Strength of Materials, Vol. 39, No. 5, 2007

LIFE CALCULATIONS FOR MATERIALS UNDER IRREGULAR NONPROPORTIONAL LOADING

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Experimental data on low-cycle fatigue of 304 steel and a VT9 titanium alloy in deformation by complex loading histories, which are the sequence of blocks of cycles of different shapes in the space of total strains, cited in the literature, are analyzed to develop adequate models for life calculations. The four damage accumulation rules and the low-cycle fatigue deformation criterion were used as basic approaches. Life prediction models were compared. It is shown that the application of a modified nonlinear damage accumulation rule can improve life prediction results, with better outcomes obtained for the programs involving nonproportional cycles.

Keywords: low-cycle fatigue, block of cycles, strain hardening, damages, life.

Introduction. Engineering structures are often subjected to complex (multiaxial) loads. In many cases those loads undergo cyclic changes in time. They can be regular, when amplitudes and cycle shapes remain constant or irregular with stepwise changes in amplitudes or cycle shapes (so-called block loading), as well as stochastic, when amplitudes or cycle shapes vary in a random manner. To simplify calculations, the latter case is reduced to irregular block loading with the application of different cycle schematization rules, e.g., method of complete cycles, "rain," etc. Studies on the material behavior under regular cyclic loading have already been carried out for a long time. And by now a sufficient body of experimental data on this problem has been published in the literature. It should also be noted that intensive development of corresponding mathematical models is performed to describe stress-strain state kinetics under regular cyclic loading within different theories of plasticity [1–4]. Considerable studies on the construction of life prediction models are also carried out [5–8]. As regards irregular nonproportional loading, it is less studied experimentally because of higher complexity of realization, especially for complex (nonproportional) loading cycles. In this respect, most representative studies, e.g., [9, 10], should be quoted, where the material behavior is investigated within complex irregular loading programs.

For life prediction under irregular loading, the Miner linear rule of damage accumulation is usually used [11], according to this rule, the damage level D at variable loading amplitudes is linearly growing with a number of cycles, and failure takes place when it reaches unity, i.e.,

$$D = \sum_i \frac{n_i}{N_i} = 1. \tag{1}$$

This approach is rather simple and much used in many cases, but it cannot always result in adequate life prediction, especially when loading has a certain tendency to a constant growth, or vice versa to a decrease. Therefore, many attempts to develop damage models, based on the nonlinear rule of damage accumulation, were quite natural. In the damage models proposed by Richart and Newmark [12] and Marco and Starkey [13], the exponential equations were already used

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$$D = \sum_i \left(\frac{n_i}{N_i} \right)^{q_i}, \qquad (2)$$

where the exponent q is dependent on the cycle stress level.

Morrow [14] proposed the rule, based on the plastic interaction work, that allows for the effect of loading cycle sequences. Here the damage accumulation rule at variable amplitudes is written as

$$D_i = \frac{n_i}{N_i} \left(\frac{\sigma_i}{\sigma_m} \right)^q, \qquad D = \sum_i \frac{n_i}{N_i} \left(\frac{\sigma_i}{\sigma_m} \right)^q, \qquad (3)$$

where σ_m is the maximum stress amplitude for the examined loading history and q is the exponent dependent on the plastic interaction work. The exponent is interpreted as material sensitivity to variable amplitudes of stress history.

Manson et al. [15] proposed the double linear rule to examine two-stage cyclic loadings. By this rule the point, where two damage lines intersect, is determined. This can be represented as

$$\frac{n_1}{N_1} = 0.35 \left(\frac{N_1}{N_2} \right)^{0.25}, \qquad \frac{n_2}{N_2} = 0.65 \left(\frac{N_1}{N_2} \right)^{0.25}, \qquad (4)$$

where N_1 and N_2 are the fatigue lives for the first and second loading stages, respectively, and n_1 and n_2 are the number of cycles for the first and second loading stages, respectively. This approach gives $D > 1$ for the two-stage loading with the sequence: smaller-larger amplitudes and $D < 1$ with the sequence: larger-smaller amplitudes.

The approach based on the damage curve [16] allows the damage level at the first and second stages to be calculated in the following way:

$$D_1 = \frac{n_1}{N_1}, \qquad D_2 = \left(\frac{n_2}{N_2} \right)^{(N_1/N_2)^{0.4}}. \qquad (5)$$

The modification of model (5) for the two-stage loading with nonproportional cycles was proposed elsewhere [10]. In this case, additional strain hardening typical of nonproportional paths, which usually reduces fatigue life, should be taken into account. According to modified approach (5) [10], the damage level is determined as

$$D_1 = \frac{n_1}{N_1}, \qquad D_2 = \left(\frac{n_2}{N_2} \right)^{(1/(1+\alpha\Phi))(N_1/N_2)^{0.4}}, \qquad (6)$$

where α is the factor of sensitivity to cycle nonproportionality and Φ is the coefficient of cycle nonproportionality.

