

On Solar-Wind Electron Heating at Large Solar Distances

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Abstract We study the temperature of electrons advected with the solar wind to large solar distances far beyond 1 AU. Almost nothing is known about the thermodynamics of these electrons from *in-situ* plasma observations at these distances, and usually it is tacitly assumed that electrons, due to adiabatic behaviour and vanishing heat conduction, rapidly cool off to very low temperatures at larger distances. In this article we show, however, that electrons on their way to large distances undergo non-adiabatic interactions with travelling shocks and solar-wind bulk-velocity jumps and thereby are appreciably heated. Examining this heating process on an average statistical basis, we find that solar-wind electrons first cool down to a temperature minimum, which depending on the occurrence frequency of bulk velocity jumps is located between 3 and 6 AU, but beyond this the lowest electron temperature again starts to increase with increasing solar distance, finally achieving temperatures of about 7×10^4 K to 7×10^5 K at the location of the termination shock. Hence these electrons are unexpectedly shown to play an important dynamical role in structuring this shock and in determining the downstream plasma properties.

1. Introduction

Unfortunately, little is known about the thermal behaviour of solar-wind electrons at solar distances beyond 3.5 AU, up to where *in situ* plasma observations still are available (see Pilipp *et al.*, 1987; Feldman *et al.*, 1975; Scime *et al.*, 1994). Obviously, however, solar-wind electrons even within this distance range seem to behave differently from what is expected from classical transport theories such as that presented by Spitzer (1969). For instance, the classical heat-conduction flow by electrons is obviously strongly suppressed, although surprisingly enough, electrons are, nevertheless, not cooling off with increasing distance, as

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could be expected in an adiabatic case (see Scime *et al.*, 1994). As explained by Chashei and Fahr (2000) and Chashei, Fahr, and Lay (2001), the observationally supported magnitude of the electron heat-conduction flow can, however, be obtained, if one takes into account that electrons transported beyond the electric potential of the coronal polarisation field have truncated Maxwellian distribution functions with an underpopulation in the sunward part of the super-escape-velocity branch.

In this article we consider in addition to the energy-processing of electrons at travelling shocks only an electron-cooling process due to the conservation of the electron magnetic moment in the comoving solar-wind frame with decreasing magnetic-field magnitudes at increasing solar distances. We do this because we are mainly interested with conditions of the solar-wind plasma at large solar distances (≥ 10 AU). Here competing processes such as Whistler-wave turbulence generation due to electron heat conduction instability as discussed by Scime *et al.* (1994) or Gary *et al.* (1994) no longer play a role, because the electron heat flow rapidly falls off as r^{-3} , and hence this process does not compete with our magnetic-cooling process beyond 10 AU.

On the other hand, other wave–electron interaction processes such as pitch-angle scattering or energy diffusion can be neglected as well at larger distances. This has to do with the fact that both processes are determined by diffusion coefficients (see Schlickeiser, Dung, and Jaeckel, 1991; Achatz *et al.*, 1993) in the form

$$\mathbf{D}_{\mu\mu, vv} \sim \left[\frac{I_+ \left(\frac{\Omega_e}{v\mu} \right)}{v|\mu|} + \frac{I_- \left(\frac{\Omega_e}{v\mu + 2v_A} \right)}{|v\mu + 2v_A|} \right], \quad (1)$$

which require turbulence power [$I_+ \left(\frac{\Omega_e}{v\mu} \right)$] at the electron gyrofrequency [Ω_e]. However, as we have discussed in Chashei and Fahr (2000), there is not enough advected power arriving at larger distances to make this process effective.

On the other hand, the self-driven turbulence generated by pick-up ions at larger distances, although it contains substantial free energy, appears to be unable to create resonances with electrons at k -vectors/frequencies. As shown by Chashei, Fahr, and Lay (2005) or Fahr and Fichtner (2012), there is simply a power cut-off already at frequencies far below the electron gyrofrequency (*i.e.* six orders of magnitude) or the plasma frequency (*i.e.* eight orders of magnitude).

The electron heat flux is known to be non-classical (*i.e.* it does not follow a Spitzer–Härm Fourier law), but drops off much more rapidly with distance. At small solar distances the so-called *strahl*-configuration of the electron distribution function is responsible for this behaviour, but this configuration, which originates from Coulomb-differentiations above the solar corona, drives instabilities and disappears at larger solar distances where it transforms into a pitch-angle-isotropic core–halo distribution (see, *e.g.*, McComas *et al.*, 1992) with an electron heat flux there falling off as $r^{-2.36}$, instead of $r^{-3.08}$ at smaller distances (Scime *et al.*, 1994). We start our considerations with electron conditions at distances beyond 10 AU, where these *strahl*-induced instabilities have stopped operating.

