EFFECTS OF SUPERTHERMAL PARTICLES ON WAVES IN MAGNETIZED SPACE PLASMAS

MANFRED HELLBERG[1,*], RICHARD MACE[1] and TOM CATTAERT[2]
[1]*School of Physics, University of KwaZulu-Natal, Durban 4000, South Africa*
[2]*Sterrenkundig Observatorium, Universiteit Gent, B-9000 Ghent, Belgium*
*(*Author for correspondence: E-mail: hellberg@ukzn.ac.za)*

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Abstract. Distributions with excess numbers of superthermal particles are common in space environments. They are well modelled by the isotropic kappa distribution, or, where magnetic effects are important, the kappa-Maxwellian. This paper presents a review of some studies of electrostatic and electromagnetic waves in such plasmas, based on the associated generalized plasma dispersion functions, Z_κ and $Z_{\kappa\mathrm{M}}$. In particular, the effects of low values of κ are considered, *i.e.* strongly accelerated distribution functions. Recently the full susceptibility tensor for oblique propagation of electromagnetic waves in a kappa-Maxwellian magnetoplasma has been established and has been applied to the study of whistler waves.

Keywords: plasma waves, magnetosphere, kappa distribution

1. Introduction

Recent observations have shown that kinetic effects on space plasma waves may be important. Although in the kinetic theory, it is often assumed that the distribution function is Maxwellian-based, other distributions are commonly observed, *e.g.* a power-law form: $4\pi v^2 f(v)dv \propto v^{-\alpha}dv$ for $|v| > v_{\mathrm{th}}$. Such distributions have a high-energy 'tail', *i.e.* more superthermal particles than a Maxwellian, but may be Maxwellian-like at low energies.

Vasyliunas (1968) modelled observed distribution functions by a 'generalized Lorentzian' or 'kappa' distribution. In its usual form, the normalized isotropic κ-distribution is written as

$$f_\kappa(v) = (\pi\kappa\theta^2)^{-3/2}\frac{\Gamma(\kappa+1)}{\Gamma\left(\kappa-\frac{1}{2}\right)}\left(1+\frac{v^2}{\kappa\theta^2}\right)^{-(\kappa+1)} \tag{1}$$

Here the modified thermal speed $\theta = (\frac{2\kappa-3}{\kappa})^{\frac{1}{2}}(T/m)^{1/2}$, and hence the distributions are defined for $\kappa > 1.5$. The 'kappa distribution' is, in fact, a family of power-law-like distributions, the real-valued parameter κ allowing one to fit to the actual distribution (Figure 1). It may vary from a Lorentzian-like form ($\kappa \simeq 1.5$), representing a hard, accelerated spectrum, to a Maxwellian ($\kappa \to \infty$). Below the thermal speed, $f(v)$ is Maxwellian-like, albeit with a slightly reduced density, whereas above the

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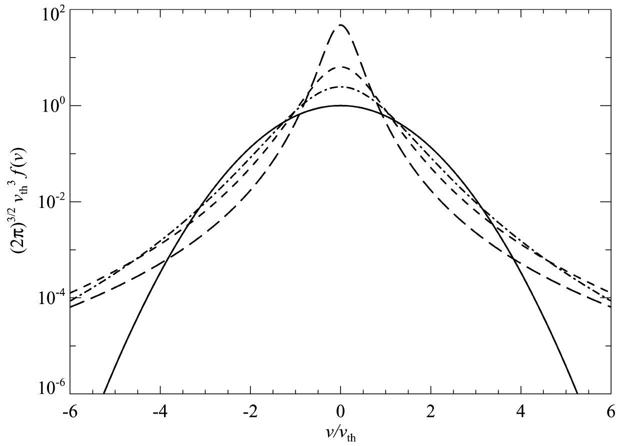

Figure 1. Representation of typical kappa distributions for $\kappa = 1.6$, 2, 3, and infinity, showing increasing 'tail' for lower κ values.

thermal speed, $f(v)$ approaches a power-law form, with $\alpha \simeq 2\kappa$. Kappa distributions have been found to fit the data from satellite experiments well, with typical values of $2 < \kappa < 6$. Some examples are as follows: Christston *et al.* (1988) matched the plasma sheet distributions with $\kappa_i = 4.7$ and $\kappa_e = 5.5$, and distant magnetotail data with $\kappa_e = 5.5$; and in the earth's foreshock, Feldman *et al.* (1982, 1983) fitted the electrons with $3 < \kappa_e < 6$, while Lemaire's group (Pierrard and Lemaire, 1996; Maksimovic *et al.*, 1997) developed a Lorentzian ion exosphere model and associated solar wind model with κ-distributed coronal electrons, using, typically, $2 < \kappa < 6$. with $\kappa_e = 4$ yielding good agreement with electron distributions observed in the solar wind.

