Strength of Materials, Vol. 42, No. 2, 2010

INFLUENCE OF THE PARAMETERS OF A LOCAL DEFECT IN A REGULAR SYSTEM ON THE RANGE OF EIGENFREQUENCIES OF VIBRATIONS AND THE LEVEL OF VIBRATION STRESSES IN ELEMENTS OF THE SAME TYPE

I. G. Tokar' and A. P. Zinkovskii

UDC 534.1;539.433;620.179.12:62-26

We describe the results of our experimental investigations aimed at the analysis of the influence of the parameters of defects in the form of open edge cracks (depth and location) on the formation of the spectrum of eigenfrequencies and the level of vibration stresses in similar elements of a regular system. It is shown that our results are in good agreement with the results of computations.

Keywords: regular system, subsystem, detuning of vibration frequencies, defect, vibration stresses.

Introduction. Numerous units of state-of-the-art machines are characterized by the presence of constructive regularity. First of all, this is true for the blade rows of turbine wheels characterized, as a rule, by s structural rotational symmetry (i.e., by a special type of regularity). However, due to the action of various technological factors, these systems are produced with inevitable deviations from the identity and periodicity of location of elements of the same type, i.e., blades in our case. These deviations are, in turn, responsible for the deviations in the elastic, inertial, and dissipative characteristics whose integral influence leads to the detuning of frequencies and damping characteristics of vibrations of similar elements in regular systems. The analysis of the results of theoretical and experimental investigations of the influence of the detuning of blade frequencies on the formation of vibrations of the blade row shows that the indicated detuning affects the fundamental properties of the spectra of natural vibrations and results in the appearance of the spread of amplitudes of the resonance vibration stresses [1–3].

Under the operating conditions, one or several similar elements of the investigated system may have defects in the form of fatigue cracks, nicks, etc., which, in fact, also lead to the violation of regularity. The analysis of the scientific and engineering literature shows that the available results of investigations deal, for the most part, with the study of the influence of defects on the regularities of vibrations of individual structural elements (e.g., rods, beams, shafts, plates, and blades) [4–7]. As to the best of our knowledge, the number of available publications devoted to the investigation of the influence of defects of this sort on the formation of vibrations of regular systems in the form of packets or blade rows of turbine wheel is fairly small [8–10]. Moreover, the works [8, 9] deal solely with numerical calculations. Some results of experimental investigations aimed at the evaluation of the influence of defects in the form open edge cracks and in the form of grooves on the frequencies and shapes of vibrations in a simple model of a regular system formed by two similar rodlike elements are presented in [10]. However, they were obtained for a broad range of depths of the groove and its fixed location along the length of the rod. Despite the small amount of the available data of investigations, their results reveal presence of a strong influence of defects on the formation of vibrations in regular systems. Therefore, the aim of the present work is to establish the regularities of the influence of two parameters of local defects (depth and location of the open crack) in the regular system on the range of eigenfrequencies of vibrations and the level of vibration stresses in elements of the same type.

Object of Investigations and Basic Concepts of the Experimental Procedure. By analogy with [10, 11], as the object of investigations, we choose specimens in the form of a tuning fork with prismatic rods regarded as a simple example of regular systems formed by two similar elements (subsystems) in modeling the open edge cracks

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Pisarenko Institute of Problems of Strength, National Academy of Sciences of Ukraine, Kiev, Ukraine.
Translated from Problemy Prochnosti, No. 2, pp. 55 – 64, March – April, 2010. Original article submitted September 18, 2008.

