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A METHOD OF VIRTUAL DESIGN OF THE FATIGUE LIFE OF A DYNAMIC STRUCTURE

Wenli Zhao,[a,1] Xiaojun Zhou,[b] and Meina Shen[a]

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The fatigue life prediction and reliability analysis of dynamic systems under random excitement are important topics in modern engineering design. However, the stress power spectra density of a component in a dynamic system must be known when one predicts its fatigue life and analyzes its reliability. With the rapid development of computer technology and numerical computation theories, it has been possible to simulate stress power spectra density of dynamic systems. Based on this, a method of simulating computation for the fatigue life prediction of a dynamic structure is presented. It depended on some simulating equations of the stress power spectra density of the structure and the fatigue life prediction which was obtained in this paper. By this way, as an example, the virtual design of the fatigue life and reliability analysis for the CW-200k vehicle truck was developed. The method is suitable not only for the running dynamic structure but also for newly designed structures.

Keywords: random vibration system, stress power spectral density, fatigue life prediction, virtual design.

Introduction. The fatigue life prediction and reliability analysis of dynamic systems under random excitement are important topics in modern engineering design [1–8]. The vibration characteristics of fatigue damage of beam-type structural components has been researched [9, 10] and a computational and experimental assessment of the sensitivity of structural materials to stress concentration under high-cycle asymmetrical loading has been performed [11]. A numerical calculation of the frequencies of natural vibrations of beams with a cross section varying linearly in height under different conditions of fixing their ends is presented [12]. However, the stress power spectral density of a component in a dynamic system must be known when one predicts its fatigue life and analyzes its reliability. A traditional way is to measure the stress power spectral density, which has played an important role in practice [4]. However, the stress power spectral density of a dynamic system will change with the alteration of the structure and dynamic parameters of a system. So the engineering application in this way is limited. Moreover, one is not able to obtain the stress power spectral density of a new dynamic system before it has been designed. With the rapid development of computer technology and numerical computation theories, it has been possible to simulate stress power spectral density of dynamic systems. Based on this, the method of simulating stress power spectral density is put forward in this paper, and as an example the fatigue life prediction and reliability analysis of CW-200k [4] vehicle truck are made.

Computational Method of Stress Power Spectral Density.

Random Response Spectra of Dynamic System Computation [1, 2]. In general, the vibration equation of a linear system can be written in matrix form as

$$\mathbf{M}\ddot{z}(t)+\mathbf{C}\dot{z}(t)+\mathbf{K}z(t)=\mathbf{F}(t), \tag{1}$$

[a]Hangzhou Dianzi University Hangzhou, ZheJiang, China ([1]zhaowlcn@163.com). [b]ZheJiang University, Hangzhou, ZheJiang, China. Translated from Problemy Prochnosti, No. 3, pp. 162 – 169, May – June, 2015. Original article submitted July 10, 2014.

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are $N \times N$ symmetric matrices representing the masses, damping, and stiffness of the system, $\ddot{z}(t)$, $\dot{z}(t)$, and $z(t)$ are column vectors representing acceleration, velocity, and displacement, respectively, and $\mathbf{F}(t)$ is matrix of excitation function.

From Eq. (1), the frequency response matrix is obtained by taking the Fourier transform as follows [3]:

$$\mathbf{H}(\omega) = \mathbf{F}(\omega)[\mathbf{K} - \omega^2 \mathbf{M} + j\omega \mathbf{C}]^{-1}, \tag{2}$$

where $\mathbf{H}(\omega) = F(\mathbf{z}(t)/F(\mathbf{F}(t)).$

Therefore, the power spectral density matrix of system can be deduced as

$$\mathbf{G}_z(\omega) = \mathbf{H}^{*T}(\omega)\mathbf{G}_x(\omega)\mathbf{H}(\omega), \tag{3}$$

where $\mathbf{H}^{*T}(\omega)$ is conjugate transpose matrix of $\mathbf{H}(\omega)$ and $\mathbf{G}_x(\omega)$ is input power spectral density matrix of a system.

