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CALCULATION OF THE RELIABILITY OF STEEL UNDERGROUND PIPELINES

S. F. Pichugin and A. V. Makhin'ko UDC 624.014.042:519.2

We present basic principles of reliability calculations for steel underground pipelines resting on stochastic elastic foundation that based on the strength and stiffness criteria using the absolute maxima of arbitrary functions. Specific numerical examples illustrating a general calculation procedure are presented.

Keywords: underground pipeline, pipeline reliability, stochastic elastic foundation, arbitrary force function.

Introduction. Among the complexes of facilities that are an integral part of enterprises in mechanical engineering, metallurgical, coal, oil, chemical and other branches of industry, hydroelectric–and-thermoelectric power plants, amelioration systems, agriculture and municipal economies, we will distinguish underground pipes of various purposes. These are used in water pressure head, sewer, oil pipeline, central heating, drainage and other systems. The pipeline strength calculation is usually made under the assumption that its properties and those of the foundation it rests upon are deterministic. However, this statement is not always valid. Firstly, the properties of foundation soil depend on many factors that cannot be directly accounted for and therefore are of a random character. Secondly, the coefficients of rigidity and strength can vary arbitrarily both when going from one point of foundation to another and when going from structure to another. Thirdly, external loads, the material properties and geometry of pipelines depend on a variety of poor-controlled and intricately interacting causes that are also of a varying random nature. Taking into account the above-mentioned notations, the introduction of probabilistic methods into the practice of calculations of underground pipelines is urgent under the state-of-the art conditions of construction.

Stresses and strains in steel underground pipelines laid in statistically heterogeneous soils are arbitrary functions of the axial coordinates. This fact should be taken into account in the strength and rigidity analysis of pipelines, and their characteristics should be specified in a such way that the probability of attaining the first and second limiting states be sufficiently low.

Bolotin was the first who applied the probabilistic approach to the strength calculation of underground pipelines by presenting it in his works [1, 2]. However, in these fundamental works, more attention was given to the solution of the differential equation for the pipeline deflection curve with arbitrary parameters rather than to direct assessment of reliability of the pipeline itself. Moreover, due to the complexity and specific nature of the mathematical apparatus used, the obtained results were inaccessible for design engineers. In view of the above, and taking into account the ideas and results obtained by Bolotin, this paper presents the procedure for just the reliability calculation of steel underground pipelines whose basis is the idea of rational tradeoff between the accuracy and the ease of performing probabilistic calculations.

Problem Formulation. As in [1, 2], we will consider a steel pipeline with the outer diameter D and wall thickness δ laid in statistically heterogeneous soils. The bending stiffness of the pipeline will be denoted by EI, the pipeline stiffness in compression or in tension will be denoted by EA, the effective width by b_{eff}, and the foundation stiffness coefficient by $c = \tilde{c}(x)$. The coordinate system $0xy$ is such as is shown in Fig. 1. According to [1, 2], we suppose that that the foundation prepared for lying the pipeline is uneven. The equation for the curve describing this initial unevenness is as follows: $u = \tilde{u}(x)$ (Fig. 1a). Moreover, we assume that the pipeline axis has initial small distortions in the plane $0xy$. These distortions are characterized by the function $w = \tilde{w}(x)$ chosen such that its mean