At such distances beyond 10 AU, up to now only cooling processes were envisioned to operate on solar-wind electrons, and thus it was expected that electrons will soon cool down to fairly low temperatures, so that their thermal pressures no longer play a dynamical role. In this article, however, we discuss processes of solar-wind electrons that react to the passage of solar-wind bulk velocity jumps in the form of travelling shocks. As we show, they thereby gain substantial amounts of thermal energy, so that it turns out that they are heated to increasing temperatures at larger distances, *i.e.* they undergo a temperature minimum and beyond that show increasing temperatures with increasing distances. This substantially changes the picture of distant solar-wind electrons and may mean that finally electrons indeed play an

important role in the dynamics of the distant solar wind near and beyond the termination shock.

2. The Downstream Centre-of-Mass Bulk System

For a shock with an inherent electric potential of $\Delta\Phi$, the bulk velocities of protons and electrons change according to

$$e\Delta\Phi = \frac{1}{2}M(U_1^2 - U_{p2}^2) \tag{2}$$

and

$$-e\Delta\Phi = \frac{1}{2}m(U_1^2 - U_{e2}^2), \tag{3}$$

respectively. Here m and M are electron and proton masses, and the suffixes 1 and 2 characterise upstream and downstream quantities.

The resulting downstream momentum flows thus are given by

$$F_{e2} = mn_{e2}U_{e2}^2 \tag{4}$$

and

$$F_{p2} = Mn_{p2}U_{p2}^2. \tag{5}$$

The centre-of-mass (COM) bulk flow velocity [$U_{2,COM} = U_2^*$] derived from both these above momentum flows is derived from the following equation:

$$F_2^* = Mn_{p2}U_{p2}^2 + mn_{e2}U_{e2}^2 = 2n_2^* \left(\frac{m + M}{2} \right) U_2^{*2}, \tag{6}$$

where n_2^* denotes the common density of electrons and protons in the COM-frame.

In all systems the particle number flow should be conserved, *i.e.* identical, and thus one finds

$$2n_1U_1 = 2n_2^*U_2^* = n_{e2}U_{e2} + n_{p2}U_{p2}, \tag{7}$$

yielding

$$MU_{p2} + mU_{e2} = 2 \left(\frac{m + M}{2} \right) U_2^*, \tag{8}$$

and finally

$$U_2^* = \frac{1}{m + M} (MU_{p2} + mU_{e2}). \tag{9}$$

Inserting from Equation (1) the above result for U_{e2} , one then finally finds

$$\begin{aligned} U_2^* &= \frac{MU_{p2} + m\sqrt{\frac{M}{m}(U_{1p}^2 - U_{2p}^2)}}{m + M} \approx U_{2p} \left(1 + \frac{m}{M} \sqrt{\frac{M}{m}(s_p^2 - 1)} \right) \\ &= U_{2p} \left(1 + \sqrt{\frac{s_p^2 - 1}{1840}} \right), \end{aligned} \tag{10}$$

where $s_p = U_{1p}/U_{2p}$ is the proton compression ratio at the shock, clearly showing that the centre-of-mass bulk flow velocity [U_2^*] is essentially identical to $U_{2,p}$.

3. The Process of Overshooting Electrons at Travelling Shocks

In a recent article on the MHD shock structure (Fahr, Siewert, and Chashei, 2012) we have shown that the unavoidable electron overshooting into the downstream plasma frame is a highly important physical process that eventually leads to strong electron heating, to entropy generation, and to high compression ratios. Starting from the consideration of a shock potential step $[\Delta\Phi]$ associated with the deceleration of the proton bulk velocity from its upstream to its downstream value, we find that electrons from just this potential jump simultaneously get, at the first moment, an accelerating velocity kick.

As we have shown in Fahr, Siewert, and Chashei (2012), the initial overshoot velocity of the electrons (*i.e.* differential velocity with respect to the downstream bulk flow) in the downstream region is given by

$$\delta U_e = \left| U_2 - U_1 \sqrt{1 + \frac{M}{m} \left(1 - \frac{1}{s^2} \right)} \right|, \tag{11}$$

where $U_{1,2}$ are the centre-of-mass plasma bulk velocities at the upstream and downstream side of the shock, respectively.

This formula clearly shows that according to the mass ratio $[m/M = \mu_{e,i} = 1/1840]$ the resulting overshoot velocity is as high as

$$\delta U_e = \left| U_2 - U_1 \sqrt{1 + \frac{M}{m} \left(1 - \frac{1}{s^2} \right)} \right| \approx (U_1/s) \left| 1 - \sqrt{\frac{M}{m} (s^2 - 1)} \right| \geq 43U_1. \tag{12}$$