There is currently no accepted theoretical explanation for the common occurrence of kappa distributions in space. However, Treumann (2001) and Treumann *et al.* (2004) have developed the statistical mechanics of stable, nonlinear (turbulent) states far from equilibrium. This work provides a heuristic explanation for κ-distributions in collisionless plasmas. Leubner (2004) has invoked 'Tsallis statistics' (Tsallis (1988) revived earlier work on non-extensive entropy representations due to Daroczy (1970)) to describe both high-energy tails and core-halo distributions, reported, for example, by IMP 6 (Feldman *et al.*, 1973). We also note that theory predicts that Fermi acceleration at collisionless shocks should yield a spectral index $\alpha = \frac{3u_1}{u_1 - u_2} = \frac{3r}{r-1} \leq 4$, where $r = u_1/u_2$ is the shock compression ratio, *i.e.* $\kappa \leq 2$ (*cf.* Mace and Hellberg, 1995).

Extensive wave studies using κ models were carried out by Summers, Thorne, and co-workers (Summers and Thorne, 1990, 1991a,b, 1992; Thorne and Summers, 1991; Meng *et al.*, 1992; Xue *et al.*, 1993). These were confined to integer values of κ only. Extending this work, Mace and Hellberg (1995) obtained a generalized plasma dispersion function, Z_κ, for arbitrary real κ (in general, α, and hence κ, is not an integer), expressed in terms of the Gauss hypergeometric function ${}_2F_1$, which is analytically well-understood and easily calculated. Some of the earlier applications of Z_κ have been discussed in a review paper (Hellberg *et al.*, 2000a). The Z_κ function for the isotropic kappa distribution has been applied to (a) Electron plasma waves (EPW) (Mace and Hellberg, 1995; Mace *et al.*, 1998); (b) Ion-acoustic waves (IAW) (Mace *et al.*, 1996, 1998); (c) Electron-acoustic waves (EAW) (Mace *et al.*, 1999; Hellberg *et al.*, 1998, 2000b) (d) Electrostatic fluctuations (Mace *et al.*, 1996, 1998) and (e) Electromagnetic waves in a magnetoplasma, propagating parallel or perpendicularly to the magnetic field, $\mathbf{B_0}$ (Mace, 1996a,b, 1998, 2003, 2004).

Unfortunately, when waves in a magnetized plasma having an isotropic κ distribution are studied, the required integrals over perpendicular velocity space prove intractable for oblique propagation, ruling out that approach. Although Mace (1996a,b) has used a Gordeyev formulation to find a general time-like integral expression for the relevant dielectric tensor elements, the result is not particularly transparent.

As there is a preferred direction in space, *viz.* along the magnetic field, the assumption of an isotropic distribution is also not ideally suited to a magnetized plasma. One would expect equilibration (isotropization) to occur in the plane perpendicular to $\mathbf{B_0}$, leading to a Maxwellian form, together with an accelerated (power-law) behaviour along the field. We have thus introduced the 'kappa-Maxwellian' distribution, $f_{\kappa\mathrm{M}}$, which is a product of a 1-dimensional kappa distribution along the field, and a Maxwellian distribution in the perpendicular plane (Hellberg and Mace, 2002). This distribution is also a better fit to data such as that of Marsch (1991), which revealed velocity distribution contours that are elongated along a preferred direction. Although solar wind plasmas have been fitted by double-κ distributions (Pierrard *et al.*), $f_{\kappa\mathrm{M}}$ is a good approximation for wave-study purposes, leads to tractable expressions, and is an improvement on an isotropic κ distribution.

