

Center-to-Limb Variability of Hot Coronal EUV Emissions During Solar Flares

E.M.B. Thiemann¹ *p* \cdot **P.C.** Chamberlin¹ *p* \cdot **F.G. Eparvier**¹ \bullet **· L. Epp²**

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Abstract It is generally accepted that densities of quiet-Sun and active region plasma are sufficiently low to justify the optically thin approximation, and this is commonly used in the analysis of line emissions from plasma in the solar corona. However, the densities of solar flare loops are substantially higher, compromising the optically thin approximation. This study begins with a radiative transfer model that uses typical solar flare densities and geometries to show that hot coronal emission lines are not generally optically thin. Furthermore, the model demonstrates that the observed line intensity should exhibit center-to-limb variability (CTLV), with flares observed near the limb being dimmer than those occurring near disk center. The model predictions are validated with an analysis of over 200 flares observed by the *EUV Variability Experiment* (EVE) on the *Solar Dynamics Observatory* (SDO), which uses six lines, with peak formation temperatures between 8.9 and 15.8 MK, to show that limb flares are systematically dimmer than disk-center flares. The data are then used to show that the electron column density along the line of sight typically increases by 1.76 × 10¹⁹ cm⁻² for limb flares over the disk-center flare value. It is shown that the CTLV of hot coronal emissions reduces the amount of ionizing radiation propagating into the solar system, and it changes the relative intensities of lines and bands commonly used for spectral analysis.

Keywords Center-limb observations · Flares, spectrum · Spectral line, intensity and diagnostics

1. Introduction

Solar flares are widely believed to be the result of magnetic reconnection in the solar corona, which directly heats coronal plasma and accelerates particles along magnetic field lines.

 \boxtimes E.M.B. Thiemann thiemann@lasp.colorado.edu

¹ Laboratory for Atmospheric and Space Physics, University of Colorado, 3665 Discovery Dr., Boulder, CO 80303, USA

² Colorado School of Mines, 1500 Illinois St, Golden, CO 80401, USA

These accelerated particles flow toward the field line footpoints until local plasma densities are sufficiently thick to stop them, resulting in intense heating and subsequent upwelling of chromospheric plasma toward the loop apices through a process known as chromospheric evaporation. The newly formed loop of dense plasma cools, in part, by radiating its energy into space at extreme ultraviolet (EUV) and X-ray wavelengths. The substantial increase in loop density that occurs during flares has implications on the common assumptions of optical depth that are typically made when coronal plasma observations are analyzed.

Coronal extreme ultraviolet (EUV) emissions are typically approximated as being optically thin because of the low densities of the non-flaring corona. As such, a coronal source located at disk center will have the same full-disk integrated irradiance if it were instead located near the limb, assuming all other properties of the disk remain constant. However, since solar flare plasma densities can be one to four orders of magnitude higher than typical quiet-Sun and active region plasma densities (see Milligan *et al.*, [2012](#page-18-0) and references therein), the optically thin assumption will break down, and a flare occurring near the limb may have significantly less EUV irradiance of coronal origin than the same flare occurring near disk center.

Multi-vantage, spectrally resolved irradiance measurements needed to spectrally characterize solar flare center-to-limb variability (CTLV) do not exist, therefore statistical analyses of how flares vary spectrally depending on their location on the solar disk must be used instead. This method has been used, for example, to characterize anisotropic emissions of solar flare hard X-rays by McTiernan and Petrosian [\(1991](#page-18-1)). Similarly, for EUV emissions, Chamberlin *et al.* (2008) used full-disk integrated solar flare observations from the *Solar EUV Experiment* (SEE) (Woods *et al.*, [2005\)](#page-18-2) on board the *Thermosphere Ionosphere Mesosphere Energetics and Dynamics* (TIMED) satellite to show that limb flares have systematically lower EUV spectral content above 35 nm for the same soft X-ray magnitude. Although the 1 nm resolution of TIMED/SEE prevented a detailed analysis of the CTLV of individual lines, Chamberlin, Woods, and Eparvier ([2008\)](#page-18-3) attributed the differences to the flare spectrum above 35 nm to absorption from chromospheric and transition region ions, which are assumed to be optically thick because of the relatively high density of these regions.

In this present paper, recent flare spectral measurements made by the *EUV Variability Experiment* (EVE; Woods *et al.*, [2010](#page-19-0)) on board the *Solar Dynamics Observatory* (SDO) are used to characterize the CTLV of hot lines originating in the solar corona. The EVE observations include a dataset of over 292 M-class solar flares measured at wavelengths ranging from 6 to 37 nm with sufficient resolution to characterize the CTLV of individual lines. This study analyzes these data to characterize the CTLV of emissions from six bright flare lines with peak formation temperatures above 7.9 MK. Before we report the observations, Section [2](#page-1-0) motivates the data analysis with radiative transfer model results, showing that CTLV is expected for hot coronal lines based on the current understanding of flare density and geometry. Sections [3](#page-3-0) and [4](#page-5-0) present observational evidence of flare CTLV for hot coronal lines. Section [5](#page-7-0) discusses the implications of the CTLV on flare irradiance and that spectral analyses of flare plasma assuming the optically thin paradigm are adversely affected.