While it is interesting to note that the ratio of the electron overshoot energy $[E_{t,e}]$ and the kinetic energy $[E_{kin,i}]$ of the downstream ion bulk flow is given by

$$\frac{E_{t,e}}{E_{kin,i}} = \frac{\frac{1}{2} m_e (U_1 \sqrt{\frac{m_p}{m_e}})^2 n_{2,e}}{\frac{1}{2} m_p U_2^2 n_{2,i}} = s^2 \tag{13}$$

and thus is purely given by the compression ratio, we did, however, show that in an electron–proton MHD shock just this resulting compression ratio $[s]$ itself strongly depends on electron pressure. This is why one can also write the overshoot velocity in the form

$$\delta U_e(\mu_{e,i}) = \left| U_2 - U_1 \sqrt{1 + \frac{1}{\mu_{e,i}} \left(1 - \frac{1}{s^2} \right)} \right| = \left| U_2 - U_2 \sqrt{s^2 + \frac{1}{\mu_{e,i}} (s^2 - 1)} \right|. \tag{14}$$

This overshoot phenomenon not only plays a role at standing shocks, for instance at the solar-wind termination shock, but is also relevant at travelling shocks such as bulk-velocity jumps in the supersonic solar wind. Therefore we examined the influence on electrons that such travelling shocks might have.

4. Travelling Solar-Wind Shocks Influencing Electrons

In a previous article (Fahr, Siewert, and Chashei, 2012) we have developed a theoretical description of travelling solar-wind bulk velocity jumps that process ions to higher energies. It was shown there that this process can be formulated as a velocity-space diffusion process that helps to energise ions while they are advected outwards with the solar wind to larger distances. Here we return to this process with the aim to analogously apply it to electrons.

As in Fahr, Siewert, and Chashei (2012), we thus simply obtain the change in time of the electron distribution function [f_e] due to jump-induced velocity diffusion in the following form:

$$\left(\frac{\partial f_e}{\partial t}\right)_{\text{jump}} = -\frac{1}{v^2} \frac{\partial}{\partial v} \left[\frac{\Delta U \delta_e}{2L} v^2 \frac{\partial}{\partial v} (v^2 f_e) \right], \tag{15}$$

where ΔU is the average magnitude of the bulk velocity jump and L the average distance between such jumps, *i.e.* the typical characteristics of the chain of consecutive jumps. Here δ_e means a diffusion coefficient with an electron-specific magnitude, which we derive below in its concrete form.

First we now expand the above term into two parts by writing

$$\left(\frac{\partial f_e}{\partial t}\right)_{\text{jump}} = -\frac{1}{v^2} \frac{\partial}{\partial v} \left[\frac{\Delta U \delta_e}{2L} v^4 \frac{\partial}{\partial v} f_e \right] - \frac{1}{v^2} \frac{\partial}{\partial v} \left[\frac{\Delta U \delta_e}{2L} 2v^3 f_e \right]. \tag{16}$$

Here the second term $\left(\frac{\partial f}{\partial t}\right)_{\text{jump},2}$, due to its conformal shape, can be taken together with the magnetic cooling term $\left(\frac{\partial f}{\partial t}\right)_{\text{jump},3}$, first derived by Fahr (2007) and subsequently studied in the frame of a general ion phase-space transport equation by Fahr and Fichtner (2011). Then one obtains

$$\begin{aligned} \left(\frac{\partial f}{\partial t}\right)_{\text{jump},2} + \left(\frac{\partial f}{\partial t}\right)_{\text{jump},3} &= \frac{1}{v^2} \frac{\partial}{\partial v} \left[\frac{v^3 U}{r} f_e - \frac{\Delta U \delta_e}{2L} 2v^3 f_e \right] \\ &= \left(\frac{U}{r} - \frac{\Delta U \delta_e}{L} \right) \frac{1}{v^2} \frac{\partial}{\partial v} [v^3 f_e]. \end{aligned} \tag{17}$$

Since we are aiming now specifically at describing the electron temperature, we multiply the whole transport equation above by $(m/3K)v^4 dv$ and integrate over velocity space. Then one finds

$$\left(\frac{\partial T_e}{\partial t}\right)_{\text{jump},2,3} = \left(\frac{U}{r} - \frac{\Delta U \delta_e}{L} \right) \frac{m}{3K} \int v^2 \frac{\partial}{\partial v} [v^3 f_e] dv, \tag{18}$$

which after partial integration leads to

$$\left(\frac{\partial T}{\partial t}\right)_{\text{jump},4} = -\left(\frac{U}{r} - \frac{\Delta U \delta_e}{L} \right) \frac{2m}{3K} \int v^4 f_e dv = -2 \left(\frac{U}{r} - \frac{\Delta U \delta_e}{L} \right) \cdot T_e. \tag{19}$$

Clearly, heating overcomes cooling when

$$\frac{UL}{\Delta U \delta_e} \leq r. \tag{20}$$

On the other hand, from the first term in the full transport equation we obtain

$$\begin{aligned} \left(\frac{\partial T_e}{\partial t}\right)_{\text{jump},1} &= -(m/3K) \int v^4 dv \frac{1}{v^2} \frac{\partial}{\partial v} \left[\frac{\Delta U \delta_e}{2L} v^4 \frac{\partial}{\partial v} f_e \right] \\ &= -\frac{\Delta U \delta_e}{2L} \frac{m}{3K} \int v^2 \frac{\partial}{\partial v} \left[v^4 \frac{\partial}{\partial v} f_e \right] dv. \end{aligned} \tag{21}$$