Using an equilibrium distribution $f_{\kappa\mathrm{M}}$, we have derived the generalized plasma dispersion function $Z_{\kappa\mathrm{M}}$ appropriate for electrostatic waves, and have used it to study obliquely propagating IAW (Hellberg and Mace, 2002) and EPW (Mace and Hellberg, 2003). We have recently found the general susceptibility tensor for the kappa-Maxwellian plasma, and this enables us to study obliquely propagating electromagnetic waves. Our first application is to the whistler mode (Cattaert *et al.*, 2005).

2. Waves in a κ Plasma

2.1. The Generalized Plasma Dispersion Function Z_κ

In the kinetic theory of waves, a pivotal role is played by the plasma dispersion function, which for a Maxwellian velocity distribution is the well-known Z-function of Fried and Conté (1961)

$$Z(\xi) = \frac{1}{\pi^{1/2}} \int_{-\infty}^{\infty} \frac{e^{-s^2} ds}{(s-\xi)}; \quad \text{Im}(\xi) > 0. \tag{2}$$

Substituting the κ distribution (Summers and Thorne, 1990), one obtains the generalized plasma dispersion function (Mace and Hellberg, 1995),

$$Z_\kappa(\xi) = \frac{1}{\pi^{1/2}\kappa^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma\left(\kappa-\frac{1}{2}\right)} \int_{-\infty}^{\infty} \frac{ds}{(s-\xi)(1+s^2/\kappa)^{\kappa+1}} \quad \text{Im}(\xi) > 0, \tag{3}$$

which is valid for arbitrary real $\kappa > 1.5$. It is seen that the integrand has branch points at $s = \pm i\sqrt{\kappa}$. A suitable deformation of the Landau contour leads to Pochhammer's integral, and hence one obtains

$$Z_\kappa(\xi) = \frac{i\left(\kappa+\frac{1}{2}\right)\left(\kappa-\frac{1}{2}\right)}{\kappa^{3/2}(\kappa+1)} {}_2F_1\left[1,\ 2\kappa+2;\ \kappa+2;\ \frac{1}{2}(1-\xi/i\sqrt{\kappa})\right], \tag{4}$$

i.e. Z_κ is proportional to the Gauss hypergeometric function, ${}_2F_1$, and can thus be easily manipulated analytically and calculated, *e.g.* using standard routines (*cf.* Press *et al.*, 1992) or *MATHEMATICA*.

One may deduce many relationships from the hypergeometric function form. For instance, the derivative relationship.

$$Z'_\kappa(\xi) = -2\frac{\left(\kappa+\frac{1}{2}\right)\left(\kappa-\frac{1}{2}\right)}{\kappa^2}\left\{1 + \frac{\kappa+1}{\kappa+\frac{1}{2}}\left(\frac{\kappa+1}{\kappa}\right)^{1/2} \xi Z_{\kappa+1}\left[\left(\frac{\kappa+1}{\kappa}\right)^{1/2}\xi\right]\right\},$$

reduces to the usual expression $Z'(\xi) = -2\{1 + \xi Z(\xi)\}$ for $\kappa \to \infty$.

2.2. Electrostatic Wave Studies

The dispersion relation for one-dimensional electrostatic waves in a κ distribution plasma is

$$\epsilon(\boldsymbol{k}, \omega) = 1 + 2\sum_j \frac{\omega_{pj}^2}{k^2\theta_j^2}\left\{\frac{2\kappa_j - 1}{2\kappa_j} + \frac{\omega}{k\theta_j} Z_{\kappa_j}\left(\frac{\omega}{k\theta_j}\right)\right\} = 0. \tag{5}$$

Using the expression for $Z'_\kappa(\xi)$, this may also be written as

$$1 - \sum_j \frac{(\kappa_j - 1)^2}{\kappa_j\left(\kappa_j - \frac{3}{2}\right)} \frac{\omega_{pj}^2}{k^2\theta_j^2} Z'_{\kappa_j - 1}\left[\left(\frac{\kappa_j - 1}{\kappa_j}\right)^{1/2} \frac{\omega}{k\theta_j}\right] = 0. \tag{6}$$

Although earlier work on this topic has been reviewed by Hellberg *et al.* (2000a), we summarize the main effects of the 'tail' particles for completeness.

