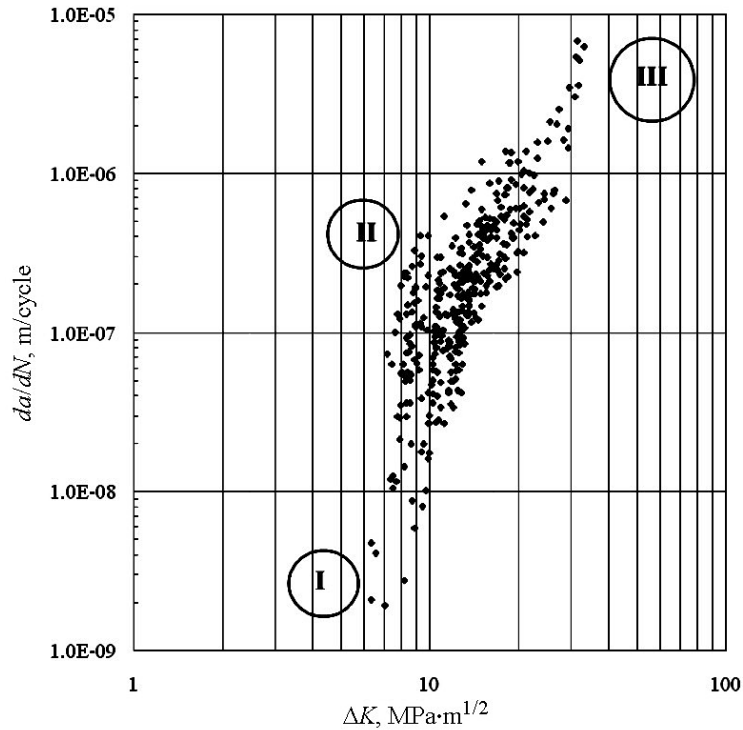| Coefficients	Region I	Region II	Region I
C	$1.9364 \cdot 10^{-17}$	$1.2246 \cdot 10^{-10}$	$1.306 \cdot 10^{-20}$
m	10.168	2.8081	9.6774
R^2	0.7805	0.6767	0.9145

Fig. 2. Kinetic fatigue crack growth curve of a D16AT aluminum alloy obtained in the testing of specimens with holes: *I*, *II*, and *III* are the regions of low, middle, and high crack growth rates, respectively.

In the SIF determination, an account was taken of the increase in the nominal stress at the cost of a decrease in the net section of the specimen caused by the presence of cracks along the row of holes.

Test Results and Their Discussion. The generalized kinetic fatigue crack growth (FCG) curve plotted from the testing results for specimens with holes made of the D16AT aluminum alloy is shown in Fig. 2. As seen, characteristic regions of the FCG curve in the log-log coordinates are approximated by the straight lines. The values of the coefficients m and C of Eq. (1) and the coefficients of the correlation R^2 in the approximation are summarized in Table 1.

A cluster of points in the characteristic regions of the generalized FCG curve (Fig. 2) corresponds to the experimental data obtained for cracks in each tested specimens.

As seen from the data obtained (Table 1), a great variation in the values of the coefficients m and C is observed on different regions of the FCG curve. However, a sufficiently close correlation between the values of these coefficients is peculiar to all the FCG curves. In a quite wide range of the values of m and C, the experimental points in the semi-log coordinates are approximated by the linear relationship

$$\log C = -6.7956 - 1.0674m \tag{7}$$

with the correlation coefficient $R^2 = 0.974$ (Fig. 3).

TABLE 2. Values of the Coefficients of Eq. (4) for Various Types of Steels [23]

