Strength of Materials, Vol. 45, No. 4, July, 2013

METHODOLOGY AND SOFTWARE PACKAGE FOR ASSESSMENT OF STRESS-STRAIN STATE PARAMETERS OF FULL-SCALE STRUCTURES AND ITS APPLICATION TO A STUDY OF LOADING LEVEL, DEFECT RATE, AND RESIDUAL STRESS LEVEL IN ELEMENTS OF NPP EQUIPMENT

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UDC 620.191.33:539.4

The paper outlines the methodological approach and the related software package, which permit determining stress-strain state and defect rate for full-scale crack-containing objects. The method is based on computational assessment of the above-mentioned parameters on the condition of minimum of the objective function that describes discrepancy between the experimental data arrays and the results of numerical solutions to model problems using the finite-element method.

Keywords: processing of experimental information, finite element method, load level, defect rate, residual stresses.

Introduction. One of the most important prerequisites for improving strength and reliability of nuclear power plant equipment in combination with increasing its performance and extending service life is to obtain reliable information about the true stress-strain state (SSS) at all stages of its operation. The state-of-the-art software packages (CAN, ANSYS, NASTRAN, ABAQUS, etc.), which are based on the finite element method (FEM), provide fairly accurate analytical models for the objects under study during the design stage. However, the more complex the structure and the higher its criticality, the higher is significance of the SSS assessments acquire during their experimental verification (startup and commissioning tests) and operation.

It is known that SSS of a full-scale structure may considerably differ from that obtained by calculations. This is due to the following factors: (i) the analytical model is always conditional to a certain extent; (ii) some errors in information about the real operating loads; (iii) the presence of fabrication-induced residual stresses (RS); (iv) creep and relaxation in the material. This is especially the case with the objects that are long in operation, with essentially worn heavy-duty zones and elements.

The problem of determination of real load levels, SSS, and damage levels for full-scale structures can be solved only through experimental investigations of real objects or specimens cut from them. Usually, the study of full-scale objects includes two stages:

(i) recording experimental data on strains, displacements and other parameters representing the deformation response to some actions: mechanical or thermal loading (unloading), removal of a certain zone from the object (drilling a small hole, cutting off a layer, etc.);

(ii) determination of sought-for parameters of the structure by processing the experimental data obtained.

The availability of parameters (maximum stresses, forces, residual stresses, fracture mechanics parameters, the presence and size of possible defects, etc.) as determined at a given stage of operation can serve as a basis for obtaining a refined assessment of strength and operability of the structural elements under consideration.

It should be emphasized that the conventional methods of SSS determination by processing experimental data are based on a common methodological approach involving the a priori known relations between the sought-for parameters and the experimental data, which are derived by means of analytical or numerical solutions to the problems close in definition. Such approaches have a limited range of application and may contain significant errors in the construction of a model for specific problems.

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In [1–3] a method was put forward which, as distinct from the conventional ones, suffers no fundamental limitations on the types of distribution of the sought-for parameters. It is based on the determination of sought-for characteristics of the particular region P_j of the structure on the basis of minimizing the "total" discrepancy (the objective function I) between the arrays of experimental data e_i^* and the results e_i of numerical solutions to the corresponding model problems such that their special features can be taken into account during the definition of these problems.

For obtaining a great body of e_i^* we employ the up-to-date experimental methods based on interference-optical recording of displacement fields under laboratory and full-scale conditions (the methods of electronic digital speckle interferometry, digital image correlation), which provide the direct digital information about SSS on the surface of the object at hand [4, 5]. The next stage of the study is to determine the above-mentioned parameters by calculations.

To implement the proposed approach, we have developed a "flexible" software package (SP) which includes programs with a graphical interface (MATLAB-hosted) and parametric macros (within ANSYS), which provide the following:

1) The on-line generation of an array of experimental data e_i^* against the background of experimental patterns of deformation response fields.

2) The FEM solution of the direct problem that consists in calculating the values of deformation responses e_i at the assigned measurement points (MP) at specified values (taken as a first approximation) of the parameters P_j. This permits calculating the deformation fields that arise due to loading of the region or due to any other action on the pre-loaded region (e.g., drilling a hole).

3) The inverse solution (determination of the parameters P_j) on the basis of interpretation of experimental data e_i^*. According to the proposed procedure, the inverse problem comes down to a problem of minimizing the objective function $I(e_i, e_i^*)$ that represents a complex discrepancy between the arrays of experimental data (e_i^*) and the corresponding calculated values (e_i). Root mean square deviation (I_{RMS}), maximum deviation ($I_{\max}$), or a function of a special form can be taken as objective function.

4) Numerical investigation of the influence of scatter of various factors on the accuracy of determination of the parameters P_j.