(i) The dispersion relation shows significant, albeit monotonic, dependence on κ, the phase velocity increasing with κ. Damping of the high-phase velocity

EPW is strongly affected by the 'tail' particles in a low-κ distribution, but the κ dependence is, overall, complicated (Mace and Hellberg, 1995):

$$\gamma \simeq -\pi^{1/2}\frac{\Gamma(\kappa_e+1)}{\Gamma\left(\kappa_e-\frac{1}{2}\right)}\omega_{pe}(2\kappa_e-3)^{\kappa_e-1/2}\left(k^2\lambda_{De}^2\right)^{\kappa_e-1/2}. \tag{7}$$

(ii) Chateau and Meyer-Vernet (1991) showed that, in a kappa-distribution plasma, the additional superthermal particles reduce the Debye length, *viz.*

$$\lambda_{\kappa\alpha} \equiv \left[\left(\frac{\kappa_\alpha-\frac{3}{2}}{\kappa_\alpha-\frac{1}{2}}\right)\frac{\epsilon_0 T_\alpha}{n_\alpha q_\alpha^2}\right]^{\frac{1}{2}}. \tag{8}$$

This was also found by Bryant (1996), and, independently, by Mace *et al.* (1996, 1998). As shielding by electrons plays an important role in the IAW, this affects both dispersion and damping. The wave dynamics are similar to those of the usual IAW, but the replacement of λ_{De} by $\lambda_{\kappa e}$ leads to a monotonic dependence of the phase velocity on κ, $\omega_{pi}\lambda_{\kappa e} \propto [(\kappa_e-\frac{3}{2})/(\kappa_e-\frac{1}{2})]^{1/2}$. The effect of low κ on damping/growth (*e.g.* in the presence of a drift (*cf.* Meng *et al.*, 1992)) is not intuitively obvious. A number of parameters have to be considered to analyse the role of resonant particles, including the wave phase speed, the drift speed and the two thermal speeds. In addition one needs to fold into the discussion the excess fast population, and the associated reduction in other parts of the distribution. There are three ranges of $k\lambda_{De}$ (Mace *et al.*, 1998): for $k\lambda_{De} < 1.5$ damping shows a monotonic decrease with increasing κ; for $k\lambda_{De} > 4$ the trend is reversed; while intermediate $k\lambda_{De}$ values provide the transition.

(iii) Electrostatic fluctuations are enhanced in low-κ plasmas (Mace *et al.*, 1998). This follows because the plasma parameter $g = 1/n\lambda_\kappa^3$, a measure of discrete particle effects, is considerably increased as the Debye length is decreased with decreasing κ.

(iv) Electron-acoustic waves (EAW) are normal modes of a two-electron-temperature plasma, with a frequency $\simeq \omega_{pe}$. Hot electron Landau damping for $k\lambda_{Dh} \ll 1$ and cool electron Landau damping for short wavelength modes leaves waves with intermediate k weakly damped if the temperature ratio, $T_h/T_c \gg 1$. The EAW, EA instability, and EA solitons have been invoked to describe, for instance, broadband electrostatic noise (BEN) in the magnetotail (Tokar and Gary, 1984), electrostatic noise in the polar cusp (Schriver and Ashour-Abdalla, 1987), BEN and hiss in the cusp/cleft region (Mace and Hellberg, 1993). Assuming cold ions, cool Maxwellian electrons and hot κ-electrons, the dispersion relation is

$$1-\frac{\omega_{pc}^2}{2k^2v_c^2}Z'\left(\frac{\omega}{\sqrt{2}kv_c}\right)-\frac{(\kappa-1)^2}{\kappa\left(\kappa-\frac{3}{2}\right)}\frac{\omega_{ph}^2}{k^2\theta_h^2}Z'_{\kappa-1}\left[\left(\frac{\kappa-1}{\kappa}\right)^{1/2}\frac{\omega}{k\theta_h}\right]=0. \tag{9}$$

Mace *et al.* (1999) found weakly-damped existence domains in the space of $k\lambda_{Dc}$ and n_h/n_e as a function of temperature ratio, T_h/T_c, and compared the results with an earlier bi-Maxwellian study (Mace and Hellberg, 1990). Superthermal particles were found to affect both dispersion and damping. The phase velocity is $\simeq\omega_{pc}\lambda_{\kappa h}$, yielding increased hot electron Landau damping, and qualitatively, a decrease in κ has similar effects to an increase in temperature. Hellberg *et al.* (1998, 2000b) revisited the earlier experiment of Karlstad *et al.* (1984) for which neither a bi-Maxwellian nor a Maxwellian-waterbag assumption fitted both dispersion and damping. The experimental data were well fitted with the cold electrons having $\kappa_c = 50$ (Maxwellian), and hot electrons with $\kappa_h = 3.8$. This showed that the technique can be used to deduce the value of $\kappa_{c,h}$, *i.e.* it is a useful diagnostic for the shape of the velocity distribution functions.

