

Sunspot Positions and Areas from Observations by Galileo Galilei

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Abstract Sunspot records in the seventeenth century provide important information on the solar activity before the Maunder minimum, yielding reliable sunspot indices and the solar butterfly diagram. Galilei's letters to Cardinal Francesco Barberini and Marcus Welser contain daily solar observations on $3 - 11$ May, 2 June – 8 July, and $19 - 21$ August 1612. These historical archives do not provide the time of observation, which results in uncertainty in the sunspot coordinates. To obtain them, we present a method that minimizes the discrepancy between the sunspot latitudes. We provide areas and heliographic coordinates of 82 sunspot groups. In contrast to Sheiner's butterfly diagram, we found only one sunspot group near the Equator. This provides a higher reliability of Galilei's drawings. Large sunspot groups are found to emerge at the same longitude in the northern hemisphere from 3 May to 21 August, which indicates an active longitude.

Keywords Sunspots · Sun: activity · Solar cycle, observations

1. Introduction

Sunspot indices are the base of our knowledge of the long-term behavior of the solar cycle. The period of the seventeenth century is covered by the group sunspot numbers $[R_{\sigma}]$ (Hoyt and Schatten, [1998](#page-20-0)), which were recently revised and recalibrated as the group numbers (Clette *et al.*, [2014,](#page-20-1) [2015](#page-20-2); Svalgaard and Schatten, [2016](#page-20-3); Vaquero *et al.*, [2016\)](#page-20-4). Interest in historical records on sunspots is due to the puzzle of solar-activity behavior in the past.

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Recently, the early telescopic observations, among them records by Marius, Reccioli, father and son Fabricius, and Saxonius, were analyzed by Neuhäuser and Neuhäuser [\(2016](#page-20-5)). Arlt *et al.* [\(2016](#page-19-0)) measured sunspot positions and areas of sunspots for 1611 – 1631 from observations by Christoph Scheiner. Another early observer who provided drawings of sunspots seen through a telescope was Galileo Galilei (Reeves, [2008](#page-20-6)).

Galilei [\(1613\)](#page-20-7) published an *Istoria e dimostrazioni intorno alle macchie solari e loro accidenti comprese in 3 lettere*, which yields continuous daily records of the solar disk from 2 June to 8 July and 19, 20, and 21 August 1612. The Galileo Project by Al Van Helden and Owen Gingerich provides the processed drawings of June and July (galileo.rice.edu). For these two months, Casas, Vaquero, and Vazquez ([2006\)](#page-20-8) obtained the heliographic coordinates of the sunspots. To evaluate the solar rotation, they measured the positions of only those groups or subgroups of sunspots that lived at least two days. The near-limb sunspots were excluded from the analysis.

Other observational reports, stored in the Vatican Library among the most precious manuscripts and books, are the originals of Galilei's letters to Cardinal Maffeo Barberini (Pope Urban VIII) and to his nephew Cardinal Francesco Barberini (Manuscript Barb.lat.6479 of the Biblioteca Apostolica Vaticana [digi.vatlib.it/view/MSS_Barb.lat.6479/](http://digi.vatlib.it/view/MSS_Barb.lat.6479/0053) [0053](http://digi.vatlib.it/view/MSS_Barb.lat.6479/0053)). One of these letters contains drawings for 3 – 11 May 1612.

In this article, we process sunspot drawings made by Galileo Galilei between May and August 1612. In Section [2](#page-1-0) we define sunspot positions in a rectangular grid. Section [3](#page-3-0) compares sunspot-group areas and counts to those found by Sakurai [\(1980](#page-20-9)) and Hoyt and Schatten ([1998\)](#page-20-0). In the following sections, we present the method that we used to deduce the heliographic positions of sunspot groups and the time of observation. In Section [6](#page-10-0) we show the resulting butterfly diagram. Then we briefly discuss an activity cluster that existed from May 3 to August 21. Section [8](#page-13-0) lists our main conclusions. The appendix explains the sorting of sunspots into groups. An Electronic Supplementary Material file provides the results.

