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Two Populations of Sunspot Groups and Their Meridional Motions

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Abstract

It is shown that the average meridional velocities of sunspot groups linearly depend on their average longitudinal velocities (solar rotation) with a high correlation coefficient of -0.95. The relationship differs for small, short-lived and large, long-lived groups. The meridional motions of sunspots do not have any pronounced global distribution law with latitude, but depend on their individual longitudinal velocities in a rotating coordinate system close to the Carrington one. The found relations indicate that the Coriolis force may play a role in driving the meridional motions of sunspot groups.

Keywords Solar activity · Sunspots · Sunspot group populations · Velocity fields

1. Introduction

The global velocity field plays a critical role in the generation and evolution of magnetic fields on the Sun. In kinematic astrophysical dynamo (e.g., $\alpha\Omega$ -dynamo), the gradient of solar/stellar rotation with latitude (differential rotation) shears the magnetic field, while the meridional flows in the North-South direction transport magnetic field from low to high latitudes. This is so-called meridional circulation (see Hanasoge, [2022](#page-11-0), for a review). In the heliographic coordinate system, the horizontal velocity field at/near the solar surface can be decomposed on zonal (in the longitudinal or East–West direction) and meridional components (in the latitudinal or North–South direction). In this coordinate system, the longitudinal component corresponds to solar rotation and the latitudinal component corresponds to meridional flow.

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Solar rotation was discovered in 1610 – 1611 by several early telescopic observers (G. Galileo, J. Fabricius, T. Harriot, K. Scheiner). Scheiner [\(1630](#page-13-0)) found a difference in the rotation rate between low and high latitudes — hence the term "differential rotation", which was coined later. In the 19th century, solar rotation was studied by tracking sunspots (Carrington, [1863;](#page-11-0) Spörer, [1894\)](#page-13-0) and faculae (Belopolsky, [1890](#page-11-0); Stratonoff, [1896](#page-13-0)) or via direct Doppler velocity measurements (Dunér, [1890,](#page-11-0) [1891](#page-11-0)). Studies conducted in the 20th century used feature tracking (tracer method, e.g., Maunder and Maunder, [1905](#page-12-0); Chevalier, [1910](#page-11-0); Newton and Nunn, [1951;](#page-13-0) Lustig, [1983;](#page-12-0) Howard, Gilman, and Gilman, [1984](#page-12-0); Balthasar, Vazquez, and Woehl, [1986;](#page-11-0) Pulkkinen and Tuominen, [1998a](#page-13-0)) or the direct Doppler velocity measurements (e.g., Halm, [1906](#page-11-0); Adams, [1907](#page-11-0), [1909;](#page-11-0) Storey and Wilson, [1911](#page-13-0); Adams, [1911](#page-11-0); Belopolsky, [1933;](#page-11-0) Howard and Harvey, [1970;](#page-12-0) Beck, [2000](#page-11-0)).

Based on 1853 – 1861 observations, Carrington ([1863\)](#page-11-0) obtained an analytical expression for the variation of angular velocity of solar rotation ω with latitude φ (here we converted coefficients expressed by Carrington in arcmin to degrees):

$$
\omega = 14.42^{\circ} - 2.75^{\circ} \sin^{7/4} \varphi. \tag{1}
$$

Later, Carrington's expression was replaced by E. Faye's formula (e.g., Chevalier, [1910;](#page-11-0) Delury, [1939](#page-11-0)):

$$
\omega = 14.37^{\circ} - 3.10^{\circ} \sin^{2} \varphi.
$$
 (2)

