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FATIGUE LIFE ASSESSMENT FOR METAL ALLOYS UNDER NONPROPORTIONAL LOW-CYCLE LOADING

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A procedure for nonproportional low-cycle fatigue life assessment is proposed for various metal alloys that differ in structure. It is based on the Pisarenko–Lebedev equivalent strain and a correcting function allowing for nonproportional loading effects such as additional hardening and life reduction in the case of nonproportional strain paths. The calculated results are compared to the experimental ones reported elsewhere for steels, titanium-, nickel-, and aluminum-based alloys. A good agreement between the results is observed for all the materials and strain paths studied. The accuracy of the calculations for nonproportional loading is found to be same as in the case of proportional loading.

Keywords: low-cycle fatigue, nonproportional loading, limiting state criteria, fatigue life assessment.

Introduction. When in operation, many critical structural elements and machine components are usually subjected to multiaxial low-cycle loading. In such a situation, the structural material in the maximum-stress zones has to work beyond elastic limits, where the effects of strain cycling come into play. Experimental studies as well as practical experience of using such structures demonstrate that a typical feature of these loading conditions is that the processes of deformation and fracture depend on a number of factors such as the kind of stress and the strain path shape.

Currently, the influence of these factors on low-cycle strength of structural materials has been given much attention by the community of deformable solid mechanics [1–11, et al.], with emphasis placed on metals and related alloys. The research findings show that materials do behave differently under low-cycle loading along proportional and nonproportional paths. In particular, a so-called "additional hardening" effect has been revealed; it shows up in the fact that, with the range of equivalent strain being the same, a material exhibits more hardening along nonproportional cyclic strain paths in comparison to proportional ones. Another finding is that there is a direct relationship between a reduction of fatigue life and an increase in the extent of the additional strain hardening in metal alloys under nonproportional low-cycle loading. Various metals that differ in structure differ also in the extent of the additional cyclic hardening under nonproportional loading, i.e., depending on their structure metals and related alloys may exhibit a higher or lower sensitivity to a factor such as the loading nonproportionality.

To the best of our knowledge, the majority of available methods for the multiaxial low-cycle fatigue life assessment does not allow for the above-mentioned peculiarities of the deformation and fracture processes in metal materials; therefore, it would be experimentally unjustified to apply them to nonproportional loading cases.

A procedure for fatigue life assessment for metal alloys under multiaxial low-cycle loading is proposed below; its basic equations include the nonproportional loading effects and can be applied to various alloys that differ in sensitivity to nonproportionality of loading.

The resistance of materials under service conditions, where they one subjected to an arbitrary system of active stresses, is conventionally assessed using equivalence conditions (strength criteria). The most widely used criteria for the evaluation of a limiting state under low-cycle loading are the deformation-type criteria.

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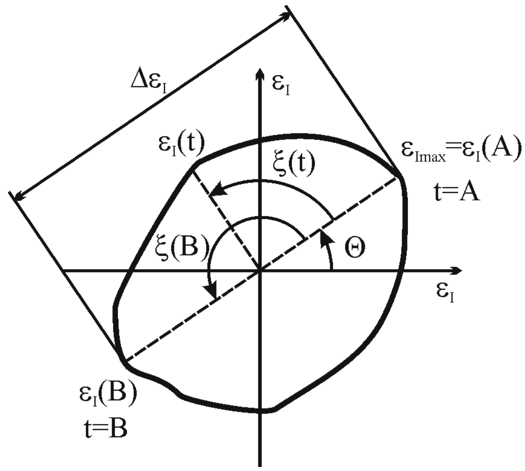

Fig. 1. Variation of the maximum absolute value of principal strain ε_I per cycle.

The above-mentioned experimentally established patterns suggest that the equivalent range of strains under nonproportional loading $\Delta\varepsilon_{NP}$ can be represented as a function of three parameters,

$$\Delta\varepsilon_{NP} = f(\Delta\varepsilon_{eq}, \alpha, f_{NP}), \tag{1}$$

where $\Delta\varepsilon_{eq}$ is the range of equivalent strain, which is to be determined by a specific theory of strength chosen, α is the coefficient of a material's sensitivity to the loading nonproportionality, and f_{NP} is the loading nonproportionality parameter which depends on the strain path shape only.

