

## SELECTION OF ANALYTICAL METHODS FOR DETERMINATION OF MECHANICAL CHARACTERISTICS OF UNIDIRECTIONAL COMPOSITES BASED ON GLASS FIBERS

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UDC 539.3

*The most common methods for determination of mechanical characteristics of composite materials are considered. The accuracy of methods for each constant of some unidirectional composite materials based on E-glass fibers and epoxy binder is found.*

**Keywords:** unidirectional glass fiber plastic, elastic and strength constants, analytical determination accuracy.

**Introduction.** Fiber composite materials (CM) are widely used in aerospace and rocket engineering, power engineering, automotive, metal mining and metallurgy industries, construction and etc. Unidirectional composite materials consist of reinforcing continuous fibers that are embedded into a polymer matrix.

Three the most commonly used methods for determination of mechanical properties of CM are known, namely: experimental, analytical and finite element modeling of mechanical properties. The most reliable among them is experimental one [1].

The papers [1–14] present the investigations on determination of mechanical characteristics using different experimental methods, finite element modeling and engineering dependencies. New methods for determination of characteristics of anisotropic materials are developed to simplify the procedure of investigation and shorten the time spent for solution of the given problem as well as to improve the accuracy of obtained results.

The objective of this paper is to select the most accurate and the least difficult approaches to prediction of both elastic and strength characteristics of unidirectional composite materials based on glass fibers.

In considering any methods it is assumed that there is a unidirectional fiber composite, which is formed by transversely isotropic materials with the plane of isotropy perpendicular to the fiber direction and the Tsai–Wu failure criterion. Such material is characterized by five independent elastic ( $E_1$ ,  $E_2$ ,  $G_{12}$ ,  $G_{23}$ , and  $\nu_{12}$ ) and five strength ( $\sigma_{u1}^+$ ,  $\sigma_{u2}^+$ ,  $\sigma_{u1}^-$ ,  $\sigma_{u2}^-$ , and  $\tau_{u12}$ ) properties [6]. Here,  $E$  and  $G$  are the Young and shear modulus, respectively,  $\nu$  is Poisson's ratio,  $\sigma_{ui}^+$ ,  $\sigma_{ui}^-$ , and  $\tau_{u12}$  are the ultimate tensile strength, compression strength, and shear strength, respectively,  $i=1, 2, 3$  (1 is the anisotropy axis in the reinforcing fiber direction, 2 and 3 are perpendicular to axis 1).

Below are given the most known methods for determination of CM mechanical characteristics.

**Determination of Elastic Characteristics of CM. Mixture Rule** [8, 15, 16] (hereinafter referred to as MR). In accordance with the given rule, the material unknown characteristic depends on the contribution of each component in proportion to its volume content in the composite:

$$E_1 = E_{1f}V_f + E_{1m}(1 - V_f), \quad \frac{1}{E_2} = \frac{V_f}{E_{1f}} + \frac{V_m}{E_{1m}}, \quad \nu_{12} = \nu_f V_f + \nu_m V_m, \quad \frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m},$$

where  $E_1$ ,  $E_2$ ,  $\nu_{12}$ , and  $G_{12}$  are unknown characteristics, subscripts  $f$  and  $m$  denote fiber and matrix, respectively, and  $V_f$  and  $V_m$  are volumetric content of reinforcing fiber and matrix, respectively.

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The expression for  $E_1$  is also true for the Halpin–Tsai method [17]. The proposed method [18] for determination of  $E_2$  is of the same form. The formula is true for the Halpin–Tsai, Barbero, and Jones methods [19].

*The Tsai Method* [6, 20]. This method is similar to MR; however, it has a correction factor  $C$ :

$$E_1 = C(E_{1f}V_f + E_{1m}(1 - V_m)),$$

$$G_{12} = (1 - C)G_m \frac{2G_f - (G_f - G_m)V_m}{2G_f + (G_f - G_m)V_m} + CG_f \frac{(G_f + G_m) - (G_f - G_m)V_m}{(G_f + G_m) + (G_f - G_m)V_m},$$

where  $C \leq 1$ .