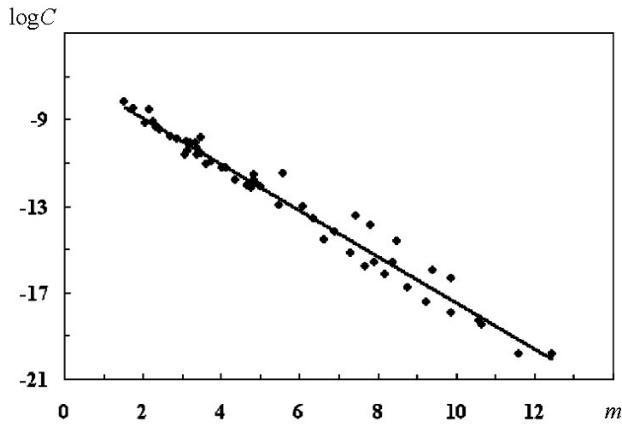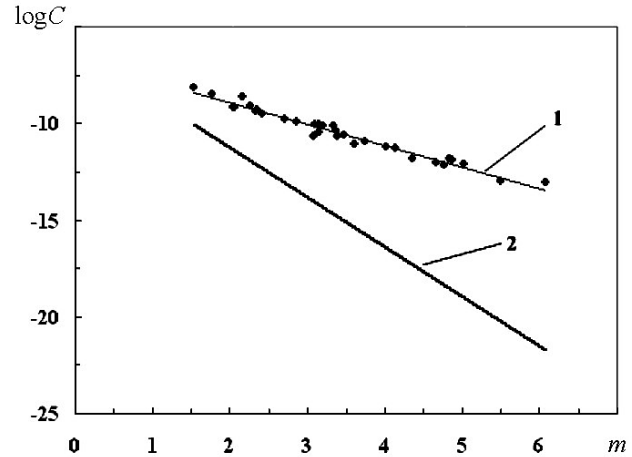| Relationship authors	p	q	m
T. R. Gurney	2.9518	−5.3794	1.8–4.0
K. Kobayashi et al.	3.0527	−5.0654	3.0–6.5
K. Tanaka and C. Matsuoka	3.0068	−5.2696	1.8–5.1
T. Kanazawa et al.	2.7618	−5.7612	2.0–5.5
P. Romvari, L.Toth, and D. Nad'	2.9900	−5.2647	1.5–11.0

Fig. 3

Fig. 4

Fig. 3. Relationship between the coefficients C and m of the Paris equation for cracks in specimens of a D16AT aluminum alloy with multiple concentrators. [The solid line shows the approximation of the obtained experimental data with Eq. (7).]

Fig. 4. Relationship between the coefficients C and m in the Paris equation for cracks in specimens of a D16AT aluminum alloy (*1*) and of various types of steels (*2*).

The presence of the close correlation relationship between the coefficients of the Paris equation is a known fact. However, the results of the systematic investigations are very limited. Based on the survey of data from the literature, four relationships derived by different authors for various steel types and also the author's own correlation expression are presented in [21]. All these relationships are described by Eq. (4), whose coefficient values are converted to $[\text{m}^{1-m/2} \cdot \text{MPa}^{-m}/\text{cycle}]$ for the dimensions of C and are presented in Table 2. From the standpoint of graphics, they are very close and can be described, with a high degree of correlation, by the generalized regression equation

$$\log C = -5.3481 - 2.9526m. \tag{8}$$

Comparison of expression (7) with (8) shows that the absolute value of the coefficient p is almost three times higher for steels than for aluminum alloy. For $2 \leq m \leq 6$, the value of the coefficient C is by 2 to 6 orders of magnitude higher for steels than for a D16AT aluminum alloy (Fig. 4).

The results of the investigation into the fatigue crack growth kinetics in alloy D16AT obtained in this study were compared with similar data for aluminum alloys 2024-T3 and 7075-T6 used in aircraft industry, which are close in properties. The study of [20] summarizes results of numerous experimental investigations into the fatigue crack growth kinetics in these alloys in the form of a set of values of the coefficients m and C in the Paris equation taken from different sources: 21 values for alloy 2024-T3 and 39 values for alloy 7075-T6. As for alloy D16AT, these data are approximated by the linear relationship given by (4), whose coefficient values are presented in Table 3 (for $m \leq 6$).