Faye's expression (Equation 2) was used by many researchers for different solar features (for reviews, see Paternò, [2010](#page-13-0); Mahajan et al., [2023](#page-12-0)). The choice of functional dependence in Faye's formula is based on theoretical models in which the angular rotation of the Sun, $ω$ in spherical coordinates, is described as a function of solar radius, R_{\odot} , and co-latitude *θ* (Vitinsky, Kopecký, and Kuklin, [1986\)](#page-14-0)

$$
\omega(R_{\odot}, \theta) = C_0 \omega_0(R_{\odot}) + \sum_{k=1}^{\infty} C_k \omega_k(R_{\odot}) P_k(\cos \theta), \tag{3}
$$

where $P_k(\cos \theta)$ is the Legendre polynomial of degree k. Assuming symmetry about the equator (Wavre, [1932;](#page-14-0) Lichtenstein, [1933](#page-12-0)) and making use of the relation sin $\varphi \equiv \cos \theta$, the solar rotation can be represented by Legendre polynomials of even degrees. Using Legendre polynomials of second degree, one will arrive to the Faye's formula (Equation 2), and the Legendre polynomials of fourth degree yield another expression used in later studies of solar rotation:

$$
\omega = a + b \sin^2 \varphi + c \sin^4 \varphi. \tag{4}
$$

Determining solar rotation using sunspots as tracers can be subject to several errors (Schroeter, [1985](#page-13-0)). For example, in the growth phase of the active region, the leading and following spots move in the opposite direction from the center of the region. Thus, using leading or following spots separately, one can arrive at rotation velocities that are faster or slower than those of active regions (Osipova and Nagovitsyn, [2022](#page-13-0)). For an overview of various errors in determining the heliographic coordinates of tracers affecting measurements of the horizontal velocity field on the Sun, see Nagovitsyn and Nagovitsyna [\(1996](#page-13-0)).

Meridional circulation is important for modeling solar and stellar activity cycles (Kitchatinov, [2016\)](#page-12-0). It has been studied using direct Doppler measurements (Duvall, [1979;](#page-11-0) Labonte and Howard, [1982;](#page-12-0) Hathaway, [1996](#page-11-0); Ulrich, [2010,](#page-14-0) and references therein), and by tracking solar features (Tuominen, [1952](#page-13-0); Becker, [1954a,b;](#page-11-0) Ward, [1964,](#page-14-0) [1965](#page-14-0); Bumba and Howard, [1965;](#page-11-0) Coffey and Gilman, [1969;](#page-11-0) Ward, [1973](#page-14-0); Tuominen, [1976](#page-13-0); Balthasar and Woehl, [1980](#page-11-0); Howard and Labonte, [1981](#page-12-0); Arevalo et al., [1982](#page-11-0); Tuominen, Tuominen, and Kyröläinen, [1982](#page-14-0); Wang, Nash, and Sheeley, [1989;](#page-14-0) Komm, Howard, and Harvey, [1993;](#page-12-0) Pulkkinen and Tuominen, [1998b](#page-13-0); Švanda, Kosovichev, and Zhao, [2007;](#page-13-0) Hathaway, [2012](#page-12-0)). The existence of a meridional circulation below the photosphere has been confirmed by helioseismology techniques (Zhao et al., [2013](#page-14-0); Choudhuri, [2021](#page-11-0), and references therein).

The earliest theoretical prediction of the meridional circulation (Eddington, [1925\)](#page-11-0) is usually associated with the Eddington-Sweet circulation (e.g., Hanasoge, [2022](#page-11-0)). We note, however, that in his work on solar rotation, Bélopolsky [\(1888](#page-11-0)) refers to a theoretical work by Prof. Zhukovsky (published in 1885 in article entitled "Sur le mouvement d'un corps solide qui a des cavités remplies par un liquide homogéne"), who suggested that the presence of radial gradient of velocity in rotating liquid sphere would lead to development of flows in the meridional direction. Specifically, if the rotation velocity decreases from the center of sphere to its surface, a meridional flow near the surface of sphere would be directed from the equator to the poles. Bélopolsky [\(1888](#page-11-0)) also state that Zhukovsky confirmed his theoretical prediction using laboratory experiments by rotating water-filled glass spheres with small filaments suspended in the water as tracers. Bélopolsky [\(1888](#page-11-0)) used photoheliographic observations carried out at the Moscow Observatory from 1877 to 1883 to study both solar differential rotation and the latitudinal displacement of sunspots. He concluded that his mea-surements were in agreement with Prof. Zhukovsky predictions. Sampson ([1895\)](#page-13-0) used early results of Bélopolsky as a starting point for his theoretical considerations. He confirmed that a negative gradient of angular velocity along solar radius was responsible for poleward drift of sunspots near solar surface. While physics behind these early experiments (the effect of drag force in rotating liquid sphere) is different from the solar plasma environment, the work of Bélopolsky [\(1888](#page-11-0)) and Sampson ([1895\)](#page-13-0) can be considered as the earliest discussions of the meridional circulation on the Sun.

