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Rigidity Dependence of the Long-Term Variations of Galactic Cosmic-Ray Intensity in Relation to the Interplanetary Magnetic-Field Turbulence: 1968 – 2002

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Abstract We studied the relationship between the power-law exponent γ on the rigidity R of the spectrum of galactic cosmic-ray (GCR) intensity variation ($\delta D(R)/D(R) \propto R^{-\gamma}$) and the exponents ν_y and ν_z of the power spectral density (PSD) of the B_y and B_z components of the interplanetary magnetic field (IMF) turbulence (PSD $\sim f^{-\nu}$, where f is the frequency). We used the data from neutron monitors and IMF for the period of 1968 – 2002. The exponents ν_y and ν_z were calculated in the frequency interval $\Delta f = f_2 - f_1 = 3 \times 10^{-6}$ Hz of the resonant frequencies ($f_1 = 1 \times 10^{-6}$ Hz, $f_2 = 4 \times 10^{-6}$ Hz) that are responsible for the scattering of GCR particles with the rigidity range detected by neutron monitors. We found clear inverse correlations between γ and ν_y or ν_z when the time variations of the resonant frequencies were derived from *in situ* measurements of the solar wind velocity U_{sw} and IMF strength B during 1968 – 2002. We argue that these inverse relations are a fundamental feature in the GCR modulation that is not restricted to the analyzed years of 1968 – 2002.

Keywords Galactic cosmic ray intensity · Interplanetary magnetic field turbulence · Long period variations · Rigidity dependence

1. Introduction

The relationship between the power-law exponent γ on the rigidity R of the spectrum of galactic cosmic-ray (GCR) intensity variations ($\delta D(R)/D(R) \propto R^{-\gamma}$) and the exponent ν

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of the power spectral density (PSD) of the interplanetary magnetic-field (IMF) turbulence (PSD $\sim f^{-\nu}$, where f is the frequency) and its long-term changes are one of the most interesting research topics on the GCR modulation in the heliosphere. Several fundamental processes such as diffusion, convection, changes in GCR particle energy due to interactions with the radial expansion of the solar wind, and drifts due to curvature and gradient of the regular IMF and on the heliospheric current sheet (HCS) are responsible for the GCR intensity variations in the heliosphere.

The roles of various processes in the GCR intensity modulation depend upon the rigidity of GCR particles and the spatial and time scales of GCR intensity variations. It is shown that, in the first approximation, about 75 – 80 % of the 11-year variation of the GCR intensity can be interpreted based on the anisotropic diffusion–convection model of GCR propagation (Dorman, 2001; Alania, 2002; Alania, Iskra, and Siluszyk, 2010; Strauss and Potgieter, 2014) for particles of rigidity range 5 – 35 GV that are detected by neutron monitors (NMs). According to the quasi-linear theory (QLT) by Jokipii (1971) and Shalchi (2009), there should be a relationship between the parallel diffusion coefficient $K_{//}$ and rigidity R of GCR particles, as $K_{//} \propto R^{\alpha}$ for the rigidity range of NMs. The parameter α is represented as $\alpha = 2 - \nu$, where ν is the exponent of the PSD of the IMF turbulence.

Comprehensive studies based on numerical solutions of Parker's transport equation and neutron monitor data (Alania and Iskra, 1995; Alania, 2002; Alania, Iskra, and Siluszyk, 2003) have shown that the exponent γ is proportional to α. We have also shown that the exponent γ demonstrates clear 11-year variations (Alania, 2002; Alania, Iskra, and Siluszyk, 2003; Siluszyk, Wawrzynczak, and Alania, 2011).

Figure 1 presents temporal variations in γ (middle panel) and sunspot number (SSN, bottom panel). The top panel shows the GCR intensity variations J from Moscow neutron monitor data rescaled to free space using the coupling coefficients by Yasue *et al.* (1982) and Alania, Iskra, and Siluszyk (2010). In the entire period of 1968 – 2002, there exists a clear negative correlation between J and γ (correlation coefficient $r = -0.62 \pm 0.12$) and a positive correlation between SSN and γ ($r = 0.70 \pm 0.11$), in spite of some time delays between the peaks and valleys in J, SSN, and γ of individual 11-year cycles.

The scattering of GCR particles in the heliosphere is generally caused by the turbulence of the B_y and B_z components of the IMF (Jokipii, 1971; Bieber, Mathaeus, and Smith, 1994), although the roles of the two components are not equal at all. The average power of the PSD of the B_y component ($\sim f \nu_y$) is more than twice as large as the average power of the B_z component ($\sim f \nu_z$). Nevertheless, there is a good correlation between the temporal variations in the exponents ν_y and ν_z (Alania, Iskra, and Siluszyk, 2009).