TABLE 3. Values of the Coefficients of Eq. (4) for Aluminum Alloys

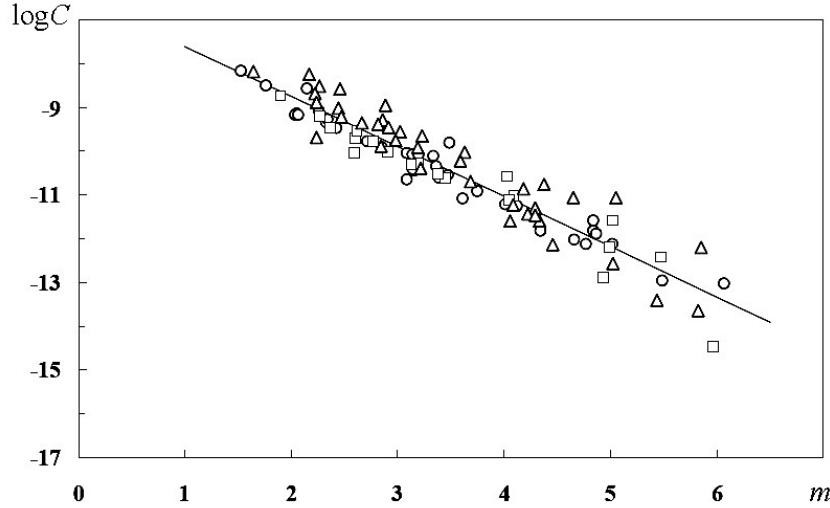| Alloy	p	q	Correlation coefficient R^2
D16AT	1.0813	−6.7757	0.9553
2024-T3	1.3050	−5.7452	0.9077
7075-T6	1.0206	−6.4447	0.9398

Fig. 5. Relationship between the coefficients C and m of the Paris equation for cracks in specimens of aluminum alloys D16AT ($\bigcirc$), 2024-T3 ($\square$), and 7075-T6 ($\triangle$). [The line shows the approximation of the obtained experimental data with Eq. (11).]

The experimental points for aluminum alloys D16AT, 2024-T3, and 7075-T6 are grouped quite compactly around the linear relationship

$$\log C = -6.4607 - 1.1442m \tag{9}$$

with the correlation coefficient $R^2 = 0.9057$ (Fig. 5), which testifies to the validity of the results obtained.

The presence of the linear relationship between the parameters $\log C$ and m is explained by that the kinetic fatigue crack growth curves have a common point, which can be referred to as the curve focus (the Gurney's point [24]), see Fig. 6.

Indeed, let the FCG curves obtained for different cracks in a specific material be a set of lines in the log-log coordinates (the solid lines in Fig. 6), which intersect at the point with the ($\log K_f$, $\log C_f$) coordinates, where K_f and C_f are the parameters of a definite class of materials (steels, aluminum alloys, etc.).

According to the accepted scheme (Fig. 6), it is possible to write for the ith curve

$$m_i = \frac{\log C_f - \log C_i}{\log K_f}. \tag{10}$$

The relationship

$$\log C_i = \log V_f - m_i \log K_f, \tag{11}$$

which is valid for any curve intersecting at the point ($\log K_f$, $\log V_f$), follows from (10).

By comparing between the relationships given by (4) and (11), we obtain

$$p = \log K_f, \qquad q = \log C_f. \tag{12}$$

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TABLE 4. Values of the Numerical Characteristics of Distribution of the Coefficient m for Aluminum Alloys

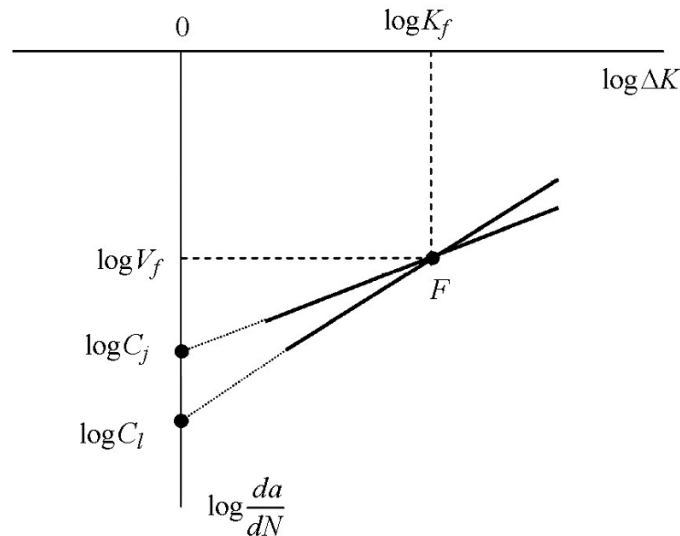| Alloy	Amount of sampling	$\mu[m]$	$\sigma[m]$	Distribution law
D16AT	36	3.4163	1.1306	Lognormal
2024-T3	21	3.7519	1.3610	
7075-T6	39	3.6102	1.2110	
2024-T3, 7075-T6	60	3.6598	1.2559	
D16AT, 2024-T3, 7075-T6	96	3.5685	1.2101	

