Strength of Materials, Vol. 42, No. 3, 2010

APPLICATION OF NONLINEAR RESONANCES FOR THE DIAGNOSTICS OF CLOSING CRACKS IN RODLIKE ELEMENTS

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The vibrations of elastic bodies with closing cracks are essentially nonlinear. As a specific feature of these vibrations, one can mention the manifestation of so-called nonlinear effects, e.g., nonlinear (i.e., sub- and superharmonic) resonances and the nonlinearity of vibrations for these resonances. The proposed method for the evaluation of the parameters of cracks (their sizes and location) is based on the analysis of the nonlinearity of vibrations in the neighborhood of a superharmonic resonance of order 2/1 and/or a subharmonic resonance of order 1/2 in the case of variation of the site of application of the driving force because, as follows from the results of numerical and experimental investigations, the level of nonlinearity of the vibrations of rods with closing cracks for nonlinear resonances depends not only on the parameters of the crack but also on the site of application of the driving forces.

Keywords: closing crack, sub- and superharmonic vibrations, diagnostics of the crack, nonlinear effects.

Introduction. Fatigue cracks form one of the most widespread types of defects in machines and structures operating under dynamic loads. As follows from the results of numerous experimental and theoretical investigations, cracks lead to a decrease in the natural frequencies of these objects and distort the form of vibrations. The relationship between the parameters of the crack (sizes and location), on the one hand, and the changes in the natural frequencies and forms of vibrations, on the other hand, is studied in numerous works [1, 2]. However, the sensitivity of the methods of diagnostics of the cracks based on the changes in the natural frequencies and forms of vibrations appeared to be to be quite low. Later, it was shown that the so-called nonlinear effects [3], i.e., the sub- and superharmonic resonances, a strong nonlinearity of vibrations at these resonances, and the characteristics of damping of vibrations, are characterized by a higher sensitivity to the presence of cracks [4].

In order to simplify the analysis of nonlinear effects, it is customary to assume that the stiffness of structures suffers instantaneous changes at the times of crack closure and opening. As a rule, this phenomenon is modeled by asymmetric piecewise-linear characteristics of the restoring forces. The analytic investigations of the forced vibrations of a mechanical system with one degree of freedom and the indicated characteristic of restoring forces make it possible to reveal sub- [5–14] and superharmonic [8, 9, 11, 13, 15, 16] resonances of different orders. In addition, as shown in [3, 7, 8], the nonlinear effects strongly depend on the dissipative properties of the investigated system: the higher the level of damping of vibrations in the system, the lower the amplitudes of nonlinear resonances and the level of nonlinearity of vibrations at these resonances.

The data of experimental investigations [4, 17–19] demonstrate that the growth of fatigue cracks is accompanied by a significant increase in the characteristics of damping of vibrations for specimens made of different materials. If this increase is neglected, then the degree of damage to the vibrating system predicted on the basis of nonlinear effects is underestimated. Hence, it is necessary to determine the relationship between the parameters of cracks and nonlinear effects with regard for the actual level of damping in the damaged system but not under the assumption that damping is independent of the sizes of defects [11].

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Translated from Problemy Prochnosti, No. 3, pp. 125 – 141, May – June, 2010. Original article submitted December 18, 2008.

The number works devoted to the investigation of forced vibrations of structures and their elements with closing cracks available from the literature is insignificant. This is explained by the fact that the dynamic behavior of mechanical systems with closing cracks is essentially nonlinear, which significantly complicates the exact analytic solution of the problem. Moreover, actual fatigue cracks are, in fact, hardly predictable and permanently changing objects.

The results of numerical investigations of the forced vibrations of rods with closing cracks show that their main specific feature is connected with the appearance of sub- [11] and superharmonic [11, 20–22] resonances of different orders. It is worth noting that nonlinear resonances may be caused by all types of nonlinearities in the vibrating system, including the symmetric elastic nonlinearity of the restoring forces [23–30], geometric nonlinearity [11], and nonlinear damping [31]. There exists a threshold value of the driving force (the power of excitation of vibrations) under which it is impossible to excite the subharmonic resonance of order 1/2 [11, 25] and the superharmonic resonances of orders 2/1 and 3/1 [11].

In analyzing the forced vibrations of damaged bodies, the place of application of the driving force and the level of damping of vibrations were regarded as invariable. However, our investigations demonstrate that the nonlinear effects strongly depend on the place of application of the driving force. This enables us to develop an efficient vibration method for the diagnostics of defects based on the evaluation of the level of nonlinearity of vibrations excited by the forces applied at different sites of the analyzed structure at the nonlinear resonances. The proposed approach enables one to avoid the influence of various factors, such as nonlinear damping, geometric nonlinearity, etc., on the nonlinear effects because these factors are definitely independent on the place of application of the driving force.

The aim of the present work is to study the influence of the place of application of the driving force on the nonlinearity of vibrations corresponding to the sub- and superharmonic resonances of the rods with closing cracks and develop, on the basis of the accumulated results, a procedure for the diagnostics of fatigue cracks.

Influence of the Place of Application of the Driving Force: Numerical Results. The numerical analyses of the influence of the place of application of the driving force on the nonlinearity of vibrations at the superharmonic resonance of order 2/1 and the subharmonic resonance of order 1/2 are carried out by using a finite-element model of a rod with closing crack [32]. The model consists of 21 elements (Fig. 1). The crack is modeled by a shortened element with lower stiffness whose length constitutes 10% of the length of an ordinary element. The application of the shortened element enables one to specify the location of the crack (L_c) more precisely. It is assumed that the cracked rod has an asymmetric piecewise-linear characteristic of the restoring force.