One can integrate the remaining expression by parts to obtain

$$\left(\frac{\partial T_e}{\partial t}\right)_{\text{jump},1} = -\frac{\Delta U \delta_e}{2L} \frac{m}{3K} \left[\int \frac{\partial}{\partial v} \left[v^6 \frac{\partial}{\partial v} f_e \right] dv - 2 \int v^5 \frac{\partial f_e}{\partial v} dv \right]. \tag{22}$$

If the function f_e , as in the case of ions, falls off as v^{-5} , then the first part disappears, and one is left with

$$\begin{aligned}
 \left(\frac{\partial T_e}{\partial t}\right)_{\text{jump},1} &= -\frac{\Delta U \delta_e}{L} \frac{m}{3K} \left[\int v^5 \frac{\partial f_e}{\partial v} dv \right] \\
 &\approx 5 \frac{\Delta U \delta_e}{L} \frac{m}{3K} \left[\int_{v_0}^{v_\infty} \frac{1}{v} dv \right] \\
 &= \frac{5m \Delta U \delta_e}{3KL} [\ln(v_\infty) - \ln(v_0)].
 \end{aligned}
 \tag{23}$$

Without deeper knowledge of the resulting electron distribution function $[f_e]$, *i.e.* specifically the lower and upper velocity borders, one cannot evaluate this expression here in more detail. Instead, we derive below an explicit form of the heating term from a slightly different basis.

5. Conversion of the Overshoot Velocities into Thermal Energy

The electrons that overshoot from the upstream side of the shock are expected first to act like an injection of electrons with high speeds to the downstream plasma bulk. This, however, generates an unstable plasma condition, creating waves that convert the overshoot kinetic energy into thermal energy. We therefore consider the Buneman instability in the downstream plasma frame of electrons, which moves with high speed relative to protons, thereby acting as instability drivers.

In principle, the modified two-stream instability as studied by Scholer and Matsukiyo (2004) also has to be considered in competition with the Buneman instability, which is studied here. The relative efficiency of these two instabilities is strongly related to the upstream Mach numbers of the shock and to the upstream electron β -value. For electron plasma β -values below $\beta = 8\pi P_{e1}/B_1^2 = 0.02$, which characterise the conditions of travelling shocks considered here, at least in the foot ramp of the shock, the Bunemann instability is clearly dominant, however.

The downstream electron overshoot speed is equal to

$$U_{2e} = U_{2p} s \{ 1 + (M/m) [1 - (1/s^2)] \}^{1/2},
 \tag{24}$$

and the differential speed $[\Delta U_2]$ of electrons relative to protons is equal to

$$\Delta U_2 = U_{2e} - U_{2p} = U_{2p} \{ s [1 + (M/m) [1 - (1/s^2)]]^{1/2} - 1 \}
 \tag{25}$$

$$\approx U_{2p} [(M/m)(s^2 - 1)]^{1/2} \approx u_{2p} s (M/m)^{1/2},
 \tag{26}$$

which in most cases is much higher than the thermal speeds of electrons $v_{th,e}$ and protons $v_{th,p}$, hence fulfilling the relation $\Delta U_2 > v_{th,e} \gg v_{th,p}$. These jump-induced conditions are similar to those of a plasma with an electric current, where the Buneman instability is triggered (see Alexandrov, Bogdankevich, and Rukhadze, 1984; or Chen, 1984). Owing to the above inequalities, this instability is strong; its highest growth rate corresponds to the resonance condition $(k\Delta U_2) = \omega_{pe}$ (see Alexandrov, Bogdankevich, and Rukhadze, 1984; Chen, 1984) and is given by

$$\gamma_{\max} = \sqrt{\frac{3}{4}} (m/2M)^{1/3} \omega_{pe},
 \tag{27}$$

where ω_{pe} is the electron plasma frequency, \mathbf{k} and $\Delta \mathbf{U}_2$ are the wave-vector of the disturbance and the vector of the relative electron-proton speed. According to Equation (27), the typical growth period of the instability is of the order of 10^{-3} seconds for typical values

of plasma density $n \approx 10^{-1} \text{ cm}^{-3}$ at solar distances of about 10 AU. Landau-damping of these oscillations at protons is very weak under these conditions and can be neglected in our subsequent considerations. The Landau-damping at electrons is described by the decrement (Alexandrov, Bogdankevich, and Rukhadze, 1984; Chen, 1984)