2. Sunspot Positions in the Rectangular Coordinate System

In order to recognize the solar disk position, the images of $3 - 11$ May, 2 June – 8 July, and 19 – 21 August 1612 were cut to the size of 1626×1626 pixels, 800 \times 800 pixels, and 1098×1098 pixels, respectively. Note that the August images were slightly asymmetrical; the horizontal size of the solar disk is wider than the vertical size. We slightly compressed these drawings in the horizontal direction.

First, the rectangular coordinate grid and solar radius [*R*] were set, see Figure [1](#page-2-0)a. Green frames denote the rightmost, leftmost, topmost, and lowermost intensity minima (red dots). These four points clearly define the rectangular coordinate system, with the origin in the center of the solar disk and the fixed radius $[R]$ of the Sun (Figure [1](#page-2-0)b). Blue and pink lines are the *x*- and *y*-axes, and the red circle is the solar limb.

Next, sunspots in the drawings have to be collected into groups. To divide paper defects from the real small sunspots, we compared four copies of the book *Istoria* (Galilei, [1613](#page-20-7)). We used the classification of sunspot groups by McIntosh ([1990\)](#page-20-10) and suggest that the linear size of a sunspot group is on average limited to $7°$ degrees of latitude and $15°$ of longitude. In several complex cases, groups were assigned arbitrarily. The separation of sunspots into groups is a non-trivial task, and therefore we detail our sorting method graphically in the appendix.

The rectangular grid with the origin in the center of the Sun provides *x*-, *y*-, and *z*coordinates of each object on the solar surface. Figure [2](#page-2-1) shows the intensity [*I*] of an image

Figure 1 Example of the definition of the rectangular coordinate grid and solar radius. (**a**) *Green rectangles* define the frames where the rightmost, leftmost, topmost, and lowermost *red points* set the rectangular coordinate grid, with the center in the solar disk. (**b**) *Blue and pink lines* represent the *x*- and *y*-axes of the rectangular coordinate system. The *red circle* is the edge of the solar disk with the fixed radius *R*.

Figure 2 Intensity of the whole image and a part of it.

in the rectangular coordinates. We calculated the *x*-coordinate of an object *s* (spot, subgroup of sunspots, or sunspot group) as the intensity-weighted value:

$$
x_s = \frac{\sum_{i,j} x_{i,j} I_{i,j}}{\sum_{i,j} I_{i,j}},
$$
\n(1)

where *i* and *j* define the position of each pixel with an intensity that is higher or lower than a certain threshold (for instance, for June – July $I > 90$, see the color bar in Figure [2\)](#page-2-1). The same was done for the *y*-coordinate. Since the Sun's surface is a sphere, the *z*-coordinate for each object *s* was defined as $z_s = \sqrt{R^2 - x_s^2 - y_s^2}$.

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3. Areas and Number of Groups

It has to be noted that the quality of Galilei's sunspot drawings in comparison to other observers of the seventeenth century is outstanding (Zolotova and Ponyavin, [2016\)](#page-20-11). He produced a new drawing each observing day; umbra and penumbra are separated in most cases, and the sunspot groups are mostly bipolar or of a complex structure.

For each object [s] of a drawing, its area in pixels $[A_s^{\text{pix}}]$ is the sum of pixels with intensity [*I*] exceeding or falling below a certain threshold value. In the drawings of May, there is a color gradient, which results in an uncertainty in determining the sunspot boundaries. Figure [3a](#page-3-1) shows a fragment of the drawing of 3 May containing a large sunspot group. For these days, the image contrast has to be adapted. The best fit is shown in Figure [3b](#page-3-1). For May, we defined an intensity threshold of *I <* 200, for August, *I <* 90.