Therefore, in spite of the important role in the GCR modulation ascribed to the B_y component of the IMF (Alania, Iskra, and Siluszyk, 2009), we here consider the behaviors of both exponents ν_y and ν_z *versus* solar activity. Our finding $\gamma \propto 2 - \nu_y$ (Alania and Iskra, 1995; Alania, 2002; Alania, Iskra, and Siluszyk, 2003) is consistent with the inverse correlation between γ and ν_y, as expected from the QLT by Jokipii (1971) and Shalchi (2009).

We have calculated the annual average values of ν_y and ν_z (Alania, Iskra, and Siluszyk 2008a, 2008b, 2009, 2010) for the period 1968 – 2002 and divided them into three subperiods – I: 1968 – 1976, II: 1977 – 1989, and III: 1990 – 2002. Period I corresponds to the second half of solar cycle 20. Period II consists of cycle 21 and the first half of cycle 22. Period III contains the second half of cycle 22 and the first half of cycle 23. We have found high inverse correlations between γ and ν_y or ν_z for period II; while the relationships between γ and ν_y or ν_z are less obvious for periods I and III.

The aim of this paper is i) to study the relationship between the power-law exponent γ of the long-term variations in the GCR intensity ($\delta D(R)/D(R) \propto R^{-\gamma}$) and the exponents

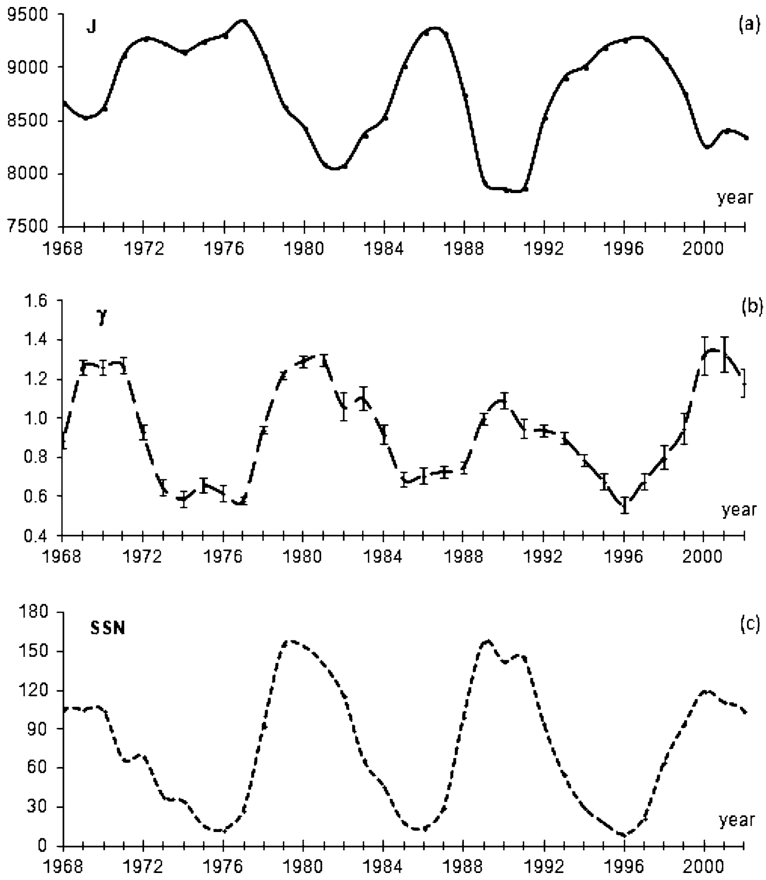

Figure 1 Temporal variations in the annual means for the period of 1968 – 2002 of (a) the GCR intensity measured by the Moscow neutron monitor, (b) the rigidity spectrum exponent γ of the GCR intensity variations, and (c) the sunspot number.

ν_y and ν_z of the PSD of the B_y and B_z components of the IMF turbulence, and ii) to show that the inverse correlations between γ and ν_y or ν_z are a fundamental feature in the GCR modulation not restricted only to the analyzed period, as expected from the QLT of GCR particles for the rigidity range of NMs.

2. Observational Data and Analysis Methods

We have used data of NMs and IMF in 1968 – 2002. Table 1 presents the list of NMs used to calculate the exponent γ (see Appendix A). Unfortunately, we need a sufficient number of NMs that functioned well for a long period to calculate γ. To provide at least an acceptable accuracy for the calculated value of γ we used carefully selected appropriate NMs for each chosen period of solar activity.

Alania, Iskra, and Siluszyk (2009, 2010) calculated the annual average values of ν_y and ν_z by the method given in Appendix B. This method (which we call the first method) assumes the same frequency interval $\Delta f = f_2 - f_1 = 3 \times 10^{-6}$ Hz and constant resonant frequencies

Table 1 Neutron monitors used to calculate γ, denoted by '+' in the analysis periods.