We study the first mode of forced flexural vibrations of the rods for different boundary conditions and locations of the crack, namely, the cantilever rods ($L_c/L = 0.1$ and 0.5), the rods placed on two supports ($L_c/L = 0.1$, 0.25, and 0.5), and the rods both ends of which are restrained ($L_c/L = 0.1$, 0.25, and 0.5). The length L of the cantilever rod, its height h, and the width b of its cross section are equal to 0.23, 0.02, and 0.004 m, respectively. The sizes of the rod placed on two supports and the rod with restrained ends are as follows: $L = 6$ m, $h = 0.5$ m, and $b = 0.5$ m. In all cases, the relative crack depth $a/h = 0.2$. The modulus of elasticity E, density ρ, and the logarithmic decrement of vibrations δ of the material are set equal to 206 GPa, 7850 kg/m^3, and 0.01, respectively.

The idea of the procedure of numerical analysis is as follows: the driving force is consecutively applied to different nodes of the model (Fig. 1) and, in each case, the spectral analysis of the vibration process of acceleration is performed for the sub- and superharmonic resonance vibrations of the rod. The process of vibrations is studied for the end of the cantilever rod and for the median sections of the rod placed on two supports and the rod whose ends are restrained.

The vibrations of the elastic bodies corresponding to the nonlinear resonances are essentially nonlinear. For these resonances, the nonlinear distortions of vibrations are caused mainly by the presence of a harmonic whose frequency coincides with the frequency of the principal resonance in the spectrum of vibrations. The amplitude of this harmonic significantly exceeds the amplitudes of the other harmonics and, hence, it is called predominant. Thus, for the second-order superharmonic resonance, the second harmonic with amplitude A_2 is predominant. At the same time, for the subharmonic resonance of order 1/2, the role of predominant harmonic is played by the subharmonic

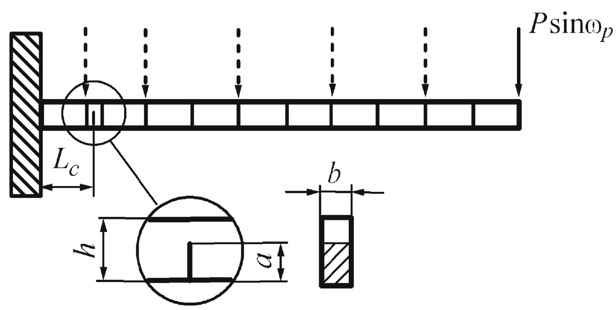

Fig. 1. Finite-element model of a cantilever rod.

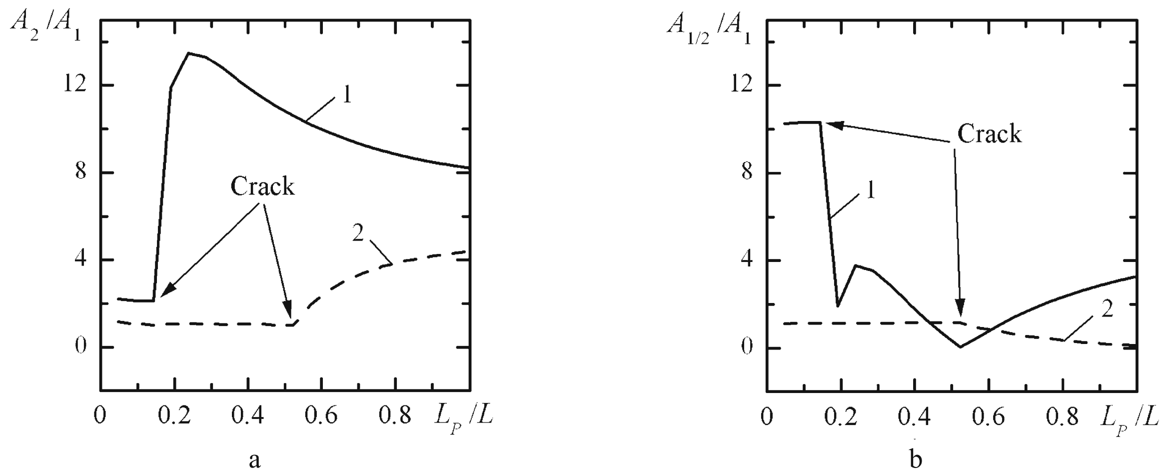

Fig. 2. Dependences of the relative amplitude of the predominant harmonic in the spectrum of acceleration for the superharmonic resonance of order 2/1 (a) and the subharmonic resonance of order 1/2 (b) on the place of application of the driving force for the cantilever rod: (*1*) $L_c/L=0.1$, (*2*) $L_c/L=0.5$.

with amplitude $A_{1/2}$. Therefore, the level of nonlinearity of vibrations in these cases is estimated by the ratios A_2/A_1 and $A_{1/2}/A_1$, respectively (A_1 is the amplitude of the fundamental harmonic).

As follows from Fig. 2, the nonlinearity of vibrations of the cantilever rod for both nonlinear resonances strongly depends on the site of application of the driving force. The kinks in these curves correspond to the location of the crack. Moreover, on finding, as indicated above, the location of the crack, one can also estimate its sizes according to the degree of nonlinearity of vibrations because, for the nonlinear resonances, there exists a direct (but nonlinear) dependence of the crack sizes on the degree of nonlinearity of vibrations.

For the rod on two supports, the location of the crack is determined according to the extrema of nonlinearity (Fig. 3). These are the maximum of nonlinearity for the superharmonic resonance and its minimum for the subharmonic resonance. The case where the crack is located in the cross section $L_c/L=0.1$ is an exception. It is easy to see that, in this case, the subharmonic resonance is weak, which is explained by the location of the crack near the "stress node" for the first mode of flexural vibrations of the rod ($L_{node}/L=0$). As a result, the degree of nonlinearity of the system is too low to induce a strong subharmonic resonance. The notion of "stress node" was introduced in [33] by analogy with the notion of "vibration node" as a cross section where the stresses are equal to zero for a given mode of vibrations. This notion becomes necessary because the coordinates of the vibration and stress nodes do not always coincide.