To calculate the area in microhemispheres [msh: A_s^{msh}], the correction for foreshortening toward the limb has to be taken into account (Meadows, [2002\)](#page-20-12). The angular distance [*α*] on the surface of the Sun from the center of the disk to a pixel (varies from 0 to $90°$) was calculated as follows:

$$
\alpha = \arcsin\left(\sqrt{\frac{x^2 + y^2}{R}}\right),\tag{2}
$$

where *x* and *y* are the coordinates of a pixel in the rectangular grid, and *R* is the solar radius. In other words, the area in msh of an object [*s*] is the sum of its pixels, where each pixel is adjusted to its *α*. Commonly,

$$
A_{\rm s}^{\rm msh} = \frac{A_{\rm s}^{\rm pix} \times 10^6}{2\pi R^2 \cos(\alpha)}.
$$
 (3)

Figure [4](#page-4-0)a shows the areas of the individual sunspot groups; the largest are the groups G1 on 3 May with 3898 msh and G42 on 26 June with 3777 msh. It should be noted that areas of individual groups exceeding 4000 msh were observed only during Cycle 18, according to the data from the Royal Greenwich Observatory. Here, we may suggest that the time of Galilei's observations fell in the secular maximum of the solar activity. On the other hand, the uncertainty is roughly 30%.

The total daily sunspot areas are shown in Figure [4](#page-4-0)b, and in Figure [4c](#page-4-0), the daily sunspotgroup numbers are compared with the results of Sakurai [\(1980](#page-20-9), green diamonds) and Hoyt and Schatten [\(1998](#page-20-0), red stars). Group counts defined by us and Sakurai ([1980\)](#page-20-9) in June – July systematically exceed those of Hoyt and Schatten ([1998\)](#page-20-0).

Figure 4 (**a**) Area of each sunspot group. (**b**) Total daily sunspot area. (**c**) *Green* denotes the number of groups according to Sakurai [\(1980\)](#page-20-9), *red* represents groups according to Hoyt and Schatten [\(1998](#page-20-0)), and *blue* shows groups of this work.

4. Heliographic Coordinates

To deduce the heliographic coordinates of sunspots, we divided the observations into three sets. The most accurate and continuous are the observations of June and July, then August, and those made in May are the least accurate. We also need the geoposition of the observer and the observation date and time.

The *Istoria* contain the correspondence of Galilei and Welser in Italian. Galileo's sunspot letters were dated 4 May 1612 at the Villa delle Selve (15 km from Florence), 14 August 1612 in Florence, and 1 December 1612 also at the Villa delle Selve. Hence, we assume that Galilei observed in the Villa delle Selve (43.77◦N and 11.08◦E).

To calculate the ephemeris of the Sun, we refer to Meeus [\(1991](#page-20-13)). For a given instant, the required quantities are the position angle $[P]$, the heliographic latitude $[B_0]$, longitude $[L_0]$ of the center of the solar disk, and the parallactic angle [*q*] (Meeus, [1991\)](#page-20-13). The difference *P* − *q* defines the diurnal clockwise rotation of the whole image of the Sun. In Figure [5](#page-5-0), the red curve shows the variation of *q* within 24 hours for mid-June for the Villa delle Selve. The sign of *q* is negative before, and positive after Noon ($q = 0^\circ$ near local Noon at 11:17 UT). In other words, at solar Noon, the orientation of the solar rotation axis is only defined by the position angle $[P]$ and the tilt of the Sun's North Pole $[B_0]$. In Figure [5](#page-5-0), the schematic solar image demonstrates the alignment of the Sun at 9:00, 11:17 (Noon), and 18:00 UT. The blue curve depicts the altitude [*h*], which is the height of the Sun relative to the horizon; the yellow shading indicates the time when *h* is positive.

For Italy, the Central European Standard Time (CET) is UTC/GMT+01. Hence, $q = 0°$ and $h = h_{\text{max}}$ near 11:17 UT or 12:17 CET in mid-June.

Figure 5 *Blue* denotes altitude and red parallactic angle within 24 hours of UT for mid-June for the Villa delle Selve. *Yellow shading* shows the time from sunrise to sunset. The position of the schematic solar disk is shown at 7:00 UT, solar Noon, and 18:00 UT.