Reference point (RP):			1965		1976		1986		1996	
No.	Stations	Cut-off rigidity [GV]	1960 – 1964	1966 – 1970	1971 – 1975	1977 – 1981	1982 – 1985	1988 – 1991	1992 – 1996	1998 – 2002
1	Apatity	0.65	–	–	–	–	–	–	–	+
2	Climax	3.03	+	+	+	+	+	+	+	+
3	Deep River	1.02	+	–	–	+	+	+	–	–
4	Goose Bay	0.52	–	–	–	+	+	+	+	–
5	Haleakala-Huancayo	13.4	+	+	+	+	+	+	+	+
6	Hermanus	4.90	–	+	–	+	+	+	–	+
7	Inuvik	0.18	–	+	+	+	+	+	+	–
8	Jungfraujoch	4.48	–	–	–	+	–	–	–	–
9	Kergulen Is	1.19	–	–	+	–	–	–	–	–
10	Kiel	2.29	+	+	+	+	+	+	+	+
11	Mc Murdo	0.01	–	–	–	–	–	–	–	+
12	Moscow	2.46	+	+	+	+	+	+	+	+
13	Mt Norikura	11.39	–	–	–	–	+	–	–	–
14	Mt Washington	1.24	–	+	+	–	+	–	+	–
15	Pic-du-Midi	5.36	–	+	+	–	–	–	–	–
16	Potchestroom	7.30	–	–	–	+	+	+	+	+
17	Rome	6.32	–	–	–	–	–	–	–	+
18	Tbilisi	6.91	–	–	–	+	+	+	–	–

($f_1 = 1 \times 10^{-6}$ Hz, $f_2 = 4 \times 10^{-6}$ Hz), corresponding to the constant solar wind velocity $U_{sw} = 432.4$ km s^{-1} and IMF strength $B = 4.6$ nT for the whole period 1968 – 2002.

On the other hand, the solar wind velocity U_{sw} and the IMF strength B might have varied significantly from year to year during 1968 – 2002 and the frequencies f_1 and f_2 might have changed correspondingly. Therefore, it is interesting to see how the correlations between γ and ν_y or ν_z change when ν_y and ν_z are calculated for the frequency interval $\Delta f = f_2 - f_1$ obtained based on the solar wind velocity U_{sw} and the IMF strength B that change from year to year (hereafter called the second method).

Figures 2a and 2b show the variations in the solar wind velocity U_{sw} and the IMF strength B in the period 1968 – 2002. The IMF strength B (Figure 2a) shows a quasi-11-year periodicity, while the solar wind velocity U_{sw} (Figure 2b) does not show a clear 11-year periodicity.

To calculate the resonant frequency f_{res} and the corresponding frequency interval Δf from the values of U_{sw} and B that vary with the solar activity, we used the formula $f_{res} = \frac{300}{2\pi} \frac{U_{sw} \cdot B}{R}$ by Jokipii and Coleman (1968) and Jokipii (1971). The temporal variations in f_{res} thus derived for GCR particles of rigidity $R = 10$ GV are presented in Figure 3.

By comparing Figures 2a, 2b, and 3, one can see that the time profile of f_{res} is mostly caused by the variations in B from year to year that clearly show quasi-11-year periodicity (correlation coefficient r between f_{res} and B is $r = 0.95 \pm 0.01$), while the role of the solar wind velocity U_{sw} is not significant (correlation coefficient r between f_{res} and U_{sw} is only $r = 0.31 \pm 0.03$). As an example, Figure 4 presents variations in f_{res} *versus* rigidity R for