Hypotheses of fatigue damage accumulation at the stage of nucleation and propagation of crack-like damages are comprehensively analyzed in a review by Fatemi and Yang [17].

Despite a great number of models for life prediction under irregular fatigue loading, described in the literature, preference is given to none of them as regards programs with nonproportional cycles Therefore, the object of the present study is to investigate the potentials of the low-cycle fatigue deformation criterion [18] and the nonlinear damage accumulation rule for life prediction under irregular loading by the programs with nonproportional cycles.

Experimental and the Damage Prediction Model. The applicability of the deformation criterion was analyzed with low-cycle fatigue data for a VT9 titanium alloy under rigid deformation with an amplitude of the Mises 1% equivalent strain for the two irregular programs, being the sequence of blocks with an equal number of cycles (20) of different shapes, viz tension–compression, torsion, circular cyclic path (Table 1) [9]. Low-cycle fatigue data for 304 stainless steel under the same deformation conditions with a 0.6% strain amplitude by 13 irregular loading programs were also used (Table 2) [10]. Those programs included the following cycles: **a** – tension–compression, **t** –

TABLE 1. Experimental Programs of Irregular Loading for a VT9 Alloy

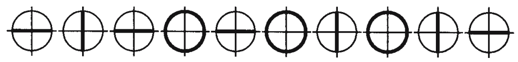
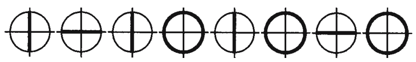
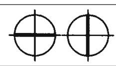
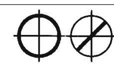
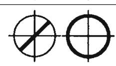
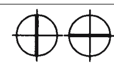
Program	Cycle sequence	Number of cycles to failure N_f
A		200
B		160

TABLE 2. Experimental Programs of Irregular Loading for 304 Steel

Program	Cycle sequence	Number of cycles to failure N_f	Program	Cycle sequence	Number of cycles to failure N_f
at		3969 (973/2994)	**oi**		3936 (364/3572)
		2927 (1946/981)			3157 (583/2574)
io		2281 (1228/1053)			3209 (728/2481)
		3190 (1965/1225)			3258 (1093/2165)
		3143 (2456/687)	**ta**		2869 (1559/1310)
		4234 (3685/549)			3942 (3117/825)
					5044 (4676/368)

Note. The number of loading cycles for the first and second blocks is cited in brackets, respectively.

alternating torsion, **i** – proportional inphase biaxial path, **o** – nonproportional circular path. In both sets of experiments the number of cycles to failure (life) N_f was determined by the instant when a 10% change in loading was registered, which is the evidence of macrocrack initiation in the material.

For comparative analysis, the four damage accumulation rules were examined: linear (1), Morrow (3), Manson (5), and Marco–Starkey nonlinear (2). The latter was modified with the dimensionless parameter, viz the equivalent amplitude of strains ε_{eq} as the exponent q in Eq. (2), for the account of complex cyclic loading. The parameter q is determined as

$$q = \frac{\bar{\varepsilon}}{\varepsilon_{eq}}. \tag{7}$$

The equivalent amplitude of strains ε_{eq} was proposed elsewhere [18] as the low-cycle fatigue deformation criterion for examining complex cyclic loading. This parameter can be written as

$$\varepsilon_{eq} = (1 + k \sin\varphi)(1 + \alpha\Phi)\varepsilon_{\mathrm{M}}, \tag{8}$$

where k is the factor of material sensitivity to a stress state and ε_{M} is the Mises maximum amplitude of cycle strain.

The amplitude of strains $\bar{\varepsilon}$ is obtained from the analysis of the Basquin–Manson–Coffin fatigue curve under uniaxial loading, it corresponds to the intersection point of asymptotes to this curve, which characterize elastoplastic and elastic deformation ranges (Fig. 1). In fact, this parameter corresponds to the conventional division of low-cycle and high-cycle fatigue ranges as regards strain amplitudes.