If the time difference τ exists in the excitation process, according to the time difference property of the Fourier transform, $\mathbf{G}_x(\omega)$ may be expressed as

$$\mathbf{G}_{x_i x_j}(\omega) = \mathbf{G}_x(\omega) \begin{bmatrix} 1 & e^{-j\omega\tau_{12}} & e^{-j\omega\tau_{13}} & \cdots & e^{-j\omega\tau_{1n}} \\ e^{j\omega\tau_{21}} & 1 & e^{-j\omega\tau_{23}} & \cdots & e^{-j\omega\tau_{2n}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ e^{j\omega\tau_{n1}} & e^{j\omega\tau_{n2}} & e^{-j\omega\tau_{n3}} & \cdots & 1 \end{bmatrix}. \tag{4}$$

Similarly, the acceleration amplitude of the system response is given by

$$\ddot{z} = 3\left[\int_{\omega_{\min}}^{\omega_{\max}} \omega^4 \mathbf{G}_z(\omega) d\omega\right]^{1/2}. \tag{5}$$

The Unit Load Stress Matrix Computation. In a linear elastic system, the stress state of the structure may be expressed as

$$\sigma(x, y, z, t) = \mathbf{S}(x, y, z)\,\mathbf{b}^T(t), \tag{6}$$

where $\mathbf{S}(x, y, z)$ is defined as unit load stress matrix. Vector $\mathbf{b}(t)$ is the load state vector. The load state vectors may be expressed as

$$\mathbf{b}(t) = (\mathbf{F}(t), \mathbf{M}(t)) = (\mathbf{W}z(t), \mathbf{V}z(t)) = \mathbf{J}z(t), \tag{7}$$

where $\mathbf{F}(t) = (\mathbf{F}_1(t), \mathbf{F}_2(t), \ldots, \mathbf{F}_n(t))$ is called the forces in the translation sense, $\mathbf{M}(t) = (\mathbf{M}_1(t), \mathbf{M}_2(t), \ldots, \mathbf{M}_n(t))$ is called the moments in rotation, $\mathbf{J} = (\mathbf{W}, \mathbf{V})^T$ is called linear operator or coefficients' matrix, and $z(t)$ is vector representing the response of each degree of freedom.

According to the theory of correlation function [3], the correlation function matrix of stress state of stationary random process can be written as

$$\mathbf{R}_s(\tau) = E[\sigma(t)\sigma^T(t+\tau)] = \mathbf{S}(x, y, z)E[b(t)b^T(t+\tau)]\mathbf{S}^T(x, y, z)$$

$$= \mathbf{S}(x, y, z)\mathbf{J}E[z(t)z^T(t+\tau)]\mathbf{J}^T\mathbf{S}^T(x, y, z) = \mathbf{S}(x, y, z)\mathbf{J}\mathbf{R}_z(\tau)\mathbf{J}^T\mathbf{S}^T(x, y, z), \tag{8}$$

where E represents mathematical expectation.

Therefore, the stress power spectral density matrix can be obtained by taking Fourier transform as

$$\mathbf{G}_s(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{R}_s(\tau)e^{-j\omega\tau}d\tau=\mathbf{S}(x,\ y,\ z)\,\mathbf{J}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{R}_z(\tau)e^{-j\omega\tau}d\tau\right]\mathbf{J}^T\mathbf{S}^T(x,\ y,\ z)$$

$$=\mathbf{S}(x,\ y,\ z)\,\mathbf{J}\mathbf{G}_z(\omega)\,\mathbf{J}^T\mathbf{S}^T(x,\ y,\ z). \quad (9)$$

Then the stress square deviation is

$$\mathbf{D}_s=\mathbf{R}_s(0)=\int_{-\infty}^{\infty}\mathbf{G}_s(\omega)d\omega. \quad (10)$$