2.3. Electromagnetic Waves in a Magnetoplasma

Early studies of parallel propagating electromagnetic waves in a magnetized κ or bi-Lorentzian plasma were carried out *inter alia* by Leubner (1983) and Summers and Thorne (1990), as well as by, for example, Thorne and Summers (1991), and Xue *et al.* (1993). Mace (1996a,b) obtained a general Gordeyev-type integral expression for the dielectric tensor in a uniform magnetoplasma. This general formalism, although complicated, enables one to recover the dispersion relations for electrostatic waves, and L and R modes propagating along the magnetic field, in terms of Z_κ. Hence Mace (1998) could model the nearly field-aligned '1 Hz' whistlers observed in Earth's electron foreshock.

More recently, Mace (2003, 2004) has carried out both electrostatic and electromagnetic calculations on electron Bernstein modes propagating perpendicularly to the magnetic field. The dispersion relation is expressed in terms of ${}_1F_2$ and ${}_2F_3$. Great accuracy was required in the numerical solution as poles are in close proximity to roots. He found significant differences in the electrostatic regime between Maxwellian and kappa distributions.

The Radio Plasma Imager (RPI) aboard the IMAGE satellite is an active experiment that stimulates short-range plasma wave echoes and plasma resonances, which occur, for instance, at nf_{ce}, f_{pe}, and f_{uh}, using the usual notation. In addition, above f_{pe}, one finds the Q_n resonances between cyclotron harmonics. Assuming these to be due to a zero-speed group of Bernstein waves enables one to determine n_e and $|\mathbf{B}|$, in a technique that is essentially the magnetospheric analogue of the topside ionospheric sounder (Benson *et al.*, 2003). Using a Maxwellian model to predict these frequencies, they could identify the Q_n resonances in the ionospheric environment, but found differences between theory and observations in the magnetospheric environment. Recently, Viñas *et al.* (2005) applied Mace's fully electromagnetic κ formalism to compare with the experimental data. The observed ratio of f_{pe}/f_{ce} was used and a single value of κ_e assumed for all resonances, the value being found by a best fit for the highest and lowest Q_n resonances. Intermediate resonances were

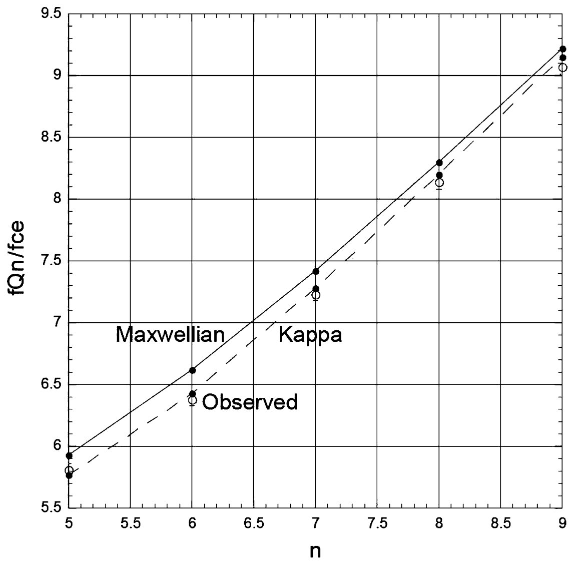

Figure 2. Plots of frequency of Q_n resonances, comparing the experimental values with Maxwellian predictions and those for $\kappa = 2.1$ (after Viñas *et al*., 2005).

then evaluated and very good agreement obtained, correcting the values obtained earlier *via* the Maxwellian assumption (Figure 2). Thus it is again seen that the κ approach can be used as a diagnostic for the velocity distribution by determining κ.