$$\gamma_{Le} = \sqrt{\frac{\pi}{8}} \omega_{pe} \left(\frac{\omega_{pe}}{k_{\parallel} v_{the}} \right)^3 \exp[-1.5 - (\omega_{pe}/k_{\parallel} v_{the})^2], \tag{28}$$

where k_{\parallel} is the component of \mathbf{k} parallel to the magnetic field. Taking into account the condition $\Delta U_2 > v_{the} \gg v_{thp}$, one can see that at the initial stage of the instability, Landau-damping is exponentially weak. This is the reason why the nonlinear stage of the instability develops very fast and leads to plasma stratifications on very small scales, *i.e.* scales $l_S \approx \Delta U_2 / \omega_{pe}$. An interaction between electron plasma bunches and oscillating electric fields then results in electron braking and proton acceleration. During this process the relative speed ΔU_2 will decrease, and in line with this Landau-damping given in Equation (28) becomes increasingly important. This finally leads to electron heating and, in turn, to an additional deceleration and heating of electrons.

This scenario can be called an instantaneous temporal relaxation. It occurs when the shock front can be considered as infinitesimally thin. In the opposite case of an extended shock front structure, however, a spatial scenario takes place in which the relaxation to the stable state is instead a continuous process acting at the instability threshold [$\gamma_{max} = \gamma_{Le}$] along the whole effective length of the shock-induced electric potential jump. The full electron speed [U_{2e}] is not fully achieved in this spatial scenario, because the work of the electric potential on electrons is continuously transferred to electron heating. This is slightly similar to the case previously considered by Verscharen and Fahr (2008) for the parallel MHD shock. The final stage of the instability for both scenarios is the motion of protons and heated electrons with one and the same speed U_2^* equal to the downstream bulk speed, but with electrons that appear to be much more heated than at a classical Rankine–Hugoniot shocks.

6. Buneman Instability in the Presence of a Magnetic Field

Now we consider the effect of the solar-wind magnetic field on the Buneman instability in the downstream plasma. We do not investigate the instability in detail, but only consider the effect of the downstream magnetic field on the growth rate near its highest value. As shown above, at the initial stage of the instability, the approximation of a cold plasma is valid. For simplicity we assume below that the k -vectors of the disturbances are located in the plane $(\tilde{\mathbf{B}}, \tilde{\mathbf{U}}_{2e,i})$, and only have k_z - and k_x -wavevector components. In this case the dispersion equation corresponding to the Buneman instability can be written in the following form (Alexandrov, Bogdankevich, and Rukhadze, 1984):

$$k_x^2 \epsilon_{\perp} + k_z^2 \epsilon_{\parallel} = 0, \tag{29}$$

where ϵ_{\perp} and ϵ_{\parallel} are the components of the cold plasma dielectric tensor with

$$\epsilon_{\perp} = 1 - \sum \frac{\omega_{pa}^2}{[(\omega - \mathbf{k} \cdot \mathbf{u}_a)^2 - \omega_{Ba}^2]} \tag{30}$$

and

$$\epsilon_{\parallel} = 1 - \sum \frac{\omega_{pa}^2}{(\omega - k u_a)^2}. \tag{31}$$

The α -indices correspond to electrons and protons, and ω_{pa} and ω_{Ba} are plasma frequencies and cyclotron frequencies, respectively. In the reference frame moving with protons we find from Equations (30) and (31) the dispersion relation

$$\omega_{pe}^2 \cos^2 \theta \left[(\omega - \mathbf{k} \cdot \Delta \mathbf{U}_2)^{-2} + \frac{m}{M} \omega^{-2} \right] + \omega_{pe}^2 \sin^2 \theta \left\{ [(\omega - \mathbf{k} \cdot \Delta \mathbf{U}_2)^2 - \omega_{Be}^2]^{-1} + \frac{m}{M} [\omega^2 - \omega_{Bi}^2]^{-1} \right\} = 1, \tag{32}$$

where $\mathbf{k} = (k \sin \theta, 0, k \cos \theta)$, and $\Delta \mathbf{U}_2$ is the downstream electron speed relative to protons, the direction of $\Delta \mathbf{U}_2$ in general does not coincide with the direction of the external magnetic field (only for a parallel shock). Below we introduce the notation $\Omega = (\omega - \mathbf{k} \cdot \Delta \mathbf{U}_2)$ and consider the dispersion equation (32) at the condition of the highest growth rate of the disturbance, *i.e.* at $\omega \ll \mathbf{k} \cdot \Delta \mathbf{U}_2 \approx \omega_{pe}$; the inequality corresponds to the aperiodic type of instability and the equality corresponds to the resonant condition. We represent Equation (32) in the form

$$\omega_{pe}^{-2} = \frac{(\Omega^2 - \omega_{Be}^2 \cos^2 \theta)}{\Omega^2(\Omega^2 - \omega_{Be}^2)} + \frac{m}{M} \frac{(\omega^2 - \omega_{Bi}^2 \cos^2 \theta)}{\omega^2(\omega^2 - \omega_{Bi}^2)}. \tag{33}$$