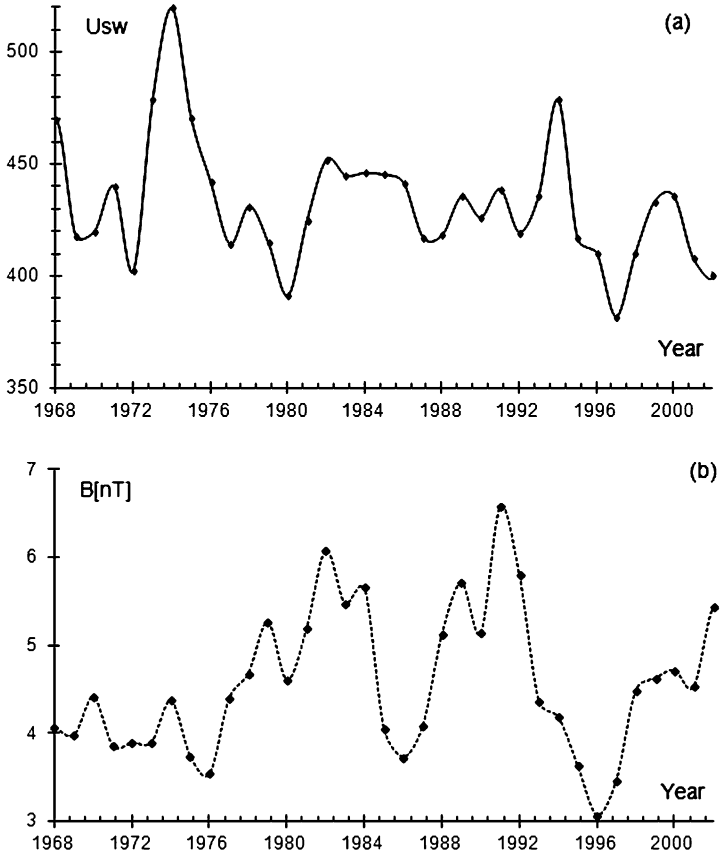

Figure 2 Temporal variations in (a) the solar wind velocity U_{sw} and (b) the IMF strength B during the period of 1968 – 2002.

given B and U_{sw} in the minimum (1986, $B = 3.7$ nT, $U_{\mathrm{sw}} = 441.6\ \mathrm{km\,s^{-1}}$) and maximum (1989, $B = 5.8$ nT, $U_{\mathrm{sw}} = 435.5\ \mathrm{km\,s^{-1}}$) epochs of solar activity.

The ranges of resonant frequencies and the corresponding intervals $\Delta f = f_1 - f_2 = (3 - 4) \times 10^{-6}$ Hz for which the exponents ν_y and ν_z are calculated for variable U_{sw} and B are presented in Table 2. For each chosen period the values of f_{res} are different, but the frequency intervals Δf remain nearly constant.

3. Results and Discussion

By using the resonant frequencies f_{res} and frequency intervals Δf given in Table 2, we have calculated the corresponding annual values of the exponents ν_y and ν_z. Figure 5a presents temporal variations in the exponents ν_y derived from the first method (dotted line) and the second method (dashed line) in periods I, II, and III, together with the variations in γ (solid line). Figure 5b presents the results for ν_z.

To carry out a more detailed study on the relations between γ and ν_y or ν_z, we evaluated the correlation coefficients $r_1(\gamma; \nu_y)$ and $r_1(\gamma; \nu_z)$ when the values ν_y and ν_z were derived

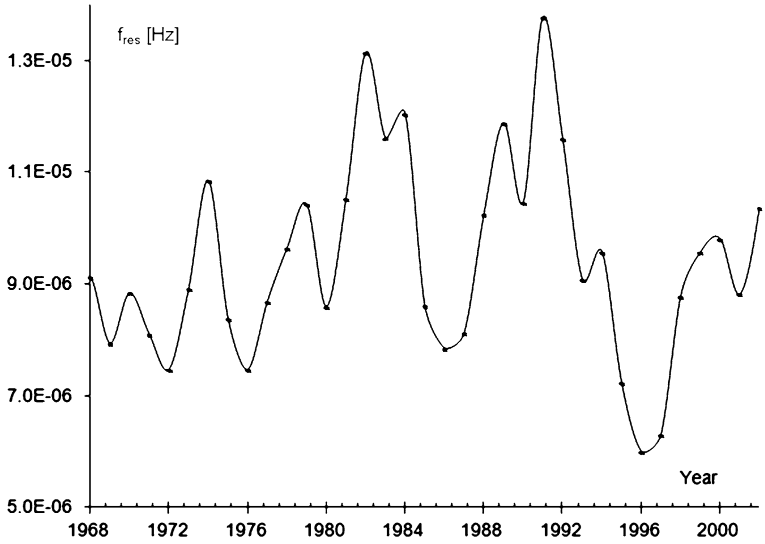

Figure 3 Annual variations in the resonant frequency f_{res} for GCR particles of rigidity $R = 10$ GV from 1968 to 2002.