3. Waves in a Kappa-Maxwellian Magnetoplasma

3.1. The Generalized Plasma Dispersion Function $Z_{\kappa\mathrm{M}}$

We have seen that the isotropic assumption is not ideal for a magnetized plasma, because there is a preferred direction in space, *viz*. along $\mathbf{B_0}$. Hence Hellberg and Mace (2002) introduced the kappa-Maxwellian distribution function

$$f_{\kappa\mathrm{M}}(v_{||}, v_\perp) = \frac{1}{\pi^{3/2}\theta_\perp^2\theta_{||}} \frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)} \left(1 + \frac{v_{||}^2}{\kappa\theta_{||}^2}\right)^{-\kappa} \exp\left\{-\left(\frac{v_\perp}{\theta_\perp}\right)^2\right\},$$

with $\theta_\perp^2 = 2T_\perp/m$ being the square of the perpendicular thermal speed, and $\theta_{||}^2 = (2-3/\kappa)(T_{||}/m)$ that of the effective parallel thermal speed. After the perpendicular velocity integrals have been carried out, the generalized plasma dispersion function for $f_{\kappa\mathrm{M}}$ is found as

$$Z_{\kappa\mathrm{M}}(\xi) = \frac{1}{\pi^{1/2}} \frac{1}{\kappa^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma\left(\kappa-\frac{1}{2}\right)} \int_{-\infty}^{\infty} \frac{ds}{(s-\xi)(1+s^2/\kappa)^\kappa}; \quad \mathrm{Im}(\xi) > 0.$$

We note that for the 1-dimensional κ distribution the power in the denominator is κ, as opposed to $\kappa + 1$ for the isotropic distribution. It follows that

$$Z_{\kappa \mathrm{M}}(\xi) = i \frac{\left(\kappa - \frac{1}{2}\right)}{\kappa^{3/2}} {}_2F_1\left[1, 2\kappa; \kappa + 1; \frac{1}{2}\left(1 - \frac{\xi}{i\sqrt{\kappa}}\right)\right]. \tag{10}$$

The general relationship between $Z_{\kappa \mathrm{M}}$ and Z_κ is complicated – it is not merely a matter of replacing κ by $\kappa - 1$ in Z_κ. As the velocity-space integral to obtain $Z_{\kappa \mathrm{M}}$ is governed by the parallel component, $Z_{\kappa \mathrm{M}}$ applies to a 1-D kappa distribution with arbitrary normalized $g(v_\perp^2)$. For application to magnetized plasmas, a 2-D Maxwellian form for g is preferred, both on physical grounds and for reasons of tractability. Using well-known properties of the hypergeometric function, one can derive many relations – special arguments, derivatives, small and large argument expansions, etc. For instance, the complicated derivative relationship

$$Z'_{\kappa \mathrm{M}}(\xi) = -2\left(\frac{\kappa - \frac{1}{2}}{\kappa}\right)\left\{1 + \xi\left(\frac{\kappa + 1}{\kappa}\right)^{1/2} Z_{(\kappa+1,\,\mathrm{M})}\left[\left(\frac{\kappa+1}{\kappa}\right)^{1/2}\xi\right]\right\} \tag{11}$$

reduces, as $\kappa \to \infty$, to the usual relationship for Z', *viz.*

$$Z'(\xi) = -2[1 + \xi Z(\xi)]. \tag{12}$$

3.2. Electrostatic Waves

Obliquely propagating electrostatic waves in a magnetized plasma satisfy

$$1 + 2\sum_\alpha \frac{\omega_{pa}^2}{k^2\theta_{||\alpha}^2} \sum_{n=-\infty}^{\infty} W_n(b_\alpha)\frac{\theta_{||\alpha}^2}{\theta_{\perp\alpha}^2} C_n = 0, \tag{13}$$

where

$$C_n = \frac{n\omega_{\mathrm{c}}}{k_{||}\theta_{||}} Z_{\kappa \mathrm{M}}(\zeta_n) - \frac{\theta_\perp^2}{2\theta_{||}^2} Z'_{\kappa \mathrm{M}}(\zeta_n), \tag{14}$$

$W_n(b) = \exp(-b)I_n(b)$, $b_\alpha = (k_\perp^2\theta_{\perp\alpha}^2)/2\omega_{c\alpha}^2$, $\zeta_{n,\alpha} = (\omega - n\omega_{c\alpha})/k_{||}\theta_{||\alpha}$, $\omega_{c\alpha} = q_\alpha B_0/m_\alpha$, and $k^2 = k_\perp^2 + k_{||}^2$. No significant simplification is achieved by relating $Z'_{\kappa \mathrm{M}}$ to $Z_{\kappa \mathrm{M}}$. We have studied ion-acoustic and ion-cyclotron-like waves propagating obliquely to $\mathbf{B_0}$ (Hellberg and Mace, 2002). Coupling between these two fundamental modes leads to two hybrid modes, *viz.* the lower frequency acoustic mode, and the ion-cyclotron-sound wave.