This approach was implemented in [6]. Its basic principles and further development are outlined below.

The Itoh–Sakane–Ohnami–Socie Criterion. The equivalent range of strain under nonproportional loading was proposed to be calculated as follows [6]:

$$\Delta\varepsilon_{NP} = (1+\alpha f_{NP})\Delta\varepsilon_I, \tag{2}$$

where $\Delta\varepsilon_I$ is the range of principle strain,

$$\Delta\varepsilon_I = \max[\varepsilon_{I\max} - \varepsilon_I(t)\cos\xi(t)]. \tag{3}$$

Figure 1 shows the parameters involved in (3) and provides their graphical interpretation.

The coefficient α which allows for the "additional hardening" effect is given by

$$\alpha = \frac{\sigma(1)-\sigma(0)}{\sigma(0)}, \tag{4}$$

where $\sigma(1)$ and $\sigma(0)$ are the equivalent stress cycle amplitudes under a circular strain path in the Mises type strain space (maximum stress) and under proportional loading (minimum stress), respectively, with the range of equivalent strain being the same.

The nonproportionality parameter f_{NP} accounts for the history of principal strains over the cycle and is determined by

$$f_{NP} = \frac{\pi}{2T\varepsilon_{I\max}}\int_0^T (\varepsilon_I(t)|\sin\xi(t)|)dt, \tag{5}$$

where $\varepsilon_I(t)$ is the absolute value of the largest principal strain at an instant of time t, $\varepsilon_{I\max}$ is the maximum value of $\varepsilon_I(t)$, T is the cycle duration, and $\xi(t)$ is the angle between the directions of $\varepsilon_{I\max}$ and $\varepsilon_I(t)$ (see Fig. 1).

Modification of the Itoh–Sakane–Ohnami–Socie Criterion. When analyzing the $\Delta\varepsilon_{NP}$ parameter [6] we have revealed its benefits and drawbacks. For instance, in the limiting case (in proportional loading) formula (2) becomes $\Delta\varepsilon_{NP} = \Delta\varepsilon_{I}$, i.e., this approach degenerates into the largest-linear-strain (LLS) criterion.

In [12, 13] the authors discussed the efficiency of available one-, two-, and three-parameter criteria as applied to multiaxial low-cycle loading in the deformation-based interpretation. It was found out that one-parameter criteria, the LLS criterion being one of them, show a considerably poorer correlation with experimental data in the case of biaxial proportional loading in comparison with two-parameter criteria such as the Pisarenko–Lebedev or the Coulomb–Mohr criteria.

Furthermore, it is known that there are discrepancies in the value of additional hardening between metals with different structures. Structural steels experience significant additional hardening, while in the case of titanium alloys this hardening is weak if it ever occurs. In the case of no additional hardening, the α coefficient is zero, and thus we have $\Delta\varepsilon_{NP} = \Delta\varepsilon_{I}$. In this way, nonproportionality of loading is disregarded, while experimental investigations demonstrate a reduction in fatigue life under nonproportional loading for titanium alloys, as well. Therefore, the α coefficient as proposed in [6] can be applied only to materials with noticeable additional hardening.

We propose to modify the approach to determination of the $\Delta\varepsilon_{NP}$ parameter as follows.

The equivalent range of strain under nonproportional loading, $\Delta\varepsilon_{NP}$, is defined using the Pisarenko–Lebedev criterion in the deformation-based interpretation,

$$\varepsilon_{PL} = \chi_{\varepsilon} \frac{\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2}}{\sqrt{2}(1+\nu^*)} + (1-\chi_{\varepsilon}) \left[\frac{\varepsilon_1 + \dfrac{\nu^*}{1-2\nu^*}(\varepsilon_1+\varepsilon_2+\varepsilon_3)}{1+\nu^*} \right] \le \varepsilon_{lim}, \tag{6}$$

where $\chi_{\varepsilon} = \dfrac{1}{\sqrt{3}-1}\left[2(1+\nu^*)\left(\dfrac{\varepsilon_{lim}}{\gamma_{lim}}\right)-1\right]$, ε_1, ε_2, and ε_3 are the values of principal strains, γ_{lim} is the limit amplitude of shear strain under simple torsion conditions for a preset number of cycles, and ε_{lim} is the limit amplitude of longitudinal strain under tension–compression conditions for the same number of cycles.