Practically the same results are obtained for the rod with restrained ends (Fig. 4). If the crack is located in the section $L_c/L=0.25$, then the subharmonic resonance of this rod is not excited because the crack is located too close to the stress node of the first mode of flexural vibrations of the rod ($L_{node}/L=0.223$) and, hence, the nonlinearity of the system is insufficient to form the subharmonic resonance.

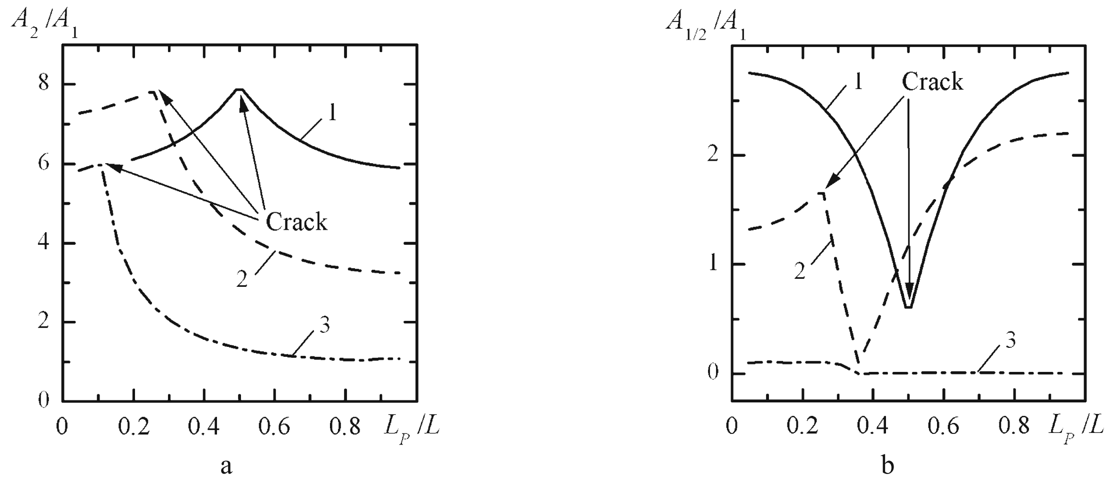

Fig. 3. Dependences of the relative amplitude of the predominant harmonic of the spectrum of acceleration for the superharmonic resonance of order 2/1 (a) and the subharmonic resonance of order 1/2 (b) on the place of application of the driving force to the rod with two supports: (*1*) $L_c/L = 0.5$, (*2*) $L_c/L = 0.25$, (*3*) $L_c/L = 0.1$.

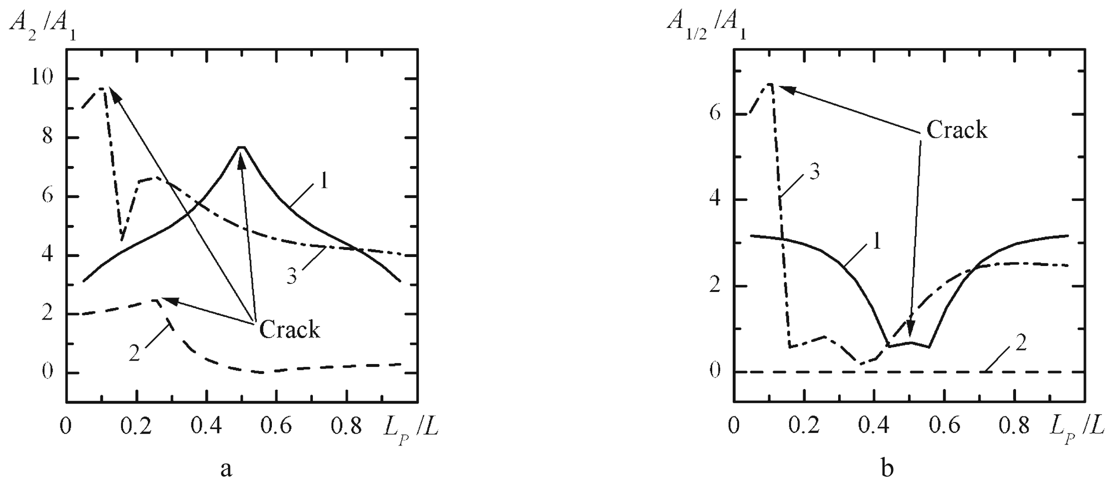

Fig. 4. Dependences of the relative amplitude of the predominant harmonic of the spectrum of acceleration for the superharmonic resonance of order 2/1 (a) and the subharmonic resonance of order 1/2 (b) on the place of application of the driving force to the rod whose ends are restrained: (*1*) $L_c/L = 0.5$, (*2*) $L_c/L = 0.25$, (*3*) $L_c/L = 0.1$.

Thus, a closing crack formed in the stress node for any mode of vibrations (flexural or longitudinal) can be detected neither by analyzing the nonlinear effects, nor by changes in the natural frequency of this mode of vibrations or in the level of damping. Hence, to improve the reliability of the vibration methods of diagnostics of defects, it is necessary to use several modes of vibrations simultaneously.

Some curves in Figs. 2–4 have more than one kink. In this case, to localize the crack, it is necessary to use the data on changes in the phase shift of the fundamental harmonic (φ_1) for the subharmonic resonance of order 1/2 or of the second harmonic (φ_2) for the second-order superharmonic resonance.