In the acoustic regime, the phase speed varies monotonically with κ because of the change in parallel Debye length. The maximum frequency is, however, not at ω_{pi}, but close to $\Omega_i \cos\theta$, because of cyclotron resonance effects ($\Omega_i \equiv \omega_{\mathrm{ci}}$ is the ion cyclotron frequency). The expression for the Landau damping dependence on κ is more complicated than in the isotropic κ, magnetic field-free case.

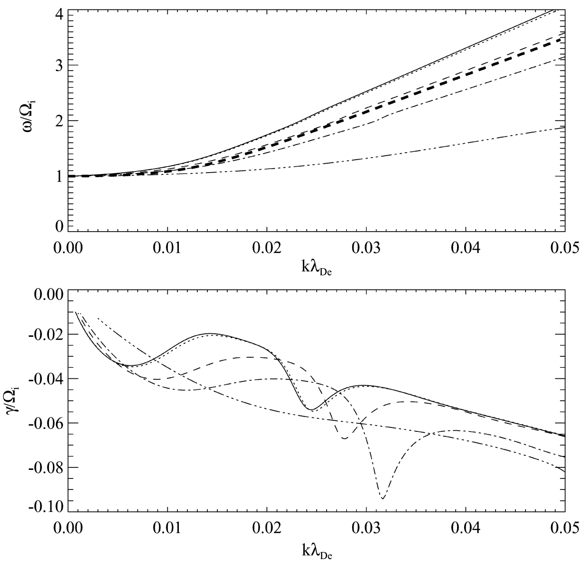

Figure 3. Frequency and damping of the ion-cyclotron-sound mode propagating at 30° to the field, showing the effect of cyclotron damping and of κ value. Here $\omega_{pe}/|\omega_{ce}| = 0.5$; the continuous curve is for $\kappa = 10$ (effectively a Maxwellian), and successive κ values are 8.5 (· · · ·), 3.5 (- - -), 2.5 (– · – · – · –) and 1.7 (– · · · –). The bold curve in the upper figure is an analytical approximation for $\kappa = 3.5$ (after Hellberg and Mace, 2002).

For the upper branch, the ion-cyclotron sound mode found for $\omega > \Omega_i$, the phase speed decreases monotonically for decreasing κ. This mode is also subject to marked cyclotron damping – enhanced damping for k values corresponding to $\omega \simeq n\Omega_i$ (Figure 3).

Mace and Hellberg (2003) have shown that the behaviour of generalized Langmuir modes is broadly similar to their Maxwellian equivalents. Oblique propagation leads to mixing of pure parallel Langmuir characteristics with Bernstein-like characteristics, giving rise to interesting variations in the frequency, $\omega_r(k)$, for nonzero propagation angle $\psi = \arctan(k_\perp/k_{||})$. But these waves are usually strongly Landau damped for $k\rho_e > 1$ where Bernstein-like characteristics are more prevalent. Nevertheless, the introduction of a source of free energy could see these wavenumbers destabilized. The most significant result on electron plasma waves is that the Landau damping rate of both upper and lower frequency modes is strongly dependent on the choice of κ. This obviously has implications for the interpretation of wave observations with $\omega \sim \omega_{pe}$ near Earth's foreshock in conjunction with a preponderance of superthermal electrons. Interpretations based on a bi-Maxwellian plasma model might lead to incorrect conclusions.

As is the case for the unmagnetized Langmuir wave in an isotropic κ-plasma, the dependence of γ on κ is complicated, especially for intermediate wavenumbers, $0.1 < k\rho_e < 0.5$. The high phase velocity of these waves for small k yields strong coupling with tail electrons and hence enhanced damping for small values of κ. Thus a highly accelerated electron velocity distribution (small κ) favours observation of shorter wavelength generalized Langmuir waves. At the other end of the κ scale, the bi-Maxwellian distribution ($\kappa \to \infty$) favours observation of the long wavelength waves where electromagnetic effects can appreciably alter the picture obtained from a purely electrostatic model.