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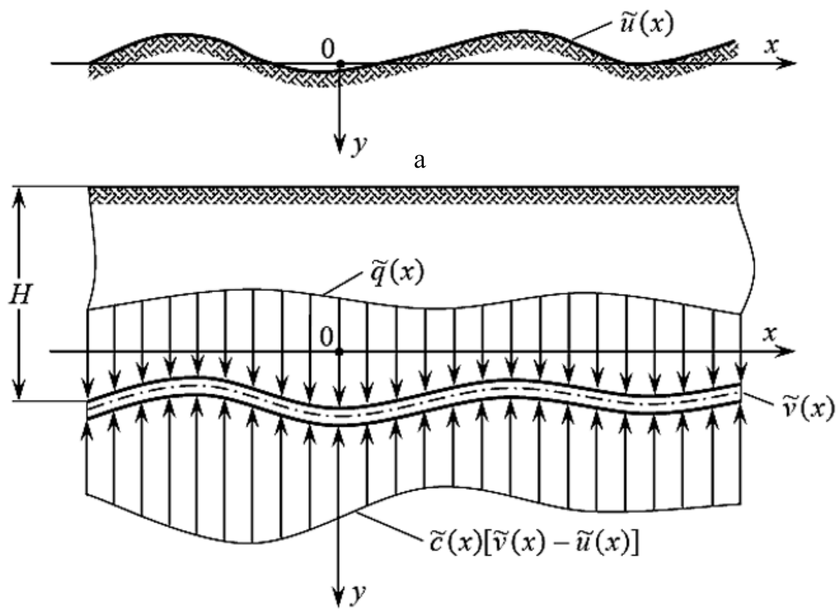

Fig. 1. Calculation scheme for the pipeline on a continuous elastic foundation with arbitrary characteristics.

value is identical with zero at a sufficiently long length. The intensity of external forces acting on the pipeline will be denoted by $\tilde{q}(x)$. In this case, in contrast to works [1, 2] wherein only the loads from the upper layers of the soil $\tilde{q}_1(x)$ were considered, we will take into account the loads from the proper weight of the pipeline $\tilde{q}_2$, hydrostatic pressure of fluid completely filling the free-flow pipeline $\tilde{q}_3$, and the temperature load $\tilde{q}_4$:

$$\tilde{q}(x) = \tilde{q}_1(x) + \tilde{q}_2 + \tilde{q}_3 + \tilde{q}_4. \tag{1}$$

Taking into account the axial force N, for the differential equation of the elastic axis of a pipeline [1] we have:

$$EI\frac{d^4 v}{dx^4} + N\frac{d^2 v}{dx^2} + cv = \tilde{q}(x) + \tilde{c}(x)\tilde{u}(x) + EI\frac{d^4 w}{dx^4}. \tag{2}$$

In the solution of this differential equation [1, 2], the following assumption is introduced that has been repeatedly verified experimentally: the function of initial distortions $w = \tilde{w}(x)$, the function of irregularities $u = \tilde{u}(x)$, the external load $\tilde{q}(x)$, and the coefficient of the foundation stiffness $c = \tilde{c}(x)$ are stationary uniform ergodic arbitrary functions of the coordinate x. Mean values of the load and the coefficient of foundation soil stiffness are denoted by $\bar{q}$ and $\bar{c}$, respectively. The mean value of the function $\tilde{u}(x)$ is taken to be equal to zero. Thus, given the stochastic boundary problem with respect to the function $\tilde{v}(x)$ together with the boundedness and infinity conditions under these assumptions and entering the heterogeneity function in soil conditions $\tilde{r}(x)$, we derive the formula for the spectral density $S_r(\cdot)$ of the function $\tilde{r}(x)$:

$$S_r(\omega) = \frac{\alpha\beta_0^2\bar{q}^2}{\pi}\left[\frac{1}{(\omega-\theta)^2+\alpha^2} + \frac{1}{(\omega+\theta)^2+\alpha^2}\right], \tag{3}$$

where α and θ are the parameters of the spectral density $S_r(\omega)$ of dimensionality m^{-1} that are dependent on the foundation soil type, and β_0 is the coefficient of heterogeneity defining the scatter of the total load on the pipeline, i.e., the sum of the pressure of the upper layers and the soil response.