As follows from Fig. 5, the curves $\varphi_1(L_P/L)$ and $\varphi_2(L_P/L)$ have a single kink corresponding to the location of the crack (in this case, in the section $L_c/L = 0.1$). This means that the other kinks of the curves $A_{1/2}/A_1\,(L_P/L)$ do not correspond to the location of the crack. The curves $\varphi_1(L_P/L)$ and $\varphi_2(L_P/L)$ can be used to detect the location of the crack independently. However, it is should be taken into account that the sensitivity of the phase shift to the presence of cracks is, in many cases, weak, e.g., for the subharmonic resonance of the rod placed on two supports and containing a crack located in the cross section $L_c/L = 0.25$ (Fig. 5b). Therefore, this

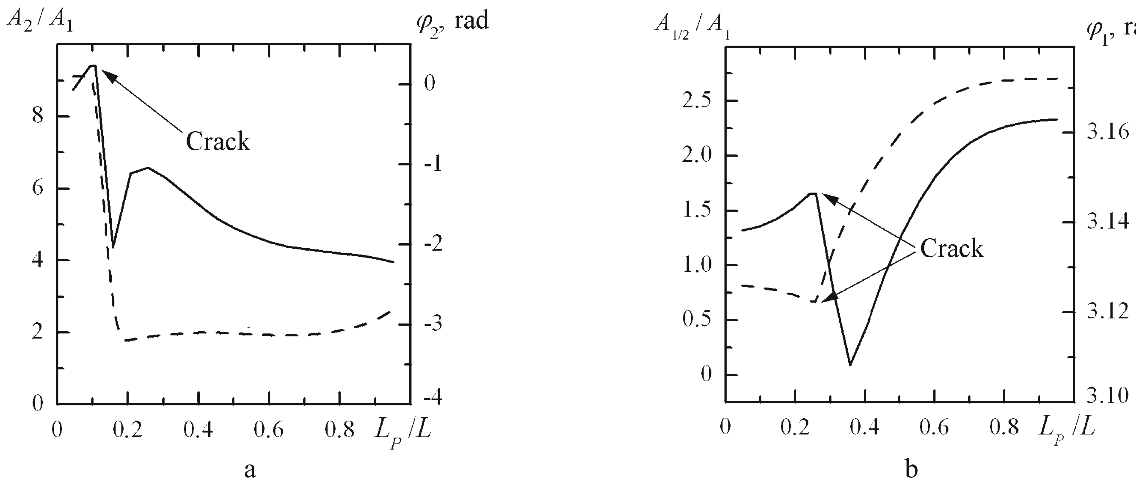

Fig. 5. Dependences of the relative amplitude of the predominant harmonic in the spectrum of acceleration (solid lines) and the phase shift (dashed lines) for the superharmonic resonance of order 2/1 in the rod whose ends are restrained (a) and for the subharmonic resonance of order 1/2 in the rod placed on two supports.

characteristic can hardly be used for the diagnostics of defects independently of the nonlinearity of vibrations at nonlinear resonances.

Mixed (Experimental-Analytic) Method for the Determination of Location of the Crack. The method described in the previous section enables one to determine the location of the crack by using solely the results of measuring the nonlinearity of vibrations, i.e., without using the model. However, it is clear that, for the exact determination of location of the crack by this method, it is necessary to perform a large volume of measurements. Furthermore, in some cases, e.g., if the crack is located not far from the restraint of the rod (or its support), it is necessary to excite vibrations whose level is sufficient for measurements by applying the driving force in the immediate proximity of these sites, which can hardly be realized in practice.

The results of investigations demonstrate that the number of measurements required for the exact determination of the location of the crack can be decreased to two measurements if we use the so-called mixed method. The method is called mixed because it is based on the simultaneous use of the data of measurements and the finite-element model of the rod. The idea of this method is based on the fact that the relative changes in the nonlinearity of the sub- and superharmonic resonance vibrations of the cracked rod determined by applying the driving force at two different sites do not depend on the crack sizes and depend solely on its location.

The nonlinearity of sub- and superharmonic resonance vibrations, e.g., of the free end of the cantilever rod, depends, in the general case, both on the crack size and its location (Fig. 6). At the same time, the relative variations of this nonlinearity are determined by the location of the crack and almost do not depend on its size (Table 1). In Table 1, the nonlinearities of vibrations of the free end of the cantilever rod computed for the cases of application of the driving force at nodes 1–20 of the model (these sites are denoted by $L_{P1} - L_{P20}$, respectively) are related to the nonlinearity of vibrations corresponding to the application of the driving force at node 21 (point L_{P21}).

Thus, to determine the location of the crack by the mixed method, it is necessary to perform two tests aimed at measuring the nonlinearity of vibrations of a certain point of the rod for the sub- or superharmonic resonance. The sole difference between these tests is the place of application of the driving force (the problem of the optimal choice of these places is a subject of separate investigation). As a result of these tests, we determine the relative variation of the nonlinearity of vibrations caused by changing the place of application of the driving force. This information is sufficient to find the location of the crack by using the model of the rod. Since, as shown above, the relative variation of the nonlinearity of vibrations is not a function of the crack sizes, this parameter is regarded as constant in our calculations. Further, we reproduce in the model the conditions of loading realized in the tests and compute two independent values of the degree of nonlinearity of vibrations at a certain point of the rod (this point corresponds to

TABLE 1. Relative Changes in the Nonlinearity of Vibrations of the End of the Cantilever Rod for the Superharmonic Resonance of Order 2/1 and the Subharmonic Resonance of Order 1/2

L_c/L	a/h	$\left(\frac{A_{2/1}}{A_1}\right)_{L_{P21}} \Big/ \left(\frac{A_{2/1}}{A_1}\right)_{L_{Pn}}$				$\left(\frac{A_{1/2}}{A_1}\right)_{L_{P21}} \Big/ \left(\frac{A_{1/2}}{A_1}\right)_{L_{Pn}}$			
		L_{P5}	L_{P10}	L_{P15}	L_{P21}	L_{P5}	L_{P10}	L_{P15}	L_{P21}
0.1	0.1	1.669	1.358	1.132	1.0	0.447	0.097	0.591	1.0
	0.2	1.646	1.349	1.129	1.0	0.413	0.111	0.597	1.0
	0.3	1.599	1.325	1.123	1.0	0.376	0.123	0.626	1.0
	0.4	1.563	1.312	1.119	1.0	0.402	0.127	0.768	1.0
0.5	0.1	0.218	0.217	0.767	1.0	–	–	–	1.0
	0.2	0.218	0.214	0.767	1.0	8.370	8.522	3.911	1.0
	0.3	0.217	0.205	0.769	1.0	9.830	10.335	3.898	1.0
	0.4	0.215	0.167	0.773	1.0	9.417	9.893	3.764	1.0

Note. The dashes mean that the subharmonic resonance does not appear for given parameters of the crack.