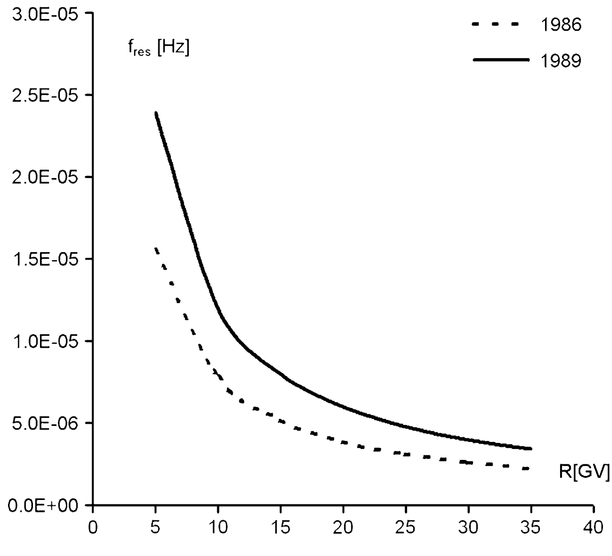

Figure 4 Temporal variations in the resonant frequency f_{res} *versus* rigidity R for given B and U_{SW} for the minimum epoch 1986 (the dotted line) and the maximum epoch 1989 (the solid line) of solar activity.

from the first method, and $r_2(\gamma; \nu_y)$ and $r_2(\gamma; \nu_z)$ derived from the second method for three periods I, II, and III separately, and for the whole period 1968 – 2002 (Table 3).

A high inverse correlation is observed in period II for both ν_y and ν_z, regardless of whether we used the first or the second method. This conclusion is valid even if the values of ν_y and ν_z were derived from different frequency ranges of the IMF turbulence shown in Table 2 over cycle 21 (1976 – 1986).

A distinction in correlations with γ for ν_y and ν_z is observed for periods I and III. A clear inverse correlation takes place between γ and ν_y or ν_z when the second method is used, namely, when the changes in the resonant frequency that depend on the solar activity level

Table 2 Frequency ranges and frequency intervals Δf calculated for different time intervals of 1968 – 2002.

Periods	Frequency range [Hz]	Δf [Hz]
1968	$7.9 \times 10^{-7} - 3.8 \times 10^{-6}$	3.0
1969 – 1970	$2.1 \times 10^{-6} - 5.8 \times 10^{-6}$	3.7
1971 – 1975	$7.9 \times 10^{-7} - 3.8 \times 10^{-6}$	3.0
1976 – 1977	$7.6 \times 10^{-7} - 3.8 \times 10^{-6}$	3.0
1978 – 1980	$1.7 \times 10^{-6} - 5.8 \times 10^{-6}$	4.1
1981 – 1982	$1.0 \times 10^{-6} - 4.0 \times 10^{-6}$	3.0
1983	$1.7 \times 10^{-6} - 5.8 \times 10^{-6}$	4.1
1984 – 1987	$7.6 \times 10^{-7} - 3.8 \times 10^{-6}$	3.0
1988	$1.0 \times 10^{-6} - 4.0 \times 10^{-6}$	3.0
1989 – 1992	$1.7 \times 10^{-6} - 5.8 \times 10^{-6}$	4.1
1993 – 1998	$7.0 \times 10^{-7} - 3.7 \times 10^{-6}$	3.0
1999 – 2000	$1.7 \times 10^{-6} - 5.2 \times 10^{-6}$	3.5
2001	$1.0 \times 10^{-6} - 4.0 \times 10^{-6}$	3.0
2002	$1.7 \times 10^{-6} - 5.2 \times 10^{-6}$	3.5

are taken into account. It is likely that odd and even 11-year solar cycles may show different turbulence structures of IMF. We do not exclude, for example, the rearrangements of the IMF turbulence depending on the sequence of the reversal of the Sun's global magnetic field; for solar cycles 20 and 22 the Sun's global field changed from negative ($A < 0$) to positive ($A > 0$) polarities, while solar cycle 21 was in the opposite situation.

The inverse correlation between γ and ν in periods I and III (during cycles 20 and 22) that was found by allowing the resonant frequencies to vary with the solar activity level should be considered as normal because it agrees with the theory of scattering of GCR particles in the interplanetary space. On the other hand, the high inverse correlations between γ and ν_y or ν_z regardless of the changes in the IMF strength could be exceptional or accidental.

Figure 6a shows the temporal variations in γ (solid line) and ν_y derived from the first (dashed line) and second method (dotted line) for the whole period 1968 – 2002. Figure 6b shows the same results for the exponent ν_z.

Figure 6 shows that the exponent γ has clear 11-year periodicity with its maxima at the maximum epochs of solar activity, while the maxima of ν_y and ν_z occur at the minimum epochs. These results indicate a significant rearrangement of the IMF turbulence structure from the maximum to the minimum epochs of solar activity, although its detailed properties may vary among individual 11-year cycles.

We found that the variations in ν_y and ν_z according to the level of solar activity play a central role in determining the exponent γ, or more generally, in characterizing long-term variations in the GCR intensity. Thus, one can conclude that the inverse relations between γ and ν are a fundamental feature in the GCR modulation, not only restricted to the analyzed years of 1968 – 2002, but found generally in any period.