If $z(t)=[z_1(t),\ z_2(t),\ \ldots,\ z_n(t)]$ represents response vector matrix of a system, then the response spectra will be of the form

$$\mathbf{G}_z(\omega)=\begin{bmatrix}\mathbf{G}_{z_1z_1}(\omega) & \mathbf{G}_{z_1z_2}(\omega) & \cdots & \mathbf{G}_{z_1z_n}(\omega)\\ \mathbf{G}_{z_2z_1}(\omega) & \mathbf{G}_{z_2z_2}(\omega) & \cdots & \mathbf{G}_{z_2z_n}(\omega)\\ \vdots & \vdots & \vdots & \vdots\\ \mathbf{G}_{z_nz_1}(\omega) & \mathbf{G}_{z_nz_2}(\omega) & \cdots & \mathbf{G}_{z_nz_n}(\omega)\end{bmatrix}. \quad (11)$$

Now, the relationship between the response spectra of the system and stress power spectral density of a structure is established. Based on this formulation, one is able to estimate fatigue life and analyze the reliability of the structure.

The Fatigue Life Prediction of the CW-200k type Truck.

The Stress Power Spectral Density Computation of the CW-200k Type Truck of a Passenger Vehicle. The stress power spectral density of the frame of a truck can be researched in the vertical vibration of six-degree-of-freedom system because, in high-speed railways, track is either straight or has a big radius. The dynamic model of vehicle is as shown in Fig. 1.

The equations of motion was obtained as follows:

$$\begin{cases} m_c\ddot{z}_c+2C_2\dot{z}_c-C_2\dot{z}_{b1}-C_2\dot{z}_{b2}+2K_2z_c-K_2z_{b1}-K_2z_{b2}=0,\\ J_c\ddot{\varphi}_c+2C_2l^2\dot{\varphi}_c-C_2l\dot{z}_{b1}+C_2l\dot{z}_{b2}+2K_2l^2\varphi_c-K_2lz_{b1}+K_2lz_{b2}=0,\\ m_b\ddot{z}_{b1}-C_2\dot{z}_c-C_2l\dot{\varphi}_c+(2C_1+C_2)\dot{z}_{b1}-K_2z_c-K_2l\varphi_c+(2K_1+K_2)z_{b1}=C_1(\dot{\eta}_1+\dot{\eta}_2)+K_1(\eta_1+\eta_2),\\ m_b\ddot{z}_{b2}-C_2\dot{z}_c+C_2l\dot{\varphi}_c+(2C_1+C_2)\dot{z}_{b2}-K_2z_c+K_2l\varphi_c+(2K_1+K_2)z_{b2}=C_1(\dot{\eta}_3+\dot{\eta}_4)+K_1(\eta_3+\eta_4),\\ J_b\ddot{\varphi}_{b1}+2C_1l_1^2\dot{\varphi}_{b1}+2K_1l_1^2\varphi_{\beta1}=C_1l_1(\dot{\eta}_1-\dot{\eta}_2)+K_1l_1(\eta_1-\eta_2),\\ J_b\ddot{\varphi}_{b2}+2C_1l_1^2\dot{\varphi}_{b2}+2K_1l_1^2\varphi_{b2}=C_1l_1(\dot{\eta}_3-\dot{\eta}_4)+K_1l_1(\eta_3-\eta_4), \end{cases} \quad (12)$$

and Eqs. (12) are written in matrix form as

$$[M]\{\ddot{z}\}+[C]\{\dot{z}\}+[K]\{z\}=[C_\eta]\{\dot{\eta}\}+[K_\eta]\{\eta\}, \quad (13)$$

where $\{z\}$ is displacement vector, $\{z\}=[z_c\ \ \varphi_c\ \ z_{b1}\ \ \varphi_{b1}\ \ z_{b2}\ \ \varphi_{b2}]^T$, $\{\eta\}$ is the input vector, $\{\eta\}=[\eta_1\ \ \eta_2\ \ \eta_3\ \ \eta_4]^T$, $[M]$, $[C]$, and $[K]$ are 6×6 symmetric matrices representing the mass, damping, and stiffness of a vehicle system, respectively, and $[C_\eta]$ and $[K_\eta]$ are matrices of coefficients. They correspond to the CW-200k type truck and 25 type passenger rail vehicle.