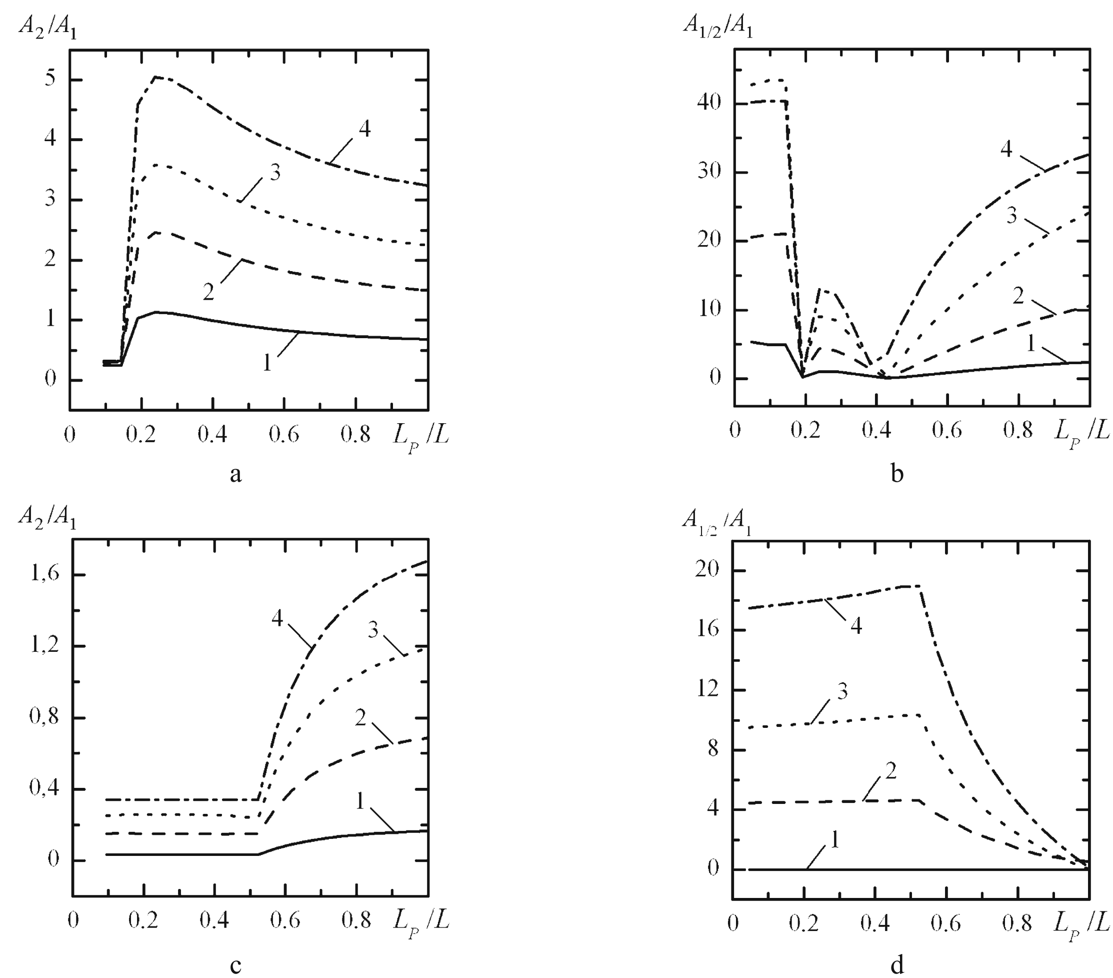

Fig. 6. Dependences of the relative amplitude of the predominant harmonic in the spectrum of displacements for the superharmonic resonance of order 2/1 (a, c) and the subharmonic resonance of order 1/2 (b, d) on the place of application of the driving force for the cantilever rod and the following locations of the crack $L_c/L = 0.1$ (a, b) and 0.5 (c, d): (*1*) $a/h = 0.1$, (*2*) $a/h = 0.2$, (*3*) $a/h = 0.3$, (*4*) $a/h = 0.4$.

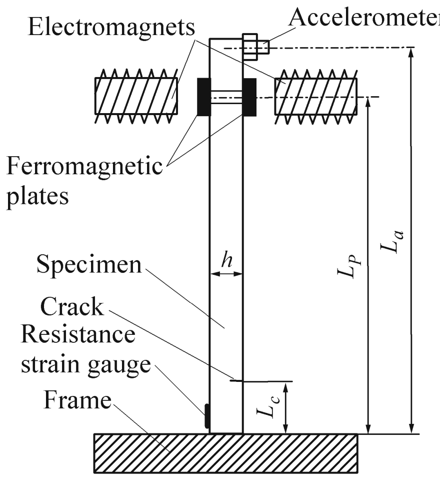

Fig. 7. Schematic diagram of the test installation.

the location of the gauge in the process measurements) for the sub- or superharmonic resonance. The sole variable parameter in these calculations is the location of the crack regarded as already found if the relative changes in nonlinearity measured in the tests and predicted by the model coincide with a prescribed accuracy.

Evaluation of the Parameters of the Crack in the Rod by the Mixed Method. The nonlinearity of vibrations is significant for both nonlinear resonances, unlike the amplitudes of vibrations at these resonances. Therefore, we take the nonlinearity of vibrations as the main characteristic of resonances under consideration in the experimental and analytic investigations.