The Poisson ratio ν^* is given by

$$\nu^* = 0.5 - 0.2\frac{\sigma_{0.2}}{E\varepsilon_{PL}}.$$

As mentioned above, the sensitivity of materials to the loading nonproportionality is associated with a reduction fatigue life. Previously, it was suggested [13] that the coefficient of sensitivity to the loading nonproportionality should be defined in terms of the fatigue life reduction effect as observed when proportional loading conditions are compared to nonproportional ones. This approach offers evident benefits in the case of materials that exhibit weak, if any, additional hardening. Here, this approach is proposed to be implemented in the form

$$\eta = \frac{1}{f_{NP}}\left[\frac{B}{B'}(N)^{k'-k} - 1\right], \tag{7}$$

where η is the coefficient of sensitivity to the loading nonproportionality, k and k' are the slopes of the fatigue life curves under tension–compression and nonproportional loading, respectively, and the values of B and B' are to be found from the data in Fig. 2.

Assuming that in the log-log chart the fatigue life curves have the same slope, i.e., $k = k'$, Eq. (7) can be rearranged to give

$$\eta = \frac{1}{f_{NP}}\left[\frac{B}{B'} - 1\right]. \tag{8}$$

TABLE 1. Physical-Mechanical Characteristics of Materials

Material	E, GPa	$\sigma_{0.2}$, MPa	σ_u, MPa	ν	Reference
1Cr–18Ni–9Ti	193.0	310	605	0.30	[9]
08Kh18N10T	203.0	320	690	0.29	[2]
304 stainless steel	185.0	260	690	0.29	[7]
SUS 304	193.0	123	403	0.30	[10]
6061 Al alloy	80.0	320	350	0.33	[7]
VT1-0	112.0	490	565	0.33	[11]
VT9	118.0	865	970	0.32	[8]
Inconel 718	208.5	1160	1420	0.34	[3]

TABLE 2. Cyclic Strength Characteristics of Materials

Material	A	n	α	η
1Cr–18Ni–9Ti	1842.70	–2.1236	0.238	0.794
08Kh18N10T	616.74	–3.6062	0.950	0.732
304 stainless steel	2342.10	–4.6904	0.900	0.795
SUS 304	2630.20	–2.9744	0.300	0.947
6061 Al alloy	1396.20	–3.1576	0.200	0.500
VT1-0	4307.00	–3.8226	0	0.140
VT9	809.31	–2.0602	0.080	0.250
Inconel 718	13818.00	–3.4341	0	0.789

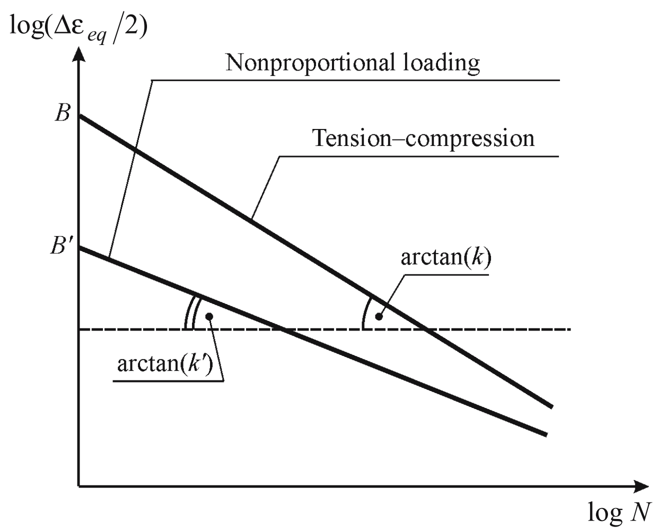

Fig. 2. A diagram for the determination of the coefficient η.

