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A PROGRAM FOR NUMERICAL CALCULATION OF DYNAMIC STRESS-STRAIN STATE AND STRENGTH OF HOLLOW MULTILAYER ANISOTROPIC CYLINDERS AND SPHERES. PART 1. PROGRAM DESCRIPTION

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The paper describes an application software package developed at the Pisarenko Institute of Problems of Strength of the National Academy of Sciences of Ukraine, for the calculation of stress-strain state and strength of multilayer composite cylinders and spheres under internal pulsed loading. The authors show its benefits and limitations in comparison with its modern foreign commercially available counterparts.

Keywords: application software package, composite materials, explosive, numerical methods, multilayer cylinders and spheres, spiral orthotropy, stress-strain state, strength.

The development of science and technology has generated interest in dynamic processes in various structures. Such structures include vessels, enclosures, and protective structures designed to retain significant hydro- and gas-dynamic loads and environmentally hazardous explosion products; containers for storing and transportation of explosive goods, toxic substances; chambers for power engineering based on explosive thermonuclear fusion, etc.

The possibility of controlling the symmetry of properties of the above-mentioned materials during their production would permit changing purposefully the stress-strain state (SSS) and strength of the structures. Among the most significant factors that govern the dynamic response and structural ability are the directions of spiral reinforcement of the layers and their mutual arrangement in a structural member, which should be taken into account during the SSS and strength analysis.

The current state of theoretical and experimental investigations of SSS and strength of multilayer anisotropic cylinders under internal explosive loading as well as some modern phenomenological strength criteria for composite materials (CM) were reviewed earlier in [1–3].

At present, the experimental studies of similar problems are detailed in the publications referred in [1]. A number of important findings have been made. Some recommendations have been put forward regarding the selection of materials, reinforcement configurations, etc. Alongside with the well-known benefits, the experimental investigations do have some drawbacks: too expensive and labor-consuming procedure, considerable difficulties in obtaining the required body of experimental data for a sufficiently full and adequate analysis of dynamic SSS and strength of a test object under explosive loading conditions. Therefore, it is rather difficult if possible to carry out a wide-scale investigation of dynamic behavior and strength and to choose an optimal design.

Nowadays the following application software packages (ASP) have been elaborated, commercialized and have found wide acceptance: ABAQUS, ANSYS, LS-DYNA and others; they enable users to calculate multi-dimensional SSS and strength of the above-mentioned structural members [4–7]. The benefits they offer are their versatility (suitability for solving a wide range of boundary value problems including those for composite cylinders and spheres), the possibility to analyze both the initial fracture and the propagating one in composite structures, the

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availability for purchasing, the detailed description of the packages and user's manuals. Noteworthy is that using these ASPs one can assess the initial strength by a number of criteria. Furthermore, a user can enter some additional subprograms, including those for strength analysis by the criteria chosen. However, the last-mentioned functionality requires some adaptation of ASP and, for that purpose, a user should have a fairly high qualification.

On the other hand, these ASPs do suffer certain drawbacks.

1. For spirally orthotropic cylinders the ANSYS and LS-DYNA packages can solve two-dimensional (2D) axisymmetric problems only as three-dimensional (3D) ones. ABAQUS makes it possible to solve them as two-dimensional problems and numerically integrate them by the finite-element method (FEM) using a 2D mesh. If applied to bodies of regular geometry, such as cylinders, this approach seems unpractical – the finite-difference methods (FDM) would be convenient for these problems [8].

2. Three-dimensional nonaxisymmetric problems are in most cases integrated by the FEM on a 3D finite-element mesh. The FEM is rather cumbersome (during the preparation of input data and discretization of a computational domain) and uneconomic (it needs large RAM volume and much computer time for handling a specific problem). Using the commercially available ANSYS package one can solve some nonaxisymmetric problems of elastic bodies of revolution by the Fourier expansion into hoop coordinates, thus lowering the problem dimension by unity; in some cases, this permits applying 2D FDMs instead of FEM [6]. However, the problems for spirally orthotropic cylinders cannot be integrated by ANSYS in this way.