2. Preliminary Analysis

Although the density of non-flaring coronal loops is typically low enough for treating their emissions as being optically thin, loop densities increase significantly during solar flares, which limits the applicability of the optically thin approximation. To illustrate this, the observed emissions from a series of simple 2D coronal loops were modeled and analyzed. The

Figure 1 Simulated loop radiance as a function of observation angle. **a**. Modeled radiance from the observation angle indicated by the *red* mark of a loop of active region density. *White* coloring indicates that the radiance is entirely visible to the observer, while shading indicates the degree to which the radiance is resonantly scattered by plasma along the line of sight. **b**. – **c**. Same as a, but for different flare densities and angles, showing that substantial portions of the loop are invisible at the indicated observation points. **d**. Simulated CTLV as a function of longitude of a flare loop with footpoints oriented on a line parallel to the solar ecliptic equator, where the CTLV is characterized by the observed integrated intensity normalized by that at disk center. For example, the loop observed at disk center is approximately 20% brighter than the same loop observed at 40◦ longitude.

modeled loop diameter and footpoint separation were 1.8 Mm and 35 Mm, respectively, and consistent with flare loop observations of 41 loops reported by Aschwanden, Nightin-gale, and Alexander ([2000\)](#page-18-4). Each point along the loop radiates at unit intensity (I_0) , and the observed intensity at a specified location (I_{obs}) was found by solving the Beer–Lambert radiative transfer equation between each radiating point and the observer,

$$
I_{\text{obs}} = I_0 \exp\biggl(-\sigma_i \int n_i \, \mathrm{d}z\biggr),\tag{1}
$$

where σ_i and n_i are the resonant scattering cross-section and density of the *i*th ion species, and the integral was taken over the line of sight along the *z* coordinate. Note that only resonant scattering was considered and multiple scattering was neglected.

Figure [1](#page-2-0) shows model results for the 13.29 Fe XXIII emission line at varying densities and observation angles, where the observation angle is measured from the loop apex. In panel a, the plasma density is 3×10^9 cm⁻³, consistent with observations of active region loops (Aschwanden, Nightingale, and Alexander, [2000](#page-18-4)). In this case, the loop is observed at a 45◦ angle; the red mark in the upper right of the figure guides the eye to the observer's location. White indicates that the incident radiance is entirely visible to the observer, while black indicates that no emission is visible, and the blue shading bridges these two extremes. The loop in panel a is entirely visible, and the integrated radiance seen by the observer is

97% of the total emitted amount, confirming that the optically thin approximation would be valid for similar active region loops. Panel b is similar to panel a, but with a plasma density of 8×10^{10} cm⁻³, which is consistent with observations for large solar flares (Milligan *et al.*, [2012](#page-18-0)); in this case, the loop is viewed from directly above the apex. Again, the entire loop structure is visible, but the underside of the loop is considerably less visible, with the inner-loop regions immediately above the foot points being completely invisible to the observer. Panel c shows the same loop as panel b, but here, the observation angle is at 45◦. In this case, a substantial portion of the left loop leg is not visible because emissions from it are completely scattered by the right loop leg. These simulations imply that the integrated intensity from a flare loop or sets of loops will depend on an observer's location because portions of the flaring structure may be obscured by other portions of the structure nearer to the observer.

The dependence of the observed integrated intensity on observation angle for the loop considered in panels b and c is shown in panel d. The vertical axis corresponds to the ratio of the intensity observed directly along the apex axis to the intensity observed from the given angle. It is apparent that the observed integrated intensity above the apex is brighter than that observed off-axis. If this loop were on the Sun with its footpoints on a line parallel with the ecliptic equator, then the angle would correspond to the loop's longitudinal location, and observations of this loop would be substantially brighter when viewed at disk center (0◦) than near the limb (*e.g.* 80◦). In conclusion, the CTLV of hot coronal flare loop line emissions is expected based on typical flare densities and geometries, even though the actual configuration is typically much more complex than that considered here. In the following sections, observations of this effect are presented.

3. Data

3.1. Flare Position

Flare positions were identified using the SDO *Atmospheric Imaging Assembly* (AIA; Lemen *et al.*, [2011](#page-18-5)) images in the 13.3 nm band rendered on the helioviewer.org website, which conveniently displays the angular distance from disk center to the user-determined cursor position. Flares that appeared to have a footpoint over the limb were discarded. The angular distance was converted into solar longitude and latitude. From these coordinates, the orthodromic angle was found between disk center and the flare location, and this angle was used to classify a flare's location on the disk. Note that the flare location data available in the Solar Events List produced and archived by the National Oceanic and Atmospheric Administration (NOAA) Space Weather Prediction Center (SWPC) were not used because some of the tabulated flare locations were found to be erroneous when independently compared with observations.