Recently Podesta (2005) has rederived $Z_{\kappa M}$, and used it to study spatial Landau damping of EPW.

3.3. Electromagnetic Waves

From the Maxwellian form of the perpendicular part of $f_{\kappa M}$, it follows that the perpendicular velocity integrals required to study oblique propagation are now easily obtained (Stix, 1992; Swanson, 2003). On the other hand, the integral over parallel velocity leads to the elements of the susceptibility tensor χ_{ij} being found in terms of $Z_{\kappa M}$ and its derivative, and hence in terms of ${}_2F_1$. Specifically, the elements can be written in terms of the function C_n defined above (Cattaert *et al.*, 2005).

We can now study dispersion and damping/growth of a wide range of obliquely propagating electromagnetic waves in a magnetized space plasma, varying a number of different parameters. Special cases, such as obliquely-propagating electrostatic

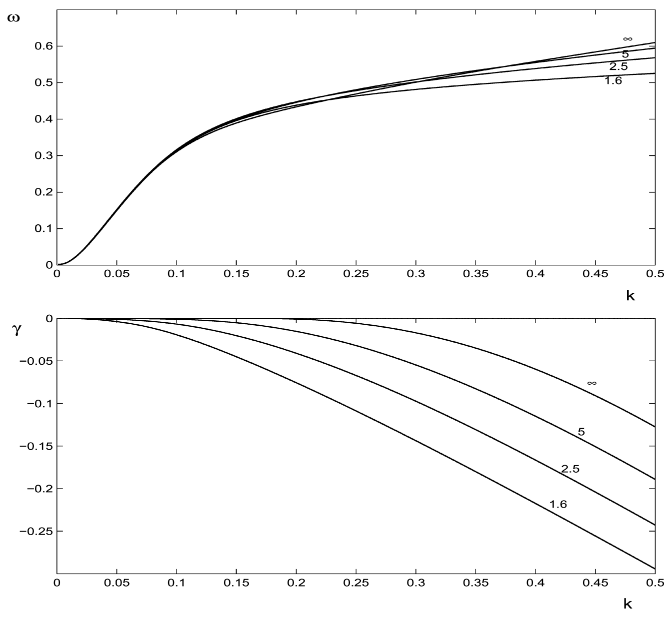

Figure 4. Frequency and damping of the whistler-like mode, propagating at 45°, showing κ dependence. Scales are normalized to $|\omega_{ce}|$ and $|\omega_{ce}|/\theta_{||}$; $(\omega_{pe}/\omega_{ce})^2 = 0.5$, $\theta_{||} = 0.1c$, $\theta_{\perp} = \theta_{||}$.

waves, and the field-free case, can be recovered. Interestingly, even for strictly parallel propagation, use of $f_{\kappa\mathrm{M}}$ leads to a result different from that found for the isotropic f_κ. That is, the perpendicular velocity distribution also plays a role in parallel propagation characteristics. The full dispersion relations have been derived for oblique and parallel propagating waves in the whistler frequency range (with $\omega \gg \Omega_i$). Figure 4 shows a preliminary result (for 45° propagation). The effect of the 'tail' particles is evident. Interestingly, the variation with κ for long wavelength waves is reversed in the case of parallel propagation.

4. Conclusion

Many space and astrophysical plasmas have power-law velocity distributions. The κ-distribution lends itself to modelling such plasmas, and the generalized plasma dispersion function Z_κ is useful for an isotropic κ distribution. Magnetized plasmas are better described by the kappa-Maxwellian distribution, $f_{\kappa\mathrm{M}}$, which leads to $Z_{\kappa\mathrm{M}}$. In both cases, the hypergeometric functions can be easily manipulated and evaluated. Both dispersion functions are useful for various waves in space plasmas. The function $Z_{\kappa\mathrm{M}}$ is needed for waves propagating obliquely to $\mathbf{B_0}$. The mathematical apparatus is now available to get away from the ubiquitous sums of Maxwellian and usual Z-function approach to kinetic wave and microinstability studies in space physics and astrophysics. As measurements provide improved accuracy, these tools can lead to a better understanding of wave data.

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