3. Three-dimensional problems including those for bodies of revolution are numerically integrated in Cartesian rectangular coordinates despite the fact that for cylindrical bodies it is most rational and convenient (for solution purposes) to write down the equations in cylindrical coordinates. In the case of cylinders, the use of Cartesian coordinates causes a number of additional rather objectionable difficulties, namely:

3.1. Intricacy of mathematical statement of boundary and contact conditions on cylindrical surfaces;

3.2. Difficulties and resulting errors arising during the 3D discretization of a cylindrical computational domain with flat-faced finite elements;

3.3. As a consequence of sub-items 3.1 and 3.2, there are a number of ghost effects that occur during the numerical calculation. In particular, there arise some ghost circumferential wave processes which fail to disappear even in the case of axisymmetric loading where no circumferential waves should occur by definition. Furthermore, these ghost error tend to accumulate with time, thus making the calculation of 3D problems over quite long time intervals almost impossible.

In spite of the considerable interest in studying the behavior of multilayer hollow cylinders and spheres of composites materials under internal explosive loading and commercial availability of ASPs, inadequate attention has been given to the theoretical investigation of SSS, especially strength, of these structural members on the basis of the use of three-dimensional equations of the dynamic elasticity theory [9] and elastoplasticity theory [10] for an anisotropic body, and modern theories of strength for anisotropic materials [2]. The relevant publications are dedicated mostly to the analysis of SSS and strength under static loading conditions, to the determination of natural frequencies and mode shapes, their dynamic response in the shell approximation [2]. A conventional approach is to substitute a homogeneous spirally orthotropic body for each spirally armored layer of the composite material.

For pulsed-loaded multilayer hollow cylinders with a spiral orthotropy of each layer, the Pisarenko Institute of Problems of Strength has undertaken research efforts to tackle the nonstationary 1D problems (infinitely long cylinders under axisymmetric loading), 2D problems (finite-length cylinders under axisymmetric loading or infinitely long cylinders under nonaxisymmetric loading), and 3D problems (finite-length cylinders under nonaxisymmetric loading) of SSS and strength analysis in linear and geometrically and physically nonlinear statements as well as to elaborate analytical (engineering) methods of SSS analysis for dynamically loaded heterogeneous spirally orthotropic hollow cylinders [11–35]. The publications [11–19] addressed only the cases of cylindrical orthotropy, while the problems in a more general statement – for spirally orthotropic layers – were solved in [20–35]. Materials with a spiral and cylindrical transtropies and with isotropy of properties were considered as particular cases in this general statement. The one-dimensional problem for a spherically transtropic multilayer hollow sphere under centrally symmetric explosion was discussed earlier in [36].

All the dynamic boundary value problems for cylinders were solved in cylindrical coordinates x, φ, r [11–35]. During the numerical integration of these problems the researchers used explicit time FDM schemes. Linear problems (small loads, small elastic strains and displacements) were solved by FDM in displacements [11, 12, 14, 15, 17] or by using a linearized two-dimensional Wilkins algorithm [14, 17, 18, 32, 33]. For nonaxisymmetric problems the researchers applied numerical-analytical techniques based either on the expansion of variables into a Fourier series [14, 17, 18, 32, 33] or on the use of the Bubnov–Galerkin method [14–17] with respect to an angular coordinate with subsequent use of the two-dimensional Wilkins algorithm [37]. The investigations [19, 31] have demonstrated that the classical two-dimensional Wilkins algorithm has proven itself in solving linear and nonlinear problems for both short and long (until a quasistatic loading mode is reached) calculation time periods. Therefore, it is this algorithm or its modifications which were used in the publications [20–35] addressing the problems in the most general statement (spiral orthotropy).