3.2. EUV Line Irradiance

Solar flare EUV line irradiances were taken from the SDO/EVE Version 6, Level 2, Spectrum data product, observed by the *Multiple EUV Grating Spectrograph* (MEGS) A channel. MEGS-A made nearly continuous observations of solar irradiance in the 6 to 37 nm range at 10-second cadence with 0.1 nm spectral resolution and 0.02 nm spectral sampling from 30 April 2010 through 26 May 2014. In this time, MEGS-A observed thousands of solar flares, including approximately 300 flares of M class or larger.

Solar flare irradiances from the following species (wavelengths) were used in this study: Fe XVIII (9.393 nm), Fe XIX (10.835 nm), Fe XXI (12.875 nm), Fe XXIII (13.291 nm), Fe XXIV (19.203 nm), and Fe XXIV (25.511 nm). These lines were selected because their flare emissions have a high signal-to-noise ratio, they contribute at least 80% of the observed peak intensity within the line as resolved by MEGS-A, and their peak formation temperature exceeds 9.0 MK, which is within the sensitivity range of temperature (T) and emission measure (EM) estimates made by the *X-ray Sensor* (XRS) on board the *Geostationary Operational Environmental Satellites* (GOES) used in the analysis and discussed later in this section.

The flare peak irradiance for each line was identified as follows. Flare line centers and line widths as measured by EVE were identified using the EVE Flare Atlas developed by Hock [\(2012](#page-18-6)). Solar flare occurrences were identified using the Solar Events List produced and archived by NOAA SWPC. For each flare, the 13.291 nm Fe XXIII light curve was plotted, and a time range representing the non-flaring background irradiance level as well as the time range of the entire flare light-curve were manually identified. An automatic routine then used these time ranges to identify the flare peaks for the seven lines we analyzed and subtracted the background irradiance. Background-subtracted light curves and selected peak levels were plotted, manually inspected for accuracy, and reprocessed or discarded as necessary when the data appeared to be spurious. Reprocessing or discarding is necessary when, for example, a spurious high data point from an energetic particle triggers a false flare maximum. The background-subtracted peak flare irradiances were used for the analysis discussed in Section [4.](#page-5-0)

The flare data set was partitioned into six subsets according to flare angle. The mean angles (number of flares) for each group are 14*.*9◦ (45), 31*.*7◦ (43), 42*.*7◦ (27), 53*.*5◦ (31), 61.5[°] (58), and 74[°] (47). The subset of flares with a mean angle near 14.5[°] was treated as the disk-center flare subset against which the other five subsets were compared.

Of the initial 292 flares considered, 64 flares were discarded after automated processing. Reasons for discarding were unphysically high (for example, due to unfiltered particle hits) or negative values (for example, due to spuriously high subtracted background values), or missing or unphysical corresponding GOES emission measures. The 228 flares analyzed in this study are listed in Table [2](#page-16-0) of the [Appendix](#page-16-1), which gives the date (in year–day format), hour, soft X-ray magnitude, and center-to-limb angle of each flare.

3.3. GOES Soft X-Rays

Soft X-ray measurements from GOES XRS (Bornmann *et al.*, [1996](#page-18-7)) were used to determine the flare emission measure. XRS measures irradiance in the $0.1 - 0.8$ nm (long) and $0.05 -$ 0.4 (short) bands. Before the analysis, flare plasma emissions were isolated by first subtracting the non-flaring background. These times were taken to be the same as those determined in Section [3.2](#page-3-1) for the Fe XXIII emission. The method of Thomas, Starr, and Crannell [\(1985](#page-18-8)) was then used to estimate the flare temperature from the ratio of the two bands. The flare temperature was used with assumed contribution functions and the long-band irradiance to estimate the emission measure. Note that the assumed spectra and contribution functions used by Thomas, Starr, and Crannell [\(1985](#page-18-8)) have been updated by White, Thomas, and Schwartz [\(2005](#page-18-9)), and are incorporated into SolarSoft (Freeland and Handy, [1998](#page-18-10)) as part of the "goes" IDL class, which was used in this study to retrieve XRS band irradiances and derived temperatures and emission measures.

Wavelength (nm)	Species	$T_{\rm X,pk}$ (MK)	$g_i/g_j \times A_{ij}$ (GHz)	Abundance (ppm)	$F_{\rm I}$	Cross-section $(x 10^{-16}$ cm ²)
9.393	Fe XVIII	8.9	34.69	39.8	0.35	9.19
10.835	Fe XIX	10.0	35.84	39.8	0.26	11.8
12.875	Fe XXI	12.6	11.67	39.8	0.22	5.74
13.291	Fe XXIII	14.1	23.26	39.8	0.2	9.97
19.203	Fe XXIV	15.8	8.70	39.8	0.27	12.6
25.511	Fe XXIV	15.8	3.44	39.8	0.27	11.7

Table 1 Atomic physics and plasma parameters. See Section [3.4](#page-5-1) for details.