Unlike the differential rotation, meridional motions of sunspots are less pronounced. Most studies show that while the center of a group slowly drifts in poleward direction, individual sunspots move away from the central latitude of activity: sunspots in the low-latitude portion of a group move towards the equator, while those in a high-latitude portion, exhibit a poleward motion (Tuominen, [1955](#page-13-0), [1966;](#page-13-0) Tuominen, Tuominen, and Kyrolainen, [1983;](#page-14-0) Kambry et al., [1991;](#page-12-0) Lustig and Woehl, [1991\)](#page-12-0). Some studies do not draw clear conclusions about the direction of meridional motions of sunspot groups (Hanslmeier and Lustig, [1986;](#page-11-0) Howard and Gilman, [1986](#page-12-0); Lustig and Hanslmeier, [1987;](#page-12-0) Balthasar and Fangmeier, [1988\)](#page-11-0).

For completeness, we also note that a number of studies have shown that the meridional motions of sunspots and their groups vary depending on the solar cycle (Lustig and Hanslmeier, [1987](#page-12-0); Kambry et al., [1991](#page-12-0); Howard, [1991](#page-12-0); Javaraiah, [1999,](#page-12-0) [2010;](#page-12-0) Javaraiah and Ulrich, [2006](#page-12-0)), the lifetime of the group (Tuominen, [1942](#page-13-0); Lustig and Woehl, [1991,](#page-12-0) [1994;](#page-12-0) Wöhl and Brajša, [2001\)](#page-14-0), and its area (Sivaraman et al., [2010](#page-13-0)). Ruždjak et al. ([2018\)](#page-13-0) found a correlation between rotation velocity residuals and meridional motions of sunspot groups, which they linked to Reynolds stress as the main driver of differential rotation.

Recently, several studies established a clear bimodality in the distribution of sunspots and sunspot groups by their size and the magnetic flux (e.g., Tlatov and Pevtsov, [2014;](#page-13-0) Muñoz-Jaramillo et al., [2015;](#page-13-0) Nagovitsyn et al., [2016;](#page-13-0) Nagovitsyn and Pevtsov, [2016](#page-13-0), [2021;](#page-13-0) Kostyuchenko, [2017;](#page-12-0) Forgács-Dajka, Dobos, and Ballai, [2021](#page-11-0); Mandal et al., [2021](#page-12-0)). The idea of bimodality originates in early work by Dmitrieva, Kopecký, and Kuklin ([1968\)](#page-11-0) who interpreted the distribution of area of sunspot groups as consisting of two distinct populations. Later, Kuklin [\(1980](#page-12-0)) found further confirmation of this idea. Nagovitsyn, Pevtsov,

and Livingston ([2012\)](#page-13-0), considering the main spots in groups, showed that their areas form two lognormal distributions. Muñoz-Jaramillo et al. ([2015\)](#page-13-0), however, suggested that larger sunspot groups are distributed lognormally, but the smaller ones follow the Weibull distribution. Nagovitsyn, Pevtsov, and Osipova [\(2018](#page-13-0)) found that sunspots of different sizes and lifetimes form two distinct groups in their rotation. In an earlier study, Howard, Gilman, and Gilman [\(1984\)](#page-12-0) found a difference in rotation rates for sunspots of different sizes without considering their lifetimes. It is currently unknown whether the meridional motions derived from sunspot tracing also exhibit bimodality similar to the differential rotation. This is the main goal of our investigation. Section 2 describes the dataset and the methodology. Sections [3](#page-4-0) and [4](#page-6-0) discuss two populations of sunspot groups and their rotation rates and the meridional motions. Section [5](#page-7-0) describes the relation between longitudinal and meridional velocities of sunspot groups, and in Section 6 we discuss our findings.