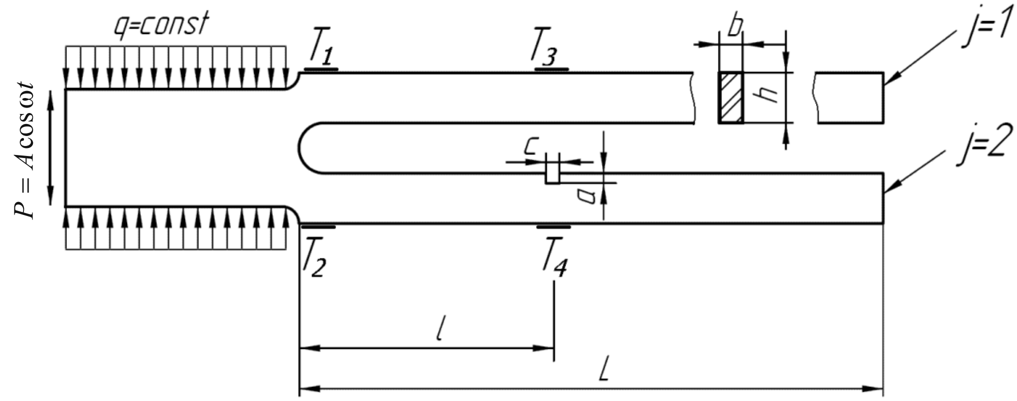


Fig. 1. Schematic diagram of fastening used for the kinematic excitation of vibrations of the specimen in the form of a tuning fork and its preparation by resistance strain gauges T_n $(n = 1, \ldots, 4)$.

by the corresponding grooves. The design of the specimen (Fig. 1) and methodical approaches to its testing are similar to those described in [10]. The specimens were made of D16 aluminum alloy in the as-delivered condition. Its modulus of elasticity is equal to $E = 0.71 \cdot 10^5$ MPa and mass density to $\rho = 2.8 \cdot 10^3$ kg/m^3 [12]. The length of the working part of the specimens $L = 175$ mm and the height h and width b of their cross section constituted 15 and 8 mm, respectively.

In the intact state, the specimen is strictly regular, i.e., the defects are absent and the eigenfrequencies of vibrations of the rods in the isolated state are identical.

A defect in the form of a rectangular groove of width $c = 1$ mm and depth a simulating an open edge crack was made in one rod of each of four tested specimens across its working part at a distance l from the root section. The depth of the groove a was measured with the help of an ICh-10 clock-type indicator mounted in a special device and varied from 0 to 8.3 mm. This corresponds to the following range of its relative values: $\bar{a} = a/h = 0$–0.56.

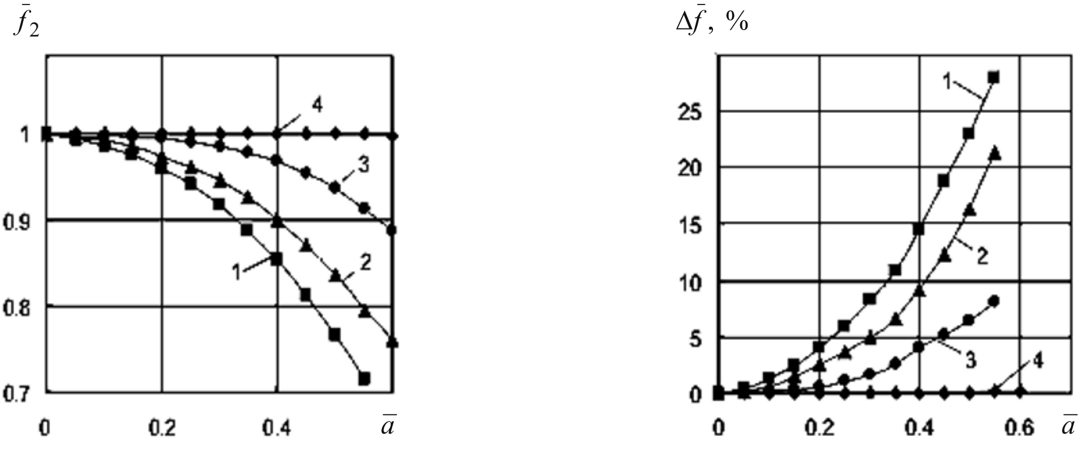
In what follows, the intact rod is marked by the subscript $j = 1$ and the damaged rod by the subscript $j = 2$.