The experiments were carried out in a KD-1M installation produced at the Pisarenko Institute of Problems of Strength of the National Academy of Sciences of Ukraine. The specimens are rigidly fastened in a massive frame (Fig. 7). Vibrations are excited by a contactless electromagnetic system formed by two electromagnets, a signal generator of, and a power amplifier. The electromagnets interact with the specimen through a special ferromagnetic plate. The measuring system includes a resistance strain gauge, an accelerometer, an analog-to-digital converter, and a computer used to record vibrations and process the accumulated data in a proper way. Fatigue cracks were grown from a sharp notch at a given site of the specimen. The crack depth was measured with the help of an optical microscope with an absolute error of ± 0.1 mm.

We tested specimens made of VT3 titanium alloy and St. 3 steel. The sizes of the specimens and the mechanical characteristics of the materials are presented in Table 2, where L is the specimen length, h and b are, respectively, the height and width of its cross section, E is the modulus of elasticity of the material, ρ is its density, and σ_{-1} is the fatigue limit. The crack depth a, the location of the crack L_c, the locations of the accelerometer (L_{ac}), resistance strain gauge (L_{str}), and the points of application of the driving force (L_{P1} and L_{P2}), and the logarithmic decrement of vibrations δ are presented in Table 3.

As shown in [3, 4, 19, 33], fatigue cracks strongly affect the damping characteristics of the specimens. Since the nonlinear effects depend on the dissipative properties of vibrating systems, special attention is given to the evaluation of the level of damping of the vibrations of cracked specimens. As a characteristic of damping, it is customary to use the logarithmic decrement of vibrations whose dependence on the amplitude of stresses σ_a is found by the method of free damped vibrations.

As follows from Fig. 8, even relatively small fatigue cracks lead to a severalfold increase in the decrement of vibrations for specimens made of the investigated materials. Clearly, the indicated significant changes in damping should be taken into account in analyzing nonlinear effects. Thus, the data of these tests are used to specify the level of damping in the model of a cracked rod.

In Fig. 9, we show the experimental and computed (by the model) ratios A_2/A_1 and $A_{1/2}/A_1$ for the vibration processes of acceleration $(A_2/A_1)_{ac}$ and deformation $(A_{1/2}/A_1)_{str}$ in specimens of VT3 titanium alloy and St. 3 steel in the vicinity of the superharmonic resonance of order 2/1 and subharmonic resonance of order 1/2.

TABLE 2. Sizes of the Specimen and the Mechanical Characteristics of the Materials

Material	L, mm	h, mm	b, mm	E, GPa	ρ, kg/m^3	σ_{-1}, MPa
VT3 titanium alloy	230	20.0	4	110	4480	480
St. 3 steel	230	19.5	4	200	7800	190

TABLE 3. Parameters of the Cracks, Gauges, and Driving Forces in the Tests

Material	Nonlinear resonance	a/h	L_c/L	L_{ac}/L	L_{str}/L	L_{P1}/L	L_{P2}/L	δ
VT3 titanium alloy	Superharmonic	0.11	0.05	0.97	–	0.78	0.42	0.009
	Subharmonic	0.24	0.05	–	0.06	0.79	0.55	0.003
St. 3 steel	Superharmonic	0.26	0.06	0.97	–	0.80	0.43	0.020
	Subharmonic	0.41	0.06	0.97	–	0.79	0.43	0.020

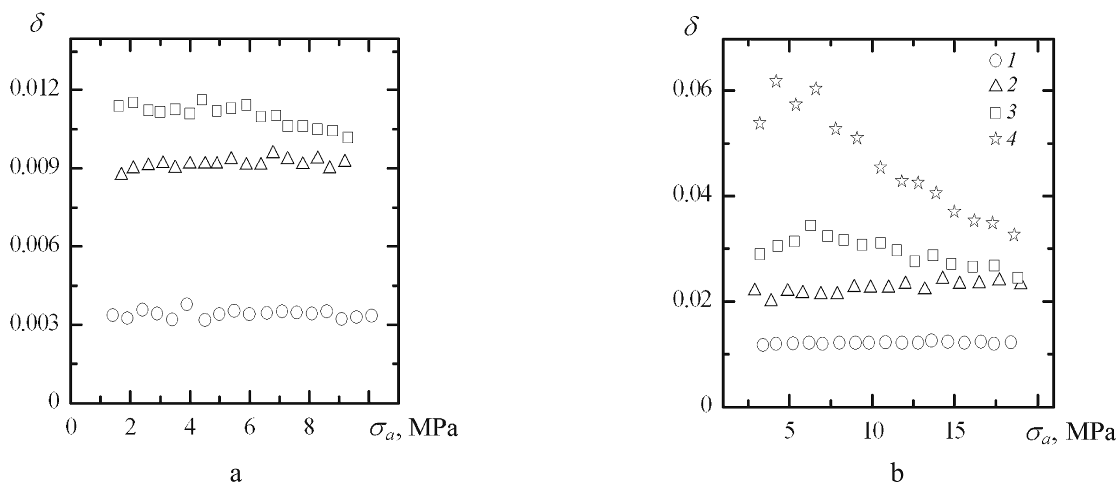

Fig. 8. Amplitude dependences of the logarithmic decrement of vibrations for specimens made of VT3 titanium alloy (a) and St. 3 steel (b) and different crack depths: (*1*) $a/h=0$, (*2*) $a/h=0.1$, (*3*) $a/h=0.2$, (*4*) $a/h=0.3$.

For each specimen, the resonance curves were obtained for two points of application of the driving force. The first point was located near the end of the rod and the second point was (approximately) placed in the middle of the rod (Table 3). In the tests, the point of application of the driving force was changed by means of displacements of the electromagnets and ferromagnetic plates along the specimen (Fig. 7). The tests were carried out for relatively low amplitudes of stresses (σ_a = 5–15 MPa) in order to avoid any noticeable crack growth and damping in the course of measurements.