3.4. Atomic Physics and Plasma Parameters

Atomic physics and plasma parameters were retrieved from the CHIANTI database, version 8.5 (Del Zanna *et al.*, [2015\)](#page-18-11). Table [1](#page-5-2) shows the wavelength, species, peak formationtemperature $(T_{X,bk})$, degeneracy ratio – Einstein *A* coefficient product – $(g_i/g_j \times A_{ii})$, elemental abundance fraction (A_i/A_H) , ionization fraction (F_I) , and resonant scattering crosssection for the six lines used in the analysis described in Section [4](#page-5-0). The 13.3 nm Fe XXIII and 9.4 nm Fe XVIII lines are both blended with Fe XX, and $g_i/g_j \times A_{ij}$ for these lines is an average of both species, weighted according to the relative line intensity contributions.

4. Methods

The optical intensity of the line emission corresponding with the $i - j$ transition (I_{ij}) observed through an absorbing medium is related to the intensity at the source $(I_{0,ij})$ by the Beer–Lambert radiative transfer equation,

$$
I_{ij} = I_{0,ij} e^{-\tau},\tag{2}
$$

where τ is the optical depth of the medium.

The flare CTLV was characterized by quantifying the difference in τ for flares observed at disk center versus those observed near the limb as follows: The flare peak emission measure (EM_{pk}) is related to the source line intensity and peak contribution function ($G_{ij}(T_{pk})$) according to

$$
I_{0,ij} = G_{ij}(T_{\text{pk}})EM_{\text{pk}}.
$$
\n(3)

Because flare plasma is to some degree likely to be multi-thermal (*e.g.* McTiernan, Fisher, and Li, [1999;](#page-18-12) Warren, Mariska, and Doschek, [2013\)](#page-18-13), the isothermally derived peak EM values from GOES were treated as a proxy for the EM of the EUV emitting plasma. The key assumption being that the EUV-emitting EM is proportional to that of the soft X-ray EM. Eliminating *I*0*,ij* from Equation [2](#page-5-3) using Equation [3](#page-5-4) and collecting terms yields

$$
I_{ij} = (G_{ij}(T_{\text{pk}})e^{-\tau})EM_{\text{pk}}.
$$
\n(4)

Note that the optical depth of GOES soft X-rays was assumed to be insignificant compared to that of the EUV line emissions because relatively small continuum cross-sections dominate the GOES bands. Furthermore, as we show later in this section, resonant scattering crosssections are proportional to the line-center wavelength and inversely proportional to the

square-root of the temperature. The resonant scattering cross-sections for any line emissions in the GOES bands are therefore much smaller than those for longer wavelengths.

Equation [4](#page-5-5) is rearranged to solve for the measured peak contribution function $(G_{M,i})$ for a set of flares with a mean orthodromic angle, *θ*, from disk center,

$$
\left\langle G_{\mathbf{M},ij}(\theta) \right\rangle \equiv G_{ij}(T_{\mathbf{p}\mathbf{k}}) e^{-\langle \tau(\theta) \rangle} = \frac{I_{ij}(T_{\mathbf{p}\mathbf{k}})}{EM_{\mathbf{p}\mathbf{k}}}.
$$
 (5)

In practice, the optical path difference was isolated by finding $G_{M,ij}$ for a set of limb flares with mean angle θ_L and separately for a set of disk-center flares with mean angle θ_C via Equation [5](#page-6-0) using EVE line irradiance measurements at the time of peak line irradiance and peak GOES EM measurements:

$$
\frac{\langle G_{\mathbf{M},ij}(\theta_{\mathbf{C}}) \rangle}{\langle G_{\mathbf{M},ij}(\theta_{\mathbf{L}}) \rangle} = e^{\Delta \tau},\tag{6}
$$

where $\Delta \tau = \langle \tau(\theta_L) \rangle - \langle \tau(\theta_C) \rangle$ is the mean optical depth difference.

Next, the optical depth is related to the emission line and plasma parameters. For an emission line, the optical depth for transition ij of species X is related to the transition's resonant scattering cross-section (σ_{ii}) and ion column density (N_X) by

$$
\tau_{ij} = \sigma_{ij} N_{\mathbf{X}}.\tag{7}
$$

The resonant scattering cross-section at line-center is related to the line atomic and plasma parameters according to

$$
\sigma_{ij} = \frac{g_i}{g_j} A_{ij} \frac{1}{4} \left(\frac{2\pi k T_{\rm X, pk}}{m_{\rm X}} + v_{\rm nth}^2 \right)^{-1/2} \lambda_{0, ij}^3 \tag{8}
$$