To find the level of strains in the vicinity of the defect in one of the rods, the third specimen was additionally prepared at a relative distance $\bar{l} = l/L = 0.5$ from the restraint by resistance strain gauges T_3 on the working surface of the intact rod and T_4 on the working surface of the damaged rod opposite to the location of the groove.

Experimental Results. The experimental investigations of the chosen specimens aimed at finding the spectrum of eigenfrequencies of vibrations and the level of vibration stresses were carried out by changing both the depth of the groove and its location.

Prior to testing, according to the statement of the problem, the specimens were subjected to the adjustment of strict regularity, i.e., to guaranteeing the equality of the eigenfrequencies of vibrations f_j $(j = 1, 2)$ of the rods in the isolated state. This was realized as follows: It is well known that, in view of the fundamental properties of vibrations in the analyzed system, the in-phase mode of vibrations of the rods is the sole possible mode of vibrations in the case of strict regularity of the system and kinematic excitations. Moreover, the resonance frequency of vibrations in this mode f_0 coincides with the eigenfrequencies of the rods, i.e., the following equality is true: $f_0 = f_1 = f_2$. To attain the indicated state of the vibrating system in the course of the tests, we modified the elastic and inertial properties of the rods to guarantee the validity of the presented equality, which was confirmed by the indications of a frequency meter. Furthermore, the exact adjustment of regularity of the system was checked by analyzing the amplitude of stresses σ_0 in the root section of each rod deformed according to the first bending mode. The frequency of vibrations f_0 and the level of stresses σ_0 were accepted as basic and then used for the comparison of the corresponding characteristics of the system obtained as a result of violation of its regularity by the damaged second rod $(j = 2)$.

Prior to the analysis of the characteristics of vibrations of the investigated system in the presence of defects, we consider their influence on the eigenfrequency of vibrations of an isolated rod. To guarantee the required isolation of the rods operating as parts of a single system, we attach a weight to one of these rods whose mass guarantees a



a b

Fig. 2. Dependences of the relative eigenfrequency of the isolated damaged rod (a) and the detuning of frequencies of the rods (b) on the relative depth of the groove for different relative locations $\bar{l}$ of the groove along the length of the rod: 0.08 (*1*), 0.25 (*2*), 0.50 (*3*), and 0.786 (*4*).

decrease in the frequency of vibrations of the rod by two orders of magnitude. As a result, the vibrations of the rods in the course of these tests become, in fact, uncoupled.

In Fig. 2, we present the dependences of the relative eigenfrequency of vibrations $\bar{f}_2 = f_2/f_0$ of the damaged rod and the frequency detuning

$$\Delta\bar{f} = \left|\frac{f_1 - f_2}{f_0}\right| \cdot 100\%$$

of the rods on the relative depth of the groove $\bar{a}$ for various values of the parameter $\bar{l}$ characterizing its location along the length of the rod. We see that the indicated dependences are smooth and continuous. Moreover, for different locations of the groove and the same values of increment of its relative depth, we get different changes in the relative frequency of vibrations $\bar{f}_2$ (Fig. 2a) whose maximum is attained for $\bar{l} = 0.08$ and minimum for $\bar{l} = 0.786$. This means that the detuning of vibration frequencies $\Delta\bar{f}$ of the rods also depends on the indicated parameters of the defect, which is important for the analysis of the regularities of formation of the resonance vibrations of detuned regular systems caused by the presence of defects in their elements of the same type.

In the course of the tests, for the chosen parameters of the defects, we determined the amplitude–frequency characteristics of the specimens (Fig. 3). Their analysis enables us to unambiguously establish the regularities of the influence of defects on the formation of vibrations in the analyzed system. Thus, we confirm the well-known fact [10, 11] that, in the case of detuning of one of two rods, either due to the tolerances accepted in the process of its manufacturing or as a result of its in-service damage, two modes close to the in-phase and antiphase vibrations appear in the spectrum of vibrations of the system, despite their in-phase excitation.

Consider the influence of the chosen parameters of damage on the characteristics of resonance vibrations of the rods.