As follows from Fig. 9, the resonance curves obtained by using the model are in good qualitative agreement with the experimental curves. Significant quantitative deviations of the numerical results from the experimental data observed for the specimens of St. 3 steel are caused by the so-called pseudosuperharmonic resonance [3], which can hardly be avoided for relatively large cracks. In addition, the model enables us to apply forces only at its nodes and, hence, the conditions of loading of the specimens are reproduced with a certain error.

The serviceability of the mixed method for the evaluation of the parameters of the crack was analyzed in the following way: The experimental data presented above were used to determine the relative variations of the maximum nonlinearities of acceleration and deformation at certain points of the specimens for their sub- and superharmonic resonances caused by the application of the force at different points (L_{P1} and L_{P2}). The maximum of nonlinearity was found directly from the experimental resonance curves (see Fig. 9) and their values are presented in Table 4.

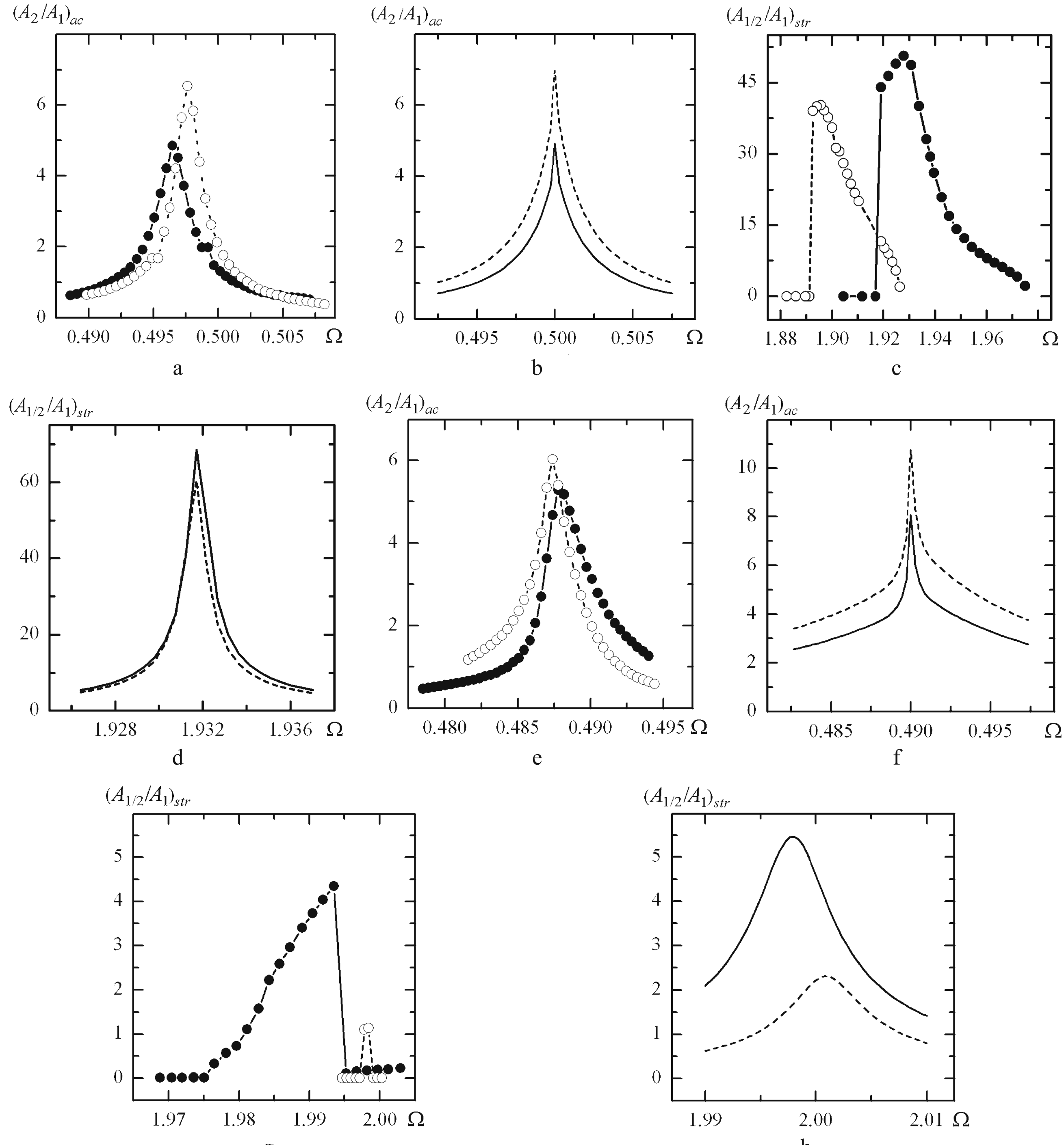

Fig. 9. Superharmonic resonance of order 2/1 (a, b, e, f) and subharmonic resonance of order 1/2 (c, d, g, h) for specimens of VT3 titanium alloy (a–d) and St. 3 steel (e–h). The solid lines correspond to L_{P1} and the dashed lines correspond to L_{P2}; Ω is the relative frequency of the driving force. The experimental results are marked by the dark (blank) symbols.