The expression for the track spectrum is given by [4]

$$\mathbf{G}_\eta(\omega)=\frac{A_v\Omega_c^2}{(\omega^2+\Omega_r^2)+(\omega^2+\Omega_c^2)}. \quad (14)$$

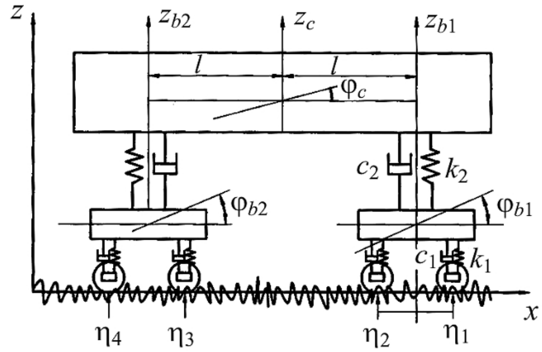
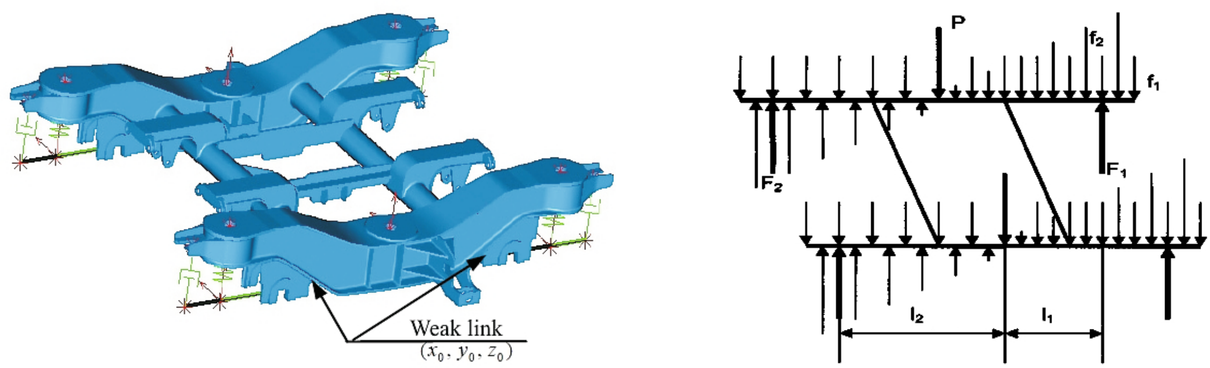


Fig. 1. The vertical vibration model of vehicle.



a b

Fig. 2. The model of the CW-200k type truck (a); loading model of the CW-200k truck (b).

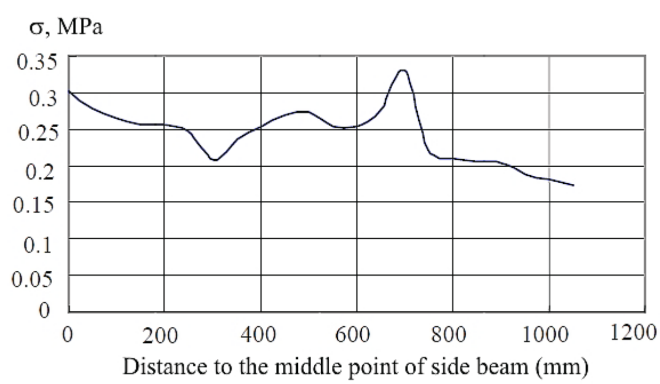


Fig. 3. The principal stress for unit load.

The track spectrum was provided by the Office for Research and Experiments of the International Union of Railways (ORE B176), the high level ORE parameters are as follows: $\Omega_c = 0.8246$ rad/m, $\Omega_r = 0.0206$ rad/m, and $A_v = 1.08 \cdot 10^{-6}$ m · rad.