As a result of processing of the amplitude-frequency characteristics obtained according to the data of strain measurements performed by using the resistance strain gauges T_1 and T_2, we determined the dependences of the relative amplitudes of resonance stresses $\bar{\sigma}_j = \sigma_j/\sigma_0$ corresponding to the excited in-phase ($\bar{\sigma}_j^{in}$) and antiphase ($\bar{\sigma}_j^{anti}$) modes of vibrations in the system on the relative depth of the groove $\bar{a}$ for various locations of the groove along the length of the rod (Fig. 4). It is easy to see that, in the case of in-phase vibrations of the system (Fig. 4a), the indicated dependences for the damaged rod are smooth and monotonically increasing functions independently of the location of the groove along the length of the rod. For the intact rod, the corresponding dependences are decreasing.

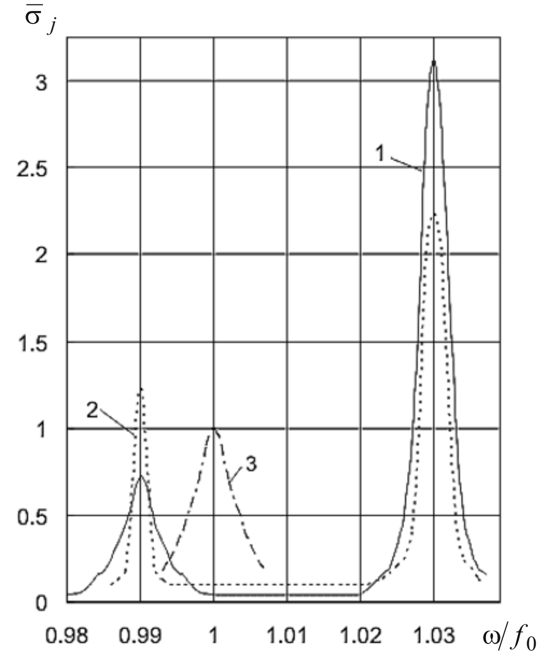
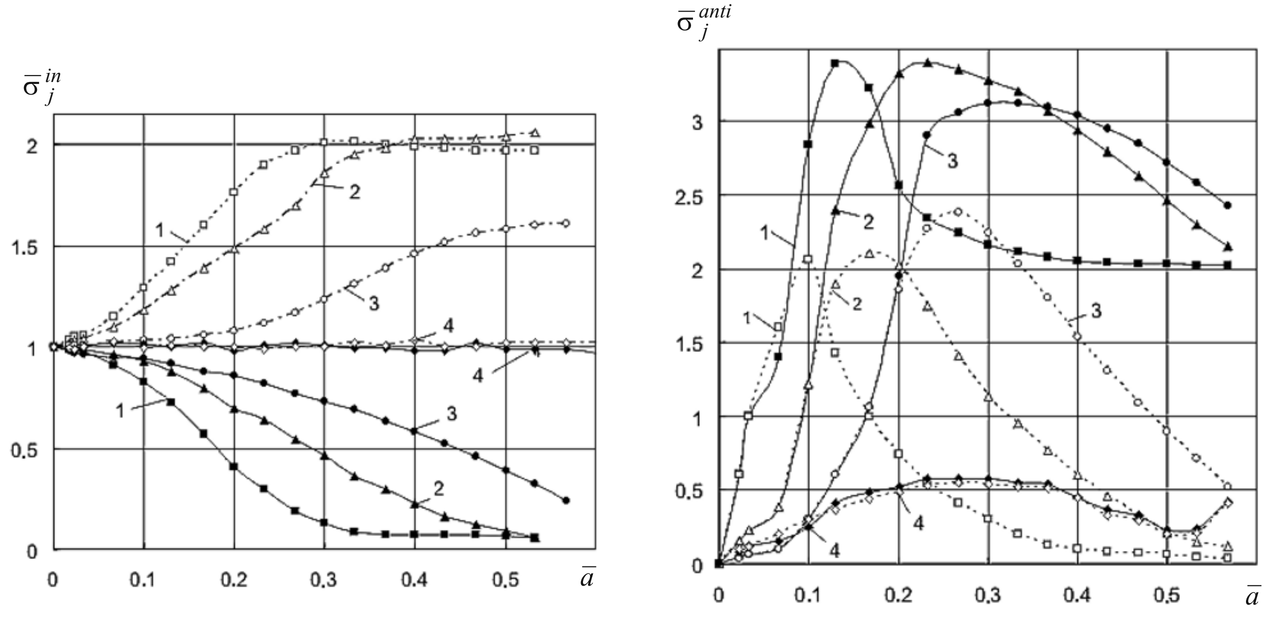