The model of the rod was also used for the evaluation of the nonlinearity of its vibrations corresponding to the nonlinear resonances. The location of the driving force, gauges, and crack was reproduced in the model with an error up to 5%. The location of the crack was the sole variable parameter in our calculations (the relative crack depth

TABLE 4. Experimental Data and Results of Prediction of the Parameters of the Crack

Material	Nonlinear resonance	Experimental data		L_c/L	a/h
		$\frac{(A_{2/1}/A_1)_{L_{P1}}}{(A_{2/1}/A_1)_{L_{P2}}}$	$\frac{(A_{1/2}/A_1)_{L_{P1}}}{(A_{1/2}/A_1)_{L_{P2}}}$		
VT3 titanium alloy	Superharmonic (acceleration)	0.74	–	$\frac{0.05}{0.07}$	$\frac{0.11}{0.11}$
	Subharmonic (deformation)	–	1.26	$\frac{0.05}{0.05}$	$\frac{0.24}{0.20}$
St. 3 steel	Superharmonic (acceleration)	0.88	–	$\frac{0.06}{0.13}$	$\frac{0.26}{0.19}$
	Subharmonic (deformation)	–	3.82	$\frac{0.06}{0.05}$	$\frac{0.41}{0.36}$

Note: The experimental and predicted data are presented in the numerators and denominators, respectively.

a/h was set equal to 0.2). Hence, this parameter was changed to guarantee the coincidence of the computed ratio of the maximum nonlinearity with its measured value with an accuracy of up to 1%. As follows from Table 4, this procedure of calculations enables us to find the location of the crack in the specimens fairly exactly on the basis of the results of analysis of both subharmonic and superharmonic vibrations. Thus, the mixed method can be used to determine the location of the crack with minimum possible number of measurements and amount of calculations.

On finding the location of the crack, we can estimate its sizes by using the model. For this purpose, one can use the procedure of numerical analysis described above with the only difference that, in this case, the location of the crack found in the previous stage of calculations is fixed and its size is varied until the measured and computed absolute values of the nonlinearity of vibrations for the sub- and superharmonic resonances coincide with the required accuracy. As follows from Table 4, in almost all cases, the crack sizes predicted by the model are smaller than the actual sizes. This can be explained by the underestimation of the level of damping in the model as compared with the actual level of damping of vibrations at the nonlinear resonances (note that it is quite difficult to evaluate this parameter experimentally). Nevertheless, the accuracy of evaluation of the parameters of cracks in the investigated specimens is acceptable for various practical applications.

Discussion of the Results. The nonlinearity of vibrations in the vicinity of sub- and superharmonic resonances of rods with various boundary conditions is sensitive to the presence of a closing crack. The sensitivity of this characteristic is higher than the sensitivity of natural frequencies or the modes of vibrations by one–three orders of magnitude. As shown both experimentally and numerically, the indicated sensitivity strongly depends on the crack sizes and location, the level of damping in the system, and the place of application of the driving force.

The excitation of vibrations of the rod by a concentrated force applied to different points of the rod leads to significant changes in the degree of nonlinearity of vibrations at the sub- and superharmonic resonances. The presence of sharp changes in nonlinearity enables one to detect the location of the crack. The degree of nonlinearity of vibrations directly (but nonlinearly) depends on the crack sizes. Hence, by measuring the nonlinearity of vibrations at the sub- and/or superharmonic resonances for the known location of the crack, we can determine its sizes according to the degree of nonlinearity of vibrations by using the model of the rod.

The mixed method for the evaluation of the parameters of the crack based on the combined application of the experimental data and the model of the rod enables us to substantially decrease the number of measurements required for the solution of the inverse problem of vibrodiagnostics. To realize this method, it is necessary to excite any nonlinear resonance of the rod by applying the force to two points of the rod (successively) and, in each case, determine the nonlinearity of vibrations of a given point of the rod. These data are sufficient to find the location of the crack with the required accuracy by using the model.

The tests of the specimens performed within the framework of the present study enable us to discover two specific features of the subharmonic resonance not detected by the model. The first feature is connected with the fact that the manifestation of the indicated resonance depends on the power of excitation of vibrations. If this power is insufficient, then the subharmonic resonance does not appear even in the presence of relatively large cracks. The power required for the excitation of subharmonic vibrations is much higher than the power required for the excitation of superharmonic vibrations. These conclusions agree with the results of investigations presented in [11, 25, 31].

The second feature is connected with the fact that the subharmonic mode of vibrations is extremely unstable: it may appear or disappear absolutely unexpectedly (Fig. 9c and g).

Although the model fails to reveal the indicated features of the subharmonic resonance and, hence, its subsequent improvement is necessary, it can be used for the investigation of nonlinear effects (with certain restrictions).

The proposed vibration methods for the diagnostics of defects guarantee the possibility of realization of the first (detection of detects) and second (determination of their location) levels of evaluation of damage according to the classification proposed by Rytter [34]. The evaluation of the sizes of defects (third level) is a more complicated problem.

Conclusions. On the examples of the investigated specimens, it is shown that the mixed method enables one to estimate the crack sizes but, most likely, these estimates contain unpredictable errors. The problem is connected with the fact that the correctness of evaluation of the crack sizes directly depends on the accuracy of modeling of the actual restoring force. In our model, the restoring force is regarded as piecewise linear. Moreover, it is assumed that the changes in the state of the crack (open or closed) occur only in the neutral position of the rod. Clearly, this representation of the elastic properties of the body in the process of opening and closing of the crack is idealized. In reality, these properties are determined by a series of factors affecting the conditions of contact between the crack lips, namely,

(1) compressive residual stresses in the vicinity of the crack tip responsible for the effect of crack closure;

(2) crushing (wear) of roughnesses caused by the contact of crack lips in the process of cyclic deformation of the structure;

(3) buckling of the crack surfaces as a result of the oxidation of the metal;

(4) partial or complete crack opening displacements caused by high plastic strains, e.g., as a result of overloading of the structure.

Even if it is possible to estimate the influence of each of these factors on the stiffness of the body in the process of opening and closing of the crack, it would be extremely difficult to take into account this influence in the model. In addition, the accuracy of evaluation of the crack sizes also depends on the accuracy of evaluation of the damping properties of the cracked rod at the nonlinear resonances. If the influence of the indicated factors is neglected, then the numerical analysis performed according to the model with regard for the nonlinear effects gives understated values of the crack size as compared with the actual values.

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