This factor  $C$  depends on the density of fibers packing:  $C = 0$  at isolated fibers with a relatively high matrix volume,  $C = 1$  at dense packing with a low binder content, at nonuniform packing  $C$  equals to the intermediate value  $0 < C < 1$ :

$$E_2 = 2(1 - \nu_f + (\nu_f - \nu_m)V_m) \left( (1 - \omega) \frac{K_f(2K_m + G_m) - G_m(K_f - K_m)V_m}{(2K_m + G_m) + 2(K_f - K_m)V_m} \right. \\ \left. + \omega \frac{K_f(2K_m + G_f) - G_f(K_m - K_f)V_m}{(2K_m + G_f) - 2(K_m - K_f)V_m} \right),$$

where

$$K_B = \frac{E_f}{2(1 - \nu_f)}, \quad K_m = \frac{E_m}{2(1 - \nu_m)}, \quad G_f = \frac{E_f}{2(1 + \nu_f)}, \quad G_m = \frac{E_m}{2(1 + \nu_m)},$$

$\omega$  is the experimental correction factor considering the factor of fibers contact,  $0 < \omega < 1$ . In [12] it is recommended to accept  $\omega = 0.2$ .

*The Hill Method* [21]. In this method it is assumed that for two-phase fibrous CM the following relation is true:

$$E_1 = E_V + \frac{4(\nu_V - \nu_m)^2}{(K_V^{-1} - K^{-1})^2} \left( \frac{1}{K_R} - \frac{1}{K_0} \right), \quad \nu_{12} = \nu_V - \frac{\nu_V - \nu_m}{K_V^{-1} - K^{-1}} \left( \frac{1}{K_R} - \frac{1}{K_0} \right),$$

where

$$E_V = E_m(1 - V_f) + E_f V_f, \quad \nu_V = \nu_m(1 - V_f) + \nu_f V_f,$$

$$K_0 = K + \frac{V_f}{(K_V - K)^{-1} + (1 - V_f)(K + G)^{-1}}, \quad G = \frac{E}{2(1 + \nu_m)},$$

$$K_R^{-1} = K^{-1}(1 - V_f) + K_V^{-1} V_f, \quad K = \frac{E}{3(1 - 2\nu)}, \quad K_V = \frac{E_1}{3(1 - 2\nu_1)}.$$

*CCA Methods* [6] (Composite cylinder assemblage model). This method takes the following form:

$$E_1 = E_{1f}V_f + E_{1m}(1 - V_f) - \frac{4E_{1m}E_{1f}(\nu_f - \nu_m)^2 G_m}{\left( \frac{V_m G_m}{K_f + G_f/3} \right) + \left( \frac{V_m G_m}{K_m + G_m/3} \right) + 1}.$$

Matrix contribution to approximate calculations, where a high accuracy is not required, is often neglected, and it is considered that  $E_1 = E_{1f}V_f$ :

$$E_2 = 2(1 + \nu_{23})G_{23},$$

where

$$\nu_{23} = \frac{K^* - mG_{23}}{K^* + mG_{23}}, \quad m = 1 + 4K^* \frac{\nu_{12}^2}{E_1}, \quad G_{23} = \frac{G_m (K_m (G_m + G_f) + 2G_f G_m + K_m (G_f - G_m) V_f)}{K_m (G_m + G_f) + 2G_f G_m + (K_m + 2G_m)(G_f - G_m) V_f},$$

$$K^* = \frac{K_m (K_f + G_m) V_m + K_f (K_m + G_m) V_f}{(K_f + G_m) V_m + (K_m + G_m) V_m}, \quad K_f = \frac{E_f}{2(1 + \nu_m)(1 - 2\nu_f)}, \quad K_m = \frac{E_m}{2(1 + \nu_m)(1 - 2\nu_m)},$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m + \frac{V_f \nu_m (\nu_f - \nu_m) (2E_f \nu_m^2 + \nu_m E_f - E_f + E_m - \nu_f E_m - 2\nu_f^2 E_m)}{(2V_f \nu_m^2 - \nu_m V_f - 1 - V_f) E_f + (2\nu_f^2 - \nu_m^2 V_f + V_f + \nu_f - 1) E_m},$$

$$G_{12} = G_m \left( \frac{G_f (1 + V_f) + G_m V_m}{G_f V_f + G_m (1 + V_f)} \right).$$

The Halpin–Tsai Procedure [3] is described by the following relations:

$$E_2 = E_m \frac{1 + \xi_1 \eta_1 V_f}{1 - \eta_1 V_f}, \quad G_{12} = G_m \frac{1 + \xi_2 \eta_2 V_f}{1 - \eta_2 V_f},$$

where

$$\eta_1 = \frac{E_f - E_m}{E_f + \xi_1 E_m}, \quad \xi_1 = 2, \quad \eta_2 = \frac{G_f - G_m}{G_f + \xi_2 G_m}, \quad \xi_2 = 1,$$

$\xi_1 = 2(a/b)$ , at hexagonal packing of fibers the factor  $\xi_1$  equals to 2, and  $a$  and  $b$  are the coefficients.