As a result of symmetry of the structure and the force, the whole model of the side frame of truck and the loading model are shown in Fig. 2a and b, respectively.

Here P is the vibration force that come from the air rings located between the center of side frame and car body, F is the vibration force that come from spring and damper of axle box, and f_1 and f_2 represent the distribution loads of the inertial force and inertial moments caused by vibration acceleration. In order to know the position of weak link of the frame, a stress chart (Fig. 3) is derived by locating the unit load (1 kN) in the center of side frame. From the chart, we find the weak link of the side frame, as shown in Fig. 2.

According to Eqs. (6), (7) and Fig. 2b, the stress function of the weak link is obtained as

$$\sigma(t) = S(x_0,\ y_0,\ z_0) J z(t) \tag{15}$$

and

$$S(x_0,\ y_0,\ z_0) = (T_1,\ T_3,\ T_3,\ T_2,\ T_3,\ T_3,\ T_3,\ T_3), \tag{16}$$

where

$$T_1 = L_1 / W_{z0}, \qquad T_2 = L_2 / W_{z0}, \qquad T_3 = L / W_{z0},$$

$$L_1 = 4l_1 - 3l_2 - l_1^2 / l_2, \qquad L_2 = -4 + 3l_1 / l_2 + l_1^3 / l_2^3, \qquad L = l_2 - l_1,$$

and W_{z0} is called the section modulus of the weak link,

$$z(t) = (\ddot{z}_{b1},\ \dot{z}_{b1},\ z_{b1},\ \ddot{\varphi}_{b1},\ \dot{\varphi}_{b1},\ \varphi_{b1},\ \dot{\eta}_1,\ \eta_1)^T, \tag{17}$$

$$J = \begin{bmatrix} m_b/4 & & & & & & & \\ & C_1/2 & & & & & & \\ & & K_1/2 & & & & & \\ & & & J_b/4 & & & & \\ & & & & C_1 l_2/2 & & & \\ & & & & & K_1 l_2/2 & & \\ & & & & & & -C_1/2 & \\ & & & & & & & -K_1/2 \end{bmatrix}, \tag{18}$$

where m_b is the mass of the truck, J_b is the moment of inertia of the truck, C_1 and K_1 are damping and stiffness located on the axle box, respectively. According to Eq. (11), the response spectra of the truck may be written as