4. Summary

1. Temporal variations in the exponents ν_y and ν_z of the PSD of the IMF obtained by *in situ* measurements of the B_y and B_z components in near-Earth space reflect the average structure of the IMF turbulence in the vicinity of the heliosphere where the long-term

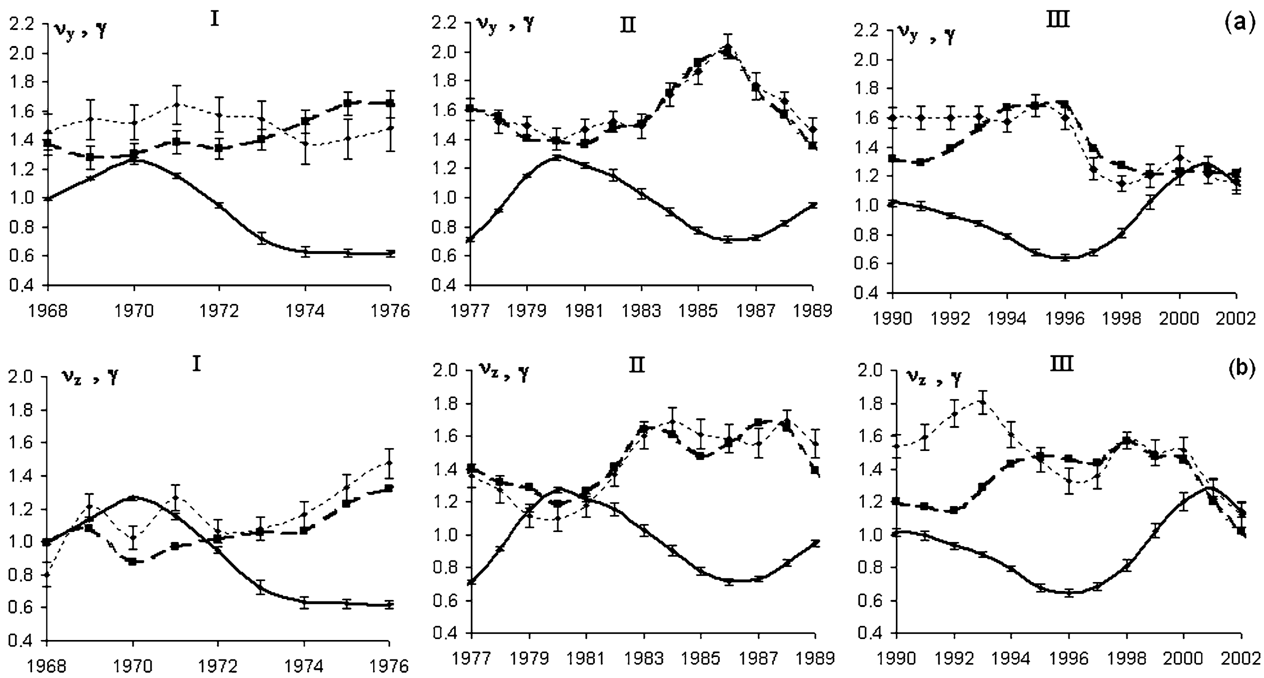

Figure 5 Panel (a) shows temporal variations in the exponents ν_y derived from the first method (dotted line) and the second method (dashed line) in three sub-periods I, II, and III. The solid curve shows the variations in γ. Panel (b) presents the results for the exponent ν_z.

Table 3 Correlation coefficients r between γ and ν_y or ν_z. The first and the second methods were used to derive ν_y or ν_z in r_1 and r_2, respectively.

	Period	$r_1(\gamma;\nu_y)$	$r_2(\gamma;\nu_y)$	$r_1(\gamma;\nu_z)$	$r_2(\gamma;\nu_z)$
I	1968 – 1976	0.64 ± 0.17	-0.86 ± 0.11	-0.43 ± 0.20	-0.78 ± 0.14
II	1977 – 1989	-0.78 ± 0.11	-0.78 ± 0.11	-0.69 ± 0.13	-0.66 ± 0.13
III	1990 – 2002	-0.43 ± 0.16	-0.78 ± 0.12	-0.19 ± 0.18	-0.54 ± 0.15
	1968 – 2002	-0.32 ± 0.17	-0.66 ± 0.13	-0.32 ± 0.17	-0.38 ± 0.16