Fig. 3. Amplitude-frequency characteristics of the investigated regular system: (*1*) intact rod; (*2*) rod damaged in the middle of working length ($\bar{l} = 0.5$) with a relative depth of the groove $\bar{a} = 0.3$ and the corresponding detuning $\Delta\bar{f} = 1.65\%$; (*3*) in the absence of defects, $\bar{a} = 0$ and $\Delta\bar{f} = 0$.



a b

Fig. 4. Dependences of the relative amplitudes of resonance stresses in the root sections of the intact (solid lines) and damaged (dashed lines) rods of the specimen for their in-phase (a) and antiphase (b) vibrations on the relative depth of the groove for its different relative locations $\bar{l}$ along the length of the rod: 0.08 (*1*), 0.25 (*2*), 0.50 (*3*), and 0.786 (*4*).

However, the degrees of increase and decrease in the resonance stresses for a given depth of the groove strongly depend on its location.

TABLE 1. Experimental Data Characterizing the Maximum Level of Vibration Stresses in the Rods of the Damaged Specimen

$\bar{l}$	Intact rod			Damaged rod		
	$(\bar{\sigma}_1^{anti})_{\max}$	$\bar{a}$	$\Delta\bar{f}$, %	$(\bar{\sigma}_2^{anti})_{\max}$	$\bar{a}$	$\Delta\bar{f}$, %
0.08	3.39	0.13	1.41	2.06	0.10	0.580
0.25	3.40	0.23	2.57	2.11	0.17	1.070
0.50	3.12	0.30	1.65	2.38	0.27	0.845

The analyzed dependences of the amplitudes of resonance stresses in the rods have an absolutely different character for their antiphase vibrations (Fig. 4b). In this case, both for the damaged and intact rods, we observe the presence of an extremum whose location is determined by the parameters of the defect.

The analysis of the accumulated results enables us to make an unambiguous conclusion that the defect playing the role of a source of the operating violation of regularity of the system leads to the formation of a spread of the levels of vibration stresses in the rods depending on the depth and location of the groove.

In view of the fact that the maximum level of vibration stresses in the analyzed vibrating system is attained for the antiphase mode of vibrations of the rods, in Table 1, we present the maximum values of the relative resonance amplitudes of vibration stresses $\bar{\sigma}_j^{anti}$, relative depth of the groove $\bar{a}$, and detuning of the frequencies of the rods $\Delta\bar{f}$ characterizing the system. These data illustrate the above-mentioned influence of the parameters of defects on the level of vibration stresses in the system. We see that the intact rod suffers the action of higher stresses because its eigenfrequency of vibrations is closer to the eigenfrequency of the antiphase mode of vibrations of the system in agreement with the fundamental regularities of oscillations in detuned systems of two elastically connected pendulums [13]. Thus, the maximum amplitudes of the resonance stresses are higher than for the damaged rod by 25–40% independently of the location of the groove relative to the site of restraint of the specimen. However, as the distance between the groove and the restraint increases, the maximum amplitudes of the resonance stresses shift to the side of higher values of the relative depth of the groove $\bar{a}$ and, hence, are attained for higher degrees of detuning of the frequencies of vibrations of the rods. In addition, the maxima of the levels of vibration stresses in the rods do not coincide. Thus, the maximum amplitude $(\bar{\sigma}_1^{anti})_{\max}$ of resonance stresses in the intact rod is recorded for higher values of $\bar{a}$ than the corresponding maximum $(\bar{\sigma}_2^{anti})_{\max}$ in the damaged rod.