If the solar rotation axis coincides with the *y*-axis, then the heliographic latitude [*B*] and longitude [L] of a sunspot are expressed in terms of $x-y-z$ Cartesian coordinates:

$$
B = \arcsin\left(\frac{y\cos(B_0) + z\sin(B_0)}{R}\right),\tag{4}
$$

$$
L = L_0 + \arctan\bigg(\frac{x}{z\cos(B_0) - y\sin(B_0)}\bigg). \tag{5}
$$

The diurnal clockwise rotation of the Sun from sunrise to sunset transforms the *x*-, *y*-, and *z*-coordinates of sunspots during a day. Unfortunately, Galilei's reports do not provide the time of observation, which in turn results in an uncertainty in the value of q and in the ephemeris of the Sun. Below we describe the method with which we deduced *B*, *L*, and the time when the drawings were made. Generally, our method is similar to that proposed by Casas, Vaquero, and Vazquez ([2006](#page-20-8)).

5. Method for the Determination of Sunspot Positions and Time of Observations

Before the invention of the projection apparatus, the initial observations of Galilei were carried out directly through the telescope at sunrise and sunset, not in the middle of the day (Drake, [1957;](#page-20-14) Biagioli, [2006\)](#page-19-1).

Parrying Shiner's arguments about the origin of sunspots, Galilei [\(1613,](#page-20-7) the first letter to Welser dated 4 May) described sunspots at sunset from 5 April to 3 May 1[6](#page-6-0)12 (Figure 6). Galilei writes (our translation): "*Sunspot A was observed on April 5 at sunset; it seemed tiny and a little shaded. The next day, this sunspot was seen at sunset as spot B. On April 7, the darkness of this sunspot increased, and the color changed (sunspot C). The position of this object has always been far from the solar limb. On the 26th of the same month, at sunset, in the upper part of the solar disk, a sunspot appears similar to D, which on the 28th day became E, 29th, F, 30th, G, 1st May, H, 3rd May, L. All these mutations occurred far from the solar limb.*"

Figure 6 Sunspot drawings from 5 April to 3 May 1612 from the *Istoria*.

The object L is exactly the same as the sunspot group G1 on 3 May that is depicted on the solar disk in the letter to Francesco Barberini (see Figure [3](#page-3-1) and Figure [15\)](#page-14-0). The difference is that Group L was drawn at sunset, while Group G1 (according to our calculations) was drawn at 13:18 UT by means of the projection apparatus.

In the very last paragraph of the first letter to Welser, Galilei [\(1613](#page-20-7)) announced that "*in a few days, I shall send him [Scheiner] some observations and diagrams of sunspots that are absolutely exact both as to their shape and their variation of position from day to day, drawn without a hairsbreadth of error in a very elegant manner discovered by a pupil of mine*" as reproduced by Drake [\(1957](#page-20-14)) or "*in a few days, I will send him [Scheiner] some observations and drawings of the solar sunspots of absolute precision, indeed the shapes of these sunspots and of the places that change from day to day, without an error of the smallest hair, made by a most exquisite method discovered by one of my students*" as translated by Biagioli [\(2006](#page-19-1)). Hence, on 4 May 1612, Galilei already knew about the projection technique.

In the second letter to Mark Welser, Galilei $(1613, p. 52)$ $(1613, p. 52)$ explains the projection technique discovered by his pupil, a monk from Cassino named Benedetto Castelli, which allows observations in the middle of the day, and hence obtaining continuous daily records of the sunspots. Galilei did not provide an image of his apparatus, but gave a verbal description; its translation is given by Biagioli ([2006,](#page-19-1) page 190).

To deduce sunspot positions and the time of observations from day to day, we divided the drawings into three parts according to the quality of drawings. First, we processed continuous observations from 2 June to 8 July 1612, then the August drawings, and finally the May drawings.