The objective of this paper is to describe a specialized ASP which has been developed recently at the Pisarenko Institute of Problems of Strength for the calculation of the dynamic one-, two-, and three-dimensional SSS and strength of multilayer hollow spirally orthotropic cylinders of finite and infinite length [20–35] as well as for solving one-dimensional problems for transtropic hollow spheres. This package whose development was started back in 2003 on the basis of algorithms [11–35] and modifications of earlier ASPs is free from the drawbacks peculiar to the commercially available ASPs.

The program was written for the Windows platform in the object-oriented programming language Object Pascal. For visualization of the problem and calculated results the package involves the use of a platform-independent library of 3D computer graphics OpenGL. The application software consists of individual visual and nonvisual modules, thus making it possible to easily modify (change, add/remove) various components, exploit new functionalities, and develop the package as a whole. Since the program has a windowing graphic interface a user can control the problem parameters in an on-line dialog mode. As a result of running the program, one obtains a complete set of SSS components and values of strength criteria at a preset instant of time at an arbitrary point of the calculation domain. There is also a demo version of the program; its only limitation is the program start count – 100 times.

The program is launched from the main window and has a dialog interface. All the parameters to run calculation of a particular problem can be set in the respective forms – “Material Base”, “Geometry”, “Load”, “Mesh” – and saved in a separate configuration file. Data are entered via editable, mutually dependent and information fields, dropdown lists and submenus, spreadsheets, and data files. The information fields have some limitations on entering incorrect information and provide tooltips (with explanations and applicability limits). For user’s convenience, some actions can be taken using a mouse or hot keys.

The pulsed loading of hollow cylinders and spheres is modeled in ASP by blasting a linear or concentrated explosive charge placed inside the cylinder hollow or at the sphere center.

The modeling of explosive loading in this ASP can be accomplished through a number of engineering procedures which were chosen on the basis of test calculations from numerous well-known methods of mathematical description of one-dimensional shock waves (SW) induced by blasting a concentrated explosive charge in air.

1. The procedure based on empirical relations proposed in [38]. This model has been widely accepted in foreign publications to describe behavior of various structural members under explosive loading. It involves the use of the similarity law implying that with an equal value of the parameter Z dependent on the explosive mass and the distance to the explosion the SWs induced will be equal. The parameter Z is given by $Z = r \big/ M_{TNT}^{1/3}$, where M_{TNT} is the explosive charge mass (in kg) of trinitrotoluol (TNT) equivalent, r is the distance (in m) to the explosive charge.

The procedure is applicable for the range $0.147 < Z < 40$ kg/m$^{1/3}$.

2. The procedure allowing for irregular SW reflection [39]. If the incident SW front forms some angle $\alpha \geq \alpha_{cr}$ with the wall surface there occurs an irregular (nonlinear) reflection which essentially implies that the wave propagating in the disturbed media overtakes the incident wave and merge with it to form a third wave called the head SW or the Mach wave. This leads to the dependence of the SW reflection coefficient C_r on the angle of incidence α and amplitude of pressure P_{int} of the incident SW. The pressure acting on the wall of the structural member P_{load} is given by $P_{load} = P_{int} C_r$.

The reflection coefficient C_r in the proposed ASP is automatically chosen from the table according to the P_{int} and α values. The accuracy of the P_{int} values which are found from the Kinney equation [40] is only slightly different from that determined by the expression presented in [38]. The other SW parameters are found in the same manner as in the procedure described in item 1 above. The procedure is applicable for the range $0.15 < Z < 20$ kg/m$^{1/3}$.

3. In the procedure proposed in [41] the pressure profile $P(r, t)$ acting on the shell inner surface during the blasting of a spherical explosive charge at the center of the shell is described by the empirical relation

$$P(r, t) = kQ_E Z^{-3} H(krQ_E^{-1/2} - t),$$

where

$$k = 0.32(3\gamma - 1)/(\gamma^2 - 1),$$

Q_E is the calorific value of the explosive, γ is the adiabatic index in the equation of state of the explosive, and $H(t)$ is the Heaviside function.