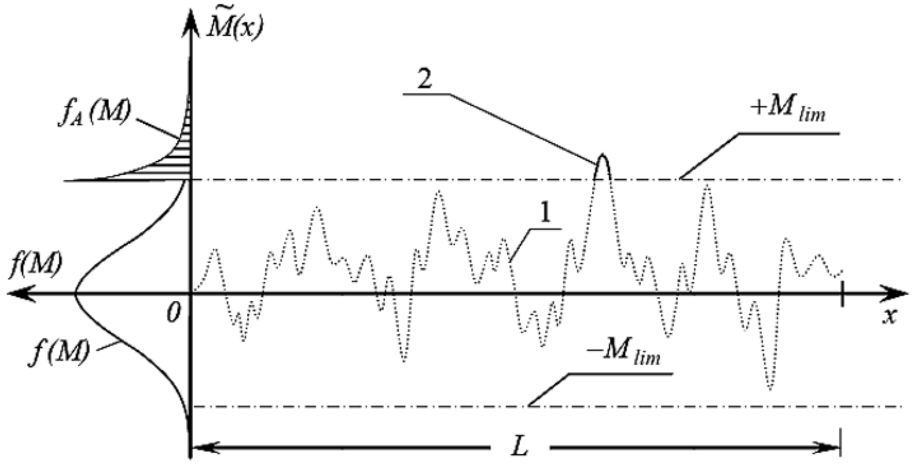

Fig. 2. Implementation of an arbitrary function of the elastic bending moment: (*1*) an arbitrary function $\widetilde{M}(x)$; (*2*) the absolute maxima of the function $\widetilde{M}(x)$ [$f(M)$ is the bending moment distribution].

Expression (3) is the main result of the above works on the calculation of the pipeline on an elastic foundation with stochastic characteristics. It is confirmed experimentally and therefore is used in what follows for assessing the reliability of steel underground pipelines.

Reliability Calculation of Pipelines Using the Strength Criterion. Let the condition of strength be reduced to the requirement that the elastic bending moment $\widetilde{M}(x)$, being the arbitrary function of the coordinate x, should not exceed (in modulus) the limiting level M_{lim} in each cross sections of the pipeline of length L. In the probabilistic formulation, this condition satisfies the requirement of a sufficiently small probabilistic event consisting in that the limiting level $\pm M_{lim}$ on the interval $0 \le x \le L$ is exceeded, in modulus, if only once (Fig. 2), i.e., $-M_{lim} < \widetilde{M}(x) < +M_{lim}$.

The function of reliability of a steel pipeline takes the form:

$$P(L) = 1 - Q(L) = P\left[\sup_{0 \le x \le L} |\widetilde{M}(x)| < M_{lim}\right], \tag{4}$$

where $\sup|\widetilde{M}(x)|$ is the upper bound of the function $\widetilde{M}(x)$ in the interval $0 \le x \le L$ and $Q(L)$ is the probability of failure of the pipeline at the length L.

Let us assume that the ordinate of the arbitrary function $\widetilde{M}(x)$ is distributed according to the normal law with zero mathematical expectation and the standard $\hat{M}$ which is uniquely determined by the bending stiffness EI of the beam and the root-mean-square curvature of the beam axis $\overline{\chi}^2$:

$$\hat{M} = EI(\overline{\chi}^2)^{1/2}. \tag{5}$$

Using the relationship between the pipeline curvature $\overline{\chi}^2$ and its spectral density $S_\chi(\omega)$, formula (5) can be rewritten in the following way:

$$\hat{M} = EI\left[\int_0^\infty S_\chi(\omega)d\omega\right]^{1/2}. \tag{6}$$

Here, $S_\chi(\omega)$, in its turn, is defined by the formula

$$S_\chi(\omega) = \frac{S_r(\omega)\omega^4}{(EI\omega^4 + b_{eff}\bar{c})^2}, \quad (7)$$

where b_{eff} is the effective width of the pipeline (in m), $\bar{c}$ is the mathematical expectation of the foundation stiffness coefficient (in N/cm^3), and $S_r(\omega)$ is the spectral density of the inhomogeneity function of deformations in a steel underground pipeline (see above).

If the load-carrying capacity of the pipeline has the Gaussian distribution, which is true in the majority of cases, then in view of the normal distribution of the elastic bending moment, the reserve of the load-carrying capacity is also distributed according to the normal law. Then the probability of failure will be defined as in [3, 4]:

$$Q(L) = \exp[-0.5(\gamma_0^2 - \beta^2)], \quad (8)$$

where γ_0 is the characteristic maximum of the arbitrary function $\widetilde{M}(x)$, and β is the safety characteristic.