If circular cyclic strain paths in the Mises strain space ($f_{NP} = 1$) are chosen for the nonproportional loading case, we have

$$\eta = \frac{B}{B'} - 1. \tag{9}$$

Thus, Eq. (1) can be written as

$$\Delta\varepsilon_{NP} = (1 + \eta f_{NP})\Delta\varepsilon_{PL}, \tag{10}$$

where $\Delta\varepsilon_{PL}$ is the maximum range of equivalent strain by the Pisarenko–Lebedev criterion, $\Delta\varepsilon_{PL} = \max[\varepsilon_{PL}^{\max} - \varepsilon_{PL}(t)\cos\xi(t)]$.

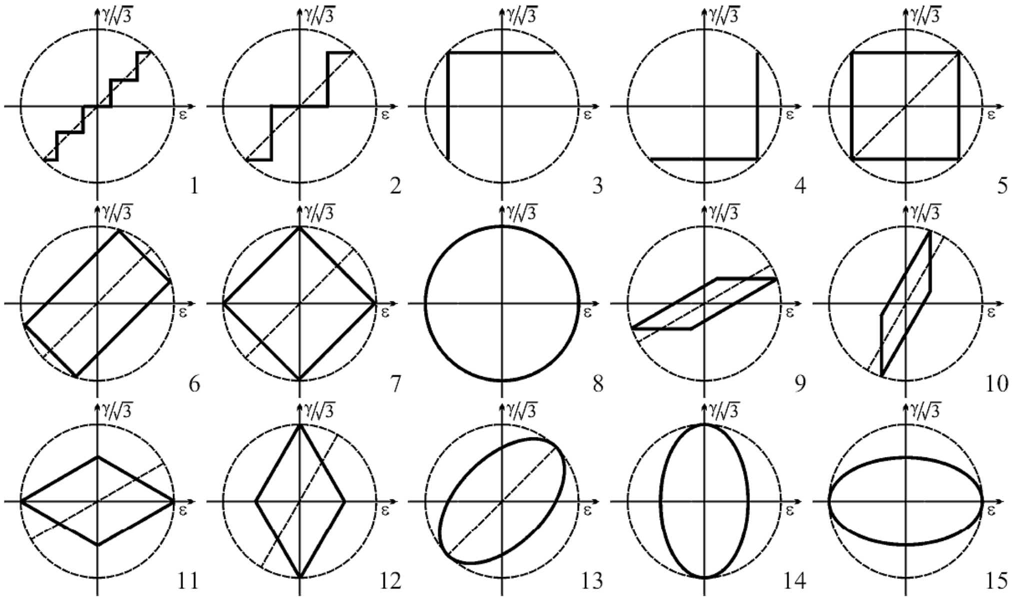

Fig. 3. Strain paths under consideration (1–15).