In the zeroth order of the value (ω/Ω) , Equation (33) evaluates to

$$(\Omega^2 - \omega_{Be}^2 \cos^2 \theta) \Omega^{-2} (\Omega^2 - \omega_{Be}^2)^{-1} = F(\Omega) = \omega_{pe}^{-2}, \tag{34}$$

and in first order to

$$\omega (dF/d\Omega) + (m/M) (\omega^2 - \omega_{Bi}^2 \cos^2 \theta) \omega^{-2} (\omega^2 - \omega_{Bi}^2)^{-1} = 0. \tag{35}$$

One can find from Equation (34)

$$dF/d\Omega = -(2/\Omega) \left[\omega_{pe}^{-2} + \sin^2 \theta \frac{\omega_{Be}^2}{(\Omega^2 - \omega_{Be}^2)^2} \right]. \tag{36}$$

We furthermore take into account that in the solar-wind plasma $\omega_{pe}^2 \gg \omega_{Be}^2$ (*i.e.* $c \gg v_A \sqrt{M/m}$) is always valid. Indeed, the ratio $\omega_{pe}^2/\omega_{Be}^2 = (m/M)(c^2/v_A^2)$ for typical solar-wind plasma parameters is sufficiently high, $\omega_{pe}^2/\omega_{Be}^2 > 10^4$. (Note that in the distant solar wind at $r > 5$ AU the ratio $\omega_{pe}^2/\omega_{Be}^2$ does not depend on solar distance either). The ratio of the second term in the brackets of Equation (36) to the first term is of the order of $\omega_{pe}^2/\omega_{Be}^2$, and hence the second term can be neglected. Then Equation (36) reduces to

$$-2(\omega/\Omega) \omega_{pe}^{-2} + (m/M) (\omega^2 - \omega_{Bi}^2 \cos^2 \theta) \omega^{-2} (\omega^2 - \omega_{Bi}^2)^{-1} = 0. \tag{37}$$

One can consider two limiting cases for Equation (37): $\omega \gg \omega_{Bp}$ and $\omega \gg \omega_{Bp}$. In the first case we have from Equation (37)

$$-2(\omega/\Omega) \omega_{pe}^{-2} + (m/M) \omega^{-2} = 0, \tag{38}$$

which is the same relation as obtained for the non-magnetised plasma. Equation (37), on the other hand, has three solutions for ω given by

$$\omega = (m/2M)^{1/3} \omega_{pe} = \omega_1; \quad \omega = \omega_{2,3} = (1/2)(1 \pm i\sqrt{3})\omega_1. \tag{39}$$

One of them is unstable with the classical growth rate

$$\gamma_{\max} = \sqrt{3/4} (m/2M)^{1/3} \omega_{pe}. \tag{40}$$

In the opposite case, *i.e.* for $\omega \ll \omega_{Bp}$ and $\omega \ll \omega_{Bp}$, we find from Equation (36)

$$-2(\omega/\Omega)\omega_{pe}^{-2} + (m/M)\cos^2\theta\omega^{-2} = 0, \tag{41}$$

leading to

$$\gamma_{\max} = \sqrt{3/4}[(m/2M)\cos^2\theta]^{1/3}\omega_{pe}. \tag{42}$$

Thus one can conclude from the estimates (40) and (42) that the Buneman instability in the solar-wind plasma is not suppressed by the magnetic field. It follows from Equation (42) that the real and imaginary parts of the unstable solution are of similar magnitude. Thus, we can estimate the frequency ω as $\omega \approx \gamma_{\max}$. The comparison of γ_{\max} given in Equation (42) and ω_{Bi} then gives

$$\omega/\omega_{Bi} \approx (M/m)^{2/3}(\omega_{pe}/\omega_{Be}) \gg 1, \tag{43}$$

showing that the classical expression for γ_{\max} in Equation (42) can be used for the downstream plasma near the shock. The growth rate of the Buneman instability presented by Alexandrov, Bogdankevich, and Rukhadze (1984), namely $\gamma_{\max} = \sqrt{3/4}(m/2M\cos^2\theta)^{1/3} \cdot \omega_{pe}$, corresponds to the limit opposite to ours here, namely to strong magnetic fields with $\omega_{pe}^2 \ll \omega_{Be}^2$. Hence in conclusion we have shown that under typical conditions of the solar wind, which are applicable to our study, the instability is unaffected by the magnetic field.

7. The Downstream Bulk Speed and the Electron Temperature Gain

Now we estimate the fraction of relative kinetic energy transferred to thermal energy of the electrons. One can find the joint bulk speed from the relation

$$mU_{2e} + MU_{2p} = (m + M)U_2, \tag{44}$$

which after taking into account Equation (8) and $m \ll M$ leads to

$$U_2 \approx U_{2p} + (m/M)U_{2e} \approx U_{2p} + s\sqrt{m/M}U_{2p} \tag{45}$$

and shows that the difference between U_2 and U_{2p} is small compared with U_{2p} .