According to [3, 4], we have

$$\gamma_0 = \sqrt{2\ln\left[\frac{\omega_\chi L}{\pi\beta_\chi}\right]}, \quad (9)$$

where ω_χ is a parameter of dimensionality m^{-1} similar to the effective frequency of the random process and describing the variance of curvature of the pipeline axis and the bending moment along its length, and β_χ is the broadbandness coefficient of the arbitrary function $\widetilde{M}(x)$. These quantities can be assessed using the following formulas [5, 6]:

$$\omega_\chi = \left[\int_0^\infty S_\chi(\omega)\omega^2 d\omega \Big/ \int_0^\infty S_\chi(\omega) d\omega\right]^{1/2}, \quad (10)$$

$$\beta_\chi = \left[\int_0^\infty S_\chi(\omega)\omega^4 d\omega \int_0^\infty S_\chi(\omega) d\omega\right]^{1/2} \Big/ \int_0^\infty S_\chi(\omega)\omega^2 d\omega. \quad (11)$$

The safety performance in formula (10) can be defined in the space of elastic bending moments by the known formula [6]

$$\beta = \frac{\bar{M}_R}{\sqrt{\hat{M}_R^2 + \hat{M}^2}}, \quad (12)$$

where $\bar{M}_R$ and $\hat{M}_R$ are, respectively, the mathematical expectation and the standard of the elastic bending moment corresponding to the load-carrying capacity of the steel underground pipeline.

Numerical Example. Based on the above material, we make the probabilistic strength calculation for the steel pipeline of length $L = 100$ m. The pipeline has the outer diameter $D = 70$ mm (the wall thickness is 3 mm, the moment of the pipeline cross-sectional resistance $W = 10.173$ cm^3) and the corresponding bending stiffness $EI =$ 73.35 kN $\cdot$ m^2. It is manufactured of a S235 steel having a calculated resistance $R_y = 220$ MPa for which the mathematical expectation of the yield strength $\bar{R} = 300$ MPa and the standard deviation $\hat{R}_y = 24$ MPa. The mathematical expectation of the load per unit of length from soil weight is $\bar{q} = 9$ kN/m. The foundation soil is fine-grained dry sand with $\bar{c} = 6.2$ N/cm^3. For the given foundation, the parameters of the spectral density are as follows: $\alpha = 2.75$ m^{-1} and $\theta = 3.65$ m^{-1}. The outer diameter D of the pipeline is taken as effective width b_{eff}. The coefficient of inhomogeneity is taken to be equal to $\beta_0 = 1$.

TABLE 1. Parameters of the Model for Absolute Maxima of Arbitrary Functions for the Internal Elastic Moment and Bending

Arbitrary function	Characteristic maxima γ_0 and f_0 for the pipeline length L, m					
	10	50	100	200	500	1000
$\widetilde{M}(x)$	1.734	2.495	2.759	3.000	3.291	3.496
$\widetilde{f}(x)$	1.052	2.080	2.390	2.664	2.988	3.212

Let us determine the root-mean square curvature of the beam axis. According to (6), we have

$$\bar{\chi}^2 = \int_0^\infty \frac{S_r(\omega)\omega^4}{(EI\,\omega^4 + b_{eff}\,\bar{c})^2}\,d\omega = 1.153\cdot 10^{-4}\ \text{m}^{-2}.$$

According to (5), the standard of the elastic bending moment in the pipeline is as follows:

$$\hat{M} = 73.35\,(1.153\cdot 10^{-4})^{1/2} = 0.787\ \text{kN}\cdot\text{m}.$$

By integrating with respect to expressions (10) and (11), for the parameter ω_χ and the broadbandness coefficient β_χ, we obtain

$$\omega_\chi = 2.433\ \text{m}^{-1}, \qquad \beta_\chi = 1.721.$$

Values of the characteristic maximum of the internal elastic moment are determined by formula (9):

$$\gamma_0 = \sqrt{2\ln\left[\frac{2.433\cdot 100}{1.721\cdot\pi}\right]} = 2.759.$$

These are presented in Table 1 for underground pipelines of various lengths.