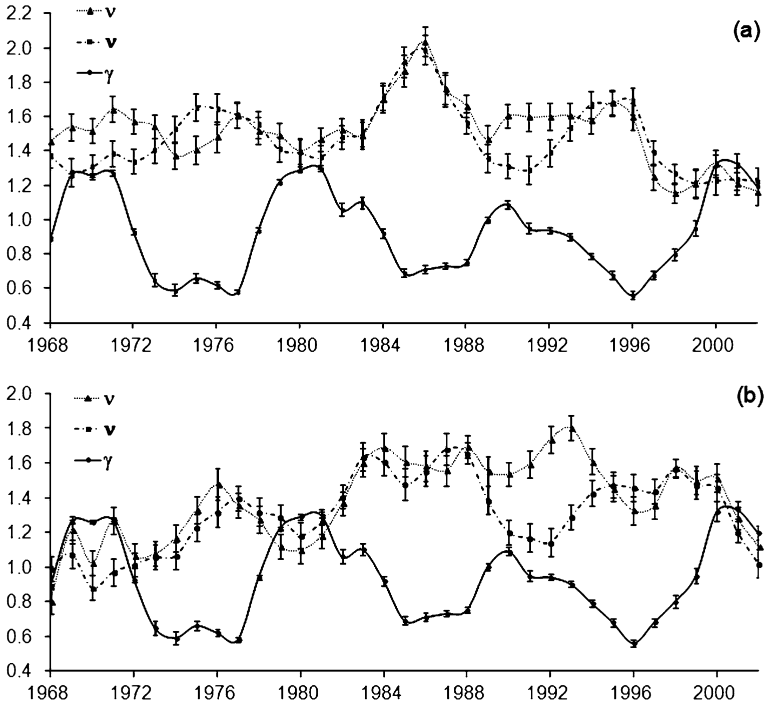

Figure 6 Panel (a) shows the temporal variations in γ (solid line) and ν_y derived from the first (dotted line) and second method (dashed line) for the whole period 1968 – 2002. Panel (b) shows the results for the exponent ν_z.

variations of the GCR modulation occur. Under this assumption, both ν_y and ν_z, which clearly show 11-year periodicity, can be considered as the central parameters controlling the modulation of GCR in the rigidity range of neutron monitor measurements.

2. We showed that for GCR particles of rigidity 10 GV, the resonant frequencies are lower in the minimum epochs than in the maximum epochs of solar activity for 1968 – 2002. These changes in the resonant frequencies of the IMF turbulence are primarily due to the quasi-11-year periodicity in the strength B of the IMF.
3. A significant high inverse correlation is observed between γ and ν_y or ν_z in period II (1977 – 1989 which includes solar cycle 21). This correlation persists even if ν_y and ν_z are calculated for constant values of B and U_{sw} corresponding to resonant frequencies averaged over the whole period of 1968 – 2002, or if the resonant frequencies are varied

according to the level of solar activity in period II. The values of ν_y and ν_z do not change if the frequency interval $\Delta f = (3-4) \times 10^{-6}$ Hz is shifted in frequency as a function of time during cycle 21. This is possibly due to a unique condition for solar cycle 21, which is characterized by nearly homogeneous and isotropic turbulence of the IMF in space.

4. On the other hand, for periods I and III (solar cycles 20 and 22) a clear inverse correlation between γ and ν_y or ν_z only occurs if the resonant frequencies of turbulence are varied according to the level of solar activity year by year.
5. We argue that the inverse correlation between γ and ν during cycles 20 and 22, which is found by changing the resonant frequencies with the level of solar activity (in our case determined by the values of IMF strength), should be considered as a normal situation, consistent with the theory of scattering of GCR particles in interplanetary space. On the other hand, the case of solar cycle 21, in which the inverse correlation between γ and ν is high and does not depend on the changes in IMF strength with solar activity, should be considered as an exceptional case.

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Appendix A: Calculation of Exponent γ

Neutron monitor data were used to calculate the exponent γ on the rigidity R of the spectrum of GCR intensity variations ($\frac{\delta D(R)}{D(R)} = AR^{-\gamma}$) in the period of 1960 – 2002 (Siluszyk *et al.*, 2005; Alania, Iskra, and Siluszyk, 2008a, 2008b, 2009, 2010). These were based on Alania, Iskra, and Siluszyk (2003), who calculated the exponent γ using thoroughly selected monthly average data of neutron monitors in the period of 1968 – 2002, including four ascending and four descending phases of solar activity in the $A > 0$ and the $A < 0$ epochs. The criterion for data selection adopted in these studies was the continuous operation of neutron monitors with different cut-off rigidities throughout the analyzed period. The magnitude J_i^k of monthly averaged GCR intensity variations from the ith neutron monitor was calculated as $J_i^k = \frac{N_k - N_0}{N_0}$, where N_k is the running monthly average count rate (months $k = 1, 2, 3, \ldots, 12$) and N_0 is the monthly average count rate for the year of highest intensity (usually in the minimum epoch of solar activity). The count rate of the highest intensity is accepted as the 100 % level. The list of selected neutron monitors is presented in Table 1. The magnitude J_i^k of monthly averaged GCR intensity variations measured by the ith neutron monitor with the geomagnetic cut-off rigidity R_i and the average atmospheric depth h_i is defined by Dorman (1975) as