Fig. 6. Scheme to justify relationship (4) for fatigue crack growth curves having a focal point (the point F).
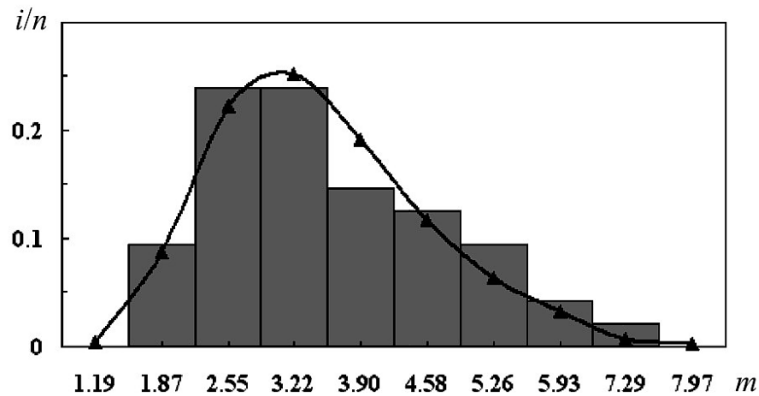

Fig. 7. Bar chart and density function of the distribution (solid line) of the coefficient m for aluminum alloys D16AT, 2024-T3, and 7075-T6 (i/n is the rate of fall within the range of values in the bar chart, n is the total number of values, $n = 96$).

Thus, the coefficients p and q (4) specify the coordinates of the focal point for the linear regions of the fatigue crack growth curves, though, as shown by the reported data, for at least two classes of materials, these parameters are invariant relative to the material grade within the given class. For example, according to relationship (9), for aluminum alloys, we have $K_f = 13.94$ MPa $\cdot$ m$^{1/2}$, $V_f = 3.4618 \cdot 10^{-7}$ m/cycle, and for steels, as it follows from formula (8), $K_f = 896.6$ MPa $\cdot$ m$^{1/2}$, $V_f = 4.4864 \cdot 10^{-6}$ m/cycle.

Paris equation (1) in view of the relationship given by Eq. (4) and with consideration of the coefficients in (12) takes the form

$$\frac{da}{dN} = 10^q (10^{-p} \Delta K)^m = V_f \left(\frac{\Delta K}{K_f} \right)^m,$$ (13)

where, as noted above, the coefficients p and q, or K_f and V_f, are common to a particular class of materials, and it is only the exponent m that depends on the specific material.

It is convenient to use Eq. (13) to statistically describe the fatigue crack growth kinetics, because only one parameter – the coefficient m – is a random quantity. The statistical distribution of this parameter for the considered aircraft aluminum alloys are described by a lognormal law with sufficiently close values of the numerical characteristics (Table 4).

The bar chart of the generalized distribution of experimental values of the coefficient m for three types of aluminum alloys is shown in Fig. 7.

CONCLUSIONS

1. Based on the performed experimental investigations of specimens with multiple concentrators in the form of holes, a generalized kinetic fatigue crack growth curve for a D16AT aluminum alloy was obtained for a great number of cracks.

2. Experimental data presented in the form of the Paris equation testify to the presence of a linear relationship between the coefficients (C and m) in semi-log coordinates. This relationship obtained for alloy D16AT agrees with results of other investigations on the fatigue crack growth kinetics in alloys with a similar behavior. This type of the relationship is indicative of the existence of a common point (the focal point) at which the kinetic fatigue crack growth curves intersect.

3. Using different types of aluminum alloys and steels as an example, the assumption has been made that coordinates of the focal points K_f and V_f are common to a particular class of materials, and it is only the exponent m that depends on the specific material.

4. Based on the generalization of the experimental data for three types of aluminum alloys, it has been shown that the distribution of the exponent m in the Paris equation is described by the log-normal distribution, and the numerical characteristics of this distribution have been determined.

5. The results obtained can be used to statistically describe fatigue crack growth in aircraft structures when solving a wide circle of problems related to the life cycle assessment of aircrafts.

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