4. The procedure proposed in [42] uses the empirical relations for the determination of the excess pressure ΔP in the near zone of explosion of the spherical charge

$$\Delta P = P_0 e^{-t/\theta},$$

where

$$\theta = 10^{-6} (r / r_0)^{1.6}, \text{ s}, \qquad P_0 = 100/(r/r_0)^{1.38}, \text{ MPa},$$

r_0 is the radius of the charge of TNT equivalent. These expressions are valid in the range $1 \le r/r_0 \le 16$. For calculating P_{int} this procedure is applied jointly with that described in item 3 above.

In all the procedures the calculations are performed for TNT. In case a different type of explosive is to be used, the explosive mass M_E should be converted to an equivalent mass M_{TNT} by formula [40]

$$M_{TNT} = M_E Q_E / Q_{TNT},$$

where Q_{TNT} is the specific heat (calorific value) of TNT.

In the commercially available application packages ABAQUS and LS-DYNA the calculation of the explosive loading parameters is carried out by the procedure described in item 1 or by using the ALE technology (the arbitrary Lagrangian–Eulerian mesh) [4, 7]. The consideration of the irregular SW reflection in the proposed ASP permits, in some case, a more accurate description of the pressure distribution over the cylinder inner surface. In addition, the use of the above procedures provides a considerable reduction of computer time for numerical solution of a problem in comparison with the ALE technology.

The cylindrical outer surface of a shell is assumed to be stress-free.

Boundary conditions at cylinder ends can be of several types. For axisymmetric problems (a 2D option, a spherical or linear explosive charge is located on the cylinder axis) the ends can be free, fixed, or sliding. For nonaxisymmetric problems (a 3D option, a spherical or linear explosive charge is radially shifted with respect to the cylinder axis) the ends can be fixed or sliding.

The analysis of infinitely long cylinders with a linear explosive charge involves the plane-strain condition; the computational domain is divided into two cells along the axis (the axial step is usually taken equal to the radial one); the sliding conditions for imaginary ends are set to be mathematically equivalent either to the plane-strain conditions or to symmetry conditions, which is the same in this case. The axisymmetric and nonaxisymmetric dynamic boundary value problems degenerate, over the space, into the one-dimensional and two-dimensional problems, respectively.

The proposed specialized ASP contains two main independent blocks as described above (2D and 3D options).

If an explosive charge is located on the cylinder axis, the SSS becomes axisymmetric, generally a two-dimensional one (except for the case of an infinitely long cylinder under the plane-strain conditions, where the problem degenerates into the one-dimensional problem) and the 2D option is used. The problem can be considered in a geometrically and physically nonlinear statement; the linear cases follow from the general statement as particular ones. The equations involved are the dynamic elastic equations for a spirally orthotropic body in cylindrical coordinates, taking into account the axial symmetry [9, 31]. Materials of the layers can also undergo plastic deformation; in this case, the equations of the no-hardening flow theory for an orthotropic medium are used [10, 30, 31] which degenerate, for isotropic materials, into the Prandtl–Reuss flow theory. Cylindrically orthotropic, spirally or cylindrically transtropic, and isotropic layers are particular cases of the spirally orthotropic ones. Numerical integration of a nonstationary two-dimensional axisymmetric boundary value problem is performed using the conventional Wilkins' algorithm [37] modified for spirally orthotropic layers [30, 31], which is known to permit including both the physical nonlinearity (plasticity) and the geometrical one (large rotations, strains, and displacements). The detailed mathematical statement, description of the numerical method and finite-difference equations, the test and model calculations of SSS and strength (including the comparison with the available experimental data) performed by means of the proposed ASP were presented earlier in [31].