(*e.g.* Foot, [2005](#page-18-14)), where m_X , and $\lambda_{0,ij}$ are the ion mass and line-center wavelength, respectively, g_i and g_j are the degeneracies of the respective levels, A_{ij} is the Einstein *A* coefficient, m_X is the ion mass, k is Boltzmann's constant, and v_{nth} is the nonthermal ion velocity. Note that other expressions for *σij* appear in the literature (*e.g.* Mariska, [1992;](#page-18-15) Schrijver and McMullen, 2000) that use the atomic oscillator strength (f_{ii}) to show $\sigma_{ij} \propto \lambda_{0,ij}$. This is because $f_{ij} \propto A_{ij} \lambda_{0,ij}^2$. It follows that it is important to treat any dependences of σ_{ij} on $\lambda_{0,ij}$ with caution because A_{ij} (or f_{ij}) also varies with $\lambda_{0,ij}$. Finally, we collected all terms on the right-hand side of Equation [8](#page-6-1), except for a factor of $\lambda_{0,ij}$ into a new variable *ξij* ,

$$
\sigma_{ij} = \xi_{ij} \lambda_{0,ij}.
$$
\n(9)

The time-dependent ion column density is related to the electron column density (N_e) by

$$
N_X = A_i / A_H F_I N_e = \eta_X N_e, \qquad (10)
$$

where A_i/A_H and F_I are the elemental abundance fraction and species ionization fraction, respectively. On the far right-hand side of Equation [10](#page-6-2), the expression is simplified by collecting all terms except for N_e into a newly defined variable, η_X .

The factoring of parameters into ξ_{ij} and η_X enables a convenient retrieval of the average electron density difference. Equations [7](#page-6-3), [9,](#page-6-4) and 10 are used to rewrite Equation [6](#page-6-5) as

$$
\ln\left(\frac{\langle G_{M,ij}(\theta_C) \rangle}{\langle G_{M,ij}(\theta_L) \rangle}\right) = \eta_X \langle \Delta N_e \rangle \xi_{ij} \lambda_{0,ij},\tag{11}
$$

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where $\langle \Delta N_e \rangle = \langle N_e(\theta_L, t_{\text{Fe18},pk}) \rangle - \langle N_e(\theta_C, t_{\text{Fe18},pk}) \rangle$ is the mean column density difference. Note that Equation [11](#page-6-6) inherently assumes constant plasma density over the period during which the EUV lines we analyzed reach their peak. In reality, this is likely not the case. However, detailed flare emission measure analysis (*e.g.* Aschwanden and Alexander, [2001;](#page-18-17) Raftery *et al.*, [2009](#page-18-18)) suggests that the emission measure is relatively constant between 10 MK and 17 MK, and begins to decrease rapidly near the formation temperature of Fe XVIII. The assumption of constant density is therefore reasonable for all lines considered, except possibly Fe XVIII, but will add uncertainty to the analysis. Finally, Equation [11](#page-6-6) is rearranged:

$$
\xi_{ij}^{-1} \eta_{\mathbf{X}}^{-1} \ln \left(\frac{\langle G_{\mathbf{M},ij}(\theta_{\mathbf{C}}) \rangle}{\langle G_{\mathbf{M},ij}(\theta_{\mathbf{L}}) \rangle} \right) = \langle \Delta N_{\mathbf{e}} \rangle \lambda_{0,ij}.
$$
 (12)

In practice, values of $G_{M,ij}$ were measured for each line at different orthodromic angles and used in Equation [12](#page-7-1) to find $\langle \Delta N_e \rangle$ by the least-squares method.

5. Results

Figure [2](#page-8-0) shows the peak emission line irradiance and peak soft X-ray EM data used to find $G_{\text{M},ii}$ by solving Equation [4](#page-5-5) for three of the five angular subsets considered in this study. The data for the two omitted angular subsets are similar. The columns correspond to different pairs of angular subsets, and the rows correspond to the six emission lines considered in this study. The disk-center observations are shown in every panel with blue diamonds, and the red triangles correspond to the limb-ward observations. Blue and red dashed lines show the zero-intercept linear fit for the disk-center and limb-ward data, respectively, where the slope of each line is $G_{\text{M},i\text{}}$ according to Equation [4](#page-5-5). We also show in each panel the Pearson correlation coefficients, where the text color matches the corresponding symbol color. In general, the peak irradiance and peak EM peak are highly correlated. The correlation increases with increasing line formation temperature, as is expected since the soft X-ray peak EM typically corresponds to 15 MK or hotter plasma (and nearest the formation temperature for Fe XXIV) for M-class and larger flares (Feldman *et al.*, [1996\)](#page-18-19).