$$G_z(\omega) = \begin{bmatrix}
G_{\ddot{z}_{b1}\ddot{z}_{b1}} & G_{\ddot{z}_{b1}\dot{z}_{b1}} & G_{\ddot{z}_{b1}z_{b1}} & G_{\ddot{z}_{b1}\ddot{\varphi}_{b1}} & G_{\ddot{z}_{b1}\dot{\varphi}_{b1}} & G_{\ddot{z}_{b1}\varphi_{b1}} & G_{\ddot{z}_{b1}\dot{\eta}_1} & G_{\ddot{z}_{b1}\eta_1} \\
G_{\dot{z}_{b1}\ddot{z}_{b1}} & G_{\dot{z}_{b1}\dot{z}_{b1}} & G_{\dot{z}_{b1}z_{b1}} & G_{\dot{z}_{b1}\ddot{\varphi}_{b1}} & G_{\dot{z}_{b1}\dot{\varphi}_{b1}} & G_{\dot{z}_{b1}\varphi_{b1}} & G_{\dot{z}_{b1}\dot{\eta}_1} & G_{\dot{z}_{b1}\eta_1} \\
G_{z_{b1}\ddot{z}_{b1}} & G_{z_{b1}\dot{z}_{b1}} & G_{z_{b1}z_{b1}} & G_{z_{b1}\ddot{\varphi}_{b1}} & G_{z_{b1}\dot{\varphi}_{b1}} & G_{z_{b1}\varphi_{b1}} & G_{z_{b1}\dot{\eta}_1} & G_{z_{b1\eta_1}} \\
G_{\ddot{\varphi}_{b1}\ddot{z}_{b1}} & G_{\ddot{\varphi}_{b1}z_{b1}} & G_{\ddot{\varphi}_{b1}z_{b1}} & G_{\ddot{\varphi}_{b1}\ddot{\varphi}_{b1}} & G_{\ddot{\varphi}_{b1}\dot{\varphi}_{b1}} & G_{\ddot{\varphi}_{b1}\varphi_{b1}} & G_{\ddot{\varphi}_{b1}\dot{\eta}_1} & G_{\ddot{\varphi}_{b1}\eta_1} \\
G_{\dot{\varphi}_{b1}\ddot{z}_{b1}} & G_{\dot{\varphi}_{b1}\dot{z}_{b1}} & G_{\dot{\varphi}_{b1}z_{b1}} & G_{\dot{\varphi}_{b1}\ddot{\varphi}_{b1}} & G_{\dot{\varphi}_{b1}\dot{\varphi}_{b1}} & G_{\dot{\varphi}_{b1}\varphi_{b1}} & G_{\dot{\varphi}_{b1}\dot{\eta}_1} & G_{\dot{\varphi}_{b1}\eta_1} \\
G_{\varphi_{b1}\ddot{z}_{b1}} & G_{\varphi_{b1}\dot{z}_{b1}} & G_{\varphi_{b1}z_{b1}} & G_{\varphi_{b1}\ddot{\varphi}_{b1}} & G_{\varphi_{b1}\dot{\varphi}_{b1}} & G_{\varphi_{b1}\varphi_{b1}} & G_{\varphi_{b1}\dot{\eta}_1} & G_{\varphi_{b1}\eta_1} \\
G_{\dot{\eta}_1\ddot{z}_{b1}} & G_{\dot{\eta}_1\dot{z}_{b1}} & G_{\dot{\eta}_1z_{b1}} & G_{\dot{\eta}_1\ddot{\varphi}_{b1}} & G_{\dot{\eta}_1\dot{\varphi}_{b1}} & G_{\dot{\eta}_1\varphi_{b1}} & G_{\dot{\eta}_1\dot{\eta}_1} & G_{\dot{\eta}_1\eta_1} \\
G_{\eta_1\ddot{z}_{b1}} & G_{\eta_1\dot{z}_{b1}} & G_{\eta_1z_{b1}} & G_{\eta_1\ddot{\varphi}_{b1}} & G_{\eta_1\dot{\varphi}_{\beta1}} & G_{\eta_1\varphi_{b1}} & G_{\eta_1\dot{\eta}_1} & G_{\eta_1\eta_1}
\end{bmatrix}, \tag{19}$$

substituting Eqs. (16), (18), and (19) into Eq. (9) and substitution of the parameters of the truck then numbering computation, the SPSD $G_s(\omega)$ of the weak link can be obtained. As an example of stress power spectral density, exhibited by the train at speed of 180 km per hour is given in Fig. 4.

The Fatigue Life Prediction of the Side Frame of The CW-200k Type Truck. After the stress power spectral density is obtained, the distribution density of stress peak can be deduced as [5, 6]

$$f(s) = \frac{\sqrt{-R''}S}{2\pi R\sqrt{R}} \exp\left(\frac{-S^2}{2R}\right), \tag{20}$$

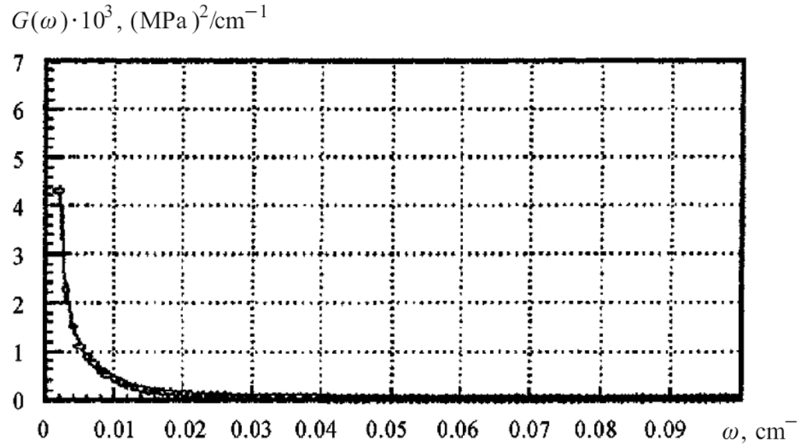


Fig. 4. Stress power spectral density (SPSD).