If the equivalent amplitude of strains $\varepsilon_{eq} < \bar{\varepsilon}$, the exponent $q > 1$. In this case, damage curve *2* (Fig. 2) is concave down and passes under straight line *1* that corresponds to the linear damage accumulation rule. It should be noted that strain amplitudes, as a rule, correspond to the elastic range, and under such loading damages at the initial stages of cyclic loading are accumulated much slower than within cycles close to failure $(n/N = 1)$. Under elastoplastic deformation, when the equivalent amplitude of strains $\varepsilon_{eq} > \bar{\varepsilon}$, the exponent $q < 1$, and damage curve *3* is concave up. Thus, the damage level is the highest at the initial loading stages, then it lowers as the material adapts to cyclic loading. High damage levels within first cycles, when cyclic deformation takes place in the elastoplastic range, are typical of many materials. As a result, e.g., a considerable gain in strain hardening at the initial low-cycle

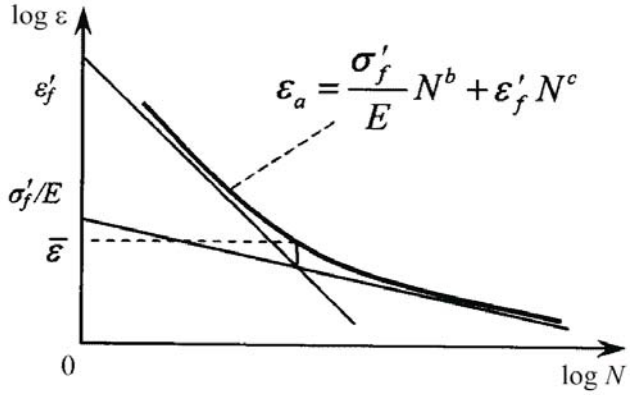


Fig. 1

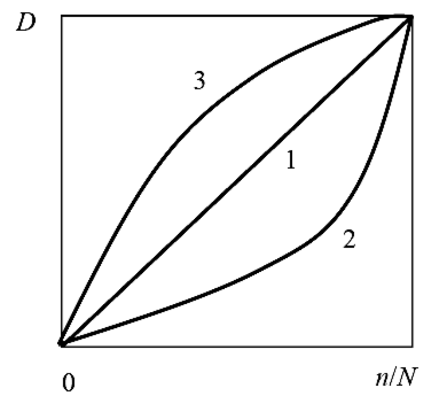


Fig. 2

Fig. 1. Scheme of determining the parameter $\bar{\varepsilon}$.

Fig. 2. Schematic representation of the linear and nonlinear damage accumulation rules: (*1*) linear for $\varepsilon_{eq} = \bar{\varepsilon}$; (*2*) nonlinear for $\varepsilon_{eq} < \bar{\varepsilon}$; (*3*) nonlinear for $\varepsilon_{eq} > \bar{\varepsilon}$.

deformation stages is observed, which decreases gradually down to stabilization of material properties and its final adaptation to cyclic loading.

With nonlinear damage accumulation rule (2) and equation for determining the exponent q (7), the following recurrence equation for calculating the damage level can be obtained

$$D_i = \left(D_{i-1}^{1/q_i} + \frac{n_i}{N_i} \right)^{q_i}, \tag{9}$$

which accounts for the effect of block sequences of cycles of lower and higher loading levels. In our case, the extent of cycle nonproportionality is allowed for through the parameter ε_{eq}. It should be noted that complex cycle programs are characterized by higher strain hardening (stress) levels as compared to proportional loading ones.

Life Prediction Results. For life prediction within the above irregular loading programs, the parameters (Table 3) in constitutive equations (7) and (8) are specified, as well as the analytical parameters for basic fatigue curves of materials under study were determined. Since a VT9 titanium alloy is deformed at very high amplitudes of elastoplastic strains, experimental low-cycle fatigue points [9] in tension–compression were approximated with the linear relation. The latter may be considered valid for life ranges of 100–10,000 cycles, viz

$$\log \varepsilon_a = 1.015 - 0.432 \log N. \tag{10}$$

For 304 stainless steel [10], the analytical parameters for the Basquin–Manson–Coffin fatigue curve were determined

$$\varepsilon_{eq} = \frac{742}{171{,}000}(2N)^{-0.081} + 0.03(2N)^{-0.28}. \tag{11}$$

Life prediction results for irregular loading (Tables 1 and 2) by linear (1) and nonlinear (2)damage accumulation rules are summarized in Table 4. Figure 3 demonstrates absolute standard deviations of life calculations within similar block loading programs for 304 steel by the four damage accumulation rules. Comparative analysis revealed that the narrowest scatter in the data, if all irregular loading programs are considered, is observed in the case of the nonlinear damage accumulation rules: Marco–Starkey modified (2) and Morrow (3). The best results were obtained for the Morrow rule. However, this rule requires knowledge of stress levels for each block, which are usually taken from the experiment (were available) and which cannot always be determined accurately for an arbitrary path in