Some advances in applied mathematics and software engineering have made it possible to greatly increase the rate of inverse solution and improve the accuracy of solving the applied problems of assessment of load level for structures.

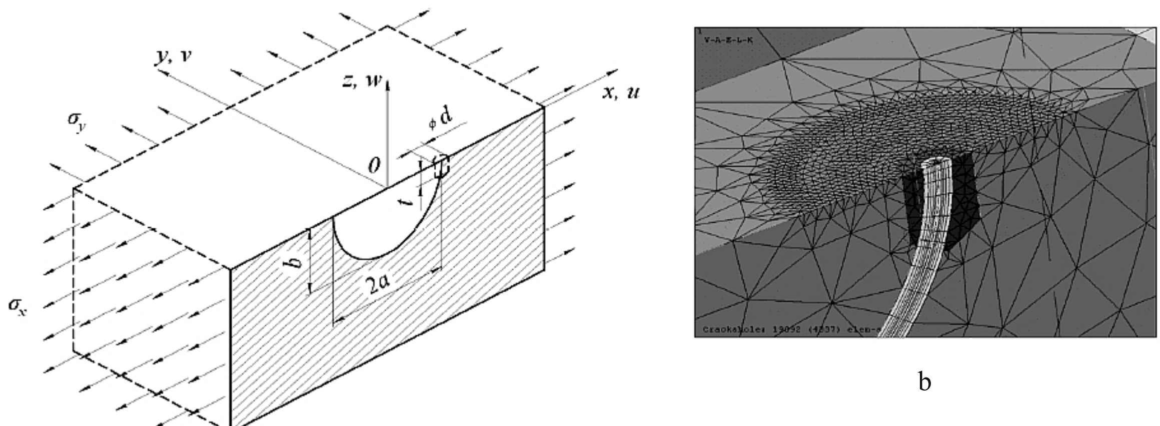
The efficiency of the method has been demonstrated by a series of numerical experiments (examples of solving typical practical problems) related to a study of the load level and rate of defects in typical NPP elements.

Load Assessment of Piping, Including Those with Crack-Like Surface Defects. In [2] the method was shown to be suitable for analyzing load level for structural elements containing surface and through-thickness cracks. To provide an action on the test zone a small hole is drilled at the crack tip. Note that this procedure is used as a structural crack arrest, which ensures reduction of stress concentration and removal of damaged material at the crack tip [6]. To further improve this method, we considered a problem of simultaneous determination of load level in the region (stresses σ_x and σ_y at some distance from the crack zone) and of the crack depth b (Fig. 1). With $P = \{\sigma_x, \sigma_y, b\}$ known, one can perform a refined calculation of fracture mechanics parameters for the fracture toughness assessment of the region. The drilling-induced u, v, and w displacement fields on the surface were used as initial experimental information. The crack length $2a$ on the surface is considered known and the crack is taken to be semi-elliptical in shape.

The method and its software implementation imply that when solving each specific problem the volume and region of localization of experimental information should be established and requirements for sensitivity of equipment for recording deformation response should be specified, in order to reliably obtain the sought-for parameters on the basis of calculation of corresponding model problems. Results of calculation of one of the model problems (Fig. 1) are given in Table 1 ($\bar{\sigma}_x = \sigma_x / \sigma_x^*$, $\bar{\sigma}_y = \sigma_y / \sigma_y^*$, $\bar{b} = b/b^*$, where b^*, σ_x^*, and σ_y^* are the true

TABLE 1. Assessment of the Influence of Experimental Conditions on the Sought-For Parameters

Experimental conditions				Mathematical expectation			Dispersion		
MP localization	N	δ_e, %	I	$\bar{b}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{b}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$
$1.0 \le \lvert\bar{r}\rvert/d \le 1.5$	55	10	I_{RMS}	1.013	0.998	0.988	0.081	0.036	0.049
			$I_{\max}$	1.010	0.996	0.988	0.109	0.061	0.067
$0.5 \le \lvert\bar{r}\rvert/d \le 1.0$			I_{RMS}	1.049	0.994	0.982	0.216	0.091	0.116
			$I_{\max}$	1.075	0.989	0.975	0.262	0.093	0.128
$1.0 \le \lvert\bar{r}\rvert/d \le 1.5$	110	10	I_{RMS}	1.003	0.992	0.992	0.051	0.023	0.030
		20	I_{RMS}	1.030	0.970	0.959	0.124	0.055	0.071
			$I_{\max}$	1.054	0.093	0.932	0.097	0.040	0.055
	60	20	I_{RMS}	0.996	0.978	0.078	0.081	0.048	0.054



a b

Fig. 1. Half-space with a surface semielliptical crack (a) and a finite-element model of the crack emerging to the surface (b).

values of the parameters, and r is the radius-vector whose origin is located at the center of the hole). It is evident that in case where large arrays of experimental information are available (where N is the number of MPs), the error of computation is within 10% for loads and 15% for crack depth, despite a significant range of scatter of experimental data ($\delta_e = 20\%$).