5.1. Drawings of June and July

The parallactic angle significantly varies between 8:00 and 14:00 UT (Figure [5,](#page-5-0) for mid-June), resulting in a strong variations of the sunspot coordinates. Similar to Casas, Vaquero, and Vazquez [\(2006](#page-20-8)), we made the assumption that the latitude of the sunspot does not change from day to day. To deduce the proper orientation of the Sun's axis and the time of the drawings, we developed a method to minimize the discrepancy between the sunspot latitudes characterized by the following:

$$
F_1(T) = \sqrt{\sum (\Delta B(T)w(\sigma))^2}.
$$
 (6)

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Here, we considered the time of observation $[T]$ to vary from 10:00 to 17:00 UT with a one-minute step (420 instances). Starting from 3 June to 8 July, the method minimizes the difference $[\Delta B]$ between the object latitude on a given day $[B(T)]$ and the average latitude of this object over the preceding days $[\overline{B}]$. An object is a sunspot, group, or subgroup of sunspots when it is defined in such a way that it was convenient to trace its trajectory across the disk of the Sun. The weighting function $[w(\sigma)]$ is defined by the standard deviation $[\sigma]$ of the object latitude in the preceding days (the function is depicted in Figure [7](#page-7-0)). The proposed method requires an initial assumption. We suggest that the sunspots on 2 June 1612 were drawn at the time T_0 . Any time can be assigned; this does not affect the final result. For each subsequent day, the time of observation and hence the sunspot coordinates were evaluated by minimizing $F_1(T)$ (the sum is taken over all objects in a given day). In this manner, we obtained the time of observation for each day from 3 June to 8 July. The procedure was repeated in reverse from 8 July to 2 June $(F_2(T))$. For $F_2(T)$, the average latitudes of objects were computed over the successive days. The initial time of the drawing on 8 July [*T*0] was found in the previous step. Thus, we determined the new time of observation on 2 June. After several runs of this iterative scheme, the result became robust.

Galilei wrote that the projection method provides the drawings of the sunspots with complete accuracy (Biagioli, [2006](#page-19-1)), but in a few cases, the latitude of a spot significantly deviates from its average value. For example, the latitude of sunspot group G42 on 26 June 1612 is found to be about 34◦, while its latitude over the other days varies near 24◦. The weighting function $[w(\sigma)]$ helps the method to focus on more "stable" sunspots that move along nearly the same latitude. One can suggest that the precision of the sunspot mapping declines toward the limbs ($\alpha > 60^\circ$), *i.e.* $w(\sigma)$ should depend on α . However, our processing of Galileo's drawings revealed that a significant portion of sunspots maintains their latitude even near the limbs. There are only few exceptions. This discovery makes the near-limb sunspots valuable for adjustment of the heliographic grid, and therefore $w(\sigma)$ is independent of α .

Possible sources of large σ could be clouds delaying the drawing process at a time when the parallactic angle changed rapidly and the adjustment of the position of the telescope following the movement of the Sun. Unfortunately, we do not know how long it took to draw sunspots, but prolonged drawing (from tens of minutes to an hour) also makes for worse sunspot mapping. In our calculations, in order to adjust the heliographic grid, we assumed certain time values. These values are some average time during which a drawing was made.

The time corresponding to the minima of $F(T)$ differs for procedures based on preceding or successive days. Figure [8](#page-8-0) shows the time of minimum F_1 in dashed black, and time of minimum F_2 in solid gray. Each of the two series defines the latitudes of the analyzed sunspots. The reliability of the results for each day can be evaluated through $F_{1,2}^{\min}/N$, where *N* is the number of objects in the analyzed drawing, and through the average dispersion of

Figure 8 *Dashed-black line* depicts the time of sunspot drawing defined in the range of preceding days T_1 , and the *solid-gray line* that on the successive days T_2 .