If an explosive charge is radially shifted with respect to the cylinder axis, the SSS becomes nonaxisymmetric, generally a three-dimensional one (except for the case of an infinitely long cylinder under the plane-strain conditions, where the problem degenerates into the one-dimensional problem) and the 3D option is used. The problem is solved in a linear statement: it is assumed that the strains and displacements are fairly small and that the materials of the layers deform elastically till failure. The equations involved are the three-dimensional dynamic elastic equations for a spirally orthotropic body in cylindrical coordinates in the general nonaxisymmetric statement [9, 32]. As with the 2D option, the cases of cylindrical orthotropy, spiral and cylindrical transtropies, and isotropy are automatically taken as particular cases. As shown earlier in [14, 17, 18, 32, 33], in solving a linear problem for bodies of revolution with elastic strains in the case where the SSS dependence on the hoop coordinate φ is only due to the nonaxisymmetric pattern of loading the numerical-analytical method based on expansion of variables into a Fourier series with respect to φ (first stage) has proven itself. As a consequence, the initial nonstationary three-dimensional boundary value problem is reduced to a set of two-dimensional one. It is this method which was used in developing the proposed specialized ASP. After trigonometric expansions the resulting two-dimensional dynamic boundary value problems are numerically integrated using the linearized two-dimensional Wilkins' algorithm. During the second stage, a complete set of data is calculated in order to determine SSS at any point of the three-dimensional computational domain for the current instant of time through summation over harmonics for a user-specified particular angular coordinate.

Where necessary, the strength verification is carried out at every finite-difference cell of the computational domain at every time step. In this case, the φ coordinate should be also varied with a sufficiently small step, thus significantly increasing the computer time in comparison to that needed for determining SSS only.

The proposed ASP permits assessment of initial strength by the phenomenological criteria which are widely accepted in the relevant applications [3]: maximum-stress, maximum-strain, the Ashkenazi, Hoffman, and modified von Mises criterion (in some publications it was called the Tsai–Wu criterion). Noteworthy is that the same criteria are also used in the above-mentioned commercially available ASPs. However, the analysis in [3] has demonstrated that the Hill's criterion conventionally involved in the applications for solving two-dimensional problems has significant drawbacks when it is applied to the three-dimensional SSS. Therefore, it is not employed in the proposed ASP. The Ashkenazi criterion is used only for the composites with equal tensile and compression strengths; all the other criteria are applied for the materials that have equal and unequal strengths.

Using the proposed ASPs, one can assess strength at the current instant of time in the entire computational domain: the maximum (over the whole volume of the test body) value of strength function Φ_{max} at every time step is monitored by a given strength criterion (or several ones simultaneously). If the condition $\Phi_{max} \leq 1$ is met, the structure is considered to be strong. If $\Phi_{max} > 1$, the strength condition is not met and the strength of the structure at a given point (a finite-difference cell) in the structural member at the current instant of time fails.

Note that when specifying strength characteristics for composites in the ASP using the Hoffman and Ashkenazi criteria one should check whether the necessary and sufficient stability conditions are met [43, 44]. For the Hoffman quadratic strength criterion and the von Mises generalized criterion the verification of invariance conditions should be added in the cases of transtropic and isotropic materials [3, 34, 43].

The mathematical statement, the description of the numerical-analytical method, the difference of the linearized Wilkins algorithm from the conventional one, the test and model calculations including the comparison with the data obtained by the commercial software package LS-DYNA were presented in [32, 33]. The numerical-analytical method including all the intermediate stages (statement of constitutive two-dimensional dynamic boundary value problems resulting from the expansion of variables into a Fourier series with respect to φ, and their finite-difference approximation) is detailed in [45].

For the purpose of monitoring the computations the program provides visualization of the time dependence of stress and strain components, displacements, and strength criteria in the form of diagrams and vector fields. This representation makes it possible to monitor values of the sought-for functions, their dynamics and distribution over the computational domain, to stop computations in case the solution becomes instable and in case the strength criteria are exceeded. This permits reducing the computation time with varying parameters. The calculation process and output are given in windows “Calculation” and “Graphics.” When the calculation is complete the selected results can be saved in a text file or exported to MS Office Excel.