The part of electron kinetic energy transformed to heat $[\Delta W_e]$ can be found from the energy conservation law related to the pair of particles

$$2\Delta W_e = mU_{2e}^2 + MU_{2p}^2 - (m + M)U_2^2. \tag{46}$$

Combining Equations (45) and (46), one can easily confirm that

$$2\Delta W_e = \left[\frac{mM}{m + M} \right] (U_{2e} - U_{2p})^2 \tag{47}$$

or

$$\Delta W_e \approx (1/2)mU_{2e}^2, \tag{48}$$

meaning that almost the entire kinetic energy of the overshooting electrons is transformed to electron heat, and the speed U_{2e} plays the role of the thermal speed of heated electrons. It should be noted that the system of Equations (46) through (48) is equivalent to the system describing an inelastic collision of two particles with masses m and M .

Hence Equation (48) allows us to estimate the jump in electron temperature after one single jump passage by

$$\begin{aligned} \Delta T_{1e} &= mU_{2e}^2/3k = (m\Delta U_1^2/3k) \left\{ 1 + \frac{M}{m} [1 - (1/s^2)] \right\} \approx (m\Delta U_1^2/3k) \frac{M}{m} [1 - (1/s^2)] \\ &= (M\Delta U_1^2/3k) [1 - (1/s^2)]. \end{aligned} \tag{49}$$

Using the typical values (see Fahr, Siewert, and Chashei, 2012) of $s \approx 1.1$ and of $\Delta U_1 \approx 50 \text{ km s}^{-1}$, we find from Equation (47)

$$\Delta T_{1e} \approx M\Delta U_1^2/18k \approx 1.5 \times 10^4 \text{ K}, \tag{50}$$

which is much lower than the electron temperature observed at solar distances of about 1 AU.

8. Heating of the Solar-Wind Electrons due to Repeated Shock Passages

Now we consider the radial dependence of the solar-wind-convected electrons taking into account their magnetic cooling and their heating by interactions with interplanetary shocks. A similar problem was previously considered by Fahr, Siewert, and Chashei (2012) for ions. As shown by these authors, the change of ion speed by the passage of two shocks with opposite compression ratios is small compared with the initial ion speed. This allows one to describe the interaction of particles with consecutive multi-shock structures of the moving solar wind as a velocity-diffusion process. In contrast to protons, the corresponding change of the electron speed, however, is much stronger and, independent of the compression ratio, as shown above, leads to a decay of the relative electron/proton kinetic motion and to electron heating. For a description of statistical electron heating in the distant solar wind we assume that statistically significant jumps in the electron temperature at multiple passages of travelling shocks are smoothed in space and time, describing multiple shock passages with an average typical distance L_{sh} between them. With these assumptions and the magnetic cooling term derived above (see Equation (18)), the equation for the radial dependence of the electron temperature can thus be written in the following form (see also Fahr and Chashei, 2002):

$$\frac{dT_e}{dr} + \frac{2T}{r} = \Delta T_{1e} \frac{\Delta U_1}{L_{sh}U_1}. \tag{51}$$

Here the second term on the left side derived in Equations (18) and (19) of this article corresponds to magnetic cooling (see Fahr, 2007; Fahr and Fichtner, 2011) and the right side describes heating by shock passages. Here ΔT_{1e} in Equation (51) is some constant value, ΔU_1 is the typical differential bulk speed relative to the travelling shock front, U_1 is the solar-wind speed, L_{sh} is the typical distance between consecutive shocks in the moving solar-wind plasma. The right-hand term of Equation (51) in accordance with Equation (19) depends only on parameters of the jump structure and sequence and does not depend on solar distance. The radial profile of the resulting electron temperature is defined by the solution of Equation (51) given in the following form (Fahr and Chashei, 2002):

$$T_e(x) = x^{-2} [T_{e0} + \Delta T_{e1} x^3 (\Delta U_1/U_1)(r_0/L_{sh})], \tag{52}$$

where we introduced the dimensionless argument $x = r/r_0$; T_{e0} is the initial value of the temperature that is defined only by coronal conditions and must be specified at $x = 1$ from the initial condition $T_e(1) = T_{e0} + \Delta T_{e1} (\Delta U_1/U_1)(r_0/L_{sh})$. The solution (52) is shown in Figure 1.