Figure [3](#page-9-0) shows results found by solving Equation [6](#page-6-5) with the $G_{\text{M},ii}(\theta)$ measurements from the five lines and six angles listed in Section [3.2.](#page-3-1) The subset of 45 flares nearest to disk center, with a mean angle of 14.9°, were used to determine $\langle G_{M,i}(\theta_{\rm C})\rangle$ and the remaining angular-partitioned subsets were used to determine $\langle G_{\mathrm{M},ij}(\theta_{\mathrm{L}}) \rangle$. The error bars correspond to the standard-deviation of the mean. Panel a shows the variation in $\exp(\Delta \tau)$ as a function of angle; the lines are distinguished by different colors. The diamond corresponds to 14*.*9◦, where $\exp(\Delta \tau)$ should equal unity. It is evident that flares that occur near disk center are systematically brighter than those occurring away from disk center, and disk-center flare line emissions are on average $10-45\%$ brighter than those from flares near the limb. It is important to note that these results show that the observed CTLV is not due to a portion of the loop lying above the limb because $exp(\Delta \tau)$ varies smoothly with decreasing angle from the limb. There is also an approximate trend of linearly decreasing EUV emission intensity with increasing orthodromic angle from disk center. Panel b shows the same data plotted versus wavelength. It is evident that the two Fe XXIV lines, with wavelengths of 19.2 and 25.5 nm, demonstrate a higher degree of CTLV. A weak correlation between the degree of CTLV and wavelength may be evident, which is suggested by Equation [8](#page-6-1). However, closer inspection shows opposite trends at the shortest and longest wavelengths, indicating that the non-wavelength parameters that determine σ_{ij} are also important.

Figure 2 EUV irradiance and soft X-ray EM data and first-order linear fits used to find $G_{\mathbf{M},ij}(\theta)$ for three of the five angular subsets considered in this study. Each *column* of *panels* corresponds to a different combination of angular subsets, with the angles and corresponding symbols shown above each column. In each *panel*, the line-center wavelength and Pearson correlation coefficients between the irradiance and EM are given, where the *text color* corresponds to the *symbol color* for the correlation coefficients.

To investigate the dependence on the atomic physics parameters listed in Table [1](#page-5-2), Equa-tion [12](#page-7-1) was solved for $\langle \Delta N_e \rangle$. The results are shown in Figure [4](#page-10-0), where each panel corresponds to a different $\langle \theta_L \rangle$. The fitted values for $\langle \Delta N_e \rangle$ are shown in each panel, along with

 $\langle \theta_L \rangle$ and the number of limb-ward flares (n_L) we used for each case. For all five cases considered, $\xi_{ij}^{-1} \eta_X^{-1} \ln(\frac{\langle G_{M,ij}(\theta_C) \rangle}{\langle G_{M,ij}(\theta_L) \rangle})$ varies approximately linearly with wavelength, as predicted by Equation [12,](#page-7-1) although for panels b through d, a non-zero intercept yields a better fit to the data. The resulting values of $\langle \Delta N_e \rangle$ range from 0*.*25 × 10¹⁹ to 1*.*76 × 10¹⁹ cm⁻³, and tend to increase with increasing $\langle \theta_L \rangle$. The 1*σ* uncertainties of $\langle \Delta N_e \rangle$ are also shown and range between 11% and 33% of the best-fit value.

The analysis shown in Figure [4](#page-10-0) assumes $v_{nth} = 0$, although this is likely not the case based on the current understanding and past observations of solar flares, which suggest rapidly flowing plasma at speeds of tens to hundreds of km*/*s (Tsuneta, [1996;](#page-18-20) Innes, McKenzie, and Wang, [2003;](#page-18-21) Mariska, [2006](#page-18-22)). However, using a non-zero value for v_{nth} does not materially change the results, and the relative magnitude and any apparent nonlinearity in $\xi_{ij}^{-1} \eta_X^{-1} \ln(\frac{\langle G_{M,ij}(\theta_C) \rangle}{\langle G_{M,ij}(\theta_L) \rangle})$ with wavelength remains unchanged, although the resulting values of $\langle \Delta N_e \rangle$ do increase. For example, assuming $v_{\text{nth}} = 50$ km/s yields $\langle \Delta N_e \rangle = 2.67 \times 10^{19}$ cm⁻² for the $\langle \theta_L \rangle = 74^\circ$ case, and the fit uncertainty is equal to 11%.

Figure [5](#page-11-0) plots the best-fit values for $\langle \Delta N_e \rangle$ versus $\langle \theta_L \rangle$ using asterisks. $\langle \Delta N_e \rangle$ is constrained to be equal to zero when $\langle \theta_L \rangle$ is equal to $\langle \theta_C \rangle$, as is indicated with the diamond. The dashed curve shown in Figure [5](#page-11-0) corresponds to the best-fit equation,

$$
\langle \Delta N_e \rangle = 0.022 \theta_L - 0.32,\tag{13}
$$

to the data. Although Equation [13](#page-9-1) is empirically determined and provides little physical insight, it is useful for estimating $\langle \Delta N_e \rangle$, which is important for understanding and predicting the CTLV of a flare spectrum, as we show in Section [6](#page-10-1).

Figure 4 Estimates of the average increase in the line-of-sight electron column density for off-center versus on-center flares found by solving Equation [12.](#page-7-1) Each *panel* corresponds to a different *(θ*_L), while *(θ*_C) is kept the same (14.5[°]) for each case. The linear regression coefficient corresponds to $\langle \Delta N_e \rangle$ and is shown in each plot along with $\langle \theta_I \rangle$ and the number of flares (n_L) in the subsets considered.