Thus, in the proposed ASP the two- and three-dimensional dynamic boundary value problems are reduced to the two-dimensional ones and are integrated in cylindrical coordinates using an explicit time scheme on a two-dimensional finite-difference mesh of the FDM without involving any three-dimensional discretization of the computational domain and dynamic three-dimensional FEM in the Cartesian coordinates as conventionally done in the modern commercially available packages [4–7]. This feature makes it significantly simpler for a user to prepare input data, reduces the computer time for solving a problem, simplify obtaining output data, provide a considerable saving in the RAM volume needed for the calculations, and thus ensure a more accurate SSS and strength calculations on fine 2D meshes in comparison to a larger 3D discretization of the computational domain with the same sizes of arrays of variables. Also, owing to the use of a global cylindrical coordinate system the program gives more accurate SSS and strength data for a cylindrical test object in comparison with the commercial ASPs [4–7]. The block structure of the package permits a user to enter additional relatively simple subprograms that do not require any high programming skills; these subprograms provide strength assessment by the criteria that differ from those described above.

The limitations peculiar to the proposed ASP in comparison to its commercial counterparts consist in the following:

(i) a narrow class of problems to handle;

(ii) the calculation of plasticity is performed using only the theory of flow with no hardening (the Prandtl–Reuss flow theory);

(iii) no possibility to calculate the propagating fracture;

(iv) nonaxisymmetric problems can be numerically calculated only in linear statements.

Finally, the proposed ASP provides also an option for solving linear one-dimensional problems for multilayer elastic hollow spheres under centrally symmetric pulsed loading [36]. The layers are spherically transtropic or isotropic. The problem is numerically solved in displacements by the explicit “cross” type scheme of FDM [11, 12, 15, 36]. The application software package enables a user to determine SSS for these objects and to assess their strength by the above-mentioned criteria.

REFERENCES

1. P. P. Lepikhin, V. A. Romashchenko, and E. V. Bakhtina, “Methods and findings of stress-strain and strength analyses of multilayer thick-walled anisotropic cylinders under dynamic loading (review). Part 1. Experimental studies,” *Strength Mater.*, **45**, No. 1, 10–19 (2013).