In view of the difficulty in determination of the value  $E_2$ , in comparison with the other elastic characteristics, there are many known methods, some of the most frequently used are given in Table 1.

Also, the Gough–Tangorra and the Akasaka–Hirano methods [25] can be used, however, they have some limitations of the minimum fiber content and, at the same time, the influence of matrix is insignificant.

Above is given the determination of the value  $G_{23}$  using CCAB model [26]. To describe  $G_{23}$ , the Chamis theory is proposed [26]:

$$G_{23} = \frac{G_m}{1 - \sqrt{V_f} (1 - G_m / G_{f23})}.$$

**Determination of Strength Properties along and across the Fiber Direction under Tension, Compression, and Shear.** There are scarce papers devoted to the problem of determination of strength properties of unidirectional fibrous CM. In general, it is proposed to use MR:

$$\sigma_i^j = \sigma_{if}^j V_f + \sigma_{im}^j V_m,$$

where  $i=1, 2$  (direction of reinforced elements),  $j=+$  (tension), and  $j=-$  (compression).

For shear strength it is as follows:

$$\sigma_{sh} = \sigma_m^u V_m.$$

TABLE 1. Determination of the Value  $E_2$  Using Different Methods

Method	Expression for $E_2$
Hopkins and Chamis [21]	$E_2 = E_m \left( 1 - V_f + \frac{\sqrt{V_f}}{1 - \sqrt{V_f}(1 - E_m/E_f)} \right)$
Kaw [22]	$E_2 = \frac{E_f \sqrt{V_f} + (1 - \sqrt{V_f})E_m}{1 - \sqrt{V_f} + V_b + (\sqrt{V_f} - V_f)(E_m/E_f)}$
Reus [21]	$E_2 = \frac{E_f E_m}{E_f V_f + E_m(1 - V_f)}$
Alfutov [23]	$E_2 = \frac{E_f E_m}{E_1(V_f E_m + V_m E_f) - V_f V_m (v_m E_f - v_f E_m)^2}$
Vasil'ev [24]	$E_2 = \frac{\pi E_m}{2V_f(1 - 2v_m \mu_m)}, \quad \mu_m = \frac{v_m(1 + v_m)}{1 - v_m^2}$

TABLE 2. Basic Characteristics of the Reinforcing Elements and Matrix of CM for Checking Calculations

Material	$E_f$ , GPa	$E_m$ , GPa	$V_f$	$V_m$	$v_f$	$v_m$	$\rho_f$ , kg/m <sup>3</sup>	$\rho_m$ , kg/m <sup>3</sup>
1 (variant 1)	73.00	3.20	0.550	0.450	0.20	0.30	2550	1100
1 (variant 2)	73.00	3.20	0.650	0.350	0.20	0.30	2550	1100
1 (variant 3)	73.00	3.20	0.400	0.600	0.20	0.30	2550	1100
2	72.52	3.20	0.476	0.524	0.28	0.33	2000	1100
3	80.00	3.35	0.620	0.380	0.20	0.35	–	–
4	74.00	3.35	0.600	0.400	0.20	0.35	–	–

TABLE 3. Mechanical Characteristics of CM used for Calculations

Material	$E_1$ , GPa	$E_2$ , GPa	$\nu_{12}$	$G_{12}$ , GPa	$\sigma_1^+$ , MPa	$\sigma_1^-$ , MPa	$\sigma_2^+$ , MPa	$\sigma_2^-$ , MPa	$\tau_{12}$ , MPa
1 (variant 1)	39.00	8.6	0.280	3.800	1080	620	39.0	128	89.0
1 (variant 2)	48.00	15.3	0.320	5.100	1297	820	27.8	150	39.2
1 (variant 3)	30.90	8.3	0.330	2.800	798	480	27.0	140	36.8
2	36.60	5.4	0.300	4.085	–	–	–	–	–
3	53.48	17.7	0.278	5.830	1140	570	35.0	114	72.0
4	45.60	16.2	0.278	5.830	1280	800	40.0	145	73.0