2. Data and Methodology

We employ historical data from the Royal Greenwich Observatory (RGO) from 1874 to 1976 (for the description of the dataset, we refer the reader to solarscience.msfc.nasa.gov/greenwch.shtml**)**. A detailed description of the RGO catalog can be found elsewhere (e.g., Willis et al., [2013;](#page-14-0) Muñoz-Jaramillo et al., [2015](#page-13-0)). For this study, we used daily whole-spot areas corrected for foreshortening and the Carrington coordinates of sunspot groups. Groups were identified on the basis of the catalog of group numbers and their Carrington coordinates. The lifetime of each group was defined as the period of observations in days when the group was present on the solar disk. Within our limitation, each disk passage of recurrent groups is treated as an independent group. For each group, for the duration of its entire lifetime on the visible disk, we determined their horizontal velocities in longitudinal (solar rotation) and latitudinal (meridional) directions using two approaches. If the groups' lifetime was longer than two days, the horizontal velocities were determined for the entire time of their existence by fitting the linear polynomial either to group's longitudes vs. time for solar rotation determination or the latitudes vs. time for meridional motion determination. If the group existed for only two days, the displacements were computed as the difference of coordinates on consecutive days divided by the difference in times of observations. Single-day groups were not included in this study. Figure [1](#page-4-0) shows examples of determination of horizontal velocities for two RGO groups: No. 15861 and No. 20270. Sometimes it seems that the considered sunspot groups show more complex dynamics than linear displacements (such as, for example, quasi-sinusoidal oscillations of group No. 15861 in the meridional direction). Such movements are beyond the scope of our consideration, and we study only the velocities in a linear approximation. For group No. 15861, the velocity in the longitudinal direction is $\alpha = 0.383 \pm 0.043$, and the one in the meridional direction is $\beta = -0.029 \pm 0.034$. For group No. 20270, $\alpha = -0.74 \pm 0.43$, $\beta = -0.030 \pm 0.056$ in degrees per day. Since calculations are made relative to the Carrington coordinates, *α* represents a deviation from Carrington rotation rate, which is $360/27.2753 = 13.1988$ degrees per day [deg/day]. *β* corresponds to meridional motions. The sidereal rotation, which we will also mention later, is greater than the synodic one by 0.9856 deg/day (Allen, [1973\)](#page-11-0).

Some authors have used the simplest way to find velocities, namely by the difference of coordinates between neighboring days. This led to a large spread of values. By applying the least-squares method, we reduce the uncertainty. Even for groups with a short lifetime of 3 – 5 days, this improves numerical estimates. Among other things, the least-squares method

Figure 1 Illustration of calculation of longitudinal velocity *α* (solar rotation relative to the Carrington rotation rate, upper panels) and the meridional motion *β* (lower panels) by the least-squares method (straight lines) for RGO sunspot groups No. 15861 (left) and No. 20270 (right).

Figure 2 Occurrence histograms of the error values of longitude (left) and latitude (right) velocities of sunspot groups, medians of their distributions, and sample size *N.* .

makes it possible to estimate the errors of the individual sunspot groups' velocities determination. The distribution of these errors is shown in Figure 2. In this figure, in addition to the histograms of various error values, the total number of measured groups *N* and the median of distributions $M\sigma_\alpha$ and $M\sigma_\beta$ are also shown. It is important that the number of individual measurements is very large, and so we can successfully apply statistical methods.

3. Two Systems of Differential Rotation and Two Populations of Sunspot Groups

Figure [3](#page-5-0) shows solar rotation rates derived for each component of bimodal distribution of sunspot groups. Nagovitsyn, Pevtsov, and Osipova [\(2018](#page-13-0)) showed that two systems of (sidereal in this case) differential rotation of sunspot groups exist simultaneously on the Sun: a fast one, T1, with

$$
\omega = (14.616 \pm 0.013) - (2.88 \pm 0.13) \sin^2 \varphi \tag{5}
$$

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and a slow one T2, with

$$
\omega = (14.3499 \pm 0.0039) - (2.869 \pm 0.043) \sin^2 \varphi, \tag{6}
$$

Sine Latitude

displayed in Figure 3 with dashed and solid lines, respectively.