Figure 9 Colored bars show *F (B)* for each day. The *black points* joined by the *yellow curve* mark the minima of *F (B)*. *On the left*, *blue* denotes the parallactic angle, and *red* the solar altitude for mid-June. The *horizontal dashed line* marks solar Noon at the Villa delle Selve.

 $B_{1,2}$ in the preceding (succeeding) days. The following weights were assigned to sunspot latitudes $[B_1 \text{ and } B_2]$ obtained in the "forward" and "backward" procedures:

$$
w'_{1,2} = w(\sigma_{1,2}) \frac{N}{F_{1,2}^{\min}}; \qquad \overline{B} = \frac{B_1 w'_1 + B_2 w'_2}{w'_1 + w'_2}.
$$
 (7)

Finally, the sunspot positions and observing time for each day were defined by minimizing the value $F'(T)$:

$$
F'(T) = \sqrt{\sum (B(T) - \overline{B})^2}.
$$
 (8)

The colored bars in Figure [9](#page-8-1) show $F'(B)$ from 10:00 to 17:00 UT for each day. The minima are shown in red (see the color bar, which in turn serves as a measure of uncertainty).

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The black points joined by the dashed-yellow curve mark these minima, which define the approximate observation time in UT. The thin horizontal dashed line is solar Noon ($q = 0$, $h = h_{\text{max}}$). In several cases (22 and 25 June; 3, 5, 7, and 8 July), $F'(B)$ does not have a noticeable minimum. For these days, the observation time is unclear. After 14:00 UT, the parallactic angle varies slowly (Figure [9,](#page-8-1) on the left); therefore the time span when sunspots were presumably drawn is much larger. Note that on 7 July, *F (B)* has two minima (black dots in Figure [9](#page-8-1)) because the parallactic angle is the same for both times. Here, we arbitrarily chose the earlier time of observation.

Moreover, sunspots were drawn after the solar Noon, therefore we can speculate that the upper culmination could be occupied by astrometric measurements. The other reason for not having a fixed time of drawing could be clouds or more than one drawing attempt.

5.2. Drawings of 19 – 21 August

In addition to the sunspot drawings of 2 June – 8 July 1612, the *Istoria* also contains drawings of a large sunspot of 19 – 21 August 1612 (Galilei [\(1613](#page-20-7), page 94): "*Disegni della Macchia grande Solare*"). At the bottom, these three drawings are accompanied by short notes, for example "*Agost. D. 19. Hor. 14*". Figure [10](#page-9-0)a shows three superimposed August drawings; sunspots on 19 August are contoured in light blue, those on 20 August in light green, and those on 21 August in red. Figure [10](#page-9-0)b shows the same, but the drawing of 20 August is rotated clockwise by 11◦. Figure [10](#page-9-0)b shows that most of the sunspot groups tend to align on the same latitude bands. Therefore we can tentatively suggest that the time of drawing on 20 August was earlier than the times on 19 and 21 August. Similarly to Casas, Vaquero, and Vazquez ([2006\)](#page-20-8), we find no relation of "*Hor. 14*" and "*H. 14*" with the horizon (altitude) or hora (hour). While the orientation of sunspots on the drawings on 19 and 21 August suggests that observations might have been carried out near sunset (14◦ altitude), the drawing on 20 August is not consistent with this suggestion. One can speculate that there were several attempts during the day, or that the observation was carried out earlier because of worsening weather.

Figure 10 (**a**) Superimposed sunspot drawings of 19 August (*light blue*), 20 August (*light green*), and 21 August (*red*). (**b**) The same, but the drawing of 20 August is rotated by 11◦.

Figure 11 *Colored bars* show the value *F (B)* for each day. *Black points* joined by the *dashed-yellow curve* mark the minima of $F'(B)$. The *horizontal dotted-black line* marks the maximum diurnal angle $P - q$ in May at the Villa delle Selve.

There are only three observations in August, and our method required more iterations to define the sunspot positions. This approach gives the result that on 20 August sunspots were drawn at 13:33 UT. For 19 and 21 August, *F (T)* does not have clear minima, which results in an uncertainty in the evaluation of time. We stress that a wide time span when sunspots could be drawn does not imply an inaccuracy of the deduced sunspot positions.