TABLE 4. Calculated Values of Modulus of Elasticity  $E_1$  (GPa) in the Direction of the Axis of CM Reinforcement

Material	Method			
	MR	Hill	Tsai (at $C = 1$ )	CCA
1 (variant 1)	41.590	41.483	41.59	41.600
	8.66	6.00	6.60	6.60
1 (variant 2)	48.570	48.460	48.570	48.579
	1.19	0.90	1.19	1.20
1 (variant 3)	31.120	31.013	31.120	31.113
	0.71	0.36	0.71	0.72
2	36.196	36.172	36.196	36.890
	1.10	1.10	1.10	0.70
3	50.873	50.538	50.873	50.890
	4.87	5.50	4.87	4.80
4	45.740	45.405	45.740	45.762
	0.31	0.40	0.31	0.35

**Note.** Here and in Tables 5–8 the calculated values are given above the line, an error between the calculated and experimental data (in %) is given below the line.

TABLE 5. Calculated Values of Modulus of Elasticity  $E_2$  (GPa) in the Direction of the Axis of CM Reinforcement

Material	Method						
	MR	Halpin-Tsai	Tsai	Hopkins-Chamis	CCA	Vasil'ev	Reus
1 (variant 1)	<u>6.749</u>	<u>6.090</u>	<u>6.119</u>	<u>9.598</u>	<u>9.89</u>	<u>12.200</u>	<u>6.749</u>
	21.0	29.0	28.00	11.61	15.0	41	21.0
1 (variant 2)	<u>8.454</u>	<u>7.990</u>	<u>7.602</u>	<u>12.380</u>	<u>13.83</u>	<u>16.700</u>	<u>8.454</u>
	44.0	47.0	50.00	19.00	9.6	17	44.0
1 (variant 3)	<u>5.181</u>	<u>4.203</u>	<u>4.618</u>	<u>8.300</u>	<u>6.62</u>	<u>10.400</u>	<u>5.181</u>
	37.0	40.9	44.00	15.20	20.3	25	37.0
2	<u>5.871</u>	<u>5.048</u>	<u>4.898</u>	<u>8.160</u>	<u>8.13</u>	<u>13.440</u>	<u>5.871</u>
	8.7	6.6	9.28	51.00	50.0	148	8.7
3	<u>8.251</u>	<u>7.770</u>	<u>7.687</u>	<u>12.014</u>	<u>13.56</u>	<u>13.610</u>	<u>8.251</u>
	53.0	56.0	56.00	32.00	23.3	23	53.0
4	<u>7.842</u>	<u>7.235</u>	<u>9.302</u>	<u>11.302</u>	<u>12.47</u>	<u>14.068</u>	<u>7.842</u>
	51.0	55.0	54.00	30.00	23.0	13	51.0

TABLE 6. Calculated Values of Shear Modulus  $G_{12}$  (GPa) CM

Material	Method			
	MR	Halpin-Tsai	Tsai	CCA
1 (variant 1)	<u>2.606</u>	<u>3.762</u>	<u>4.239</u>	<u>3.764</u>
	31.0	1.00	16.10	0.9
1 (variant 2)	<u>3.270</u>	<u>4.911</u>	<u>5.802</u>	<u>4.914</u>
	35.8	3.77	13.77	3.6
1 (variant 3)	<u>4.997</u>	<u>2.667</u>	<u>2.871</u>	<u>2.669</u>
	78.0	4.70	2.56	4.6
2	<u>2.290</u>	<u>2.201</u>	<u>3.520</u>	<u>3.072</u>
	43.0	46.00	13.80	24.0
3	<u>3.078</u>	<u>1.984</u>	<u>5.289</u>	<u>4.604</u>
	47.0	-	9.27	21.0
4	<u>2.925</u>	<u>2.030</u>	<u>4.960</u>	<u>4.317</u>
	49.0	-	14.80	25.0

TABLE 7. Calculated Values of Poisson's Ratio  $\nu_{12}$  CM

Material	Method		
	MR	Hill	CCA
1 (variant 1)	<u>0.2450</u>	<u>0.190</u>	<u>0.236</u>
	16.00	31.0	15.0
1 (variant 2)	<u>0.2350</u>	<u>0.190</u>	<u>0.227</u>
	41.00	39.0	29.0
1 (variant 3)	<u>0.2600</u>	<u>0.190</u>	<u>0.251</u>
	18.00	39.0	21.0
2	<u>0.3038</u>	<u>0.330</u>	<u>0.302</u>
	1.26	10.0	0.6
3	<u>0.2570</u>	<u>0.186</u>	<u>0.248</u>
	7.50	32.0	10.0
4	<u>0.2600</u>	<u>0.185</u>	<u>0.250</u>
	6.40	33.0	10.0