2. P. P. Lepikhin and V. A. Romashchenko, "Methods and findings of stress-strain state and strength analyses of multilayer thick-walled anisotropic cylinders under dynamic loading (review). Part 2. Theoretical methods," *Strength Mater*., **45**, No. 2, 144–153 (2013).
3. P. P. Lepikhin and V. A. Romashchenko, "Methods and findings of stress-strain state and strength analyses of multilayer thick-walled anisotropic cylinders under dynamic loading (review). Part 3. Phenomenological strength criteria," *Strength Mater*., **45**, No. 3, 271–283 (2013).
4. *ABAQUS 6.14. Theory Manual*, Dassault Systèmes (2014).
5. *ABAQUS 6.14. Analysis User's Guide*, Vol. II: *Analysis*, Dassault Systèmes, The 3D EXPERIENCE Company (2014).
6. *ANSYS Documentation. Release 15.0*, ANSYS Inc.
7. *LS-DYNA. Theory Manual*, Livermore Software Technology Corporation, Livermore (2014).
8. V. N. Kukudzhanov, "Numerical modeling of dynamic processes of deformation and fracture in elastic-plastic media," *Usp. Mekhan.*, **8**, No. 4, 21–65 (1985).
9. S. G. Lekhnitskii, *Theory of Elasticity of Anisotropic Body* [in Russian], Nauka, Moscow (1977).
10. N. N. Malinin, *Applied Theory of Plasticity and Creep* [in Russian], Mashinostroenie, Moscow (1975).
11. Sh. U. Ga1iev, V. A. Romashchenko, and Z. G. Alpaidze, "Effect of anisotropy and viscosity on the propagation of waves in multilayer cylinders," *Strength Mater*., **15**, No. 9, 1236–1241 (1983).
12. Sh. U. Galiev and V. A. Romashchenko, "Nonstationary dynamics and strength of hollow viscoelastic anisotropic multilayer cylinders of finite length," *Mekh. Kompoz. Mater.*, No. 4, 681–685 (1984).
13. V. A. Romashchenko, "Viscoelastoplastic waves in the vicinity of nodal points of anisotropic shells," *Strength Mater*., **16**, No. 10, 1456–1459 (1984).
14. Sh. U. Galiev, Z. G. Alpaidze, V. A. Romashchenko, et al., *Procedures of Numerical Pulsed-Loading Calculation for Anisotropic Hollow Bodies of Revolution* [in Russian], Preprint, Institute of Problems of Strength, Academy of Sciences of Ukr.SSR, Kiev (1984).
15. Sh. U. Galiev, *Nonlinear Waves in Bounded Continua* [in Russian], Naukova Dumka, Kiev (1988).
16. Sh. U. Galiev and V. A. Romashchenko, "Numerical investigation of three-dimensional nonlinear waves in combined bodies of revolution," *Mekh. Kompoz. Mater.*, No. 1, 136–141 (1989).
17. Sh. U. Galiev, Yu. N. Babich, S. V. Zhurakhovskii, et al., *Numerical Modeling of Wave Processes in Bounded Media* [in Russian], Naukova Dumka, Kiev (1989).
18. V. A. Romashchenko, "Method of solving three-dimensional nonlinear dynamic problems for compound bodies of rotation," *Strength Mater*., **25**, No. 6, 442–450 (1993).
19. P. P. Lepikhin, V. A. Romashchenko, S. A. Tarasovskaya, and V. G. Korbach, "Range of the applicability of the Wilkins method to the investigation of the dynamic stress-strain state of anisotropic elastic axisymmetric shells," *Strength Mater*., **35**, No. 1, 52–59 (2003).
20. P. P. Lepikhin, V. A. Romashchenko, S. A. Tarasovskaya, and V. F. Demenko, "Numerical study of dynamics of cylindrical spirally reinforced thick-walled shells," *Aviats.-Kosm. Tekhn. Tekhnol.*, Issue 5 (40), 56–60 (2003).
21. P. P. Lepikhin, V. A. Romashchenko, and S. A. Tarasovskaya, "Modification of the Wilkins method to study the dynamics of axisymmetric thick-wall shells with a spiral orthotropy," *Strength Mater*., **36**, No. 2, 119–124 (2004).
22. V. A. Romashchenko, S. A. Tarasovskaya, and V. F. Demenko, "Numerical modeling of dynamics of thick-walled multilayer spirally reinforced cylindrical shells," *Otkr. Inform. Komp. Integr. Tekhnol.*, Issue 23, 170–182 (2004).
23. P. P. Lepikhin, V. A. Romashchenko, and S. A. Tarasovskaya, "Modification of Wilkins method for the investigation of dynamics of axisymmetric orthotropic thick-walled shells," *Nadezhn. Dolgovechn. Mashin Sooruzh.*, No. 1 (22), 136–141 (2004).
24. V. A. Romashchenko and S. A. Tarasovskaya, "Numerical studies on the dynamic behavior of multilayer thick-walled cylinders with helical orthotropy," *Strength Mater*., **36**, No. 6, 621–629 (2004).
25. V. A. Romashchenko and S. A. Tarasovskaya, "Numerical study of dynamics of thick-walled cylindrical spirally reinforced shells," *Mekh. Kompoz. Mater.*, No. 2, 225–236 (2005).