TABLE 3. Parameters of Equations (7) and (8)

Material	$\bar{\varepsilon}$	α	k
VT9 alloy	0.0034	0.17	0.26
304 steel	0.0039	0.90	0.20

TABLE 4. Life Prediction Results

Path	Linear rule (1)		Nonlinear rule (2)		Experimental life
	Damage level D	Calculated life N_{calc}	Damage level D	Calculated life N_{calc}	N_{exp}
304 stainless steel					
at01	0.730890	5643	0.838868	5426	3967
at02	0.657826	5058	0.784421	4848	2927
ta01	0.587119	4475	0.702826	4605	2869
ta02	0.712589	5060	0.793057	5195	3942
ta03	0.845418	5645	0.890581	5731	5044
io01*	0.830269	2568	0.867121	2701	2281
io02*	1.057089	3093	0.981700	3251	3190
io03*	0.823038	3443	0.852514	3605	3143
io04*	0.950544	4318	0.932029	4456	4279
oi01*	0.822144	4982	0.942723	4446	3936
oi02*	0.781719	4441	0.918454	3878	3157
oi03*	0.851487	4082	0.962963	3540	3209
oi04*	1.013186	3180	1.052353	2778	3258
VT9 titanium alloy					
A	1.063528	194	1.023201	192	200
B	0.884664	172	0.948349	173	160

* Programs with nonproportional cycles.

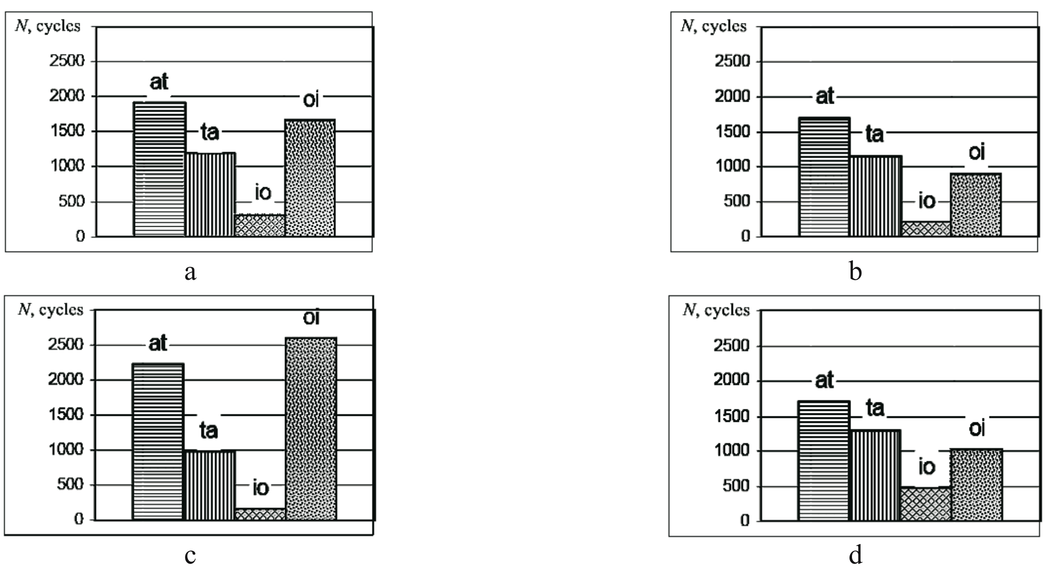


Fig. 3. Absolute standard deviations in life prediction for similar irregular loading programs of 304 steel with different damage accumulation rules: (a) linear (1); (b) Morrow (3); (c) Manson (5); (d) Marco–Starkey modified (2).

theoretical prediction. At the same time the accuracy of life prediction was better for the programs with nonproportional cycles. This suggests that 304 steel experiences an essential influence of such a factor as the sequence of blocks of cycles of lower and higher loading levels.

CONCLUSIONS

1. The low-cycle fatigue deformation criterion used in life calculations under irregular loading can provide the account of the effect of nonproportional cycles that usually reduce the life of materials.

2. Comparison of different damage accumulation rules demonstrated the advantage of the modified nonlinear rule used in all investigated irregular loading programs for 304 steel. Moreover, the accuracy of prediction was the best for the programs with nonproportional cycles.

3. Irregular loading of a VT9 titanium alloy demonstrated practically equal validity of the models. It can be explained by the fact that this alloy belongs to the materials, which exhibit low sensitivity to additional cyclic strain hardening under complex cyclic loading. Thus, stress levels in proportional and nonproportional cycles for this material are similar, which determines the absence of essential differences when different models are used.

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