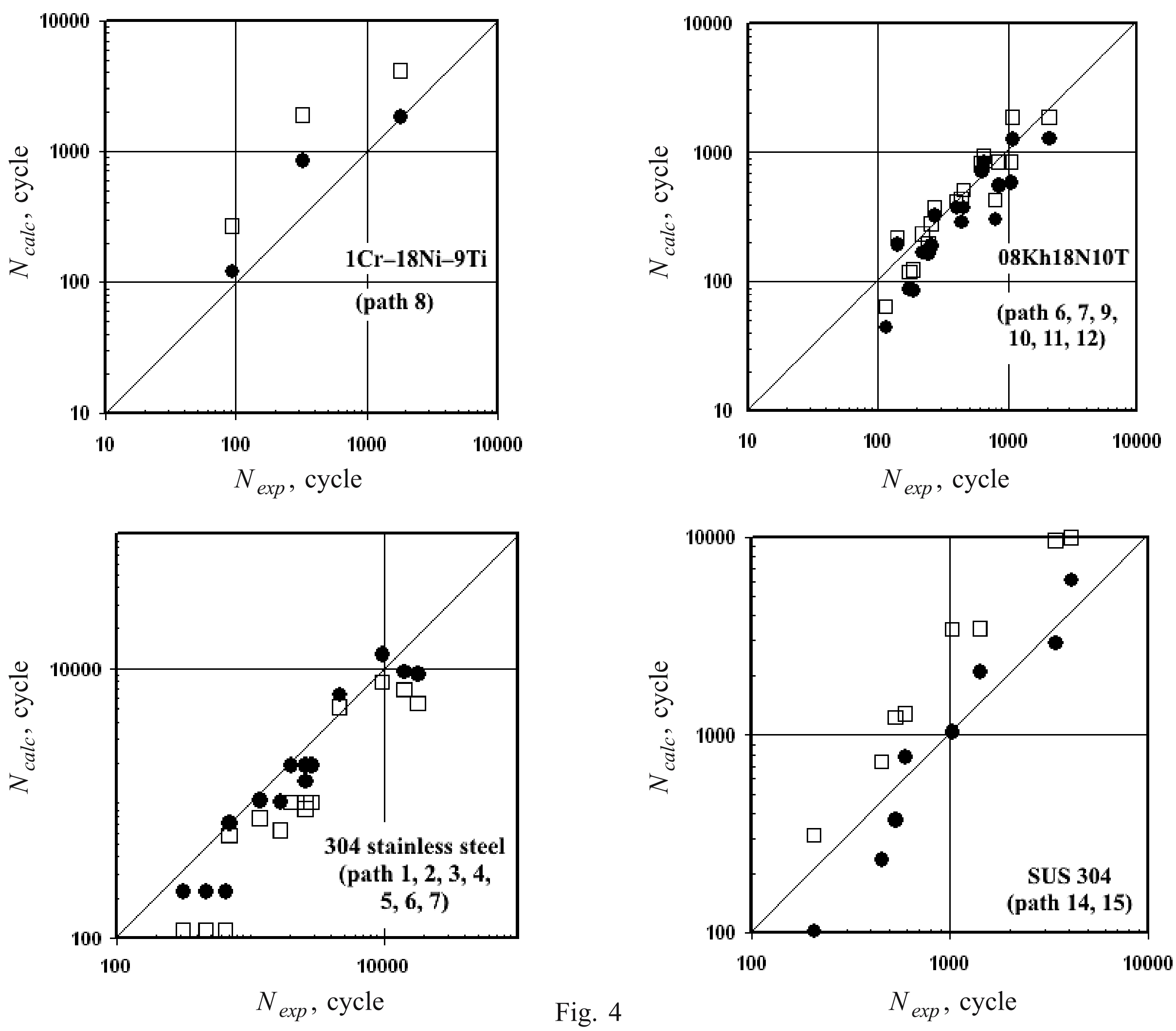

Fig. 4

continued Fig. 4

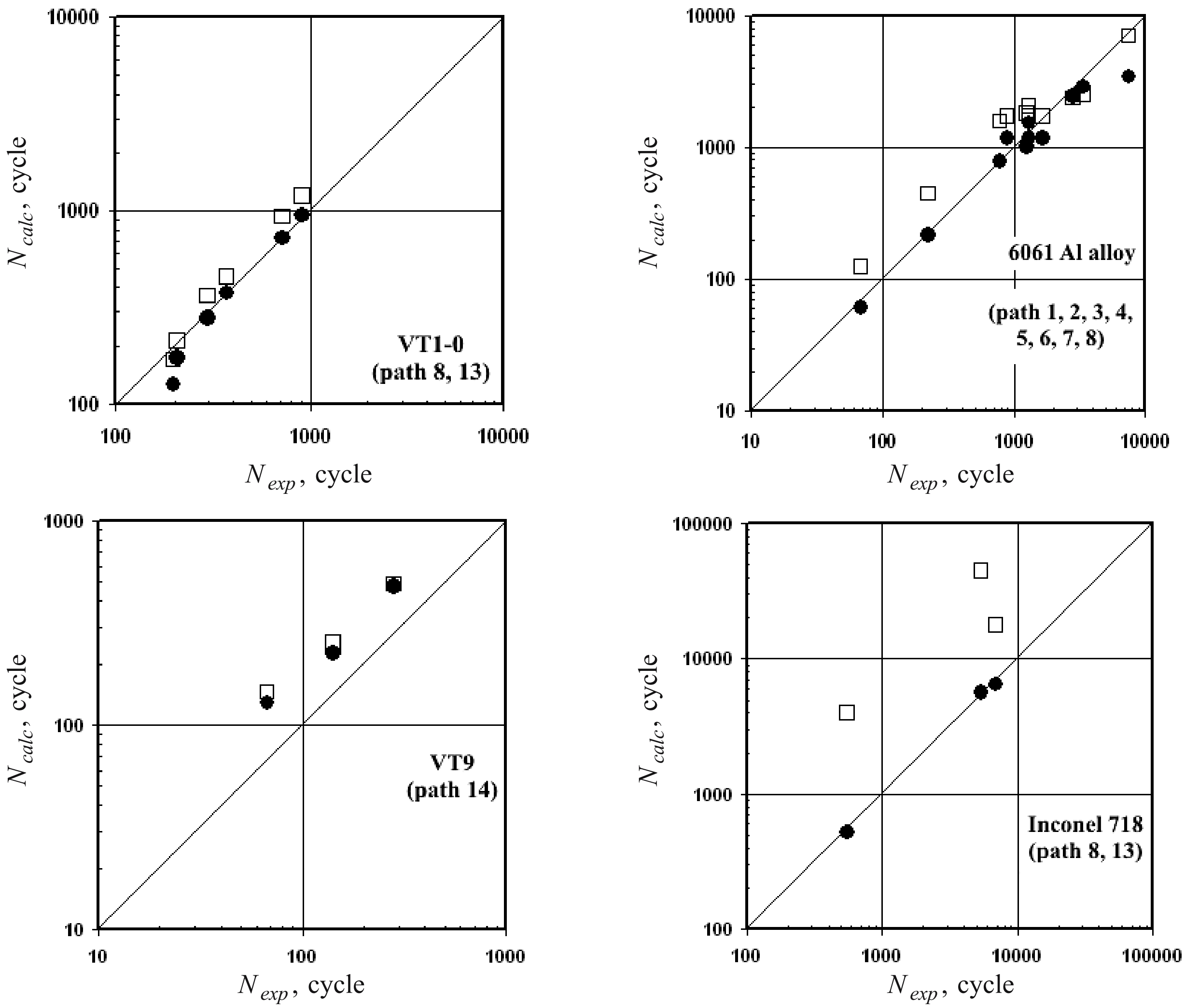

Fig. 4. Prediction of fatigue life of the materials under study by the original Itoh–Sakane–Ohnami–Socie criterion (□) and modified criterion (●).

Characteristics of Materials under Study and Strain Paths. For the purpose of verification of the proposed modification of the Itoh–Sakane–Ohnami–Socie approach [6], we processed experimental data for metal alloys that differ in structure [9–14] as well as in their behavior under nonproportional elastoplastic strain cycling.

Tables 1 and 2 summarize physical-mechanical and fatigue characteristics of the materials under study.

Figure 3 shows the nonproportional cyclic strain paths under study.

Note that no consideration has been given here to paths with asymmetry of the loading cycle parameters. Therefore, it would not be reasonable as yet to claim that the proposed procedure is applicable to such paths as well. Further research is needed.

Comparison between Experimental and Calculated Results. The calculations were carried out using the power dependence of the number of cycles to failure N_f on the range of equivalent strain $\Delta\varepsilon_{eq}$,

$$N_f = A(\Delta\varepsilon_{eq})^n, \tag{11}$$

where A and n are constants in the fatigue life equation, which are to be determined from the tension–compression test results, and $\Delta\varepsilon_{eq}$ was taken to be equal to the equivalent range of strain under nonproportional loading $\Delta\varepsilon_{NP}$.

The calculations were performed by the modified equation (10) as well as by the original Itoh–Sakane–Ohnami–Socie equation (2). Figure 4 compares the calculated results and shows their correlation with the experimental data.

CONCLUSIONS

1. A fatigue life assessment procedure for metal materials under complex elastoplastic strain cycling is put forward which is based on the modified Itoh–Sakane–Ohnami–Socie criterion.

2. It has been demonstrated that the procedure is suitable for fatigue life assessment for metal alloys with different structures under multiaxial elastoplastic strain cycling with either proportional or nonproportional loading.

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