5.3. Drawings on 3 – 11 May

For the drawings in May, we found that the deviation of the best-fitted solar rotation axis from the vertical line (direction to Zenith) exceeds the maximum diurnal variation of $P - q$. This means that the $x-y$ -orientation of the solar disk in the earlier letters of Galilei is not the same as was seen in the telescope. We therefore slightly modified the method. We removed the limit on the diurnal rotation of the Sun in the $x-y$ -plane. The colored bars in Figure [11](#page-10-1) show $F'(B)$ whose minimum value (black points joined by the dashed-yellow line) defines the best fit of the sunspot trajectories along the latitude bands. The horizontal dotted-black line marks the maximum diurnal angle $P - q$. The rotation angle of the solar image on 4, 5, 8, and 9 May exceeds the acceptable $P - q$ value, hence the observation time on these days is unknown.

In Figure [12a](#page-11-0), the drawings of May are overlaid; each day is plotted in a different color. Figure [12](#page-11-0)b shows the same after the images were processed, including the heliographic grid of 7 May. The thin gray curves schematically mark the trajectories of several sunspots.

6. Butterfly Diagram

Figure [13](#page-11-1) compares the butterfly diagram from Galilei's drawings deduced by Casas, Vaquero, and Vazquez ([2006\)](#page-20-8) (blue stars) and of this work (red points); the sunspot positions are nearly the same.

Figure 12 (**a**) Overlap of schematic colored sketches of Galilei's observations on 3 – 11 May. (**b**) The same, but each image is derotated to minimize the function *F (B)*. The *thin-gray curves* schematically mark trajectories of several sunspots, the heliographic grid on 7 May is underlaid.

Figure 13 *Blue stars* show the sunspot positions according to Casas, Vaquero, and Vazquez [\(2006](#page-20-8)), and *red points* the positions derived here.

The other observer who reported on sunspots at the end of 1611 and 1612 is Scheiner. Heliographic positions of sunspots were recently extracted by Arlt *et al.* [\(2016\)](#page-19-0). Note the significant difference in time–latitude distribution of sunspots from drawings by Galilei in comparison to that by Scheiner reported by Arlt *et al.* ([2016,](#page-19-0) Figure 6 therein). According to Scheiner, from 21 October 1611 to 11 January 1612, sunspots were located near the Equator, but on Galilei's drawings, the near-equatorial region is almost spotless except for group G7, whose latitude varies from −1*.*2◦ to −6*.*6◦. The error of the sunspot latitudes on the drawings by Galilei reaches approximately 10◦.

The appearance of sunspots at the Equator in drawings by Scheiner may be caused by the difference of techniques and purposes of observations in comparison to Galilei. By Scheiner's own admission, his earlier drawings are not very accurate, without certain and precise measurement (Biagioli, [2006](#page-19-1)). Early observations by Scheiner were aimed to prove that the sunspots are not an optical artifact of the lenses in the telescope or of the atmosphere. Therefore, we presume that the emerging of sunspots on the Equator results from the uncertainty of the heliographic coordinates of sunspots.

Similar deviations were found in Staudacher's drawings from 1749 to 1796 (Arlt, [2008](#page-19-2), [2009\)](#page-19-3). In Cycles 0 and 1, sunspots were near the Equator; in Cycle 2 (1768), he changed the drawing method, and almost no sunspots can be found at the Equator any more.

7. Active Longitudes

The time interval between the drawings in May and early June 1612 equals one synodic rotation. It is quite likely that the large sunspot group G1 (Figure [14\)](#page-12-0) lived for more than one rotation. The position of this group in May significantly varies from day to day (Figure [14b](#page-12-0)). Table [1](#page-12-1) shows average latitudes [*B*] and longitudes [*L*] of sunspots A, B, and C, which are parts of G1, on $3-7$ May and those of sunspots 37, 38, and 39 on 2 June. Almost the same heliographic position after one rotation leads to the conclusion that the distant sunspots 37, 38, and 39 represent the same active region.