The advantage of the applied method proposed by Nagovitsyn, Pevtsov, and Osipova ([2018\)](#page-13-0) is that for each latitude interval, we do not look for some average velocity value, but statistically identify real modes for two subgroups. As it turned out, the general pattern of the rotation rate is represented as a superposition of Equations [5](#page-4-0) and 6.

Note that when constructing histograms in a logarithmic grid with a small step, with finite accuracy of area measurements (for RGO, this is 1 millionth of the solar hemisphere), gaps with zero values arise. To mitigate the effect of such gaps, Nagovitsyn and Pevtsov ([2021\)](#page-13-0) employed a "precision randomization" approach, based on the assumption that all measured areas have a random component within the measurement uncertainty. The approach improved the separation of two populations of sunspot groups. In an earlier paper, Nagovitsyn and Pevtsov (2016) found that the parameter that separates the populations is the lifetime: $LT \le 5$ days, and $LT > 5$ days, corresponding to smaller and larger sunspot groups. The first of these populations was named SSG (Small Short-lived Groups), and the second LLG (Large Long-lived Groups). The division of populations by the lifetime parameter should not be surprising: the Gnevyshev–Waldmeier rule (Gnevyshev, [1938](#page-11-0); Waldmeier, [1955\)](#page-14-0) links the area and the lifetime of groups.

Returning to the horizontal velocity field of sunspot groups, we note that Nagovitsyn, Pevtsov, and Osipova ([2018\)](#page-13-0) have shown that two populations of sunspot groups also have a different type of rotation: the SSG population has a single-mode fast rotation, and the LLG population has a two-mode rotation – both fast and slow. These modes correspond to the "fundamental" modes T1 and T2 – see Figure 3. The "splitting" of LLG rotation into two modes was studied in the article by Osipova and Nagovitsyn ([2022\)](#page-13-0).

Regarding the physical nature of two populations of sunspot groups, Nagovitsyn, Pevtsov, and Livingston [\(2012](#page-13-0)) noted that it may be indicative of the action of a distributed dynamo on the Sun (Brandenburg, [2005\)](#page-11-0), with the LLG population associated with a deepseated dynamo, and the SSG population with a more shallow dynamo (Garcia de La Rosa, [1981;](#page-11-0) Hiremath, [2002](#page-12-0); Sivaraman and Gokhale, [2004](#page-13-0)).

4. Two Populations of Sunspot Groups and the Dependence of Meridional Velocities on Latitude

Let us now analyze the statistical distributions (grouped sample) of the rate of meridional displacement (meridional velocity) β , expressed in deg/day in the Carrington rotation system, separately for SSG and LLG populations.

The resulting distribution of areas of selected groups deconvolved into two (SSG and LLG) components is shown in Figure 4. By its properties, the distributions are close to lognormal, in agreement with Nagovitsyn and Pevtsov [\(2016](#page-13-0), [2021](#page-13-0)). There are some deviations from the pure lognormal distribution for the SSG component, which we attribute to the absence of groups with $LT = 1$ day in the statistics.

First, using our approach, we consider the dependence of meridional motions on latitude $\beta = \beta(\varphi)$: for samples of sine values of the heliographic latitude with a step of 0.05, we construct histograms of the occurrence of various values. Then, using the Levenberg-Marquardt algorithm (Levenberg, [1944](#page-12-0); Marquardt, [1963\)](#page-12-0) LMA, implemented in the ORIGIN software ([www.originlab.com\)](http://www.originlab.com), we approximate the histograms with a Gaussian and find the position of its maximum with its standard error. Note that the LMA is an iterative algorithm that allows solving nonlinear problems of the least-squares method; its detailed description and advantages can be found in the book by Madsen, Nielsen, and Tingleff ([2004\)](#page-12-0). We will carry out the procedure both for all groups and for LLG and SSG populations separately – see Figure [5a](#page-7-0). Positive and negative values indicate poleward and equatorward motions, respectively. We constructed similar dependencies for the absolute values of latitudes – Figure [5](#page-7-0)b.