To determine the most accurate analytical methods of determination of the material mechanical characteristics using the above relations, let us calculate mechanical characteristics of unidirectional composites and compare the obtained data with the known ones [13, 21, 27, 28]. For this purpose, four composite materials based on glass fibers and epoxy binder were selected, which are denoted as 1, 2, 3, and 4. For these materials, the elastic and strength properties, as well as volume content of components, are known.

TABLE 8. Calculated Values of the Ultimate Tensile Strength and the Ultimate Compression Strength in the Direction of the Reinforcing Axis and Along It As Well As Shear Yield Stress Values for CM

Material	$\sigma_1^+$ , MPa	$\sigma_1^-$ , MPa	$\sigma_2^+$ , MPa	$\sigma_2^-$ , MPa	$\tau_{12}$ , MPa
1 (variant 1)	$\frac{1076.0}{-}$	$\frac{879}{-}$	$\frac{31.5}{-}$	$\frac{158}{-}$	$\frac{83}{-}$
1 (variant 2)	$\frac{1259.0}{2.89}$	$\frac{1017}{24.0}$	$\frac{24.5}{12.5}$	$\frac{158}{5.0}$	$\frac{96}{-}$
1 (variant 3)	$\frac{802.0}{0.50}$	$\frac{672}{-}$	$\frac{42.0}{-}$	$\frac{158}{12.0}$	$\frac{64}{-}$
3	$\frac{1360.3}{9.60}$	$\frac{944}{-}$	$\frac{30.4}{13.0}$	$\frac{158}{-}$	$\frac{89}{24}$
4	$\frac{1322.0}{3.28}$	$\frac{918}{14.7}$	$\frac{32.0}{20.0}$	$\frac{160}{8.9}$	$\frac{87}{19}$

Material 1 was studied for three variants (1, 2, and 3) at different values of fiber-matrix volumetric ratio:  $V_f = 0.55, 0.65, \text{ and } 0.4$ . Table 2 presents the input data for materials.

Strength characteristics of the composite components are the following: for material 1 ( $\sigma_m^+ = 70$  MPa,  $\sigma_f^+ = 1900$  MPa,  $\sigma_m^- = 120$  MPa,  $\sigma_f^- = 1500$  MPa); for materials 3 and 4 ( $\sigma_m^+ = 80$  MPa,  $\sigma_f^+ = 2150$  MPa,  $\sigma_m^- = 120$  MPa, and  $\sigma_f^- = 1450$  MPa).

Table 3 presents the characteristics of composites known from independent sources, which will be used to determine the accuracy of one or another method.

Tables 4–8 provide the results of calculations for each characteristic separately with an indication of value of deviation from the experimental data given in literature references.

**Conclusions.** Since the calculations were conducted on the materials with reinforcing elements (glass fibers) of the same type, the following conclusions would be true for CM based on E-glass fibers.

The most accurate methods for determination of the value  $E_1$  are the Hill method with an error of 0.4–6%, and simpler MR method with an error of 0.31–6.6%. In the calculation of the value  $E_2$  for the material with fiber content less than 50% the Halpin–Tsai method with an error of 6.6% is the most accurate. It should be noted that in [6] for CM with fiber content of 37% the most accurate is the Tsai method (in the above-mentioned calculations the error is 9.28%); for the material with fiber content more than 60% the most accurate results were obtained using the Vasiliev method with an error of 7–25%.

In the determination of the value  $\nu_{12}$ , the most accurate is CCA method with an error of 0.6–29%, and MR method with an error of 1.26–41%.

For calculation of the value  $G_{12}$  the most accurate is the Tsai method with an error of 2.56–16.1%.

In the calculation of strength characteristics, the error varies within the range from 0.5 to 13% and only determination of compression stresses along the reinforcement axis demonstrates a low accuracy (24–65%). Use of MR to determine the values  $E_1$  and  $\nu_{12}$  and of the Tsai method for CM is generally in agreement with the results obtained in [29].

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