$$J_i^k = \int_{R_i}^{R_{\max}} \left(\frac{\delta D(R)}{D(R)} \right)_k W_i(R, h_i)\, \mathrm{d}R, \qquad (1)$$

where $(\delta D(R)/D(R))_k$ is the rigidity spectrum of the GCR intensity variations for the kth month; $W_i(R, h_i)$ is the coupling coefficient for the neutron component of GCR by Dorman (1975) and Yasue *et al.* (1982); $R_{\max}$ is the upper limit in rigidity beyond which the GCR intensity variation vanishes. For a power-law rigidity spectrum $(\delta D(R)/D(R))_k = AR^{-\gamma_k}$ one can write

$$J_i^k = A_i^k \int_{R_i}^{R_{\max}} R^{-\gamma_k} W_i(R, h_i)\,\mathrm{d}R, \tag{2}$$

where A_i^k is the magnitude of the GCR intensity variations rescaled to the heliosphere (free space). From Equation (2) we obtain

$$A_i^k = J_i^k \Big/ \int_{R_i}^{R_{\max}} R^{-\gamma_k} W_i(R, h_i)\,\mathrm{d}R. \tag{3}$$

The values of A_i^k should be the same (within the accuracy of the calculations) for any ith neutron monitor if the values of parameters γ_k and $R_{\max}$ are properly determined. A similarity of the values of A_i^k for various neutron monitors is an essential argument to affirm that the data from a particular neutron monitor and the method of calculations for γ_k are reliable. To find the temporal variations in the exponent γ_k (months $k = 1, 2, 3, \dots, 12$), a minimization of the expression

$$\varphi = \sum_i^n \left(A_i^k - A^k\right)^2$$

(where $A^k = \frac{1}{n}\sum_i^n A_i^k$ and n is the number of neutron monitors) was provided by Siluszyk *et al.* (2005) and Alania, Iskra, and Siluszyk (2008a, 2008b, 2009, 2010). The values of the expression $\int_{R_i}^{R_{\max}} R^{-\gamma_k} W_i(R, h_i)\,\mathrm{d}R$ for ranges of $R_{\max}$ (from 30 GV up to 200 GV with a step of 10 GV) and γ (from 0 to 2 with a step of 0.05) were calculated based on the method presented in Dorman (1975) and Yasue *et al.* (1982). The upper limit in rigidity, $R_{\max}$, is taken to be 100 GV. This assumption was regarded as reasonable for the 11-year variation of the GCR intensity by Gushchina *et al.* (2008). The minimization of φ for the smoothed monthly means (with the interval of 13 months) of the GCR intensity variations was carried out with respect to γ_k for the neutron monitors given in Table 1.

Appendix B: Calculation of Exponent ν

We collected the data of the B_y component of the IMF for the period of 1968 – 2002 from http://spidr.ngdc.noaa.gov. Then we calculated the power spectrum density (PSD) and its exponent ν on the frequency f by the method of Blackman and Tukey by Lyons (1996). First of all, we calculated the autocorrelation function; if $\{B_{y_i}\}$ $(i = 1, \dots, N)$ is a series of daily values of B_y of the IMF, the autocorrelation function R_r $(r = 0, 1, \dots, m)$ is given as

$$R_r = \frac{1}{N-r} \sum_{i=1}^{N-r} B_{y_i} B_{y_{i+r}},$$

where $N = 365$ and $m = 182$. We then calculated the discrete Fourier transform of the autocorrelation function, PSD_k, $(k = 0, 1, \dots, m)$ from the formula

$$\mathrm{PSD}_k = 2\Delta t \left(R_0 + 2\sum_{r=1}^{m-1} R_r \cos\frac{\pi k r}{m} + R_m \cos\frac{\pi k}{m} \right).$$

Here PSD_k corresponds to the power spectral density for the frequency $f_k = \frac{k}{2m\Delta t}$ [Hz]. The series R_r and $\{B_{y_i}\}$ are given with a time interval of $\Delta t = 1$ day. The whole frequency range $(0, \frac{1}{2\Delta t})$ is divided into m sub-intervals of length $\frac{1}{2m\Delta t}$ [Hz]. Therefore, the frequency data points are $0, \frac{1}{2m\Delta t}, \frac{2}{2m\Delta t}, \ldots, \frac{m}{2m\Delta t}$. We approximated the dependence of PSD_k on the frequency f_k by a power-law function as $\mathrm{PSD} = Pf^{-\nu}$. The values of P and ν are derived by using the least-squares method.

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