The results of our analysis presented above are based on the data of strain measurements performed by using the resistance strain gauges T_1 and T_2 (Fig. 1) and, hence, the maximum amplitudes of resonance stresses $(\bar{\sigma}_1^{anti})_{\max}$ and $(\bar{\sigma}_2^{anti})_{\max}$ correspond to the maximum amplitudes in the root sections of the corresponding rods. This fact does not enable us to decide whether these data determine the maximum relative level of vibration stresses in the specimen in the presence of a local defect in one of the rods.

To solve this problem, we used the data of strain measurements carried out by using the resistance strain gauges T_3 and T_4 to establish the dependences of the relative amplitudes of resonance stresses in the rods for the excited in-phase and antiphase modes of vibrations of the system on the relative depth of the groove similar to the dependences presented in Fig. 4. In Fig. 5, the corresponding dependences are presented for the damaged rod. They are plotted both for its root section and for the section corresponding to the location of the groove. The comparison of these dependences demonstrates that their character is independent of the site of recording of the resonance stresses but their level increases at the site of location of the defect. Thus, the maximum level of the relative resonance amplitudes of stresses for the antiphase mode of vibrations of the analyzed rod is attained near the groove and is equal to 3.59 for the relative depth $\bar{a} = 0.23$. For the in-phase mode of vibrations, the corresponding maximum level is equal to 1.71. The levels of these stresses in the root section of the rod are equal to 2.39 and 1.14, respectively, for the same depth of the groove.

Thus, the results of our experimental investigations reveal the existence of a strong influence of the parameters of the model of an open edge crack on the level of vibration stresses in the regular system.

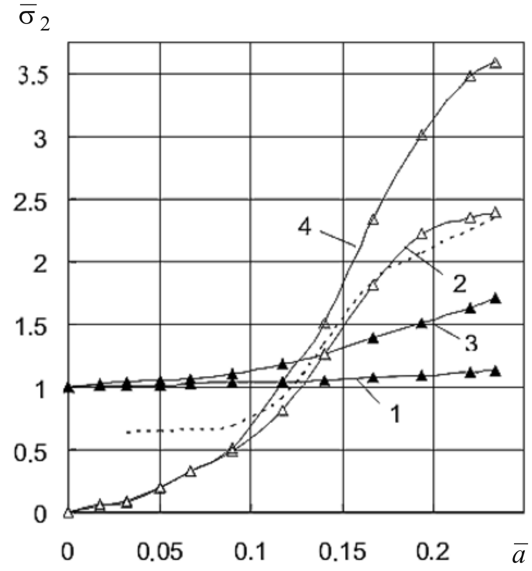


Fig. 5. Dependences of the relative amplitudes of resonance stresses in the root section (*1* and *2*) and at the site of the groove (*3* and *4*) for the in-phase (▲) and antiphase (△) vibrations on the relative depth of the groove for $\bar{l} = 0.5$. (The dashed line marks the computed dependence [14] corresponding to the experimental curve *2*).

Comparison of the Experimental Data with Numerical Results. We compare the results of our experimental investigations aimed at evaluation of the influence of the parameters of the model of an open edge crack on the characteristics of vibrations of the analyzed regular system with the data of numerical experiments carried out at the Podgornyi Institute for the Problems of Machine Building of the National Academy of Sciences of Ukraine [14].

The numerical analyses were carried out in the linear statement (by analogy with our experiments) within the framework of a finite-element model of the specimen in the form of a tuning fork (Fig. 1). Both the depth of the groove and the site of its location along the length of the second rod ($j = 2$) were variable.

We now analyze the results of numerical and experimental evaluation of the frequency characteristics of the investigated vibrating system (Table 2). The comparison of the absolute values of the eigenfrequencies of the excited forms of vibrations of the specimen within the chosen range of depths of the groove shows that their disagreement does not exceed 4% for the in-phase vibrations and 5% for the antiphase vibrations. Moreover, their relative values practically coincide because they exclude the influence of fastening of the specimen, the errors of recording instruments, and other factors.