Larger confidence intervals in Figure [5](#page-7-0) for the SSG can be explained by the fact that a smaller number of daily values were used to calculate the velocities due to the shorter lifetime of this population. There is no statistically pronounced regular difference for the values between the populations in Figure [5](#page-7-0). As well as the expressed law $\beta = f(\sin \varphi)$: values differ significantly from zero ($>$ 3 σ) for latitudes in the ranges $\sin \varphi \in [-0.45, -0.40]$, [0*,* +0*.*10] and only for the LLG population. There is a tendency: closer to the equator, the spots move towards the equator, at high latitudes – towards the pole. But the trend is weak. The conclusion that we draw from this is the following: in contrast to the differential rotation, meridional motions depend weakly on latitude. That is, there is no pronounced dependence of meridional velocities on latitude, determined from sunspot groups as tracers (as was also

Figure 5 Changes in the average meridional velocities *β* of sunspot groups: (a) depending on the sine of latitude; (b) as a function of the sine of the absolute value of the latitude. Filled squares correspond to LLG groups, SSG are shown by open circles. The error bars correspond to the standard errors of the means. Positive and negative values indicate poleward and equatorward motions, respectively.

found in Hanslmeier and Lustig, [1986](#page-11-0); Howard and Gilman, [1986;](#page-12-0) Lustig and Hanslmeier, [1987;](#page-12-0) Balthasar and Fangmeier, [1988\)](#page-11-0).

Let us look at the distribution of linear velocities (in m s^{-1}) velocities of individual meridional motions (Figure 6). We see that, in comparison with Figure 5, the spread is very large, and the values reach tens of m s⁻¹. The most frequent values of average velocities in absolute value are no more than 5 m s⁻¹. Their contribution to the distribution of individual velocities is 22.7% for the LLG population, 10.1% for the SSG, and 17.2% without division into populations. The number of groups is $N(SSG) = 9341$, $N(LLG) = 12121$.

5. Relation of Meridional and Longitudinal Velocities of Sunspot Groups

How are meridional and longitudinal velocities related? Let us construct empirical dependencies of meridional velocities *β* on longitudinal velocities *α* in the Carrington rotation sys-tem for SSG and LLG populations separately (Figures [7a](#page-9-0) and b). Extreme values $|\alpha| > 2.0$ deg/day and |*β*| *>* 1*.*5 deg/day were removed from consideration (4.6% of the total sample turned out to have such values).

For SSG, the dependence has the form (*r* is the Pearson's correlation coefficient):

$$
\beta = (1.72 \pm 0.52) \cdot 10^{-2} - (9.89 \pm 0.63) \cdot 10^{-2} \alpha, \quad r = -0.16_{-0.14}^{-0.18}.
$$
 (7)

For LLG:

$$
\beta = (0.87 \pm 0.14) \cdot 10^{-2} - (7.66 \pm 0.31) \cdot 10^{-2} \alpha, \quad r = -0.22_{-0.20}^{-0.24}.
$$
 (8)

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The confidence intervals in Equations [7](#page-7-0) and [8](#page-7-0) were found using the Fisher transformation and correspond to the probability $p = 0.95$. The correlation coefficients are small; however, we understand that Equations [7](#page-7-0) and [8](#page-7-0) describe ungrouped empirical values containing random errors. At the same time, the probability of randomness of the dependence $β$ from $α$ in Equations [7](#page-7-0) and [8,](#page-7-0) determined by the Student's *t*-test, is $p < 0.0001$.

Therefore, to derive the global law $\beta = f(\alpha)$, we went the same way as for $\omega = f(\sin \varphi)$ in Figure [3](#page-5-0), considering the maximums of statistical distributions β for α in intervals with a step of 0.1 deg/day for the SSG and LLG populations separately. The obtained result is illustrated in Figure 8.

Regression lines (data weighted with confidence intervals) are: for SSG

$$
\overline{\beta} = (1.50 \pm 0.34) \cdot 10^{-2} - (9.79 \pm 0.58) \cdot 10^{-2} \overline{\alpha}, \quad r = -0.95^{0.98}_{0.99},\tag{9}
$$

and for LLG

$$
\overline{\beta} = (0.426 \pm 0.071) \cdot 10^{-2} - (7.77 \pm 0.18) \cdot 10^{-2} \overline{\alpha}, \quad r = -0.95^{0.98}_{0.90} \tag{10}
$$

(compare Equations $7-8$ $7-8$ $7-8$ with Equations $9-10$).