A Study of Residual Stresses in Joints of Heterogeneous RPV Materials. Consider a problem of bind-hole investigation of 3D high-gradient fields of residual stresses in bimetallic structural elements containing a stress step due to the difference in thermomechanical characteristics of the materials of the bimetallic composite. By way of example, we considered a problem of RS distribution in a bimetallic cylindrical shell of WWER-1000 reactor pressure vessel (RPV), for which the law of RS distribution in the joint zone of heterogeneous materials (Fig. 2) is known from [7–9, et al.].

Determine RS field in a two-layer thin-walled cylinder where normal stresses are distributed in a piecewise-linear manner in depth and stepwise manner at the boundary of the joint of heterogeneous materials. The laws of stress distribution along the directions θ and z are considered similar (the similarity coefficient α). Thus, the stress state can be represented in terms of four parameters: $P = \{s_\theta^1,\ s_\theta^2,\ s_\theta^3,\ s_\theta^4\}$ (Fig. 2b).

When performing the numerical "experiment," we assumed the hole to be sequentially deepened in five steps; the ratio of its depth t to the thickness of the top layer h (overlay) is taken to be 0.33, 0.67, 1.22, 1.44, and 1.67. Also, at each step the hole diameter d increases so that $t/d \approx 1$. This is due to the fact that the function that relates RS to the parameters of deformation response "measured" on the surface quickly diminishes with increasing parameter t/d. The fields of displacements u, v, and w are recorded at each step. The inverse problem is solved

TABLE 2. Assessment of the Influence of Experimental Condition Error on the Parameters s_θ^1, s_θ^2, s_θ^3, and s_θ^4

Experimental conditions			Dispersion				
I	N^*	δ_e	$\overline{s_\theta^1}$	$\overline{s_\theta^2}$	$\overline{s_\theta^3}$	$\overline{s_\theta^4}$	$\overline{\Delta s}$
I_{max}	3	5	0	0.01	0.17	0.55	0.04
I_{max}	10	5	0	0	0.07	0.21	0.02
I_{RMS}	10	5	0	0.02	0.31	0.81	0.07
I_2	3	10	0.01	0.03	0.05	0.05	0.04
I_2	10	10	0.01	0.03	0.05	0.04	0.03

Note: N^* is the number of measurement points at each interference band.

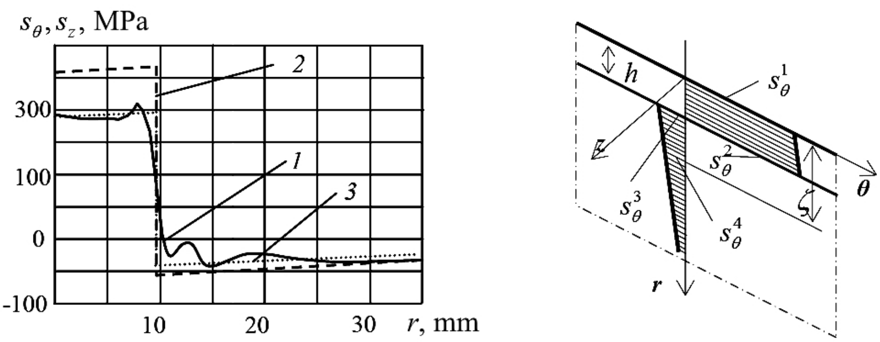


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Fig. 2. For the RS analysis of WWER-1000 reactor pressure vessel shell: (a) data given in [7–9] [(*1*) hoop stress diagram (s_θ); (*2*, *3*) schematic diagrams of hoop stresse (s_θ) and axial stresses (s_z)]; (b) schematic representation of the model problem.

accordingly: the parameters s_θ^1 and s_θ^2 are determined at the first step and s_θ^3 and s_θ^4 at the second step. The obtained values of the sought-for (dimensionless) parameters are summarized in Table 2.

The results of analysis of the values of parameters s_θ^1 and s_θ^2 have confirmed the high accuracy of determination of linearly varying stress fields. However, the rms deviation and maximum deviation did not provide the adequate accuracy of determination of the parameters s_θ^3 and s_θ^4 even with initial data error being small. Therefore, we studied the possibility of using objective functions of a different form. In particular, two functions of special form have been derived:

$$I_1 = \sqrt[3]{\frac{\sum_m \widetilde{e}_m \sum_m d_m^2}{N}}, \qquad I_2 = \sqrt{\frac{\sum_m \{(N - m + 1) d_m^2\}}{N}},$$

where $\widetilde{e}_m$ is the vector of $\widetilde{e}_i$ vector components (relative deviations of e_i and e_i^*) and d_m is the vector of difference between values of adjacent components of the vector $\widetilde{e}_m$.