The dependences of the relative amplitudes of the resonance stresses computed within the framework of the finite-element model of the specimen in the form of a tuning fork for the excited forms of vibrations on the relative depth of the groove for the same parameters of the defect as in the tests reveal their good agreement with the experimental data. This is confirmed by a dependence of this sort presented as an example in Fig. 5 for the damaged rod and antiphase vibrations in the case where the groove is located at a relative distance $\bar{l} = 0.5$ from the restraint.

Thus, the presented comparison of the experimental data with the accumulated numerical results reveals their good agreement. This fact enables us to use these results for the analysis of the regularities of vibrations of regular actual systems, e.g., of the packets of coupled blades in the presence of defects in the form of open edge cracks.

CONCLUSIONS

1. The results of the performed experimental investigations of vibrations in a simple regular system with specimen in the form of a tuning fork confirm the possibility of excitation of two modes of vibrations (close to the in-phase and antiphase vibrations) in the presence of an open edge crack.

TABLE 2. Numerical (numerators) and Experimental (denominators) Values of the Eigenfrequencies of Vibrations of the Specimen

$\bar{l}$	Frequency characteristics of the specimen	Frequency characteristics of the specimen for various values of $\bar{a}$					
		0	0.067	0.133	0.267	0.400	0.533
0.08	f_{in}, Hz	$\frac{354.50}{344.88}$	$\frac{353.35}{343.58}$	$\frac{349.95}{339.14}$	$\frac{332.64}{326.43}$	$\frac{300.50}{299.96}$	$\frac{254.54}{257.12}$
	$\bar{f}_{in}$	1.03	1.03	1.03	1.02	1.00	0.99
	f_{anti}, Hz	$\frac{373.30}{357.20}$	$\frac{372.16}{356.63}$	$\frac{369.86}{354.10}$	$\frac{366.10}{352.36}$	$\frac{364.62}{351.51}$	$\frac{364.04}{350.80}$
	$\bar{f}_{anti}$	1.04	1.04	1.04	1.04	1.04	1.04
0.25	f_{in}, Hz	$\frac{354.50}{346.15}$	$\frac{353.89}{344.59}$	$\frac{352.19}{343.87}$	$\frac{343.39}{335.39}$	$\frac{324.74}{318.74}$	$\frac{290.57}{283.29}$
	$\bar{f}_{in}$	1.02	1.03	1.02	1.02	1.02	1.03
	f_{anti}, Hz	$\frac{373.30}{357.45}$	$\frac{372.67}{357.62}$	$\frac{371.19}{356.62}$	$\frac{367.63}{353.69}$	$\frac{365.42}{352.45}$	$\frac{364.38}{351.94}$
	$\bar{f}_{anti}$	1.04	1.04	1.04	1.04	1.03	1.03
0.50	f_{in}, Hz	$\frac{354.50}{341.58}$	$\frac{354.38}{341.09}$	$\frac{353.98}{340.48}$	$\frac{352.19}{339.02}$	$\frac{347.77}{334.16}$	$\frac{336.98}{323.06}$
	$\bar{f}_{in}$	1.04	1.04	1.04	1.04	1.04	1.04
	f_{anti}, Hz	$\frac{373.30}{355.53}$	$\frac{373.18}{355.17}$	$\frac{372.75}{352.61}$	$\frac{371.15}{351.83}$	$\frac{368.82}{349.81}$	$\frac{366.44}{347.80}$
	$\bar{f}_{anti}$	1.05	1.05	1.05	1.05	1.05	1.05

2. We established the regularities of the influence of the parameters of defects, such as the depth of the groove and its location along the length of the rod (as one of similar elements of the regular system), on its frequency characteristics and the spread of levels of vibration stresses in the rods. The reliability of the accumulated experimental and numerical results is confirmed by their good agreement.

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