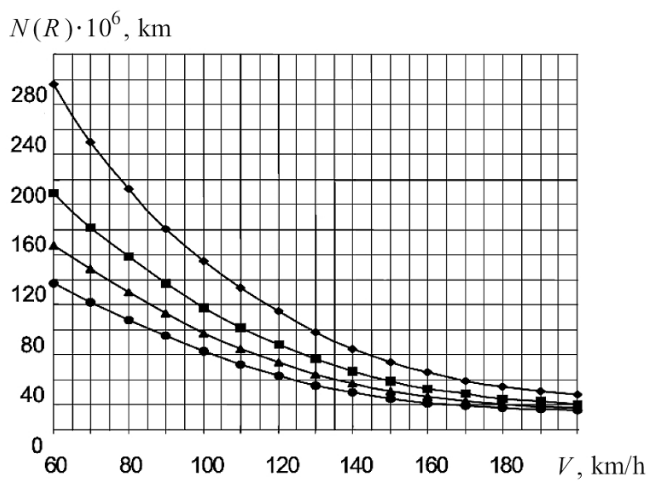


Fig. 5. The fatigue life curve of the side frame of the CW-200k type truck for different reliabilities [(◆) $P = 50\%$; (■) $P = 90\%$; (▲) $P = 99\%$; (●) $P = 99.9\%$].

where

$$R = \int_0^\infty G_s(\omega)d\omega, \qquad R'' = \int_0^\infty -\omega^2 G_s(\omega)d\omega,$$

S is random stress when the average value is zero. Generally, the fatigue curve can be described as follows:

$$S^m N = C, \tag{21}$$

where S is reversed stress amplitude, N is number of cycles to failure, and m and C are material constants of the structure.

According to the Miner cumulative damage law [7, 8], the fatigue life computational equation can be obtained as follows:

$$T_f = \left[\int_0^\infty \frac{f(S)S^m}{C} dS\right]^{-1}. \tag{22}$$

From statistics, the fatigue life of structure accords with log normal distribution. In the light of this, the fatigue life equation for different reliabilities can be deduced as

$$N(R)=\exp\left\{\sqrt{\ln[1+\eta^2(N)]}\,\Phi^{-1}[1-R(N)]+\ln\overline{N}-\frac{1}{2}\ln[1+\eta^2(N)]\right\}, \tag{23}$$

where

$$R(N)=1-\Phi\left[\frac{\ln N-\ln(\overline{N}/\sqrt{1+\eta^2(N)})}{\sqrt{\ln(1+\eta^2(N))}}\right], \tag{24}$$

$\eta(N)=\sigma_N/\mu_N$ is called change coefficient, μ_N and σ_N are average value and standard deviation, and $\overline{N}$ is the life when the reliability is 50%. Its value is equal to T_f.

In this paper, we computed the fatigue life of a side frame for four reliabilities, when the train runs at 60 to 200 km/h, as shown in Fig. 5.

Conclusions. A method for calculating the stress power spectral density of a general linear dynamic system on the basis of the unit load stress matrix is put forward. The relationship between the stress power spectral density of dynamic components and the response spectrum of dynamic system is established, and the corresponding formulas are derived. This is very important for the numerical emulation in dynamic fatigue design and reliability fatigue life prediction by computer. The dynamic response computation and fatigue life prediction of the CW-200k type vehicle truck are performed using the method and the high level ORE track spectrum.

The proposed method is suitable not only for the running dynamic system but also for new design dynamic systems.

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