These functions have yielded a result with acceptable accuracy; specifically, with $\delta_e = 10\%$ the variance of the parameter s_θ^4 which is the least "stable" to the initial data error is within 11.0% in the case of using the fields of absolute responses and 4.8% in the case of relative responses. The error of determination of the most important parameter – the stress "step" $\Delta s = |s_\theta^2| + |s_\theta^3|$ – ranges between 5 and 7%.

TABLE 3. Results of Calculation for Model Problems

Experimental conditions								Maximum deviation, %	
True values of the parameters			Initial deviations, %		Response fields	I	N		
b/h	a/b	σ_y/σ_x	δ_σ	δ_e				a	b
0.6	2	2.0	0	10	u, v	I_{RMS}	81	4.3	9.0
0.6	2	2.0	5	10	u, v	I_{RMS}	81	7.3	14.6
0.6	2	2.0	5	10	u, v	$I_{\max}$	81	9.5	20.5
0.6	2	2.0	5	10	u, v	I_2	81	4.7	6.2
0.6	2	2.0	5	10	u, v, w	I_2	81	4.0	4.8
0.6	2	2.0	10	20	u, v, w	I_2	81	7.2	6.4
0.6	2	2.0	10	20	u, v, w	I_{RMS}	81	16.6	22.6
0.6	2	0.5	5	10	u, v, w	I_2	81	3.8	5.4
0.6	2	1.0	5	10	u, v, w	I_2	81	2.3	4.5

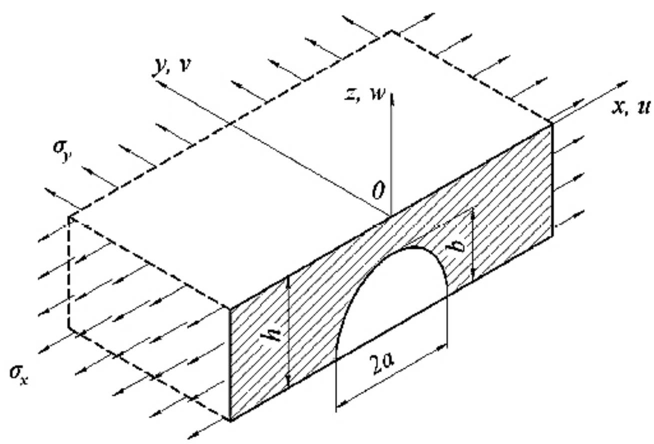

Fig. 3. A thin-walled element with an internal crack.

Assessment of Crack Dimensions on the Piping Inner Surface. The successful application of the proposed method for the determination of a surface crack depth has enabled us to state a more complex problem: determine the dimensions of a crack located on the piping inner surface by the fields of strains (disturbances) on the outer surface, which arise due to the internal-pressure loading (or partial load relief) of the piping. The following model problem was considered in order to assess the potentialities of the proposed approach. A thin-walled element contains an elliptical crack which is located orthogonally to the surface and has dimensions b and a to be determined (Fig. 3). At some portion of the surface (plane $z = 0$) we record the fields of displacements u, v, and w induced by the stresses σ_x and σ_y whose magnitude is known.

Results of calculations for some model problems are summarized in Table 3. It is evident that the use of rms deviation and maximum deviation as an objective function leads to a relative error of about 15–20% in the determination of the sought-for parameters $P_1 = a$ and $P_2 = b$, with the initial data error being small. On the other hand, even with a significant error in determination of deformation responses we have $\delta_e = 20\%$ (and with the magnitude of scatter of load values σ_x and σ_y we have about $\delta_\sigma = 10\%$); the use of the proposed function of special form I_2 the error in determination of the parameters a and b is less than 7.5%.

It has been demonstrated that when using I_2 the tensile load ratio $k = \sigma_y/\sigma_x$ has no essential effect on the accuracy of the solution. Note that the investigations were carried out for the defects whose depth exceeds $0.4h$. In the case of smaller defect dimensions, the deformation response at the surface is faint but becomes more as the load level in the element grows).

Conclusions. Thus, the present calculations performed for various model problems have demonstrated the high effectiveness of using the methodology and related software package for the study of load level, detection of defects, and assessment of residual stresses. The software package seems to be suitable also for the analysis of elements of NPP equipment. Considering that the elaborated algorithms and programs are quite versatile, the range of problems of diagnostics of NPP elements can be further extended, including elastoplastic problems, analysis of degradation of material properties. For such cases, we propose to consider indentation as a way of action on the test object.

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