To a first approximation, we assume that

$$\bar{M}_R = \bar{R}_y W = 30\cdot 10.173 = 3.052\ \text{kN}\cdot\text{m},$$

$$\hat{M}_R = \hat{R}_y W = 2.4\cdot 10.173 = 0.244\ \text{kN}\cdot\text{m},$$

and determine the safety characteristic (12):

$$\beta = \frac{3.052}{\sqrt{0.244^2 + 0.787^2}} = 3.702.$$

As a result, for the probability of failure of the steel pipeline we have

$$Q(L) = \exp[-0.5\,(2.759^2 - 3.702^2)] = 0.048.$$

Reliability Calculation for a Pipeline Using the Stiffness Criterion. The stiffness calculation from the limiting deflections is performed similarly. As in the reliability assessment according to the strength criterion, we will consider the arbitrary deflection function $\widetilde{f}(x)$ with the ordinate distribution by the centered normal distribution. The standard of this distribution [6] is as follows:

$$\hat{f} = \left[\int_0^\infty S_f(\omega) d\omega \right]^{1/2}, \qquad (13)$$

where $S_f(\omega)$ is the spectral density of the beam deflection,

$$S_f(\omega) = \frac{S_r(\omega)}{(EI\omega^4 + b_{eff}\bar{c})^2}. \qquad (14)$$

According to [3, 4], the probability of failure is as follows:

$$Q(L) = \exp[-0.5(f_0^2 - \delta_f^2)], \qquad (15)$$

where f_0 is the characteristic maximum of the arbitrary function $\tilde{f}(x)$, and δ_f is the normalized deviation of the limiting deflection of a beam f_u. Taking into account that the mathematical expectation of the beam deflection is equal to zero, we have

$$\delta_f = f_u / \hat{f}. \qquad (16)$$

The characteristic maximum f_0 is determined by formula (9) where ω_f and β_f defined by formulas (10) and (11) are substituted for ω_χ and β_χ. In formulas (10) and (11), the spectral density of curvature of beam deflection $S_\chi(\omega)$ appears instead of the spectral density of curvature of the pipeline axis $S_f(\omega)$.

Numerical Example. Let us perform the probabilistic reliability calculation for a steel underground pipeline using the stiffness criterion and the data of the previous example.

For the deflection standard we have

$$\hat{f} = \left[\int_0^\infty \frac{S_r(\omega)}{(EI\omega^4 + b_{eff}\bar{c})^2} d\omega \right]^{1/2} = 7.039 \cdot 10^{-3} \text{ m}.$$

For the parameter ω_f and the coefficient of broadbandness β_f by integrating expressions (10) and (11), respectively, we obtain

$$\omega_f = 0.941 \text{ m}^{-1}, \qquad \beta_f = 1.722.$$

As in the previous case, values of the characteristic maximum of deflection are determined by formula (9):

$$f_0 = \sqrt{2 \ln \left[\frac{0.941 \cdot 100}{1.722 \cdot \pi} \right]} = 2.39.$$

For underground pipelines of different length, the characteristic maxima of deflection are given in Table 1.

Using the recommendations [7], we assume the limiting deflection of the pipeline $f_u = 3$ cm, which corresponds to the normalized deviation from the center of distribution:

$$\delta_f = 3/0.704 = 4.26.$$

As a result, for the probability of failure of a steel underground pipeline we have

$$Q(L) = \exp[-0.5(2.39^2 - 4.26^2)] = 0.002.$$

CONCLUSIONS

1. A sufficiently convenient procedure has been proposed for calculating steel underground pipelines resting on statistically inhomogeneous soil.

2. The procedure and numerical examples are presented that illustrate the simplicity of obtaining assessments of reliability using the probabilistic representation of loads in the form of absolute maxima of arbitrary functions. The examples have also shown the difference in reliability estimates using the criteria of strength and stiffness and the influence of the broadbandness coefficients on the reliability of steel pipelines.

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