6. Discussion

The spatial scale of the obscuring volume of plasma, Δz , was estimated from the value $\langle \Delta N_e(73.8°) \rangle = 1.8 \times 10^{19}$ cm⁻³ inferred in Section [5](#page-7-0) using the approximation $\langle \Delta N_e \rangle =$ $n_e \Delta z$: Assuming n_e in the range of $10^{11} - 10^{12}$ cm⁻³ (Milligan *et al.*, [2012\)](#page-18-0), Δz was found to be in the range of $0.18 - 1.8$ Mm. This range of values is at the bottom end of estimates of flare loop diameters, which have been found to range from 1.5 to 6 Mm (Aschwanden, Nightingale, and Alexander, [2000](#page-18-4)). This is to be expected when considering that the pre**Figure 5** Variation in $\langle \Delta N_e \rangle$ with $\langle \theta_I \rangle$. The data are fit to the logarithmic function given by Equation [13](#page-9-1).

ceding simple spatial scale estimate assumes that the entire volume of plasma is obscured equally, when in reality it is not. For example, the configuration in Figure [1](#page-2-0)c results in neartotal extinction of the loop leg farthest from the observer, but little extinction of the loop leg nearest the observer. The derived spatial scales will therefore be biased to low values and are likely only useful for order-of-magnitude estimates. Furthermore, the assumption of $v_{\text{nth}} = 0$ implies that 1.8×10^{19} cm⁻³ is a lower bound of $\langle \Delta N_e(73.8°) \rangle$, also biasing the estimates of Δz to low values. Additionally, Figure [1c](#page-2-0) almost certainly oversimplifies the scenario. In reality, a flare loop is probably not oriented perfectly equatorially (even though Joy's law implies there will likely be some equatorial component), and any deviation from an equatorial orientation will reduce the amount of a loop that is self-obscured. Future studies using 3D magnetohydrodynamic models of solar flare loops should enable further progress in understanding the factors contributing to the observed CTLV.

The optically thick nature of solar flare EUV line emissions has consequences both for estimating flare irradiance and spectroscopically deriving physical properties of flare plasma, such as temperature and density, from available measurements. These consequences were investigated by simulating the evolution of the EUV spectrum for a limb flare observed by GOES XRS using the CHIANTI database and the $\langle \Delta N_e(73.8°) \rangle$ value inferred in Section [5](#page-7-0), where CHIANTI was used both to synthesize flare spectra and to compute $\Delta \tau_{ij}$.

The flare we used to drive the simulation was an M6.9 flare that occurred on 8 July 2012. Inputs and results of this simulation are shown in Figure [6](#page-12-0). Panels a and b show the GOESlong flux and flare temperature for the flare considered. The flare temperature, initially found by the instrument response and spectral contribution functions described by White, Thomas, and Schwartz ([2005\)](#page-18-9), was decreased by a factor of 0.71 to account for the average hot bias between GOES and AIA found by Ryan *et al.* ([2014](#page-18-23)). This temperature and the GOES XRS derived emission measure were used as inputs to CHIANTI to compute the corresponding isothermal solar flare spectrum. The emission measure was also used to scale (ΔN_e) , which was taken to be equal to be 1.8×10^{19} cm⁻³ at the EM peak and to vary proportional to

Figure 6 Simulated effects of flare CTLV on sample measurements of aeronomic and solar physics interest. **a**. The 0.1 – 0.8 nm measured GOES XRS irradiance, and **b**. the derived isothermal temperature used to drive the simulation. **c**. The percent decrease in integrated 10 – 40 nm and 0.1 – 100 nm irradiance for the flare occurring at the limb compared to near disk center. **d**. Similar to c., but for the five hottest AIA bands. **e**. Difference in the Fe XXI 14.573 nm*/*12.875 nm line ratio for the flare occurring at the limb from its value when observed near disk center.

 \sqrt{EM} otherwise. For each line, $\Delta N_e(t)$ was used to find $\Delta \tau_{ij}$, which in turn was used to determine by how much the line was attenuated relative to the disk-center spectrum using the Beer–Lambert law. Note that only the intensity of lines with peak formation tempera-

Figure 7 Increases in optical density for the simulated flare in Figure [6](#page-12-0) at five different times. The time indicated in each panel corresponds to the seconds shown on the *x*-axes of Figure [6.](#page-12-0)

tures at or above 1.3 MK were modified to account for the CTLV because lines that form below this temperature can exhibit coronal dimming (Woods *et al.*, [2011\)](#page-19-1), which may have a different CTLV. The CTLV of cooler forming chromospheric and transition region lines are also expected to cause limb darkening. This simulation therefore captures only the hot coronal component of spectral CTLV effects.