Evidently, the resulting function $T_e(x)$ given by Equation (52) attains its minimum at a heliospheric distance $[x_m]$ given by

$$x_m = \frac{1}{3} \left[\frac{2T_{e0}}{\Delta T_{e1}} \frac{U_1}{\Delta U_1} \left(\frac{L_{sh}}{r_0} \right)^{-1} \right], \tag{53}$$

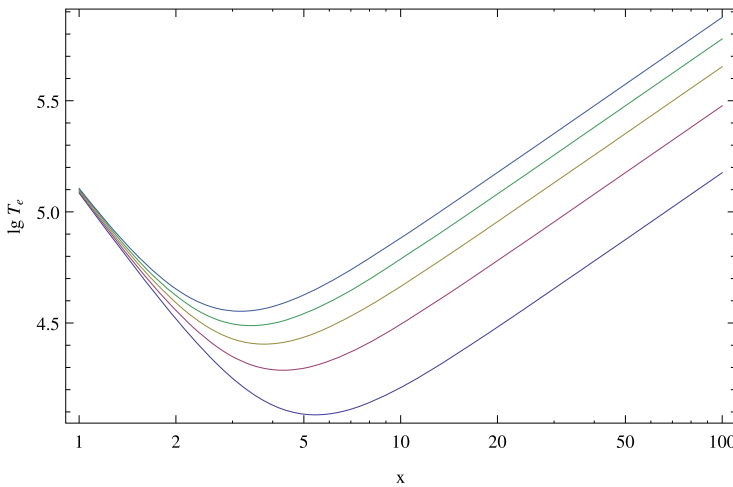


Figure 1 Radial profiles of electron temperature calculated by Equation (52) at a compression ratio $s = 1.1$ for several values of L_{sh} , the distance of consecutive shocks (*i.e.* shock occurrence period): $L_{sh} = 1; 1/2; 1/3; 1/4; 1/5$ AU from bottom to top. This shows that the position $[x_m]$ of the temperature minimum decreases and the minimum temperature increases with a decrease of the mean distance $[L_{sh}]$ between shocks.

and thus heating dominates cooling at solar distances $x \geq x_m$. When heating is stronger than cooling at $x > x_m$, this results in temperature increases with increasing distance. Equation (53) shows that the dependence on the location of the temperature minimum at x_m depends on the parameters of the problem, but is relatively weak.

Assuming that the initial solar distance $x = 1$ is located at $r_0 = 1$ AU and using the set of parameters $T_e(x = 1) = 1.2 \times 10^5$ K (Burlaga, 1971), $\Delta U_1/U_1 = 0.1$, $r_0/L_{sh} = 3$, we find from Equation (53) $T_e(1) = T_{e0} + T_{e1} = 1.2 \times 10^5$ K.

Taking $T_{e0} \approx 1.2 \times 10^5$ K then gives the estimate for x_m : $x_m \approx 4$, *i.e.* the electron-temperature increase starts at a solar distance of about 4 AU. Correspondingly, the electron temperature $[T_{e,min}]$ at the minimum of the radial profile in accordance with Equation (50) is approximately equal to $T_{e,min} \approx 2 \times 10^4$ K, which is considerably higher than in the case of pure magnetic cooling. The electron temperature according to solution (52) attains high values of about $T_e(100) = 4 \times 10^5$ K near the termination shock, *i.e.* at values of about $x \approx 100$. This value is even higher than that of the electron temperature at 1 AU, *i.e.* heating by travelling shocks overcompensates for the magnetic cooling. Similar estimates can be found at $L_{sh} = 1$ AU. In this case $T_{e0} \approx 1.2 \times 10^5$ K, $x_m \approx 5$, $T_{e,min} \approx 1.2 \times 10^4$ K; $T_e(100) \approx 1.5 \times 10^5$ K, showing that the shock effect is still strong and approximately compensates for the magnetic cooling. Obtaining the above numerical estimates, we assumed that the distance from the adjacent shocks is about $L_{sh} = (1/3)$ AU or 1 AU. These values appear to be relatively reasonable, because shocks and discontinuities are observed in the solar wind sufficiently frequently, sometimes even several shocks per day (Burlaga, 1971). At $L_{sh} \gg 1$ AU occurrence of the shocks is rare and electron heating can be expected to be fairly negligible; at $L_{sh} \ll 1$ AU the electron pressure will be too high with effective sound speeds $c_s = (kT_e/M)^{1/2} > 100 \text{ km s}^{-1}$, making the existence of the termination shock fairly questionable.

9. Conclusions

We have shown in this article that contrary to hitherto conventional thinking, solar-wind electrons do not simply cool off with increasing distance, but due to their interactions with travelling shocks undergo permanent heating processes while being advected out to larger distances with the solar wind. This leads to the occurrence of a temperature minimum in the radial electron temperature profile beyond which electron temperatures again increase. We speculate here from examining this effect of jump-induced heating that electron temperatures may be dependent on the solar-activity cycle, and that more pronounced electron heating occurs at higher activity conditions. It may also be possible to predict that electron temperatures at higher ecliptic latitudes, where high-speed solar-wind streams with low fluctuation amplitudes dominate at least during minimum conditions, electron temperatures may increase less effectively because of the reduced occurrence frequencies of travelling shocks in these regions.

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