26. *Strength of Materials and Structures* [in Russian], Second edition, Akademperiodika, Kiev (2006).
27. V. A. Romashchenko, "Dynamical problem for an incompressible laminated cylinder with helical anisotropy. Part 1. Theory," *Strength Mater*., **38**, No. 2, 189–195 (2006).
28. V. A. Romashchenko, "Dynamical problem for an incompressible laminated cylinder with helical anisotropy. Part 2. Numerical analysis," *Strength Mater*., **39**, No. 1, 80–84 (2007).
29. V. A. Romashchenko, "Solutions of dynamic problems for incompressible and weakly compressible spirally orthotropic heterogeneous thick-walled cylinders," *Prikl. Matem. Mekh*., No. 1, 56–65 (2007).
30. V. A. Romashchenko, "A numerical study of the nonlinear dynamics of multilayer spirally orthotropic cylinders," *Strength Mater*., **40**, No. 6, 678–687 (2008).
31. V. T. Troshchenko (Ed.), *Strength of Materials and Structures* [in Russia], Vol. 3: P. P. Lepikhin and V. A. Romashchenko, *Strength of Thick-Walled Rotational Shells under Pulsed Loading*, Pisarenko Institute of Problems of Strength, National Academy of Sciences of Ukraine, Kiev (2010).
32. V. A. Romashchenko, O. S. Beiner, and Yu. N. Babich, "Numerical-analytical method for 3D dynamics study of spirally orthotropic multilayer cylinders," *Strength Mater*., **43**, No. 4, 438–446 (2011).
33. V. A. Romashchenko and O. S. Beiner, "Numeric simulation of three-dimensional dynamics and strength of multilayered spirally orthotropic cylinders," *Strength Mater*., **44**, No. 2, 187–195 (2012).
34. V. A. Romashchenko, "Strength assessment for composite and metal-composite cylinders under pulse loading. Part 1. Rules of choosing various strength criteria for anisotropic material and comparative analysis of such criteria," *Strength Mater*., **44**, No. 4, 376–387 (2012).
35. V. A. Romashchenko, Yu. N. Babich, and E. V. Bakhtina, "Strength assessment for composite and metal-composite cylinders under pulse loading. Part 2. Numerical evaluation of strength for multilayer cylinders of finite length under internal explosion," *Strength Mater*., **44**, No. 5, 502–511 (2012).
36. V. A. Romashchenko and S. A. Tarasovskaya, "The elastic theory dynamic problem for a transtropic multilayer sphere," *Strength Mater*., **43**, No. 2, 178–189 (2011).
37. M. L. Wilkins, "Calculation of elastic-plastic flow," in: *Computational Methods in Hydrodynamics* [Russian translation], Mir, Moscow (1967), pp. 212–263.
38. G. Randers-Pehrson and K. A. Bannister, *Airblast Loading Model for Dyna-2D & Dyna-3D*, Army Research Laboratory, ARL-TR-1310 (1997).
39. M. K. Chock, *Review of Methods for Calculating Pressure Profiles of Explosive Air Blast and Its Sample Application*, Blacksburg, Virginia (1999).
40. G. F. Kinney and K. J. Grahm, *Explosive Shocks in Air*, Springer, Berlin (1985).
41. V. V. Adishchev, V. M. Kornev, and L. A. Talzi, *Assessment of Maximum Stresses in Closed Cylindrical Vessels under Axisymmetric Explosive Loading* [in Russian], Deposited in VINITI, No. 6588-83, Novosibirsk (1983).
42. V. V. Adushkin, "On the formation of a shock wave and dispersion of explosion products in air," *Prikl. Mekh. Teor. Fiz.*, No. 5, 107–115 (1963).
43. V. A. Romashchenko, "Particularization of quadratic strength criterion for an orthotropic material," *Strength Mater*., **45**, No. 5, 534–541 (2013).
44. V. A. Romashchenko, "On six-dimensional quadric hyperspace of strength for orthotropic materials," *Strength Mater*., **46**, No. 1, 57–63 (2014).
45. P. P. Lepikhin and V. A. Romashchenko, *Strength of Heterogeneous Anisotropic Hollow Cylinders under Pulsed Loading* [in Russia], Naukova Dumka, Kiev (2014).