Figure [6c](#page-12-0) shows the simulated decrease in limb flare irradiance relative to irradiance from the same flare observed near disk center for two bands of aeronomic interest; the solid curve corresponds to the $10 - 40$ nm wavelength range, while the dashed curve corresponds to the $0.1 - 100$ nm range. It is evident that owing to CTLV in hot coronal lines alone, the limb flare is substantially dimmer in both bands during the course of the flare, with the limb flare being near 10% (30%) for the $0.1 - 100$ nm (10 – 40 nm) range. The irradiance difference rapidly increases as the loop density increases, reaching a maximum at the time of peak EM. The difference then begins to decrease with decreasing flare density, but reverses course and increases again after approximately 1300 s. This later increase corresponds to the time when cooler forming, longer wavelength emissions that have larger resonant scattering cross-sections reach their temperatures of peak formation. This was confirmed by inspecting the evolution of the simulated $\Delta \tau_{ij}$ values during the flare, a sample of which is shown in Figure [7](#page-13-0), where the three panels correspond to three different times during the flare. The corresponding time is printed in each panel. Note that $\Delta \tau_{ij}$ for emissions that peak later in the flare (panel c) are $5 - 10$ times greater than those occurring during the rise and peak of the flare (panels a and b).

Measurements from multiple AIA bands have frequently been combined to estimate the plasma temperature, emission measure, and differential emission measure (*e.g.* Schmelz *et al.*, [2011](#page-18-24); Aschwanden *et al.*, [2013\)](#page-18-25). Therefore it is important to understand the relative CTLV that is expected in the AIA bands. Figure [6d](#page-12-0) shows the simulated limb darkening for the five temperature bands with peak temperature sensitivity above 1.3 MK. The 21.1 nm and 19.3 nm bands have comparable CTLV after approximately 600 s, when they are sensitive to

the relatively cool plasma emitting during the later flare phase. However, the 19.3 nm band shows a limb darkening of about 20% preceding the flare soft X-ray peak, when the Fe XXIV emission line dominates the pass band. The limb darkening for the 13.1 nm and 33.5 nm bands evolves in a similar way until after approximately 1300 s, when the limb darkening in the 33.5 nm band begins to increase and eventually exceed 40%. The limb darkening in the 9.4 nm band differs from all of the other bands, gradually increasing throughout the flare and reaching 40% during the flare declining phase. It follows that quantities (density, temperature, DEM, *etc*.) derived from the AIA bands for this simulated flare will be different if the flare occurs at the limb versus disk center, and care should be taken when using AIA images to characterize flare plasma to ensure that the optically thin approximation is valid for the regions of the images being considered. It should be noted that the brightest loops of large flares typically saturate the AIA detector, preventing them from being combined for flare spectral analyses, regardless of CTLV effects.

Measurements of EVE line ratios have been used to estimate flare density using the optically thin assumption (Mason *et al.*, [1979;](#page-18-26) Milligan *et al.*, [2012\)](#page-18-0), and different CTLV in the lines used will result in different derived temperatures, depending on whether the flare is observed at disk center or the limb. The ratio of the Fe XXI 14.573 nm to 12.875 nm line is sensitive to plasma density in the $10^{11} - 10^{13}$ cm⁻³ range, but differences in the resonant scattering cross-sections of the line causes CTLV in the line ratio. Figure [6e](#page-12-0) shows simulation results for the difference in the 14.573*/*12.875 ratio when the flare is observed at the limb from when it is observed at disk center. The maximum differences are approximately $+/- 10\%$. For densities near 3×10^{11} cm⁻³, a 10% error in the line ratio results in a 15 – 20% error in the derived densities.

7. Conclusions

- i) The CTLV in hot coronal emissions is expected based on the current understanding of flaring loop densities and geometries. Specifically, emissions with formation temperatures near 10 MK will cause limb darkening. For example, observations show that the observed flare irradiance will be approximately 10% lower for flares located 45◦ from disk center.
- ii) The flare CTLV can be used to estimate the increase in column density along the line of sight for limb flares versus disk-center flares. This analysis of EVE flare measurements showed the typical increase in column density for limb flares to be 1.76×10^{19} cm⁻². This column density increase is consistent with that of a loop diameter, suggesting that limb darkening occurs as farther (with respect to the observer) portions of a loop are obscured by nearer portions of the loop, an effect that is exacerbated as the loop approaches the limb.
- iii) The optically thin approximation is not valid for analyses of flares of M class or larger; further study is needed to verify if this is true for smaller flares as well. The optically thin approximation becomes increasingly invalid for flares located farther away from disk center, where portions of the flaring loop can be obscured by high-density plasma. Because resonant scattering cross-sections vary substantially from line to line, the degree of limb darkening is not uniform as a function of wavelength. Analyses that rely on relative measurements, such as line ratios, will therefore be subject to error if the plasma is assumed to be optically thin.
- iv) The CTLV of hot coronal emissions can cause a substantial reduction in the ionizing radiation that impacts planetary atmospheres. Further study is needed to compare the

relative magnitudes of the CTLV that occurs in the transition region and chromosphere with the CTLV that occurs in the corona.

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