The correlation coefficients are high, which means that Equation 9 and Equation 10 express the law of average meridional motions. Comparing Figure [8](#page-10-0) with Figure [5](#page-7-0), we can conclude that the meridional motions of sunspots do not have any pronounced global distribution law with latitude, but depend on the individual longitudinal velocity in a rotating coordinate system close to the Carrington one.

Comparing Equations 9 and 10, we find that both the intercepts and the slopes differ by more than 3*σ* . Thus, in addition to our previous work (Nagovitsyn, Pevtsov, and Livingston, [2012;](#page-13-0) Nagovitsyn et al., [2016](#page-13-0); Nagovitsyn, Pevtsov, and Osipova, [2017,](#page-13-0) [2018;](#page-13-0) Nagovitsyn

meridional velocities of individual sunspot groups on their longitudinal ones in the Carrington grid for (a) SSG population; (b) LLG population.

and Pevtsov, [2016](#page-13-0), [2021](#page-13-0)), we found that the LLG and SSG populations differ in one more parameter – the dependencies of average latitudinal velocities of sunspot groups in different 0.05 sine-latitude intervals on their corresponding longitudinal velocities, $\beta = f(\alpha)$.

6. Summary

We believe that the most likely reason for the significant correlation between meridional and longitudinal motions of sunspot groups is the role of Coriolis force. The movements of spots in the Carrington grid ahead of its rotation should lead to a component of the force directed to the equator, and vice versa, lagging behind in its rotation – directed to high latitudes. If so, then by the value $\overline{\alpha}$ for $\overline{\beta} = 0$ in Equation [9](#page-8-0) and Equation [10](#page-8-0) we can check the deviation of the rotation of our reference grids, in which the Coriolis force develops, from the Carrington one. We get:

SSG: $\overline{\alpha}(\overline{\beta} = 0) = 0.153 \pm 0.036$ deg/day, LLG: $\overline{\alpha}(\overline{\beta} = 0) = 0.0548 \pm 0.0092$ deg/day.

As we can see, the reference rotation systems of both populations differ significantly from the Carrington one: for SSG at the level of 4*.*25*σ* , and for LLG 6*.*0*σ* . Thus, the synodic reference rotation in our assumption will be 13.352 deg/day for SSG and 13.254 deg/day for LLG. The corresponding sidereal velocities will be 14.338 and 14.240 deg/day. Heliographic positions are used here.

Comparing Equations [\(9\)](#page-8-0) and [\(10\)](#page-8-0), we found that both the free terms and the slopes of the populations differ by more than 3*σ* . Thus, in addition to our previous results, we found that the LLG and SSG populations differ significantly in one more parameter, namely, the dependence of the mean latitudinal velocities of sunspot groups on the rotation speed in the Carrington grid. This does not contradict our earlier hypothesis that populations may be born in different parts of the Sun in depth.

The results obtained in this article seem to us quite unexpected, and their relation to the processes responsible for the meridional circulation (see Choudhuri, [2021;](#page-11-0) Hathaway, Upton, and Mahajan, [2022](#page-12-0)) still needs to be clarified. Our results show that the direction of longitude motions of sunspot groups in the Carrington grid also determines the direction of their meridional motions. For example, for stars whose equator rotates more slowly than the poles, the meridional flow from the poles to the equator. Our finding that the meridional flow is determined by the differential rotation and that the meridional velocity may differ for features of different magnetic flux (SSG and LLG populations) seem to be in agreement with the recent successful reproduction of solar-like differential rotation (fast equatorial rotation, slow polar rotation) by high-resolution computer simulations (Hotta and Kusano, [2021;](#page-12-0) Hotta, Kusano, and Shimada, [2022\)](#page-12-0), which may be the first step in resolving the so called convective conundrum (O'Mara et al., [2016](#page-13-0)).

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Declarations

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