Figure 14 (**a**) Schematic sunspot group G1 on 3 May 1612 (*red curve*) and its sunspots A (*brown*), B (*blue*), and C (*purple*). (**b**) Presumably, the same sunspot group G1 on 2 June 1612 consists of three sunspots.

It has to be noted that the large sunspot group G47 (from 21 June to 1 July) and the large group G73 in August also share almost the same heliographic position as group G1. We can suggest that this sunspot cluster was formed in a region that is called an active longitude. The areas of the groups mentioned are 3898 msh for G1 on 3 May, 2424 msh for G47 on 26 June, and 2193 msh for G73 on 20 August. Vaquero [\(2004\)](#page-20-15) obtained an area of 2000 msh for group G73.

8. Conclusions

We processed historical sunspot drawings by Galileo Galilei from May to August 1612. The drawings in May are archived in the Vatican Library in letters of Galilei to Cardinal Francesco Barberini. Those of June, July, and August were published in the *Istoria e dimostrazioni intorno alle macchie solari e loro accidenti comprese in 3 lettere*.

Galilei's drawings do not provide any observation time information. We developed a method that minimizes the discrepancy between the sunspot latitudes from day to day. Our approach yields the heliographic positions of sunspot groups and the observation time. Most of the drawings were made an hour or two after Noon.

Sunspots were combined into groups in order to obtain the sunspot-group number. Group counts found in this work for June – July systematically exceed those obtained by Hoyt and Schatten ([1998\)](#page-20-0). We also calculated areas of sunspot groups and their positions in the rectangular coordinate system. The areas of some large groups exceed 3500 msh. According to Scheiner's observations at the end of 1611 and the beginning of 1612, sunspots seem to be at the Equator, while in Galilei's drawings, the near-equatorial region is almost spotless. This in turn indicates the quality of the reports by Galilei.

The accuracy of the May drawings is lower than that of those from June – July. On 4, 5, 8, and 9 May, the deviation of the rotation axis of the Sun from the Zenith in the $x - y$ -plane exceeds the maximum angle of the diurnal clockwise rotation of the Sun from sunrise to sunset; for these days, the observation time is therefore not clear.

From the comparison of the heliographic positions of large sunspot groups, we propose an activity cluster in the northern hemisphere. The final result is tabulated in the Electronic Supplementary Material.

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Appendix

Figures [15](#page-14-0), [16,](#page-15-0) [17](#page-16-0), [18,](#page-17-0) [19](#page-18-0), and [20](#page-19-4) show sunspots sorted into groups. Black rectangles with numbers denote individual sunspots or subgroups of sunspots. Green ovals mark sunspot groups. Note that "not a sunspot" shows paper defects. For convenience, blue and pink lines represent the *x*-and *y*-axes in the rectangular coordinate system, and the solar limb is shown as a red circle. Note that on 14 June, one tiny spot is missing in the image provided by the Galileo Project; this spot was added by us for completeness.

Figure 16 Schematic sketches of sunspot drawings of 11 May and solar images of 2, 3, $5 - 9$ June 1612. Note that "not a sunspot" shows paper defects. *Black rectangles with numbers* denote individual sunspots or subgroups of sunspots. *Green ovals* mark sunspot groups. The solar limb is shown as a *red circle*.

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Figure 19 Solar images of 26 – 29 June and 1 – 4 July 1612. *Black rectangles with numbers* denote individual sunspots or subgroups of sunspots. *Green ovals* mark sunspot groups. The solar limb is shown as a *red circle*.

Figure 20 Solar images of $5 - 8$ July and $19 - 21$ August 1612. *Black rectangles with numbers* denote individual sunspots or subgroups of sunspots. *Green ovals* mark sunspot groups. The solar limb is shown as a *red circle*.

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